A new non-local neutral diffusion operator

MOM6 Webinar Series
11 May 2020

Andrew Shao
University of Victoria

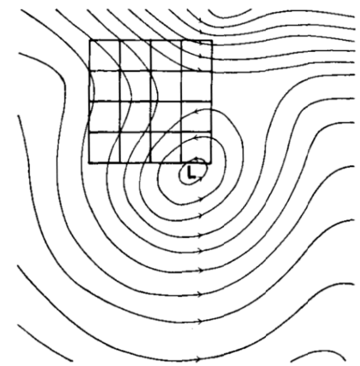
Alistair Adcroft, Bob Hallberg, and Steve Griffies

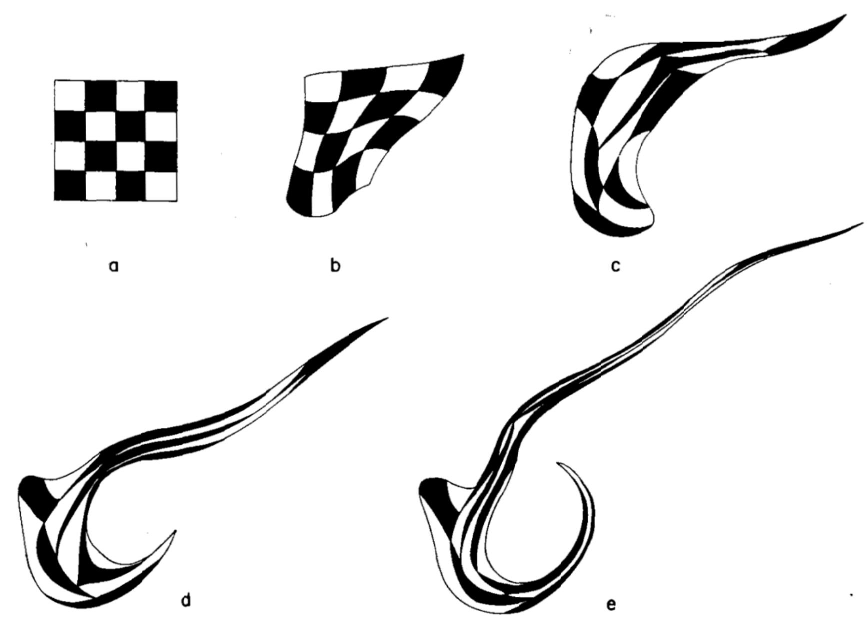
Parameterizing the mesoscale eddies

Eddy 'mixing'

- Eddy flow field **deforms** scalar tracer fields
- **Sharpens** tracer gradients
- Leads to **irreversible** mixing by molecular diffusion
- Boussinesq Hypothesis (adapted for tracers):

  Effect of turbulence can be modelled as a **down-gradient diffusion**





Welander [Tellus, 1955]

The basics of rotated diffusion

- Earliest models assumed diffusion along model surfaces
  - **Not** oriented along isopycnals (effectively diapycnal mixing)
- "Redi" Operator [1982]
  Calculate the fluxes by **rotating** the diffusion tensor along isopycnal directions
- Cox [1987]
  Significant improvements in model skill, **but** still needed background horizontal diffusion
- Griffies et al. [1998]:
  Calculate triads to ensure no net buoyancy flux and discretize to ensure tracer variance variance decreases
- Groeskamp et al. [2019]:
  Emphasize inaccuracy of calculating slopes using only local quantities

$$\mathbf{K}_\rho \equiv \kappa(x_\rho, y_\rho)\nabla_\rho$$

Along isopycnal diffusion operator

$$\mathbf{K}^{\text{small}} = A_I \begin{pmatrix} 1 & 0 & S_x \\ 0 & 1 & S_y \\ S_x & S_y & \epsilon + S^2 \end{pmatrix}$$

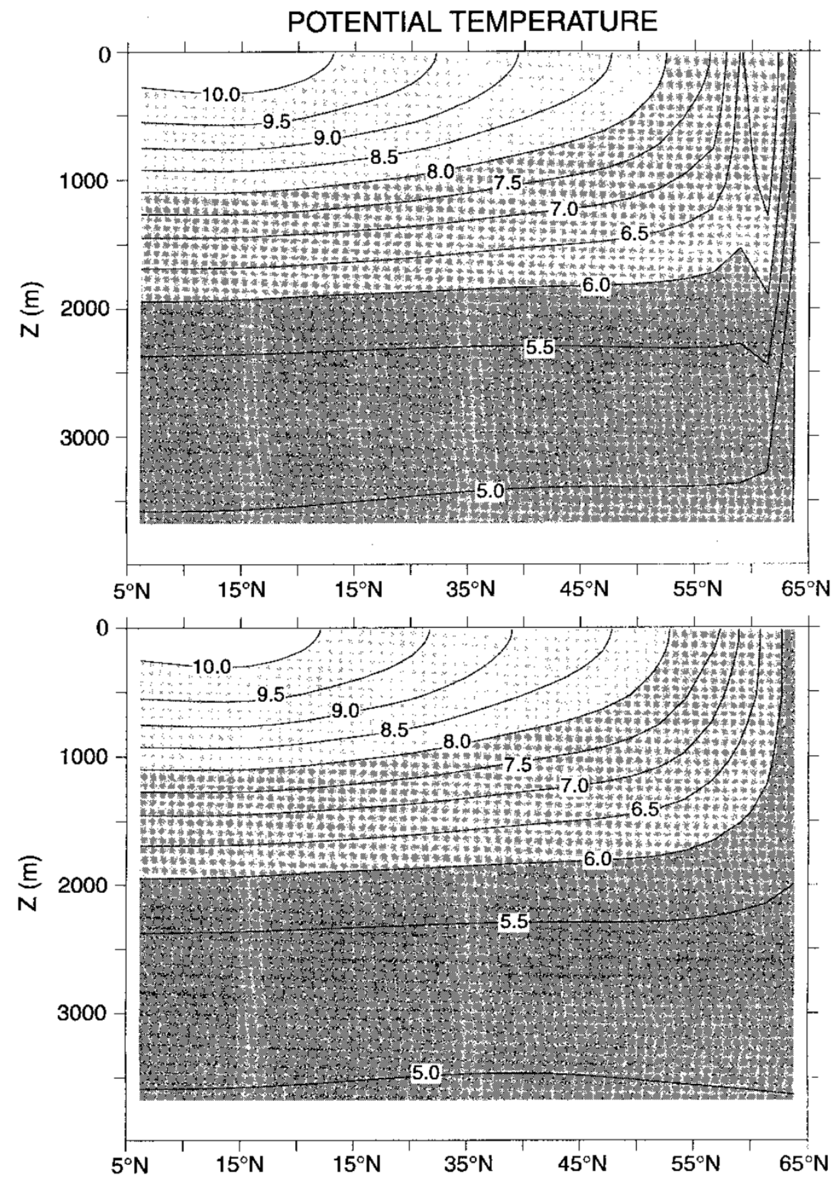
Small-slope approximation rotated onto model surfaces

$$\alpha\kappa_\rho \nabla T = \beta\kappa_\rho \nabla S$$

Necessary condition for no buoyancy gain/loss

Rotated diffusion approach has inherent numerical deficiencies

- Bleckers et al. [1998], no local, rotated tensor scheme can be guaranteed to be positive-definite
- Griffies [1998]: Demonstrated that the original Redi operator is susceptible to instability
- For all such schemes, slopes need to be tapered and limited
  - “implications for wintertime temperatures in key regions, the distribution of precipitation, and the vertical structure of heat uptake”
    Gnanadesikin et al. [2007]
  - Purely a **numerical** choice



Goal:

A discretized, epineutral diffusion operator

- Suitable for general coordinate models
- Preserves extrema
- Has no need for regularization or tapering

The way forward:

Calculate neutral fluxes within a refinement of the space between two columns using high-order polynomial reconstructions of tracers

Epineutral diffusion is appropriate for the **adiabatic, interior** ocean.

Arguments have been made that eddy fluxes **are NOT** epineutral in boundary layers

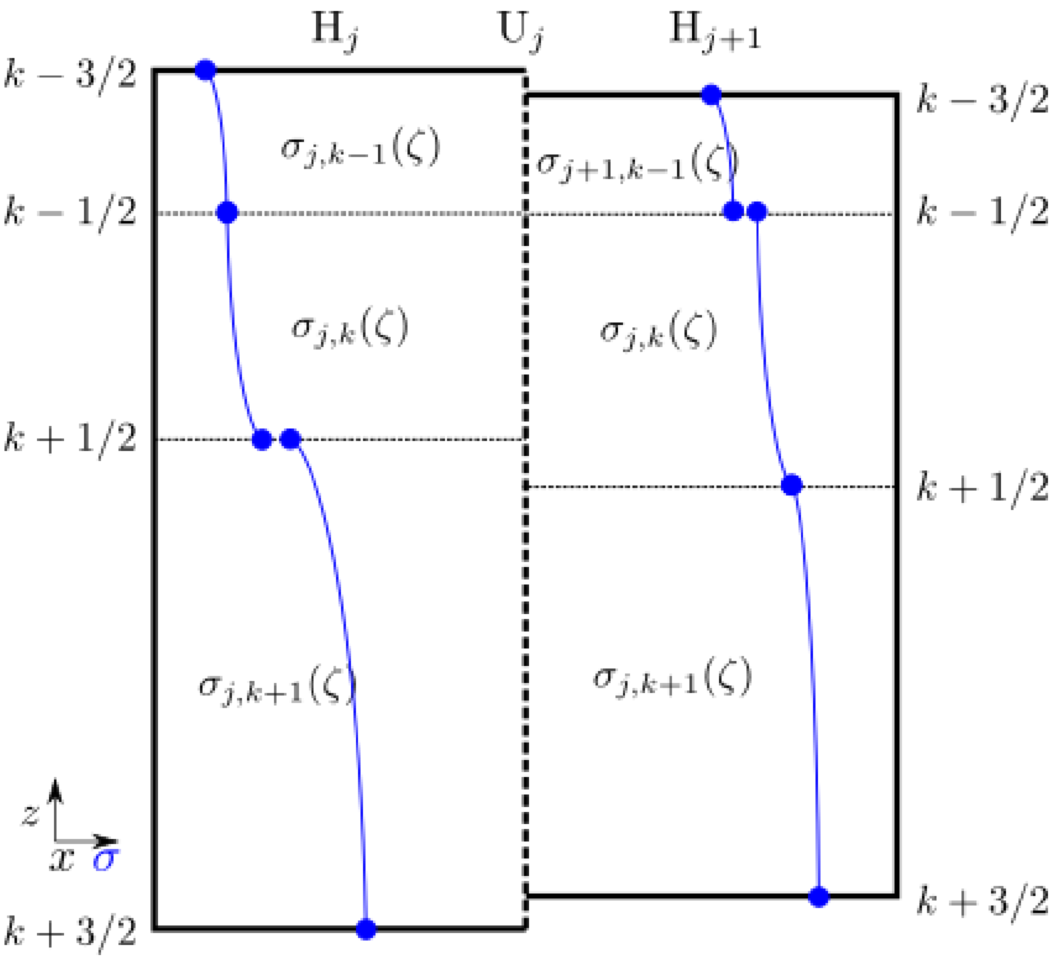
See Gustavo Marques' talk for dealing with diffusion at surface and bottom boundary layers

Primer on polynomial reconstructions

- Tracer concentration represent cell-averages in vertical discretization
- Piecewise constant
  - Finite jumps in density
- Piecewise polynomial reconstructions
  - Linear (PLM)
  - Parabolic (PPM)
- Must be monotonic and introduce no new extrema
- Discontinuous reconstructions desirable to limit intracell-slopes

Three Phases of the Algorithm

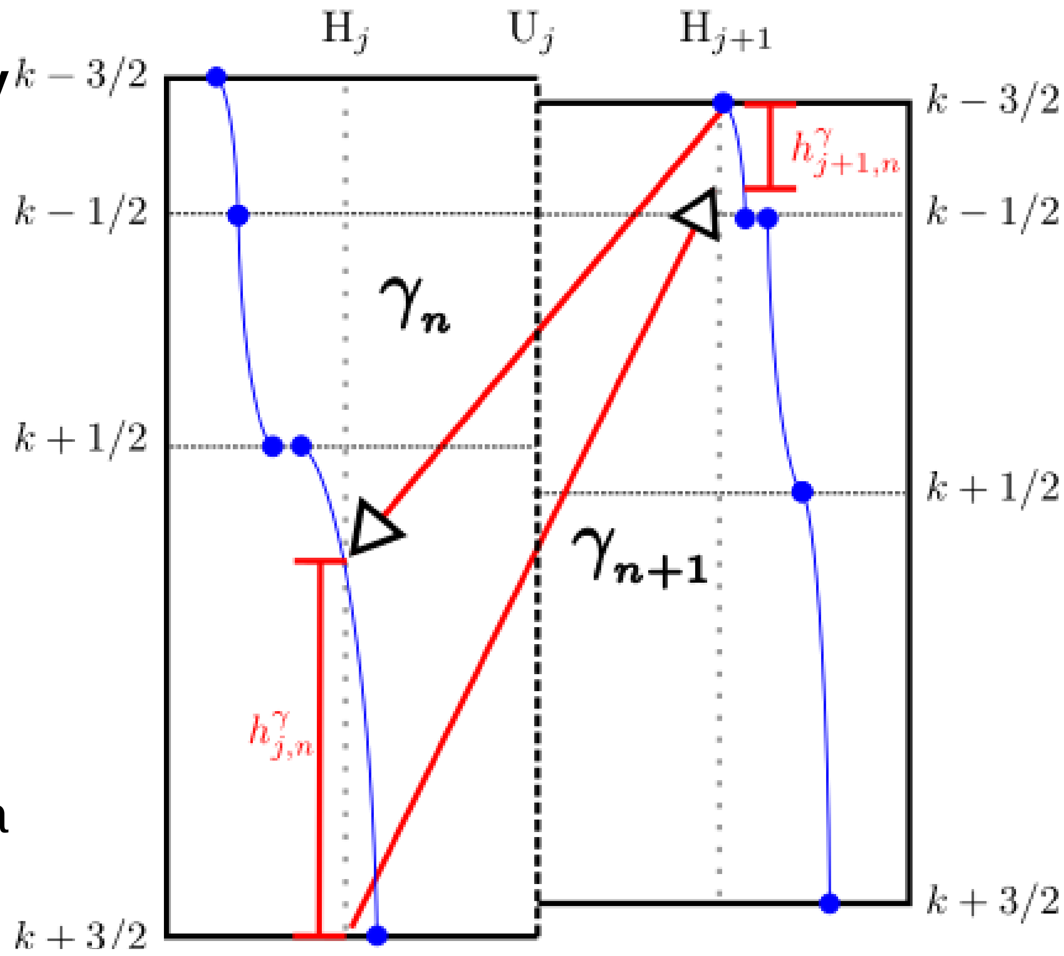
- Initialization
  - Polynomial reconstructions
    - Piecewise-linear for examples
  - α and β at interfaces
  - Filter out unstable parts of the water column
- Sorting phase
  - Create sub-layers bounded by neutral surfaces
- Flux-calculation
  - Calculate fluxes in sub-layers
  - Limit fluxes to prevent parent grid upgradient fluxes



Finding surfaces of neutral density

$$\Delta\rho = \rho_1 - \rho_2 = \frac{\alpha_1 + \alpha_2}{2}(T_1 - T_2) + \frac{\beta_1 + \beta_2}{2}(S_1 - S_2)$$

- When calculating α and β:
  - Midpoint pressure: **neutral density**
  - Reference pressure: **isopycnal**
- Starting from top two interfaces, start with the lighter one
- Search other column to find the layer whose top interface is lighter and bottom is denser
  - If one of the interfaces matches exactly, point to it
  - If top interface is lighter but bottom interface is denser, there must be a neutral surface within the layer



Finding neutral position within a layer

$$\Delta\rho = \rho_1 - \rho_2 = \frac{\alpha_1 + \alpha_2}{2}(T_1 - T_2) + \frac{\beta_1 + \beta_2}{2}(S_1 - S_2)$$

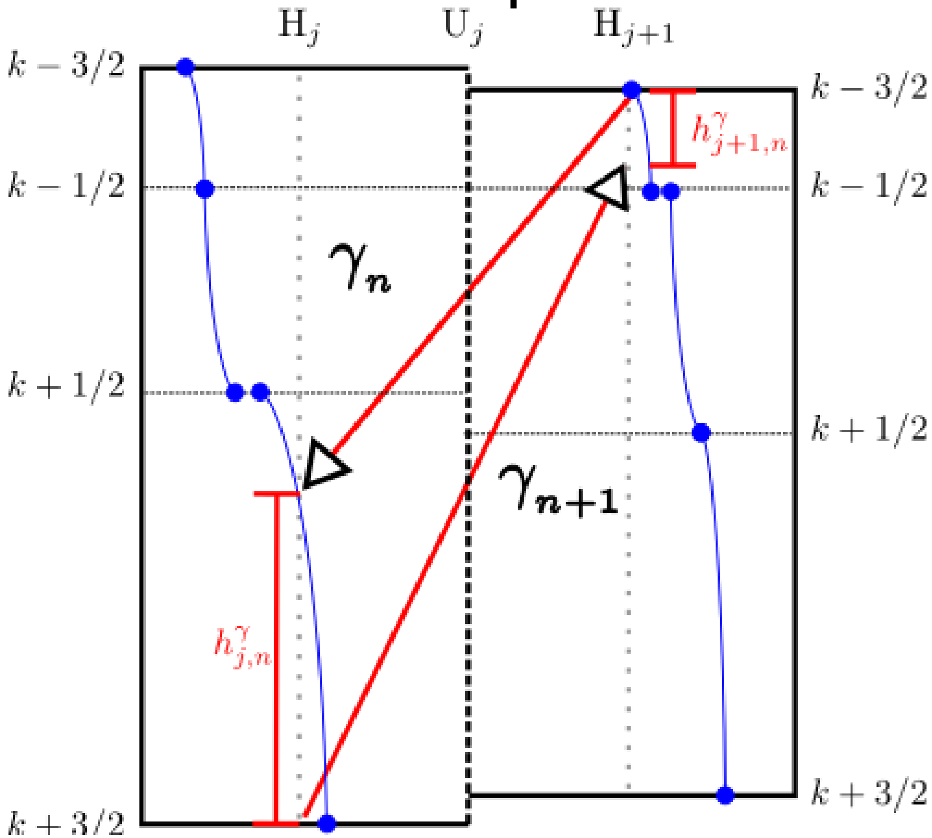
- Equivalent to finding the root of a high-order polynomial
  - Polynomial reconstructions further increase the degree
  - Use Newton's method to find the root
    - Requires first and second derivatives to be recalculated **every** iteration
- Possible simplifications
  - Assume α and β vary linearly from top to bottom
    - Cubic function in vertical (if T and S are parabolic)
  - Equation of state is linear from top to bottom interface
    - Parabolic function in vertical (if T and S are parabolic)
  - Δρ is linear in the vertical

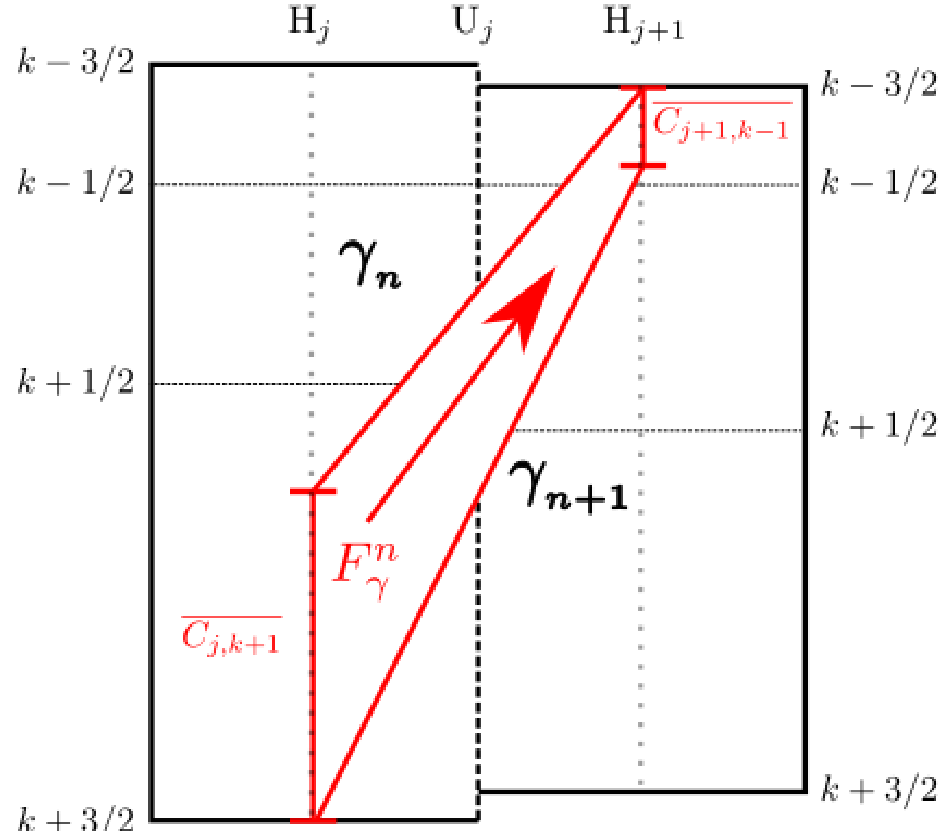
Calculating fluxes in a sublayer

- Calculate notional flux as

$$F = K\ h_{\text{eff}} \frac{\overline{C_{j,k+1}} - \overline{c_{j+1,k-1}}}{\Delta x} \Delta t \qquad \mathrm{h}_{\text{eff}} = \frac{h^{\gamma}_{j,n} h^{\gamma}_{j,n+1}}{h^{\gamma}_{j,n} + h^{\gamma}_{j,n+1}}$$

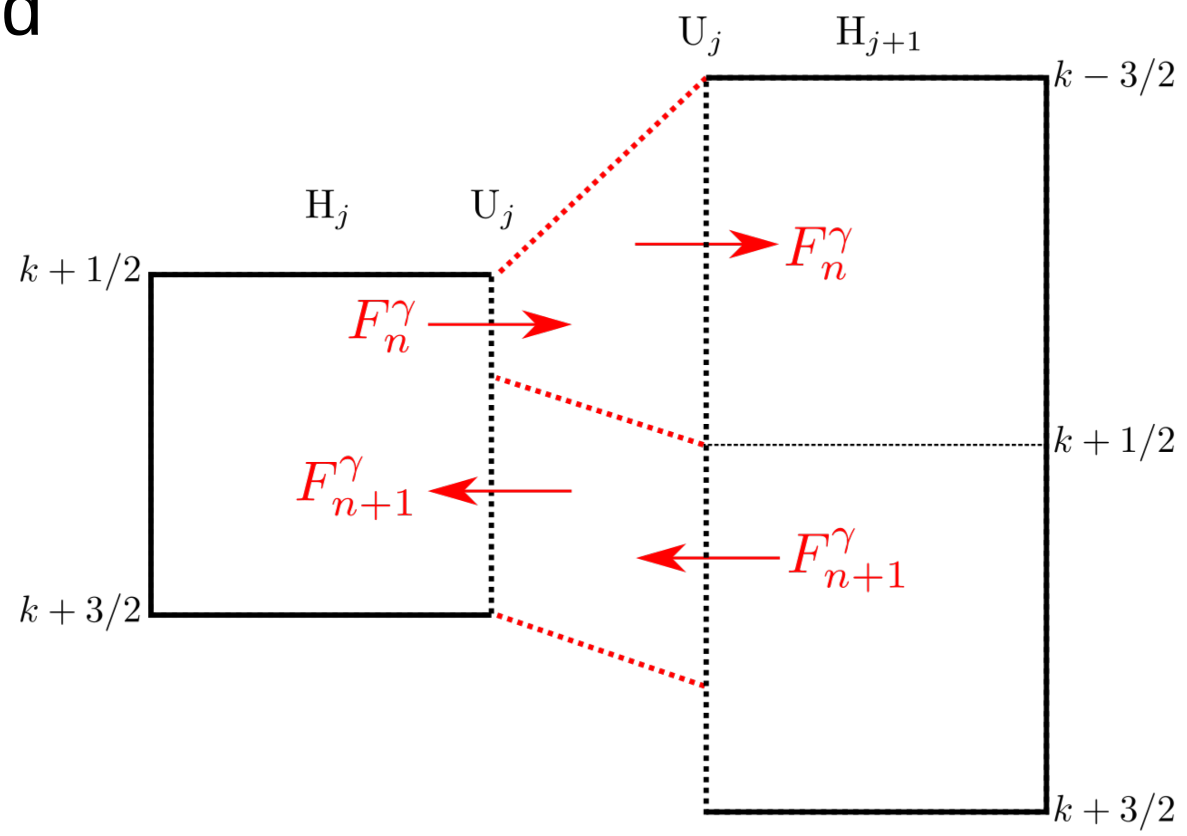
Flux required to be in the same direction as the lower-order model representation





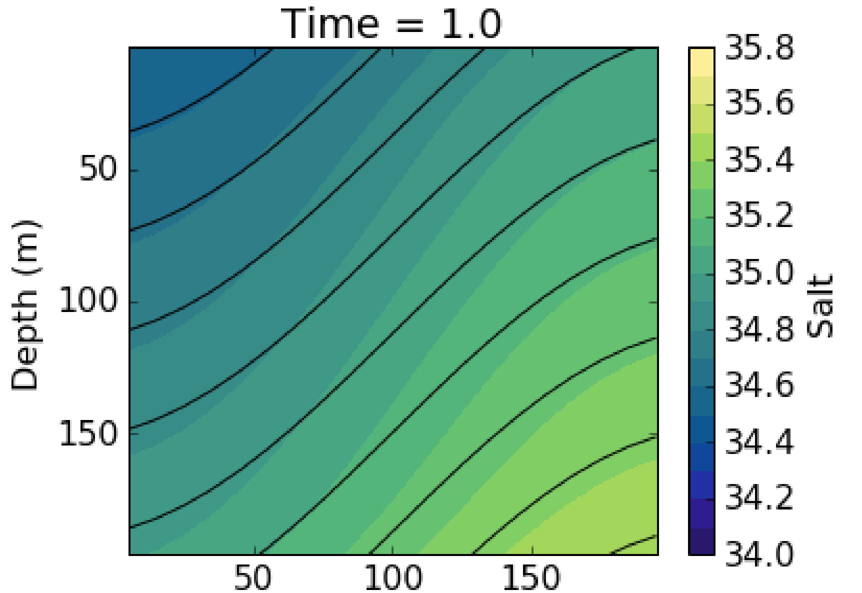
Updating the tracer state

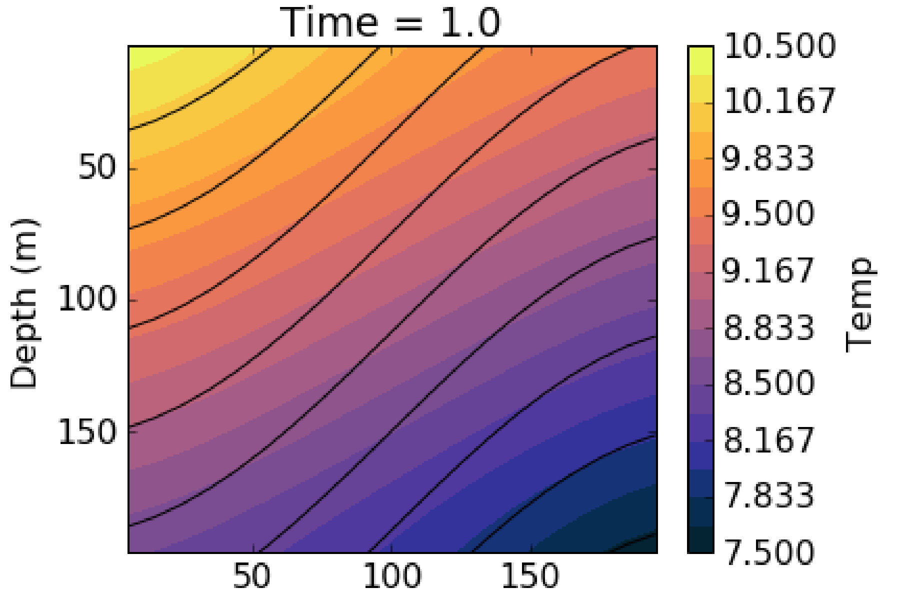
- Sublayer fluxes through the face are accumulated
- Fluxes can be non-local in the vertical
- Multiple layers can contribute fluxes to a single layer in another column
  - Entire column can diffuse into one layer!



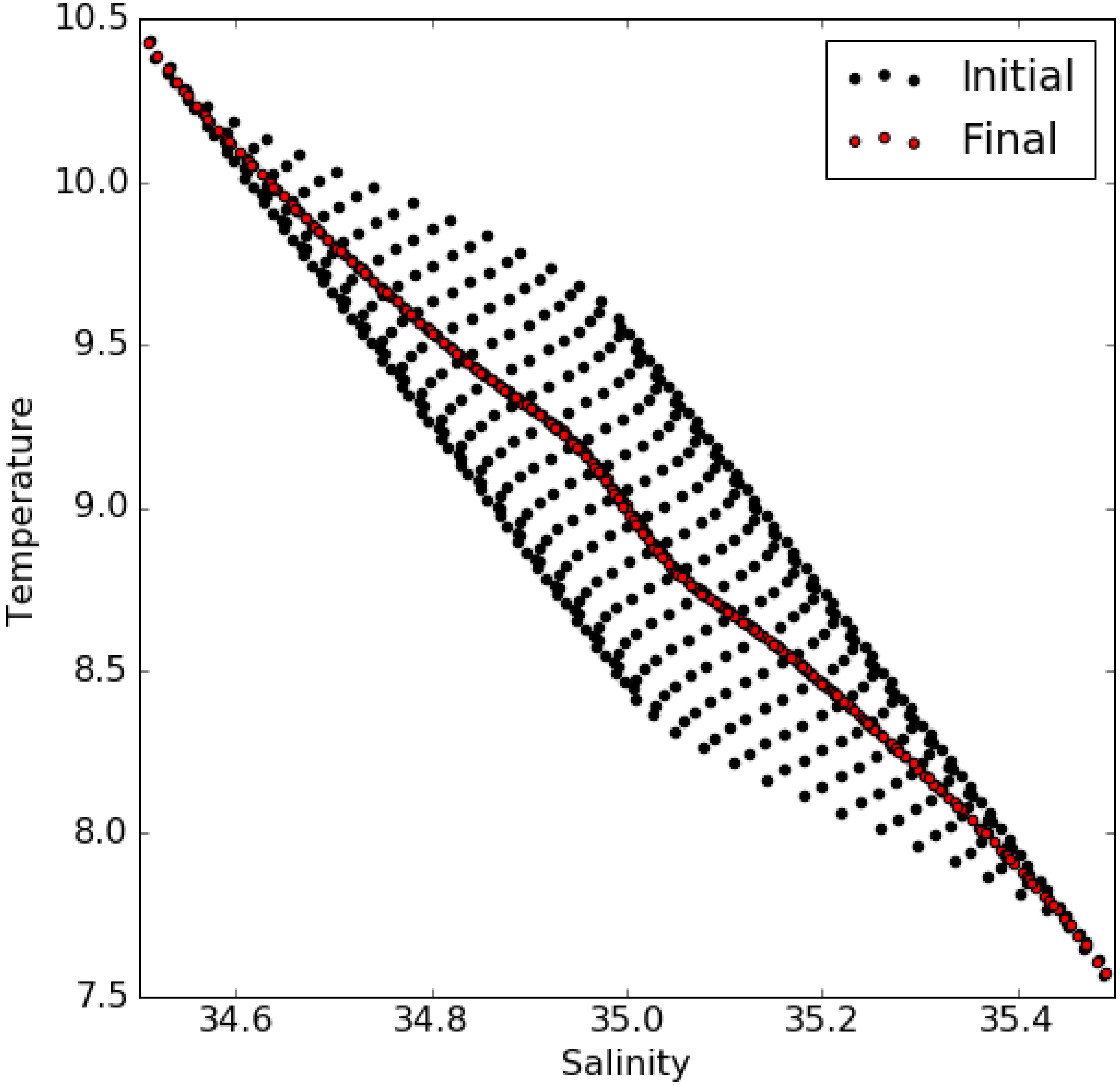
Testing the algorithm in an idealized case

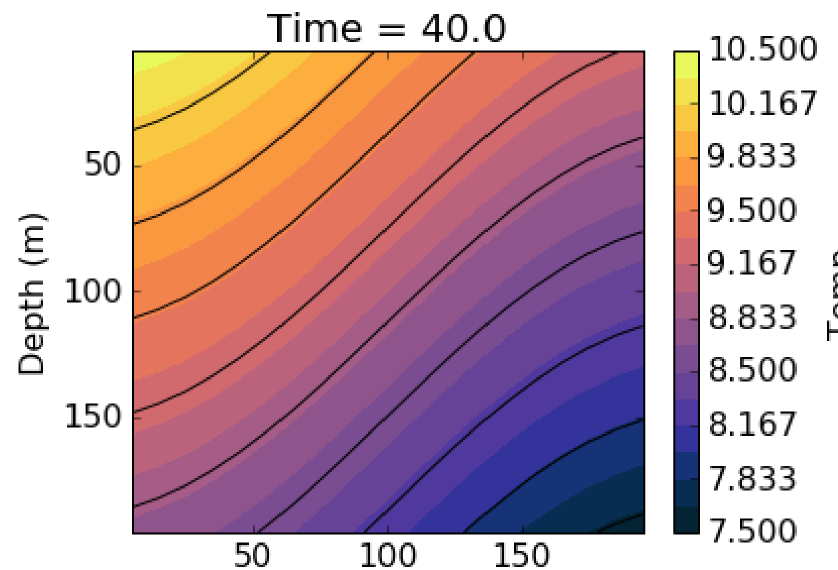
- TEOS-10 equation of state
- MOM6 (diffusion only)
- Two coordinates
  - Continuous isopycnal
    - Diffusion along layer
  - Z* coordinate
    - Neutral diffusion





Final state of the test case



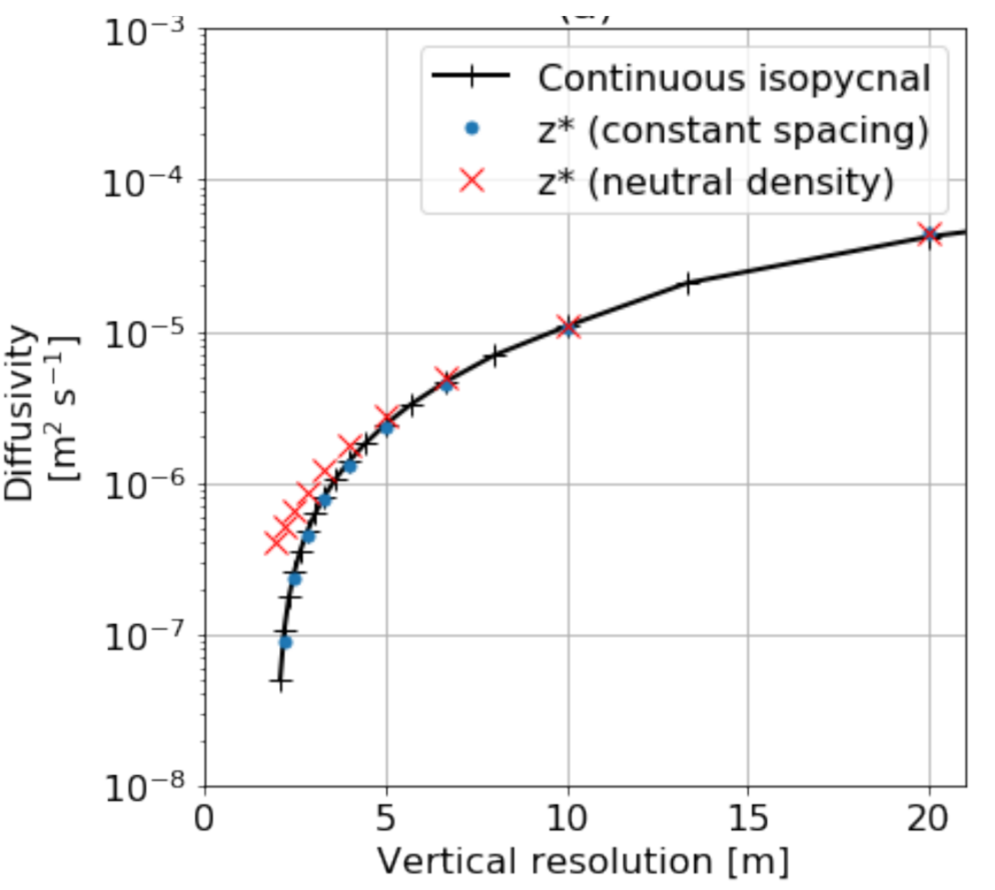


- Isotherms parallel to isopycnals
- Homogenization of both *T* and *S* on isopycnals

Diagnosing spurious dianeutral diffusion

$$\Delta\mathrm{APE} = \frac{g\int_0^L\int_0^D[\rho(t_1,x,z)-\rho(t_0,x,z)]\,z\,dz\,dx}{\int_0^L\int_0^D\rho(t_0)\,dz\,dx} \qquad \kappa_\delta = \frac{\Delta\mathrm{APE}_{\mathrm{cabbeling}}-\Delta\mathrm{APE}_{\mathrm{case}}}{N^2\Delta t}$$

- Assume any spurious mixing weakens stratification
- Density should change very slightly due to cabbelling
  - Estimated using continuous isopycnal case with 100 layers and along-layer diffusion
- z* test case converges to continuous isopycnal case



Closing comments

- Demonstrated a new way of representing the neutral diffusion within MOM6
- New scheme overcomes inherent shortcomings of the more common ‘rotated operator’
- Algorithm resulted in limited dianeutal diffusion in an idealized test case