Ocean Modeling II

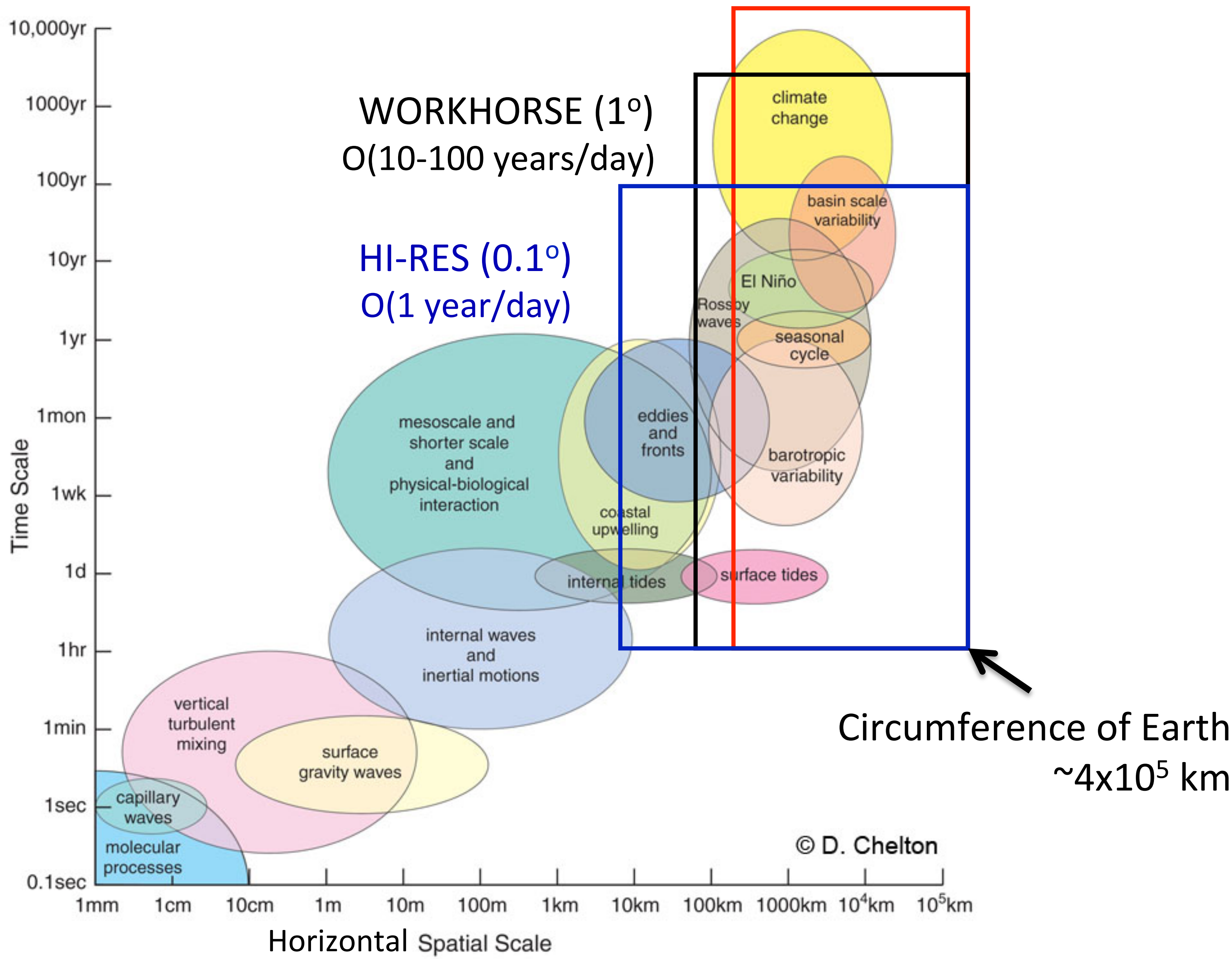
Parameterized Physics

Steve Yeager
Oceanography Section
Climate and Global Dynamics Division
National Center for Atmospheric Research

NCAR is sponsored by the National Science Foundation

Ocean Modelling Challenges

LO-RES (3°)
O(100+ years/day)



MODEL EQUATIONS

3-D primitive equations in spherical polar coordinates with vertical z-coordinate for a thin, stratified fluid using hydrostatic & Boussinesq approx (Smith et al. 2010):

Momentum equations:

1
$$\frac{\partial}{\partial t}u + \mathcal{L}(u) - (uv\tan\phi)/a - fv = -\frac{1}{\rho_0 a\cos\phi}\frac{\partial p}{\partial \lambda} + \mathcal{F}_{Hx}(u,v) + \mathcal{F}_V(u) \quad (2.1)$$

2
$$\frac{\partial}{\partial t}v + \mathcal{L}(v) + (u^2\tan\phi)/a + fu = -\frac{1}{\rho_0 a}\frac{\partial p}{\partial \phi} + \mathcal{F}_{Hy}(u,v) + \mathcal{F}_V(v) \quad (2.2)$$

$$\mathcal{L}(\alpha) = \frac{1}{a\cos\phi}\left[\frac{\partial}{\partial\lambda}(u\alpha) + \frac{\partial}{\partial\phi}(\cos\phi\, v\alpha)\right] + \frac{\partial}{\partial z}(w\alpha) \quad (2.3)$$

$$\mathcal{F}_{Hx}(u,v) = A_M\left\{\nabla^2 u + u(1-\tan^2\phi)/a^2 - \frac{2\sin\phi}{a^2\cos^2\phi}\frac{\partial v}{\partial\lambda}\right\} \quad (2.4)$$

$$\mathcal{F}_{Hy}(u,v) = A_M\left\{\nabla^2 v + v(1-\tan^2\phi)/a^2 + \frac{2\sin\phi}{a^2\cos^2\phi}\frac{\partial u}{\partial\lambda}\right\} \quad (2.5)$$

$$\nabla^2\alpha = \frac{1}{a^2\cos^2\phi}\frac{\partial^2\alpha}{\partial\lambda^2} + \frac{1}{a^2\cos\phi}\frac{\partial}{\partial\phi}\left(\cos\phi\frac{\partial\alpha}{\partial\phi}\right) \quad (2.6)$$

$$\mathcal{F}_V(\alpha) = \frac{\partial}{\partial z}\mu\frac{\partial}{\partial z}\alpha \quad (2.7)$$

3 *Continuity equation:*

$$\mathcal{L}(1) = 0 \quad (2.8)$$

4 *Hydrostatic equation:*

$$\frac{\partial p}{\partial z} = -\rho g \quad (2.9)$$

5 *Equation of state:*

$$\rho = \rho(\Theta, S, p) \to \rho(\Theta, S, z) \quad (2.10)$$

6,7 *Tracer transport:*

$$\frac{\partial}{\partial t}\varphi + \mathcal{L}(\varphi) = \mathcal{D}_H(\varphi) + \mathcal{D}_V(\varphi) \quad (2.11)$$

$$\mathcal{D}_H(\varphi) = A_H\nabla^2\varphi \quad (2.12)$$

$$\mathcal{D}_V(\varphi) = \frac{\partial}{\partial z}\kappa\frac{\partial}{\partial z}\varphi, \quad (2.13)$$

"The art of parameterization is to augment the resolved dynamics…with mathematical operators that accomplish the necessary physical effects by unresolved SGS processes, all of which involve turbulence…"

McWilliams, 1998, "Oceanic General Circulation Models", Ocean Modelling and Parameterization, NATO Science Series.

MODEL EQUATIONS

3-D primitive equations in spherical polar coordinates with vertical z-coordinate for a thin, stratified fluid using hydrostatic & Boussinesq approx (Smith et al. 2010):

Momentum equations:

1
$$\frac{\partial}{\partial t}u + \mathcal{L}(u) - (uv\tan\phi)/a - fv = -\frac{1}{\rho_0 a\cos\phi}\frac{\partial p}{\partial\lambda} + \mathcal{F}_{Hx}(u,v) + \mathcal{F}_V(u) \quad (2.1)$$

2
$$\frac{\partial}{\partial t}v + \mathcal{L}(v) + (u^2\tan\phi)/a + fu = -\frac{1}{\rho_0 a}\frac{\partial p}{\partial\phi} + \mathcal{F}_{Hy}(u,v) + \mathcal{F}_V(v) \quad (2.2)$$

$$\mathcal{L}(\alpha) = \frac{1}{a\cos\phi}\left[\frac{\partial}{\partial\lambda}(u\alpha) + \frac{\partial}{\partial\phi}(\cos\phi\, v\alpha)\right] + \frac{\partial}{\partial z}(w\alpha) \quad (2.3)$$

$$\mathcal{F}_{Hx}(u,v) = A_M\left\{\nabla^2 u + u(1-\tan^2\phi)/a^2 - \frac{2\sin\phi}{a^2\cos^2\phi}\frac{\partial v}{\partial\lambda}\right\} \quad (2.4)$$

$$\mathcal{F}_{Hy}(u,v) = A_M\left\{\nabla^2 v + v(1-\tan^2\phi)/a^2 + \frac{2\sin\phi}{a^2\cos^2\phi}\frac{\partial u}{\partial\lambda}\right\} \quad (2.5)$$

$$\nabla^2\alpha = \frac{1}{a^2\cos^2\phi}\frac{\partial^2\alpha}{\partial\lambda^2} + \frac{1}{a^2\cos\phi}\frac{\partial}{\partial\phi}\left(\cos\phi\frac{\partial\alpha}{\partial\phi}\right) \quad (2.6)$$

$$\mathcal{F}_V(\alpha) = \frac{\partial}{\partial z}\mu\frac{\partial}{\partial z}\alpha \quad (2.7)$$

3 *Continuity equation:*

$$\mathcal{L}(1) = 0 \quad (2.8)$$

4 *Hydrostatic equation:*

$$\frac{\partial p}{\partial z} = -\rho g \quad (2.9)$$

5 *Equation of state:*

$$\rho = \rho(\Theta, S, p) \rightarrow \rho(\Theta, S, z) \quad (2.10)$$

6,7 *Tracer transport:*

$$\frac{\partial}{\partial t}\varphi + \mathcal{L}(\varphi) = \mathcal{D}_H(\varphi) + \mathcal{D}_V(\varphi) \quad (2.11)$$

$$\mathcal{D}_H(\varphi) = A_H\nabla^2\varphi \quad (2.12)$$

$$\mathcal{D}_V(\varphi) = \frac{\partial}{\partial z}\kappa\frac{\partial}{\partial z}\varphi, \quad (2.13)$$

"The most common parameterization hypothesis about turbulent processes is that they mix material properties, hence the most common operator form is *eddy diffusion* (e.g. by spatial Laplacians) with an *eddy diffusivity* as the free parameter."

McWilliams, 1998, "Oceanic General Circulation Models", Ocean Modelling and Parameterization, NATO Science Series.

MODEL EQUATIONS

3-D primitive equations in spherical polar coordinates with vertical z-coordinate for a thin, stratified fluid using hydrostatic & Boussinesq approx (Smith et al. 2010):

Momentum equations:

1 $$\frac{\partial}{\partial t}u + \mathcal{L}(u) - (uv\tan\phi)/a - fv = -\frac{1}{\rho_0 a\cos\phi}\frac{\partial p}{\partial\lambda} + \mathcal{F}_{Hx}(u,v) + \mathcal{F}_V(u) \quad (2.1)$$

2 $$\frac{\partial}{\partial t}v + \mathcal{L}(v) + (u^2\tan\phi)/a + fu = -\frac{1}{\rho_0 a}\frac{\partial p}{\partial\phi} + \mathcal{F}_{Hy}(u,v) + \mathcal{F}_V(v) \quad (2.2)$$

$$\mathcal{L}(\alpha) = \frac{1}{a\cos\phi}\left[\frac{\partial}{\partial\lambda}(u\alpha) + \frac{\partial}{\partial\phi}(\cos\phi\, v\alpha)\right] + \frac{\partial}{\partial z}(w\alpha) \quad (2.3)$$

$$\mathcal{F}_{Hx}(u,v) = A_M\left\{\nabla^2 u + u(1-\tan^2\phi)/a^2 - \frac{2\sin\phi}{a^2\cos^2\phi}\frac{\partial v}{\partial\lambda}\right\} \quad (2.4)$$

$$\mathcal{F}_{Hy}(u,v) = A_M\left\{\nabla^2 v + v(1-\tan^2\phi)/a^2 + \frac{2\sin\phi}{a^2\cos^2\phi}\frac{\partial u}{\partial\lambda}\right\} \quad (2.5)$$

$$\nabla^2\alpha = \frac{1}{a^2\cos^2\phi}\frac{\partial^2\alpha}{\partial\lambda^2} + \frac{1}{a^2\cos\phi}\frac{\partial}{\partial\phi}\left(\cos\phi\frac{\partial\alpha}{\partial\phi}\right) \quad (2.6)$$

$$\mathcal{F}_V(\alpha) = \frac{\partial}{\partial z}\mu\frac{\partial}{\partial z}\alpha \quad (2.7)$$

3 *Continuity equation*:

$$\mathcal{L}(1) = 0 \quad (2.8)$$

4 *Hydrostatic equation*:

$$\frac{\partial p}{\partial z} = -\rho g \quad (2.9)$$

5 *Equation of state*:

$$\rho = \rho(\Theta, S, p) \to \rho(\Theta, S, z) \quad (2.10)$$

6,7 *Tracer transport*:

$$\frac{\partial}{\partial t}\varphi + \mathcal{L}(\varphi) = \mathcal{D}_H(\varphi) + \mathcal{D}_V(\varphi) \quad (2.11)$$

$$\mathcal{D}_H(\varphi) = A_H\nabla^2\varphi \quad (2.12)$$

$$\mathcal{D}_V(\varphi) = \frac{\partial}{\partial z}\kappa\frac{\partial}{\partial z}\varphi, \quad (2.13)$$

"…a good parameterization is…as simple as possible, with as few free parameters as possible, consistent with achieving both the hypothesized effect and a significant impact on the solution."

McWilliams, 1998, "Oceanic General Circulation Models", Ocean Modelling and Parameterization, NATO Science Series.

PARAMETERIZATIONS IN CESM1 POP2

- Vertical mixing (momentum and tracers)
  - surface boundary layer,
  - interior
- Horizontal viscosity (momentum)
- Lateral mixing / mesoscale eddies (tracers)
- Overflows
- Submesoscale eddies (tracers)
- Diurnal cycle for short-wave heat flux
- Solar absorption
- Langmuir circulation
- Near-inertial wave mixing

PARAMETERIZATIONS IN CESM1 POP2

- Vertical mixing (momentum and tracers)
  - surface boundary layer,
  - interior
- Horizontal viscosity (momentum)
- Lateral mixing / mesoscale eddies (tracers)
- Overflows
- Submesoscale eddies (tracers)
- Diurnal cycle for short-wave heat flux
- Solar absorption
- Langmuir circulation
- Near-inertial wave mixing

Sub-grid scale (SGS) Closure

- Consider some hydrodynamic variable (e.g. **v**) decomposed into large-scale (long-period) and small-scale (short-period) components, using some averaging operator $(\overline{\cdot})$:

"Reynolds decomposition"

$$\mathbf{v} = \overline{\mathbf{v}} + \mathbf{v}'$$ = (resolved flow) + (unresolved SGS eddy perturbation)

- Consider a Navier-Stokes-like nonlinear system and attempt to construct the evolution equation for large-scale flow:

$$\frac{du}{dt} + uu + ru = 0$$

$$\overline{\left(\frac{du}{dt} + uu + ru = 0 \right)}$$

$$\frac{d\overline{u}}{dt} + \overline{uu} + r\overline{u} = 0$$

$$\overline{uu} = \overline{(\overline{u} + u')(\overline{u} + u')}$$
$$= \overline{u} \cdot \overline{u} + \overline{u'u'}$$

- Therefore, evolution of first-order moments ($\overline{u}$) depends on second-order moments ($\overline{u'u'}$, "Reynolds stresses"). An evolution equation for second-order moments can be written, but it will depend on third-order moments ($\overline{u'u'u'}$), etc.
- A "first-order closure" for the large-scale flow ($\overline{u}$) **parameterizes** the second-order moments in terms of large-scale fields, e.g.: $\overline{u'u'} = -\kappa \frac{\partial \overline{u}}{\partial x}$

VERTICAL MIXING SCHEME: K-PROFILE PARAMETERIZATION (KPP)

OCEANIC VERTICAL MIXING: A REVIEW AND A MODEL WITH A NONLOCAL BOUNDARY LAYER PARAMETERIZATION

W. G. Large
J. C. McWilliams
S. C. Doney
National Center for Atmospheric Research
Boulder, Colorado

8755-1209/94/94RG-01872*$15.00*

Reviews of Geophysics, 32, 4 / *November 1994*
pages 363–403
Paper number 94RG01872

- Unresolved turbulent vertical mixing due to PBL eddies parameterized as a vertical diffusion.
- Guided by extensive study/observations of ABL, applied to OBL

- $$\partial_t X = -\partial_z \overline{w'X'} \quad \xrightarrow{\text{parameterize}} \quad \overline{w'X'} = -K_x \partial_z X$$

"First order turbulence closure"

where K_x represents an "eddy diffusivity" or "eddy viscosity"
and X = { active/passive scalars or momentum }

VERTICAL MIXING SCHEME:
K-PROFILE PARAMETERIZATION (KPP)

- KPP is non-local: $$\overline{w'X'} = -\underline{K_x}(\partial_z X - \underline{\gamma_x})$$

Diffusivity throughout BL depends on surface forcing, depth of BL, and interior diffusivity

"non-local transport" or "countergradient" term for scalars

- KPP involves three high-level steps:
  1. Determination of boundary layer (BL) depth: h
  2. Calculation of interior diffusivities: ν_x
  3. Evaluation of boundary layer (BL) diffusivities: K_x

- KPP distinguishes 3 vertical regimes:
  1. surface layer
  2. boundary layer
  3. interior

Modelled pure convection

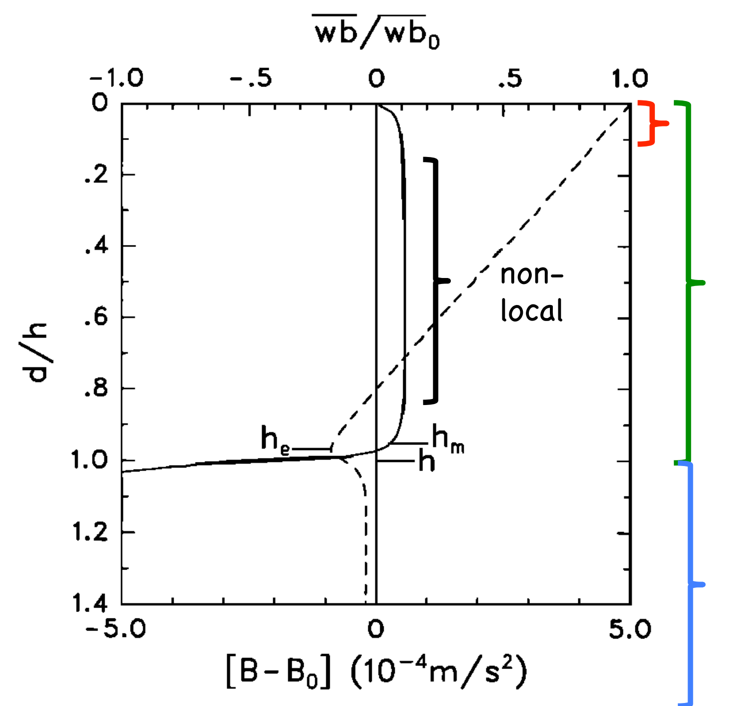


Figure 1. Relative buoyancy (solid trace, bottom scale) and buoyancy flux (dashed trace, top scale) profiles after 3.0 days of convective deepening into an initially uniformly stratified water column of $\partial_z T = 0.1°\mathrm{C\ m^{-1}}$, $N = 0.016\ \mathrm{s^{-1}}$, under the action of a steady cooling, $Q_t = -100\ \mathrm{W\ m^{-2}}$. Axes have been normalized with a boundary layer depth, $h = 13.6$ m and a surface buoyancy flux, $\overline{wb}_0 = 6.3 \times 10^{-8}\ \mathrm{m^2\ s^{-3}}$. Also shown are the entrainment depth, h_e, and the mixed layer depth, h_m.

VERTICAL MIXING SCHEME:
K-PROFILE PARAMETERIZATION (KPP)

1. BL depth, h, is minimum depth (d) where the bulk Richardson number (Ri_b) referenced to the surface equals a critical Richardson number (Ri_{cr}=0.3).

$$Ri_b(d) = \frac{\overbrace{\left[B_r - B(d)\right]d}^{\text{Stabilizing buoyancy difference}}}{\underbrace{\left|\mathbf{V}_r - \mathbf{V}(d)\right|^2 + V_t^2(d)}_{\text{Destabilizing velocity shear}}}$$

unresolved shear

B_r : near-surface reference buoyancy
$B(d)$: boundary layer buoyancy at depth d
$\mathbf{V}_r$: near-surface reference horizontal velocity
$\mathbf{V}(d)$: boundary layer horizontal velocity at depth d
$V_t(d)$: velocity scale of (unresolved) turbulent shear at depth d

➔ Ri measures the stability of stratified shear flow. "Boundary layer eddies with mean velocity $\mathbf{V}_r$ and buoyancy B_r should be able to penetrate to a depth h where they first become stable relative to the local buoyancy and velocity."

VERTICAL MIXING SCHEME:
K-PROFILE PARAMETERIZATION (KPP)

2. Calculation of interior diffusivities

$$\upsilon_x(d) = \upsilon_x^s(d) + \upsilon_x^w(d) + \upsilon_x^d(d) + \upsilon_x^c(d) + \upsilon_x^t(d)$$

ν_x : interior diffusivity at depth d (below the boundary layer)
ν_x^s : (unresolved) shear instability
ν_x^w : internal wave breaking*
ν_x^d : double diffusion
ν_x^c : local static instability (convection)
ν_x^t : tidal mixing**

➔ Superposition of processes sets interior vertical diffusivity, ν, below the surface boundary layer.

* Jochum et al., 2013: The impact of oceanic near-inertial waves on climate, *J. Climate*, **26**, 2833-2844.

** Jayne, S, 2009: The impact of abyssal mixing parameterizations in an ocean general circulation model, *J. Phys. Oceanogr.*, **39**, 1756-1775.

VERTICAL MIXING SCHEME:
K-PROFILE PARAMETERIZATION (KPP)

3. Calculation of boundary layer (eddy) diffusivity

$$K_x(\sigma) = h w_x(\sigma) G(\sigma)$$

σ : non-dimensional depth parameter, d/h
h : boundary layer depth
$w_x(\sigma)$: turbulent vertical velocity scale
G(σ) : vertical shape function (cubic poly)

- Monin-Obukhov similarity theory in surface layer (σ<ε)
- For all σ, K_x depends on h, surface forcing, & interior diffusivity at the base of the BL

➔ fundamentally non-local & interior can force OBL through G(σ) term

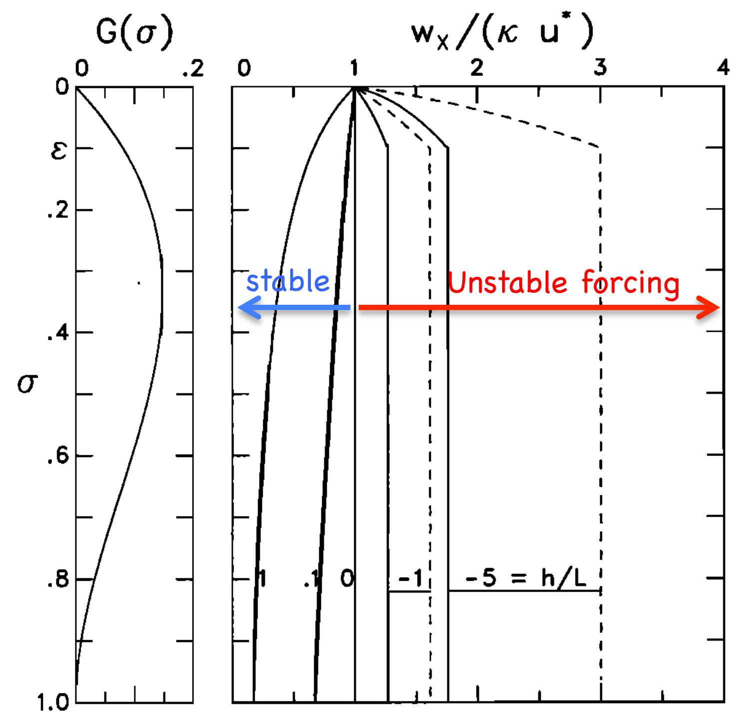


Figure 2. (left) Vertical profile of the shape function $G(\sigma)$, where $\sigma = d/h$, in the special case of $G(1) = \partial_\sigma G(1) = 0$. (right) Vertical profiles of the normalized turbulent velocity scale, $w_x(\sigma)/(\kappa u^*)$, for the cases of $h/L = 1, 0.1, 0, -1$, and -5. In unstable conditions, $w_s(\sigma)$ (dashed traces) is greater than $w_m(\sigma)$ (solid traces) at all depths, but for stable forcing $h/L \geq 0$, the two velocity scales are equal at all depths.

VERTICAL MIXING SCHEME:
K-PROFILE PARAMETERIZATION (KPP)

Verification example @ OWS Papa (50°N, 145°W):

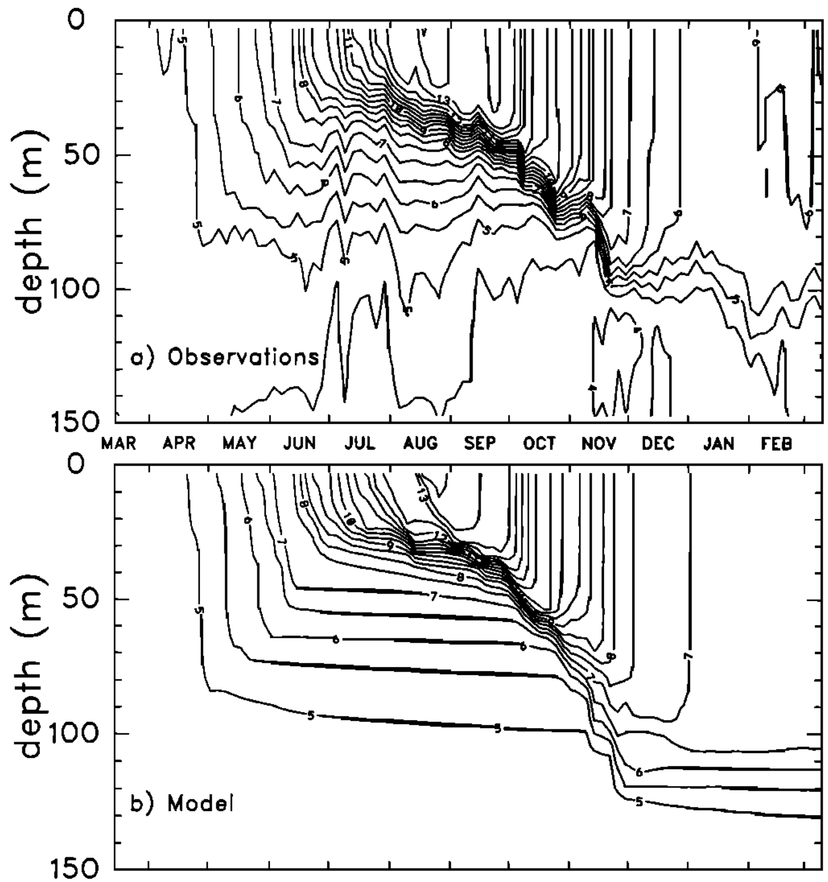


Figure 9. Time-depth sections of 4-day averages of observed temperatures in degrees Celsius (a) from ocean weather station (OWS) Papa during the ocean year March 15, 1961, to March 15, 1962 and (b) from the standard KPP simulation of OWS Papa.

Large et al (1994)

VERTICAL MIXING SCHEME:
K-PROFILE PARAMETERIZATION (KPP)

1-D experiments (equatorial region) forced identically...

Large and Gent (JPO, 1999)

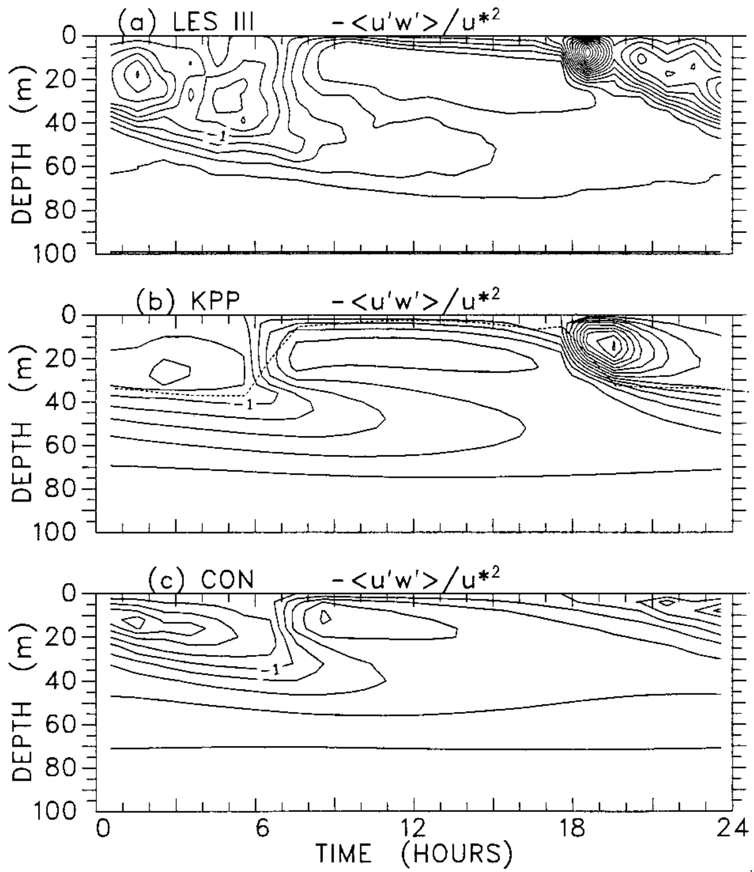


FIG. 4. The day 6 diurnal cycle of momentum flux plotted as a function of depth and normalized by $u^{*2} = 4.1 \times 10^{-5}$ m² s⁻², with a contour interval of 0.2. (a) LES III solution, (b) KPP solution, with boundary layer depth shown as the the dotted trace, and (c) CON.

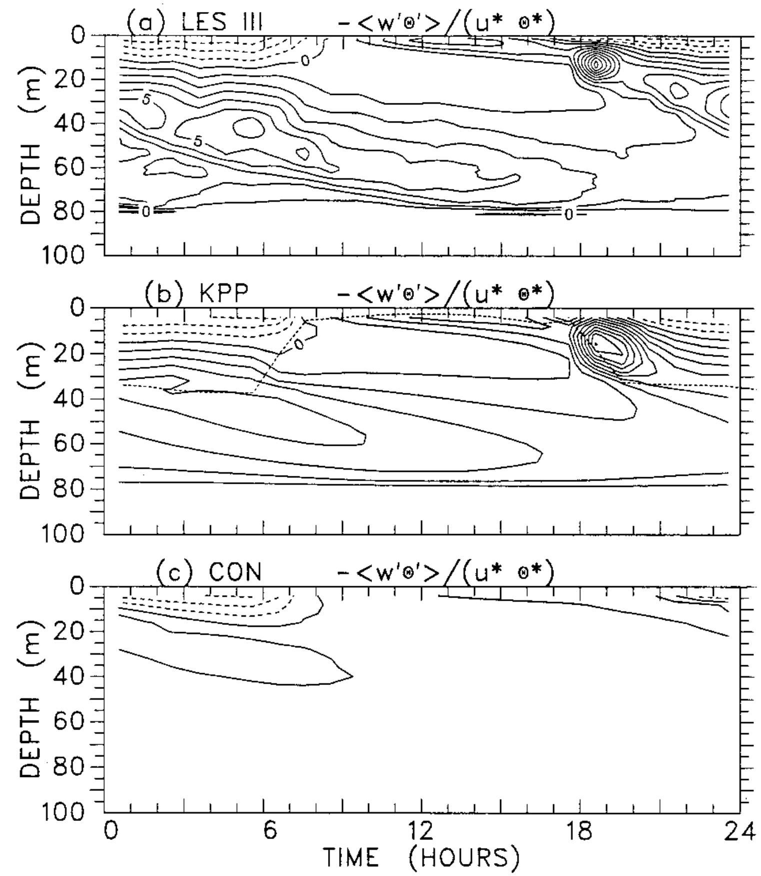


FIG. 5. As in Fig. 4 but for heat flux normalized by $u^*\theta^* = 1.15 \times 10^{-5}$ m s⁻¹ K and a contour interval of 1.0.

Mesoscale eddy mixing of tracers: the Gent-McWilliams (GM) parameterization

150 JOURNAL OF PHYSICAL OCEANOGRAPHY VOLUME 20

1990

Isopycnal Mixing in Ocean Circulation Models†

PETER R. GENT AND JAMES C. MCWILLIAMS
National Center for Atmospheric Research, Boulder, Colorado*

20 March 1989 and 14 August 1989

ABSTRACT

A subgrid-scale form for mesoscale eddy mixing on isopycnal surfaces is proposed for use in non-eddy-resolving ocean circulation models. The mixing is applied in isopycnal coordinates to isopycnal layer thickness, or inverse density gradient, as well as to passive scalars, temperature and salinity. The transformation of these mixing forms to physical coordinates is also presented.

Ocean Modelling 39 (2011) 2–9

Contents lists available at ScienceDirect

Ocean Modelling

journal homepage: www.elsevier.com/locate/ocemod

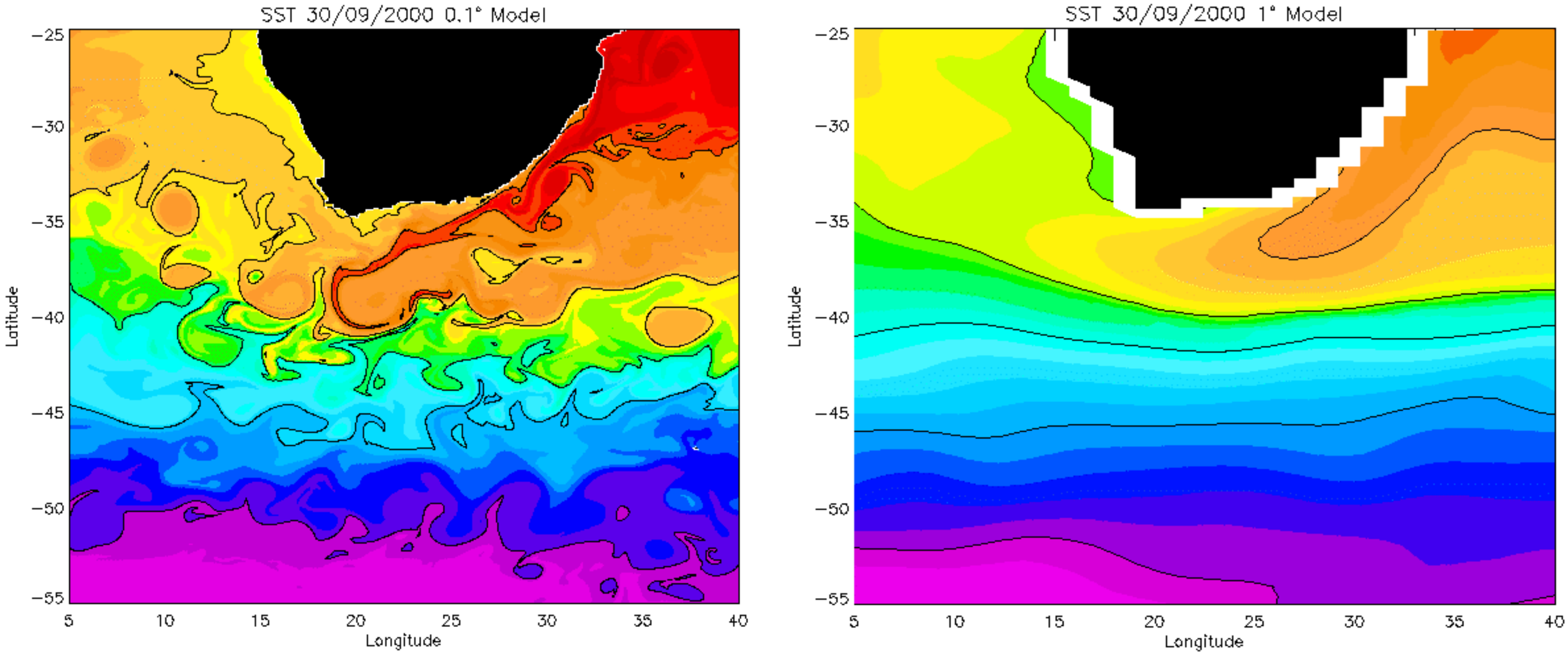
2011

The Gent–McWilliams parameterization: 20/20 hindsight

Peter R. Gent *
National Center for Atmospheric Research,[1] Boulder, CO, USA

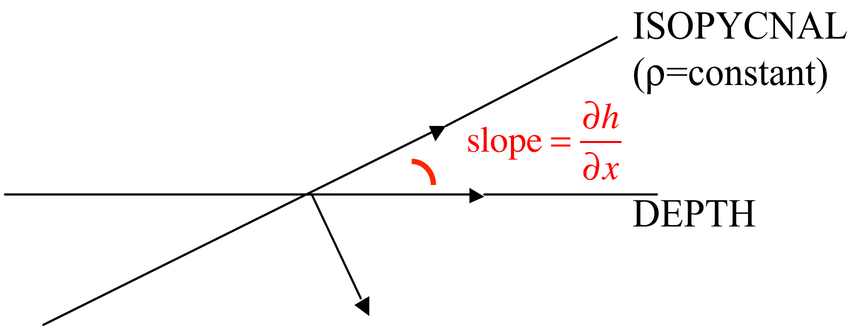
Why is GM needed? Some background…

Agulhas Retroflection



O(1°) models do not resolve the 1st baroclinic deformation radius, and hence lack the mesoscale turbulence which mixes tracers (T, S, …) in the real ocean.

Why is GM needed? Some background…



Ocean Observations suggest mixing along isopycnals is ~10^7 times larger than across isopycnals.

PRIMITIVE EQUATIONS

$$\frac{D}{Dt}\mathbf{u} + f\mathbf{k}\times\mathbf{u} + \nabla p = \nu_H\ \nabla^2\mathbf{u} + \nu_V\ \mathbf{u}_{zz}$$

where

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \mathbf{u}\cdot\nabla + w\frac{\partial}{\partial z}$$

Hydrostatic	$p_z + g\rho/\rho_0 = 0$
Continuity	$\nabla\cdot\mathbf{u} + w_z = 0$
Temperature	$\frac{D}{Dt}\theta = \kappa_H\ \nabla^2\theta + \kappa_V\ \theta_{zz}$
Salinity	$\frac{D}{Dt}S = \kappa_H\ \nabla^2 S + \kappa_V\ S_{zz}$
Eqn of State	$\rho = \rho(p, \theta, S)$

- Early ocean models "parameterized" the stirring effects of (unresolved) mesoscale eddies by using Laplacian horizontal diffusion with unrealistically large $K_H = O(10^3\ m^2/s)$.

- Horizontal mixing results in excessive diapycnal mixing which degrades the ocean solution. E.g., Veronis (1975) showed that it produces spurious upwelling in WBC regions where the primary balance becomes: $w\rho_z = \kappa\rho_{xx}$. This "short circuits" the Atlantic MOC.

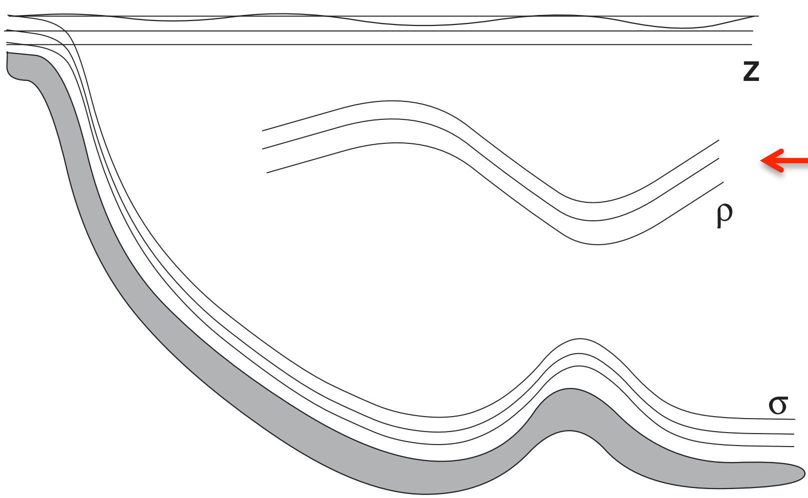
- A recognized need to orient tracer diffusion in z-coordinate models along isopycnal surfaces, to be consistent with observed ocean mixing rates.

GM 101

In isopycnal coordinates (small slope limit: $h_x, h_y << 1$):

$$\frac{\partial}{\partial t} h_\rho + \nabla \cdot (\mathbf{u} h_\rho) = 0$$ Mass continuity $h(x,y,\rho,t)$ isopycnal height

$$\frac{\partial}{\partial t}\tau + \mathbf{u}\cdot\nabla\tau = \nabla\cdot(\mu h_\rho \nabla\tau)/h_\rho$$ Passive tracer eqn $h_\rho = \frac{\partial h}{\partial \rho}$ thickness



GM is applicable here, in the adiabatic interior where isopycnal coordinates are most natural.

Fig. 1. Schematic of an ocean basin illustrating the three regimes of the ocean germane to the considerations of an appropriate vertical coordinate. The surface mixed layer is naturally represented using z-coordinates; the interior is naturally represented using isopycnal ρ-coordinates; and the bottom boundary is naturally represented using terrain following σ-coordinates.

GM 101

In isopycnal coordinates (small slope limit: $h_x, h_y \ll 1$):

$$\frac{\partial}{\partial t} h_\rho + \nabla \cdot (\mathbf{u} h_\rho) = 0$$

Mass continuity

$$h(x, y, \rho, t)$$

isopycnal height

$$\frac{\partial}{\partial t}\tau + \mathbf{u} \cdot \nabla \tau = \nabla \cdot (\mu h_\rho \nabla \tau) / h_\rho$$

tracer eqn

$$h_\rho = \frac{\partial h}{\partial \rho}$$

thickness

➔ Redi (1982) implemented a rotation in z-level model so that Laplacian diffusion of tracers occurred along isopycnals with (an enhanced) diffusivity μ, but with unsatisfactory results.

GM 101

$$\frac{\partial}{\partial t} h_\rho + \nabla \cdot (\mathbf{u} h_\rho) = 0$$ Mass continuity

$$\frac{\partial}{\partial t} \tau + \mathbf{u} \cdot \nabla \tau = \nabla \cdot (\mu h_\rho \nabla \tau) / h_\rho$$ tracer eqn

Gent & McWilliams (JPO,1993); Gent et al. (JPO, 1995):

Reynolds decomposition of mass continuity in steady-state gives the following balance (overbar = spatiotemporal averaging to filter eddy scales):

$$\nabla \cdot (\overline{\mathbf{u}}\overline{h_\rho}) + \overline{\nabla \cdot (\mathbf{u}' h_\rho')} = 0$$

Therefore, parameterize eddy effects in non-eddy-resolving models by adding an eddy-induced advection (u*) term in addition to Redi diffusion:

$$\frac{\partial}{\partial t} h_\rho + \nabla \cdot (\mathbf{u} h_\rho) + \nabla \cdot (\mathbf{u}^* h_\rho) = 0 \qquad \mathbf{u}^* = \overline{\mathbf{u}' h_\rho'} / h_\rho$$

➔ Large-scale tracers are not advected by the large-scale velocity alone, but by the "effective transport velocity" which has an eddy-induced component!

$$\frac{\partial}{\partial t} \tau + \mathbf{U} \cdot \nabla \tau = \nabla \cdot (\mu h_\rho \nabla \tau) / h_\rho \qquad \mathbf{U} = \mathbf{u} + \mathbf{u}^*$$

GM 101

Now, back to z-coordinates...

7.1.3 The Gent-McWilliams Parameterization

The transport equation of tracer φ is given by

$$\frac{\partial}{\partial t}\varphi + (\mathbf{u}+\mathbf{u}^*)\cdot\nabla\varphi + (w+w^*)\frac{\partial}{\partial z}\varphi = R(\varphi) + \mathcal{D}_V(\varphi), \qquad (7.2)$$

where the bolus velocity induced by mesoscale eddies is parameterized, from Gent and McWilliams (1990), as

isopycnal slope

$$\mathbf{u}^* = \left(\nu\frac{\nabla\rho}{\rho_z}\right)_z, \qquad w^* = -\nabla\cdot\left(\nu\frac{\nabla\rho}{\rho_z}\right), \qquad (7.3)$$

"GM coefficient" or "thickness diffusivity"

where ν is a thickness diffusivity and subscripts x, y, z on ρ and tracers φ denote partial derivatives with respect to those variables (this convention will be followed below). The Redi isoneutral diffusion operator (Redi, 1982) for small slope can be written as

$$R(\varphi) = \nabla_3\cdot(\mathbf{K}\cdot\nabla_3\varphi), \qquad (7.4)$$

where the subscript 3 indicates the three-dimensional gradient or divergence, i.e., $\nabla_3 = (\nabla, \frac{\partial}{\partial z})$. The symmetric tensor $\mathbf{K}$ is defined as

$$\mathbf{K} = \kappa_I\begin{pmatrix} 1 & 0 & -\rho_x/\rho_z \\ 0 & 1 & -\rho_y/\rho_z \\ -\rho_x/\rho_z & -\rho_y/\rho_z & |\nabla\rho|^2/\rho_z^2 \end{pmatrix}, \qquad (7.5)$$

"Redi coefficient" or "isopycnal diffusivity"

This tensor describes along-isopycnal diffusion that is isotropic in the two horizontal dimensions. The general anisotropic form of (7.5) is given in Smith (1999). The isopycnal diffusivity κ_I is in general a function of space and time, and a parameterization for variable κ_I will be described at the end of this section. In POP, we write the bolus velocity in the skew-flux form (Griffies, 1998):

GM impacts

Gent et al. (JPO, 1995): v* = (u*,w*) acts to flatten isopycnals and minimize potential energy.

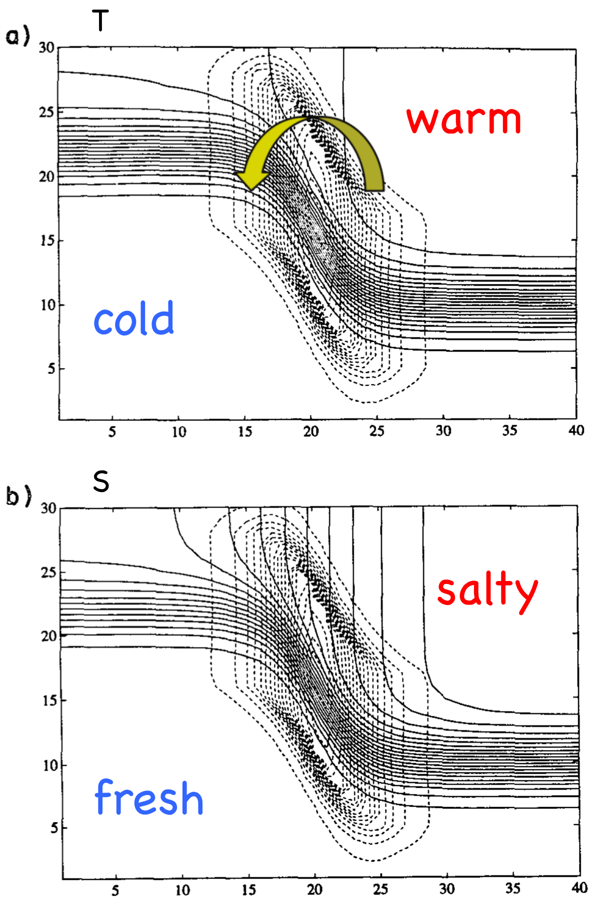


FIG. 3. Initial states of (a) temperature and (b) salt [contour interval one-quarter that of (a)]. Both panels also show the streamfunction $\kappa\rho_x/\rho_z$ for the parameterized eddy-induced transport velocity.

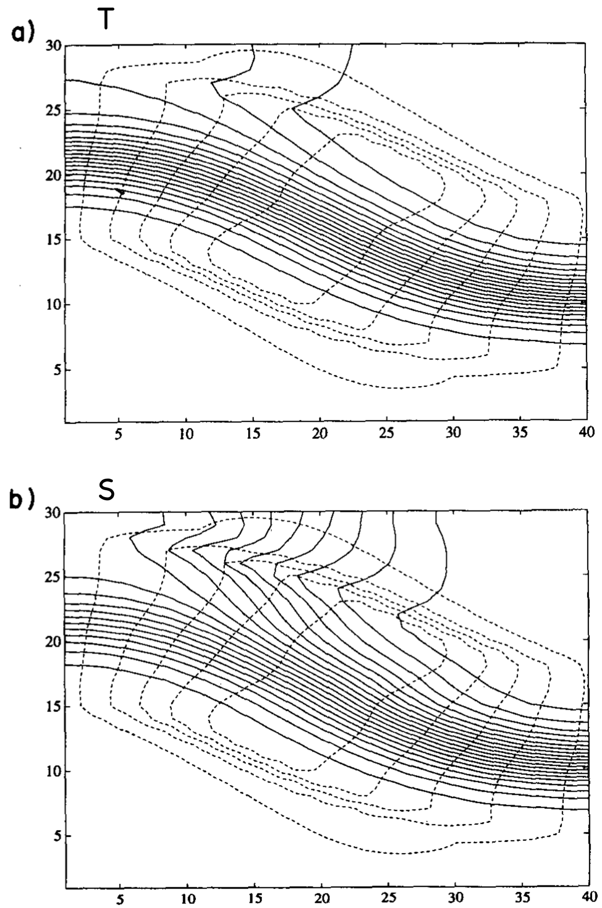


FIG. 4. Distributions of (a) temperature and (b) salt after an integration time of $20\Delta s^2/\kappa$. Both panels also show the streamfunction $\kappa\rho_x/\rho_z$ for the parameterized eddy-induced transport velocity. Contour intervals are the same as in Fig. 3.

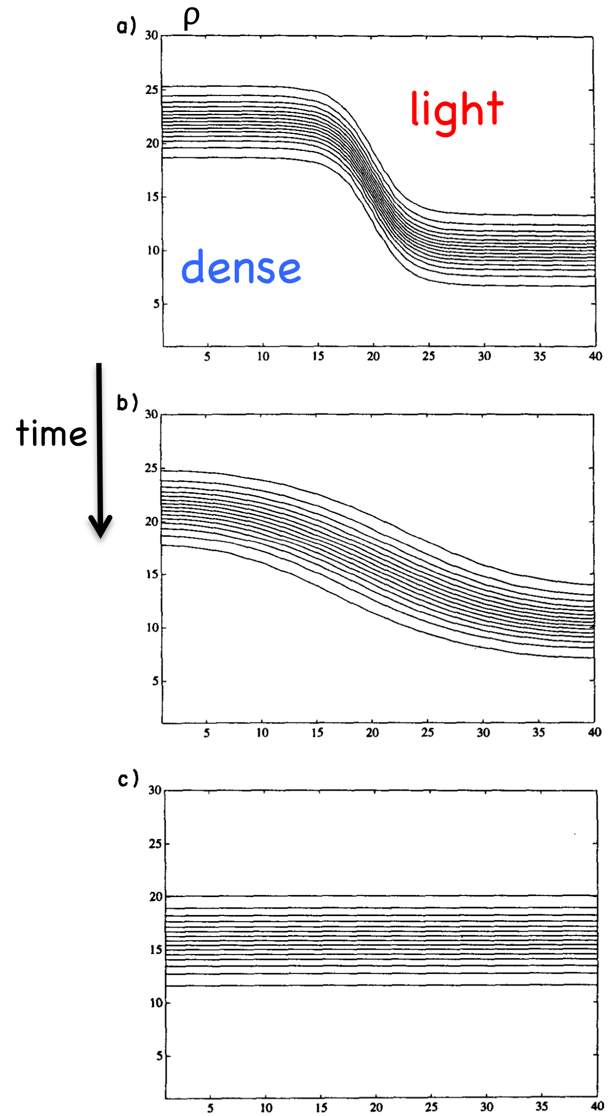


FIG. 5. Density distribution at various times of the integration. (a) Initial, (b) $20\Delta s^2/\kappa$, and (c) $1000\Delta s^2/\kappa$.

GM in a nutshell

Mimics effects of unresolved mesoscale eddies as a sum of

- diffusive mixing of tracers along isopycnals (Redi),
- an additional advection of tracers by a divergence-free, eddy-induced velocity : u^*

Quasi-adiabatic and valid for the ocean interior.

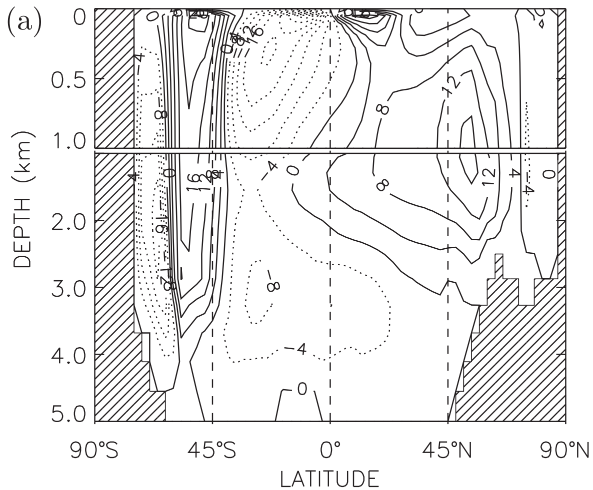
Acts to flatten isopycnals, thereby reducing potential energy.

Eliminates any need for horizontal diffusion in z-coordinate OGCMs

➔ eliminates Veronis effect.

Implementation of GM was a major factor in enabling in stable coupled climate model simulations without "flux corrections".

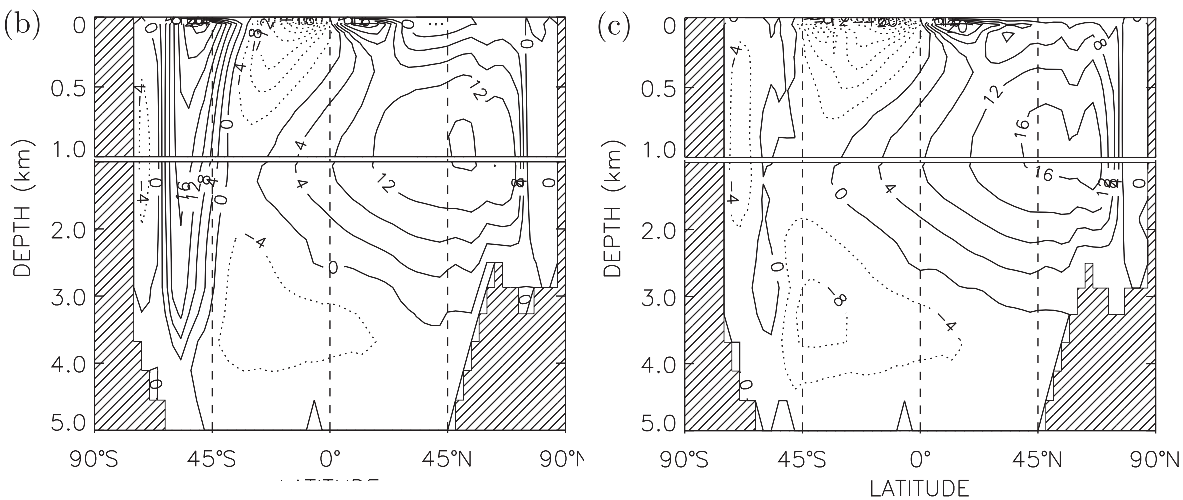
Impacts of the GM parameterization



(a) Horizontal Diffusion , MOC(**u**)

(b) GM , MOC(**u**)

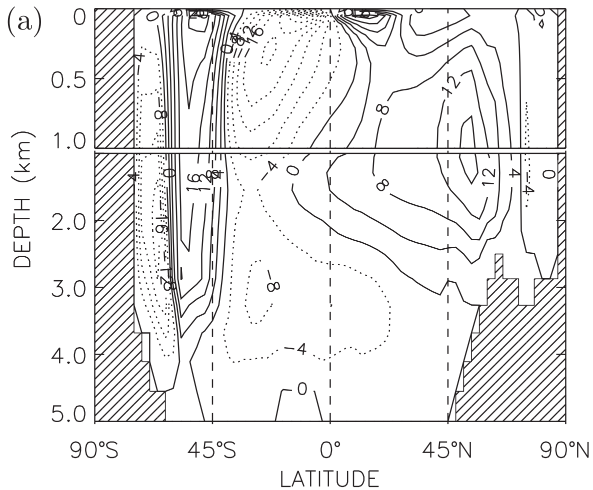
(c) GM, MOC(**u+u***)



Danabasoglu et al. (1994, Science)

4^ox3^ox20L ocean model

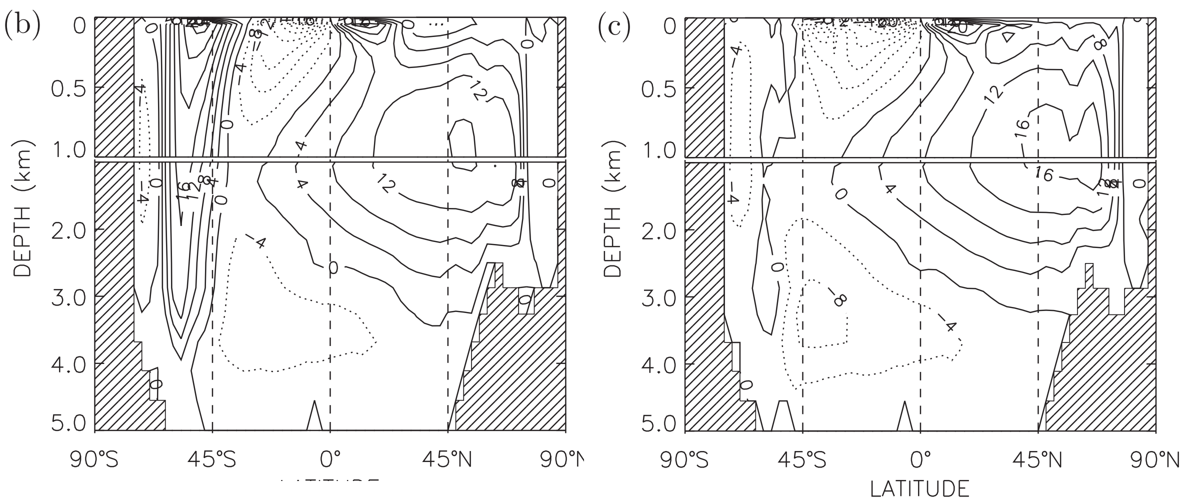
Impacts of the GM parameterization

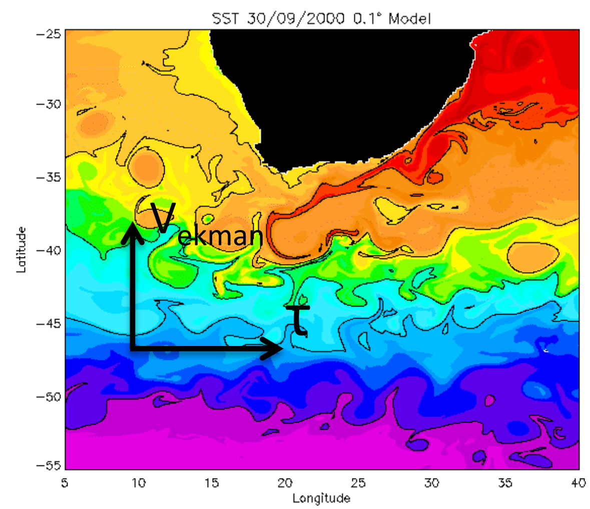


(a) Horizontal Diffusion , MOC(**u**)

(b) GM , MOC(**u**)

(c) GM, MOC(**u+u***)

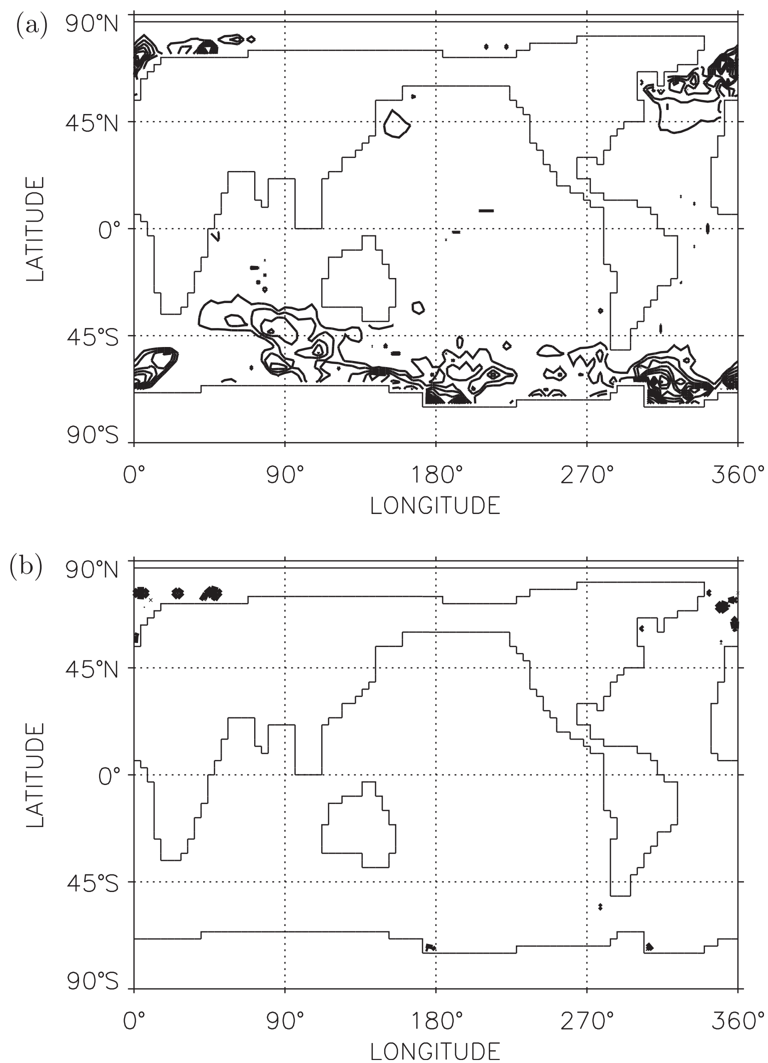




Danabasoglu et al. (1994, Science)

4°x3°x20L ocean model

Impacts of the GM parameterization



(a) Horizontal Diffusion

(b) GM

Percentage of all times and model levels where convective adjustment occurred.

Danabasoglu et al. (1994, Science)

4°x3°x20L ocean model

Improvements to GM: the near-surface eddy flux (NSEF) scheme

GM90 is valid only in the quasi-adiabatic ocean interior, therefore the usual practice has been to taper both κ_I and ν to zero as the surface is approached.

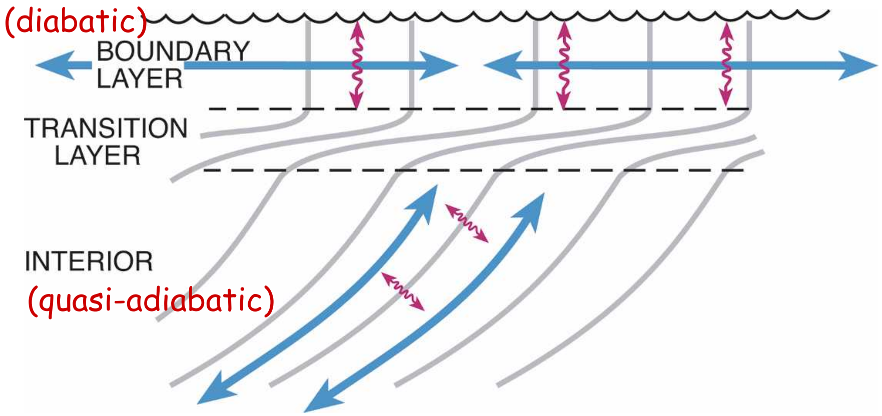


FIG. 2. A conceptual model of eddy fluxes in the upper ocean. Mesoscale eddy fluxes (blue arrows) act to both move isopycnal surfaces and stir materials along them in the oceanic *interior*, but the fluxes become parallel to the boundary and cross density surfaces within the *BL*. Microscale turbulent fluxes (red arrows) mix materials across isopycnal surfaces, weakly in the interior and strongly near the boundary. The interior and the BL regions are connected through a *transition layer* where the mesoscale fluxes rotate toward the boundary-parallel direction and develop a diabatic component.

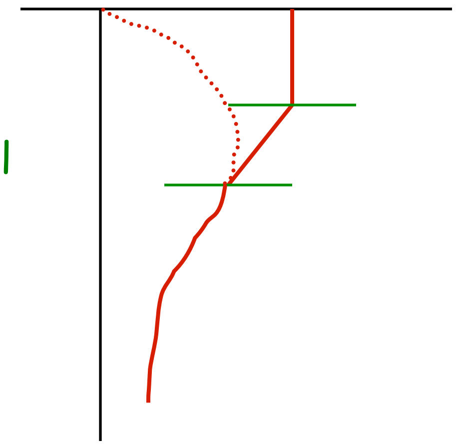
Diffusivities

Horizontal

Horizontal/Isopycnal

Isopycnal

Eddy-induced velocity profile



NSEF replaces the usual approach of applying near-surface taper functions for the diffusivities.

Ferrari et al. (2008, J. Climate)

Improvements to GM: the near-surface eddy flux (NSEF) scheme

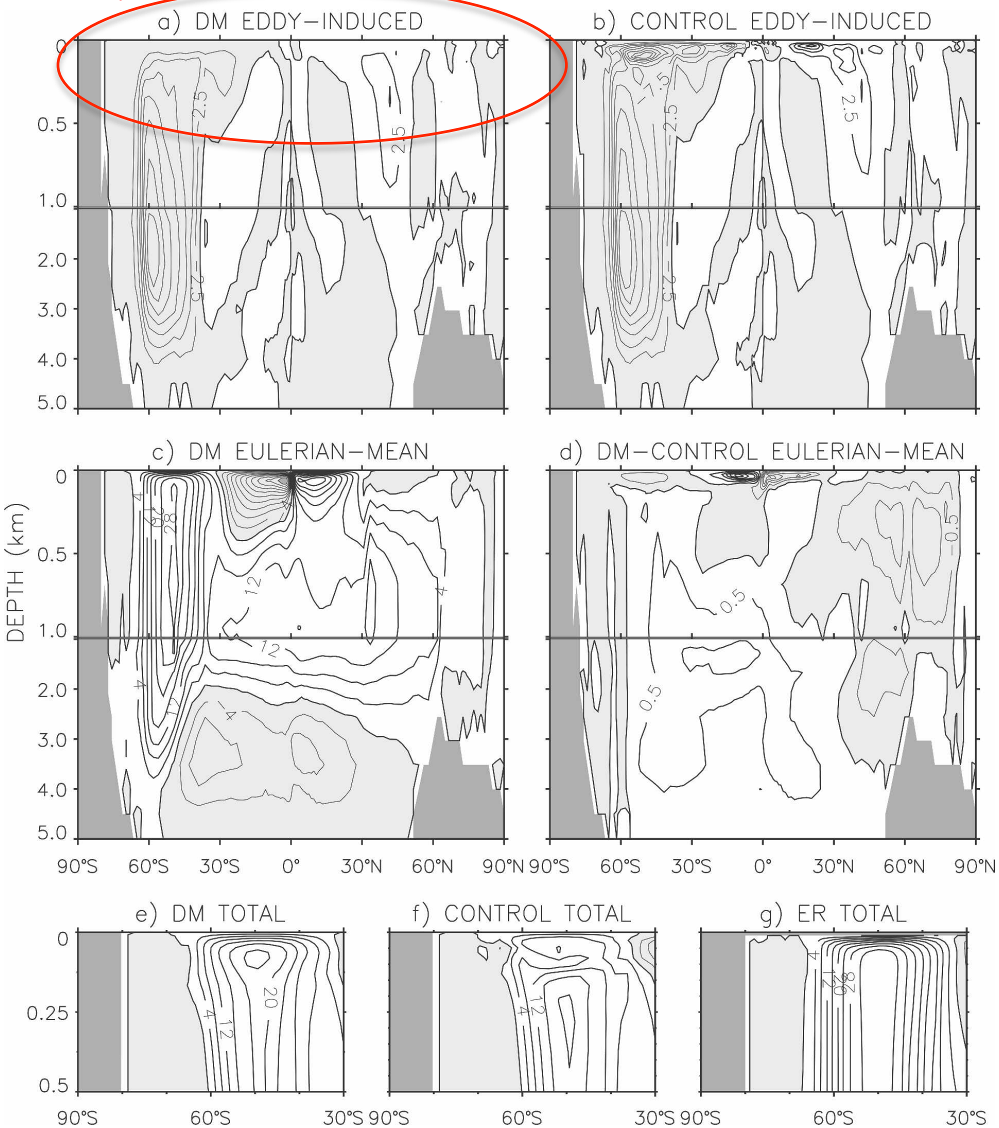


FIG. 3. Time-mean zonally integrated meridional overturning streamfunction obtained with (a) eddy-induced velocity from DM, (b) eddy-induced velocity from CONTROL, (c) Eulerian-mean velocity from DM, (d) Eulerian-mean velocity for the DM − CONTROL difference, (e) total (i.e., the sum of Eulerian mean and eddy induced) velocity from DM, (f) total velocity from CONTROL, and (g) total velocity from ER. The contour intervals are 2.5 Sv in (a) and (b); 4 Sv in (c), (e), (f), and (g); and 0.5 Sv in (d). The thin lines and shading indicate counterclockwise circulation in all panels except (d) where they indicate negative differences: (e), (f), and (g) are for the Southern Hemisphere upper-ocean high latitudes.

NSEF greatly reduces eddy-induced circulation in near surface from GM, improving comparison with 0.1° simulation.

Danabasoglu et al. (2008, J. Climate)

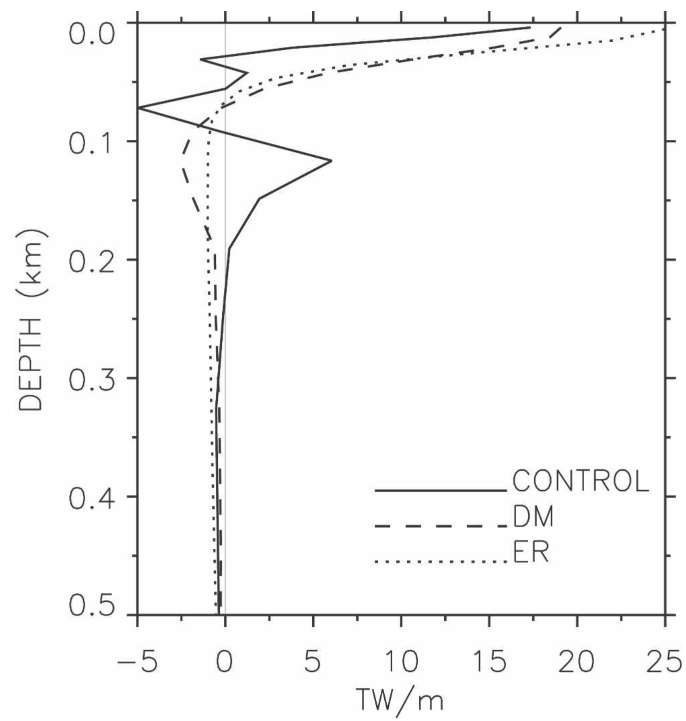


FIG. 5. Upper-ocean vertical profiles of zonally integrated, time-mean total advective (Eulerian mean + eddy induced) heat transport at 49.4°S from CONTROL, DM, and ER.

Improvements to GM: vertically-varying eddy diffusivity

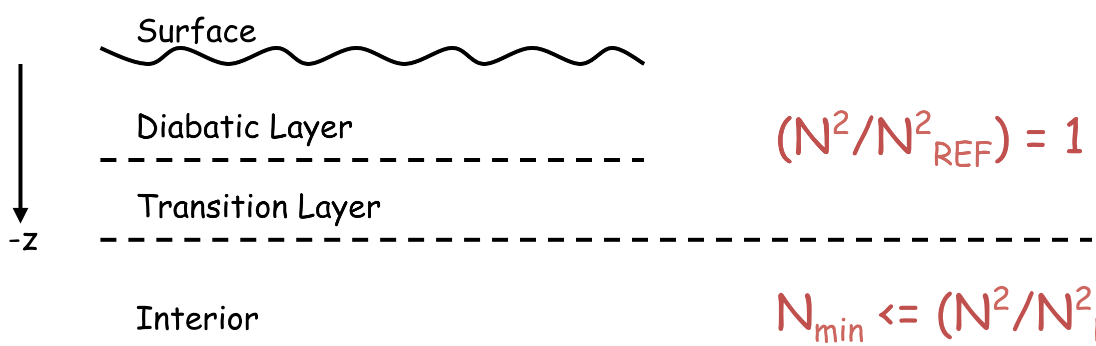
Following Ferreira et al. (2005), the GM diffusivities ($A = v$) are specified as

$$A = A_{REF} \left(N^2 / N^2_{REF} \right)$$

N^2: Local buoyancy frequency,

N^2_{REF}: Reference buoyancy frequency just below the transition layer,

A_{REF}: Constant reference value of A within the surface diabatic region.



$(N^2/N^2_{REF}) = 1$; A_{REF}

$N_{min} <= (N^2/N^2_{REF}) <= 1$

N_{min} is a lower limit, $N_{min} = 0.1$.

Ferreira et al. (2005, JPO)

Improvements to GM: vertically-varying eddy diffusivity

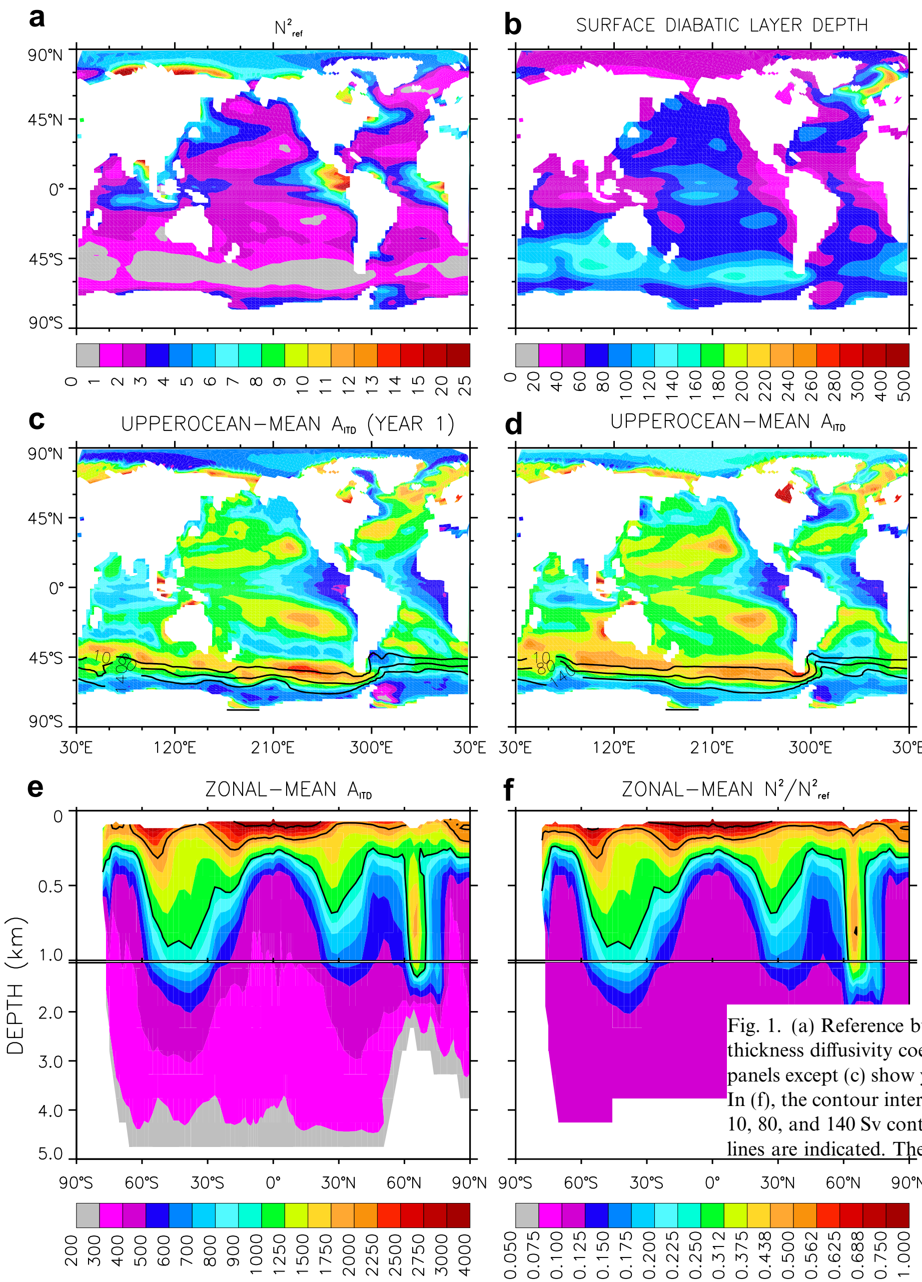


Fig. 1. (a) Reference buoyancy frequency squared, N^2_{ref}, in 10^{-4} s^{-2}; (b) Surface diabatic layer depth in m; Upper-ocean [0–945 m]-mean thickness diffusivity coefficient, A_{ITD}: (c) year 1 mean and (d) years (1981–2000) mean; Zonal-mean, global (e) A_{ITD} and (f) N^2/N^2_{ref}. All panels except (c) show years (1981–2000) mean. All panels are from TN2. Panels (c–e) are in m^2 s^{-1} and share the same contour intervals. In (f), the contour intervals are obtained by normalizing the contour intervals used in panels (c–e) by $[A_{ITD}]_{ref}$ (see Eq. (1)). In (c) and (d), 10, 80, and 140 Sv contour lines are drawn for the Antarctic Circumpolar Current transport. In (e), 1000, 2000, and 3000 m^2 s^{-1} contour lines are indicated. The corresponding N^2/N^2_{ref} contours of 0.25, 0.50, and 0.75 are also drawn in (f).

Danabasoglu & Marshall (2007, Ocean Modelling)

Improvements to GM: vertically-varying eddy diffusivity

➔ N^2-dependence of GM coeff improves comparison w/ observed eddy-induced transports at 22ºN in the Pacific:

Danabasoglu & Marshall (2007, Ocean Modeling)

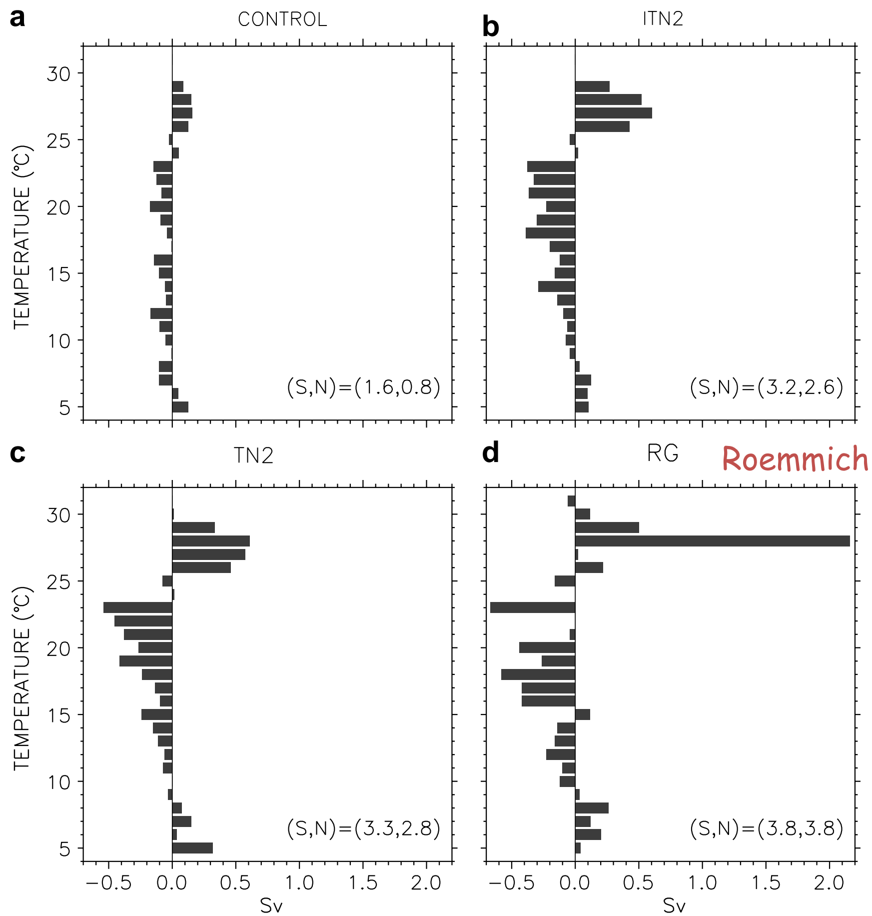
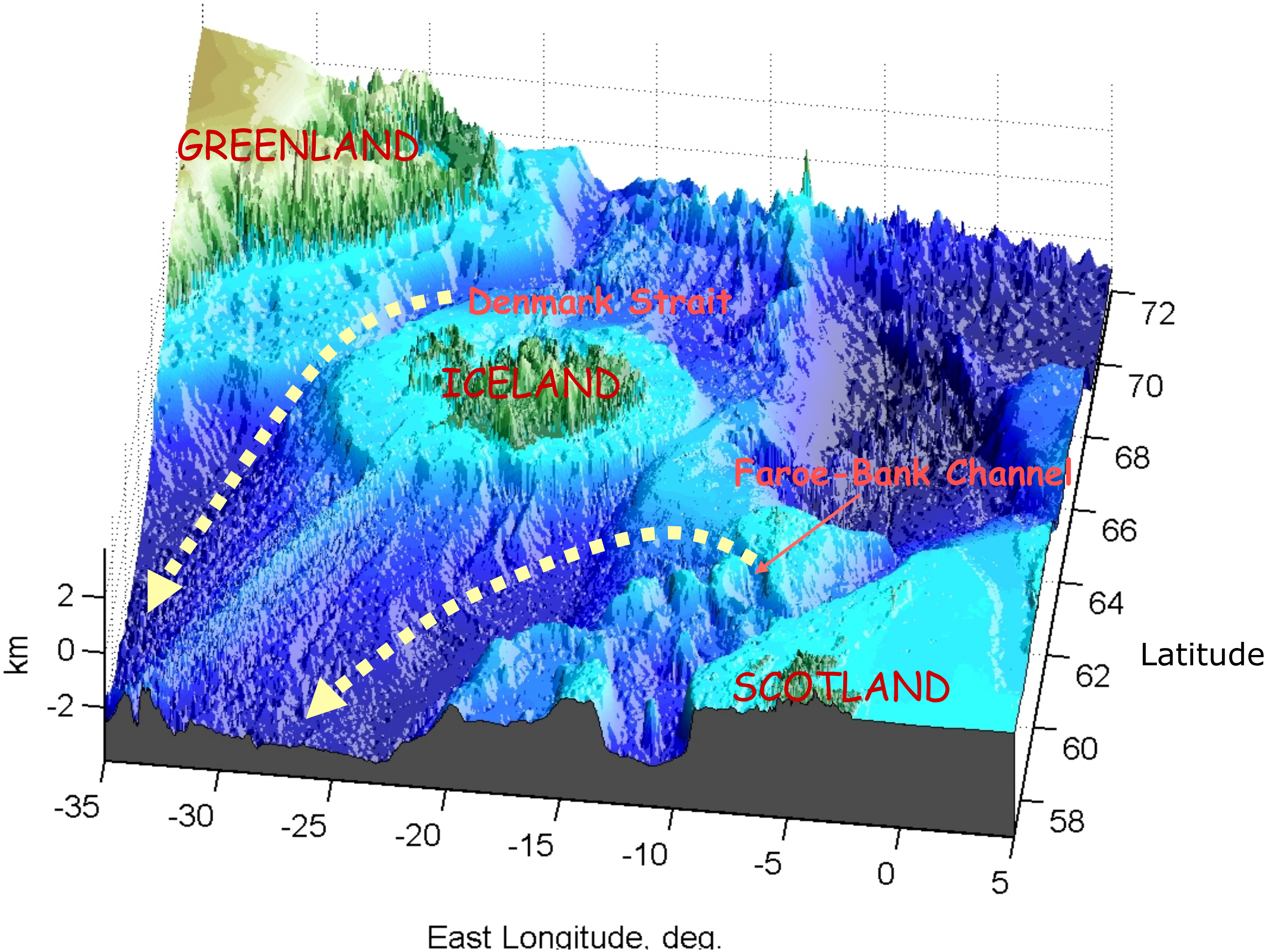


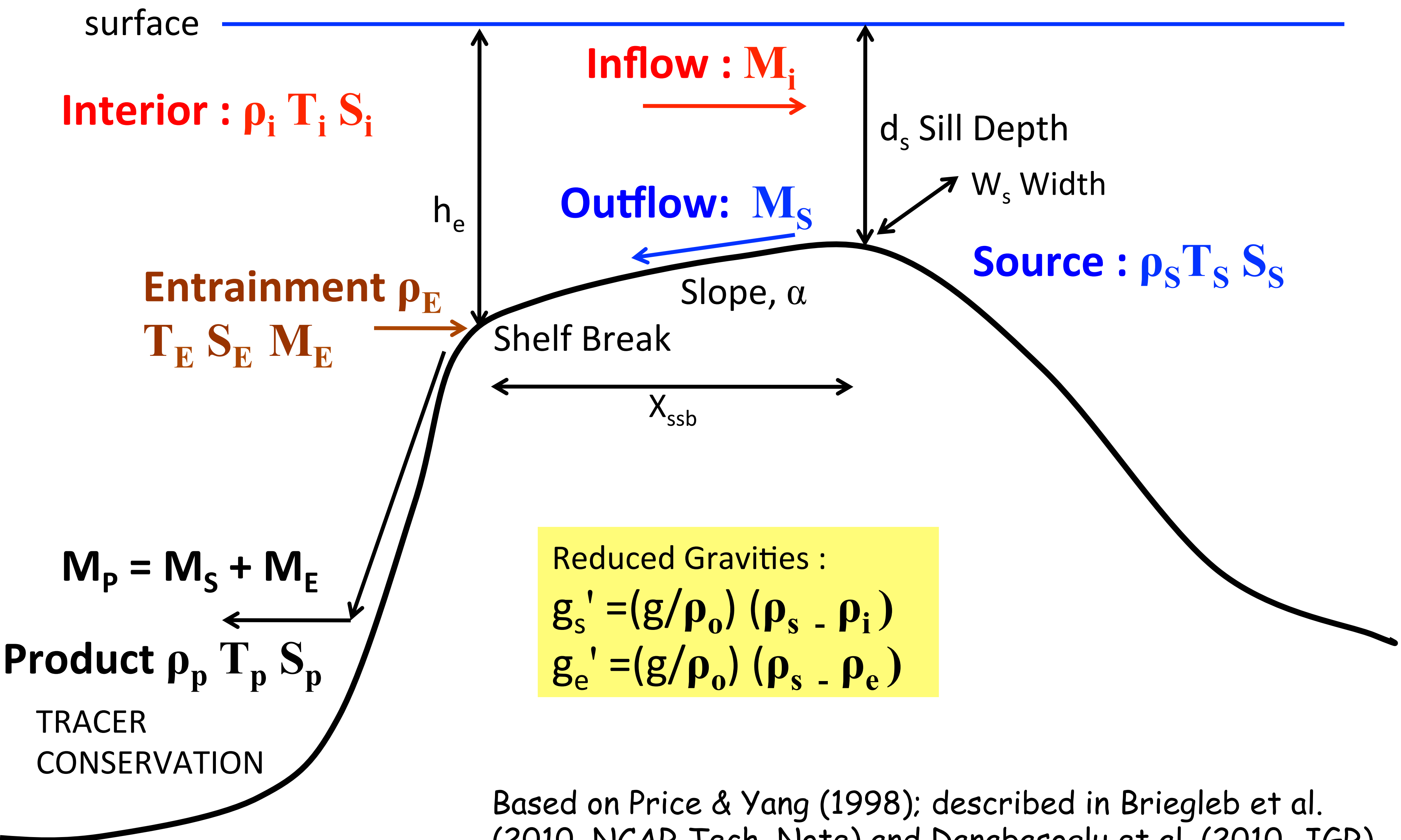
Fig. 2. Time-mean, eddy-induced normal transport in potential temperature bins (1 °C bin interval) in the upper-ocean integrated along the repeated hydrographic ship track in the tropical North Pacific from (a) CONTROL, (b) ITN2, (c) TN2, and (d) measured by Roemmich and Gilson (2001). The numbers denoted by S and N indicate the cumulative southward and northward transports in the upper ocean, respectively. The integrals extend to 945 and 800 m in the model and observations, respectively.

GRAVITY CURRENT OVERFLOWS



from J.Price

GRAVITY CURRENT OVERFLOW PARAMETERIZATION



Based on Price & Yang (1998); described in Briegleb et al. (2010, NCAR Tech. Note) and Danabasoglu et al. (2010, JGR)

OVERFLOW PARAMERERIZATION MODEL SCHEMATIC

Danabasoglu et al. (2010, JGR)

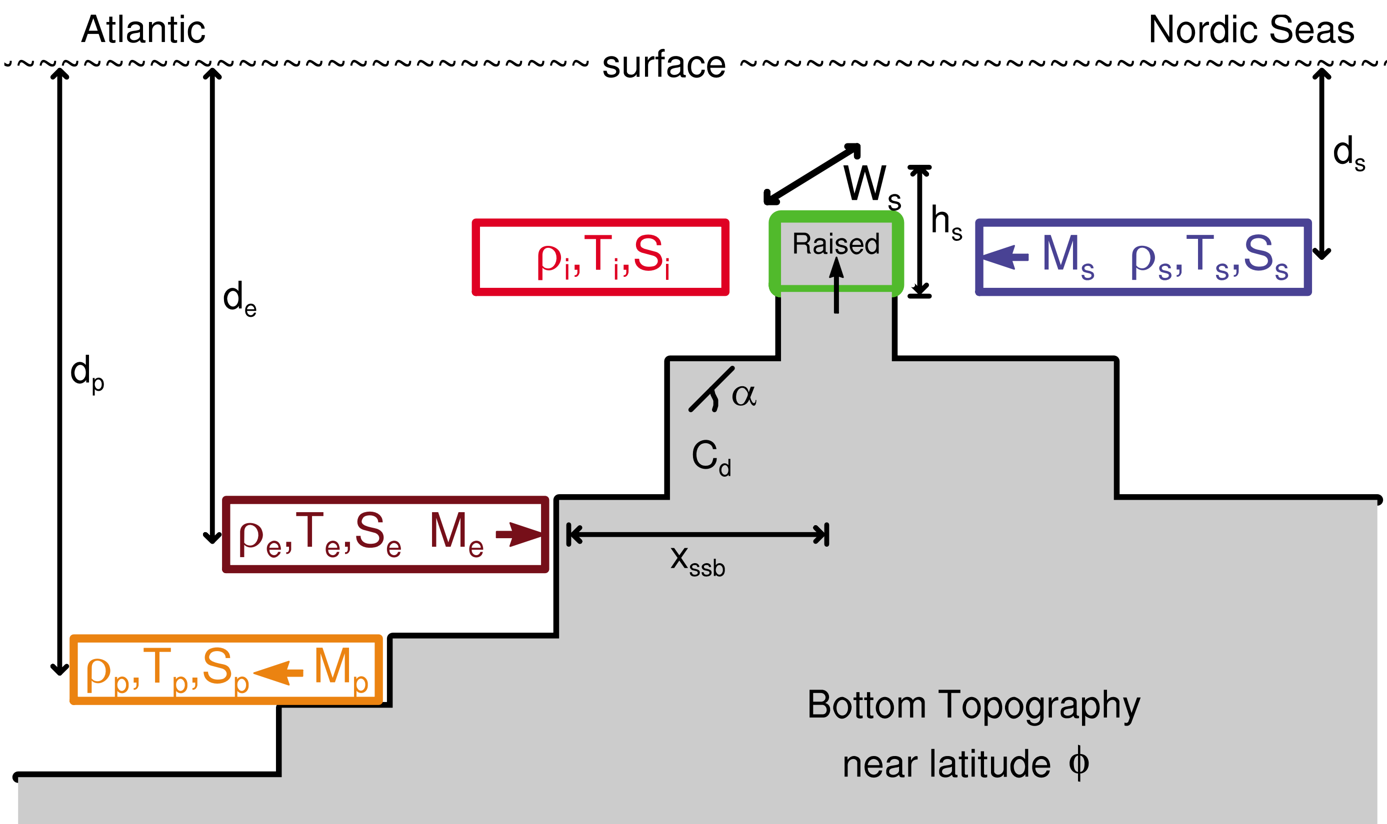


Figure 1. A schematic of the Nordic Sea overflows. T, S, ρ, and M represent potential temperature, salinity, density, and volume transport, respectively. The subscripts s, i, e, and p refer to properties of the overflow source water at the sill depth, the interior water at the sill depth, the entrainment water, and the product water, respectively. The thick, short arrows indicate flow directions. The sill depth lies within the green box of raised bottom topography. The other boxes (except the orange product box) represent the regions whose T and S are used to compute the necessary densities. All parameters shown in black are constants specified for a particular overflow (Table 1). See section 2 for further details.

BOTTOM TOPOGRAPHY OF THE x1 RESOLUTION OCEAN MODEL

Danabasoglu et al. (2010, JGR)

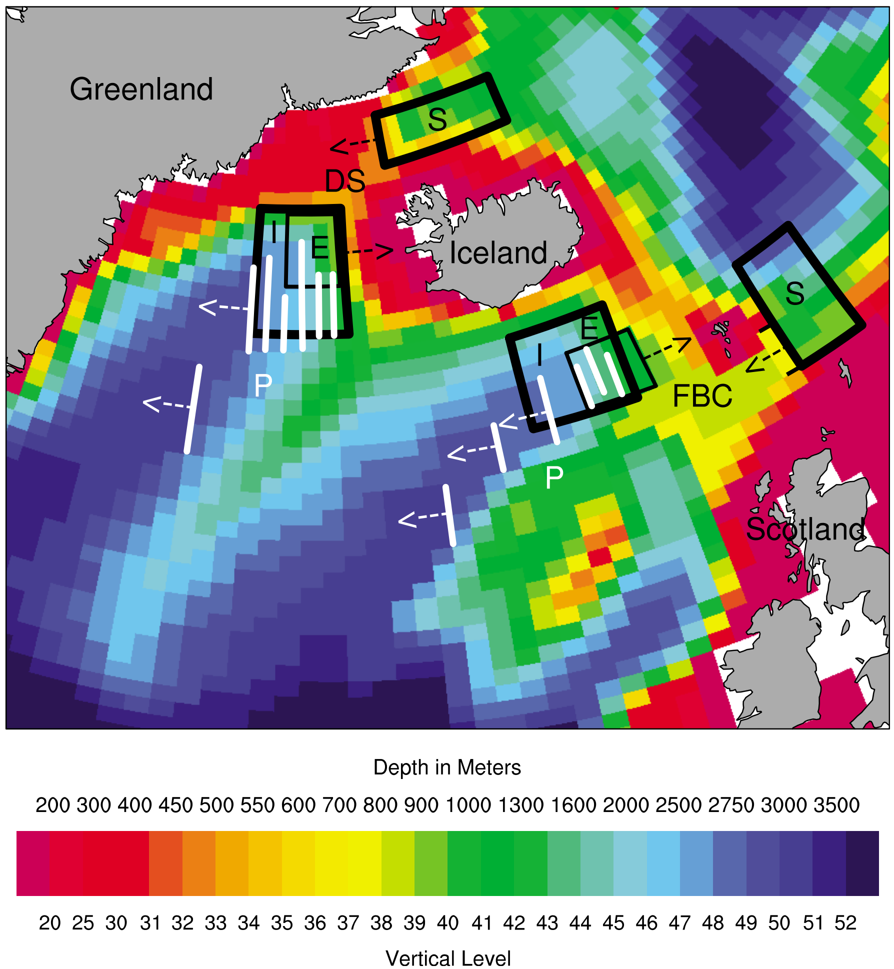


Figure 2. Bottom topography as represented in the model in the vicinity of the Denmark Strait (DS) and Faroe Bank Channel (FBC) overflows. The colors indicate the model vertical levels. The corresponding depths are given above the color bar. The boxed regions denoted by I, E, and S indicate the interior, entrainment (thin box), and source regions in the horizontal, respectively, whose *T* and *S* properties are used to compute the necessary densities. The source and entrainment box edges at which the respective water properties and transports are imposed as side boundary conditions in the OGCM are indicated by the black arrows, showing directions corresponding to flows out of the OGCM domain. The white lines denoted by P show the prespecified product water injection locations into the OGCM domain. All product water sites have the same injection direction as denoted by the white arrows drawn at only a few of the sites for clarity.

VERIFICATION AND IMPACTS OF THE OVERFLOW PARAMETERIZATION

	UNCOUPLED	COUPLED
Control (no overflows)	OCN	CCSM
With overflows	OCN*	CCSM*

Each experiment is run for 170 years.

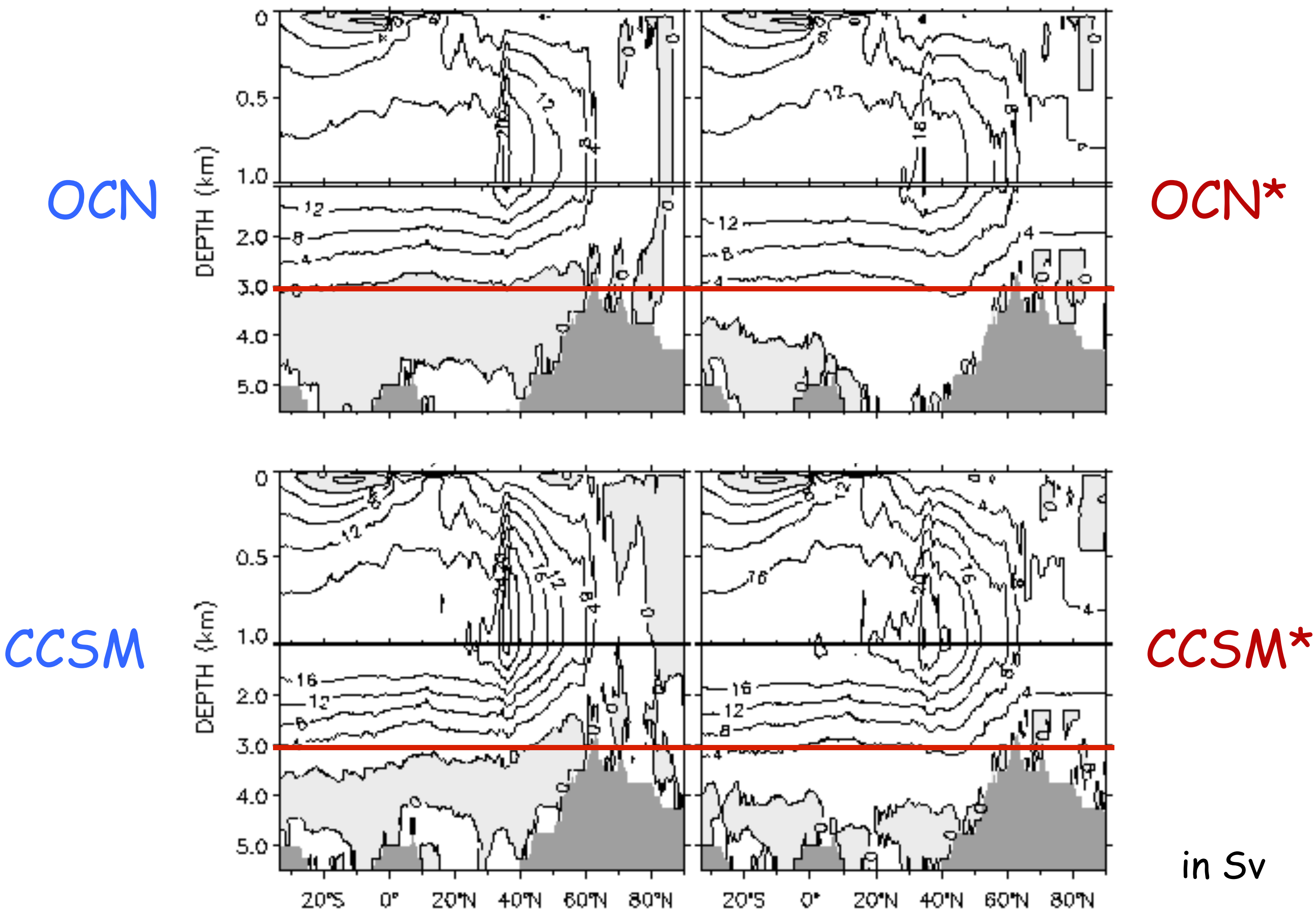
NORDIC SEA OVERFLOW TRANSPORTS

Verification Steps

All in Sv

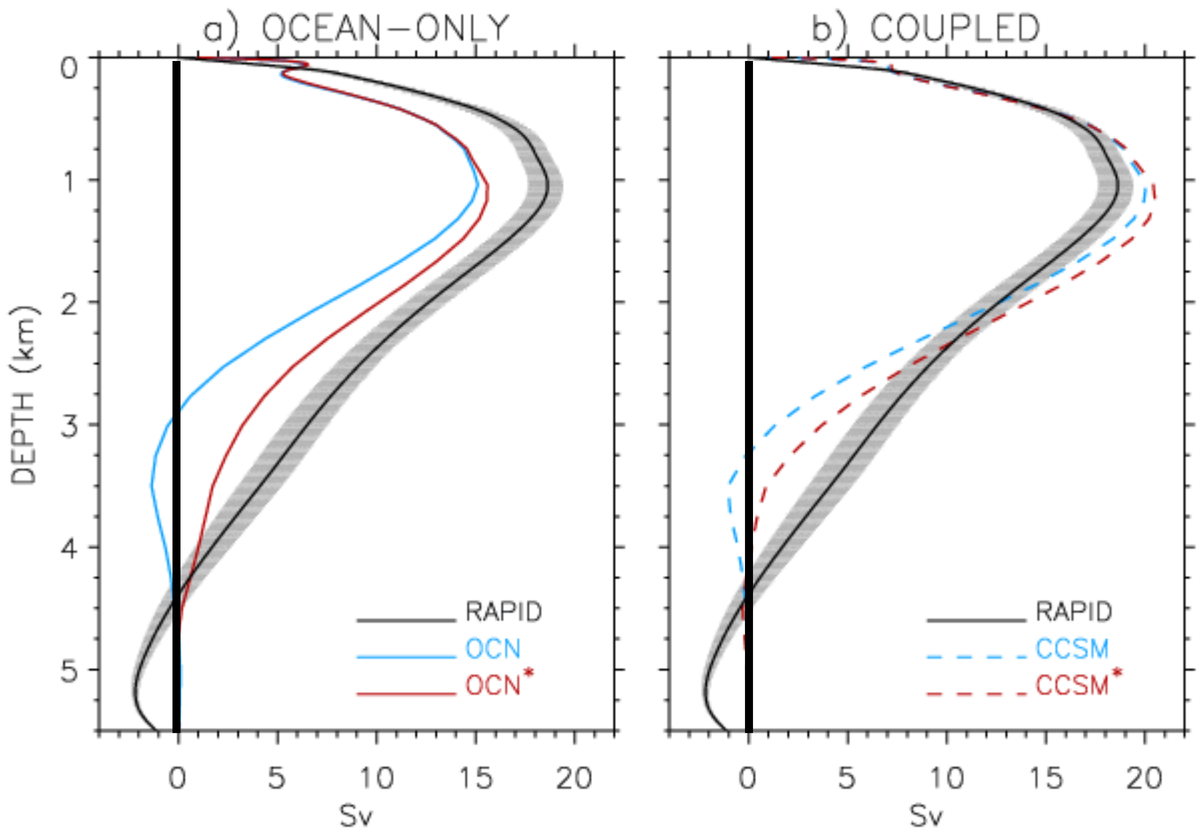
		1	2	3
	Observed	Diagnostic model (offline)	OCN*	CCSM*
M_I				
M_S	4.1 - 7.5	5.2	4.7	4.4
M_E	1.5 – 3.7	1.2	0.9	1.1
M_P	6.4 – 9.4	6.4	5.6	5.5

ATLANTIC MERIDIONAL OVERTURNING CIRCULATION (AMOC)



Danabasoglu et al. (2010, JGR)

AMOC TRANSPORT AT 26.5°N



RAPID is observational data

Danabasoglu et al. (2010, JGR)

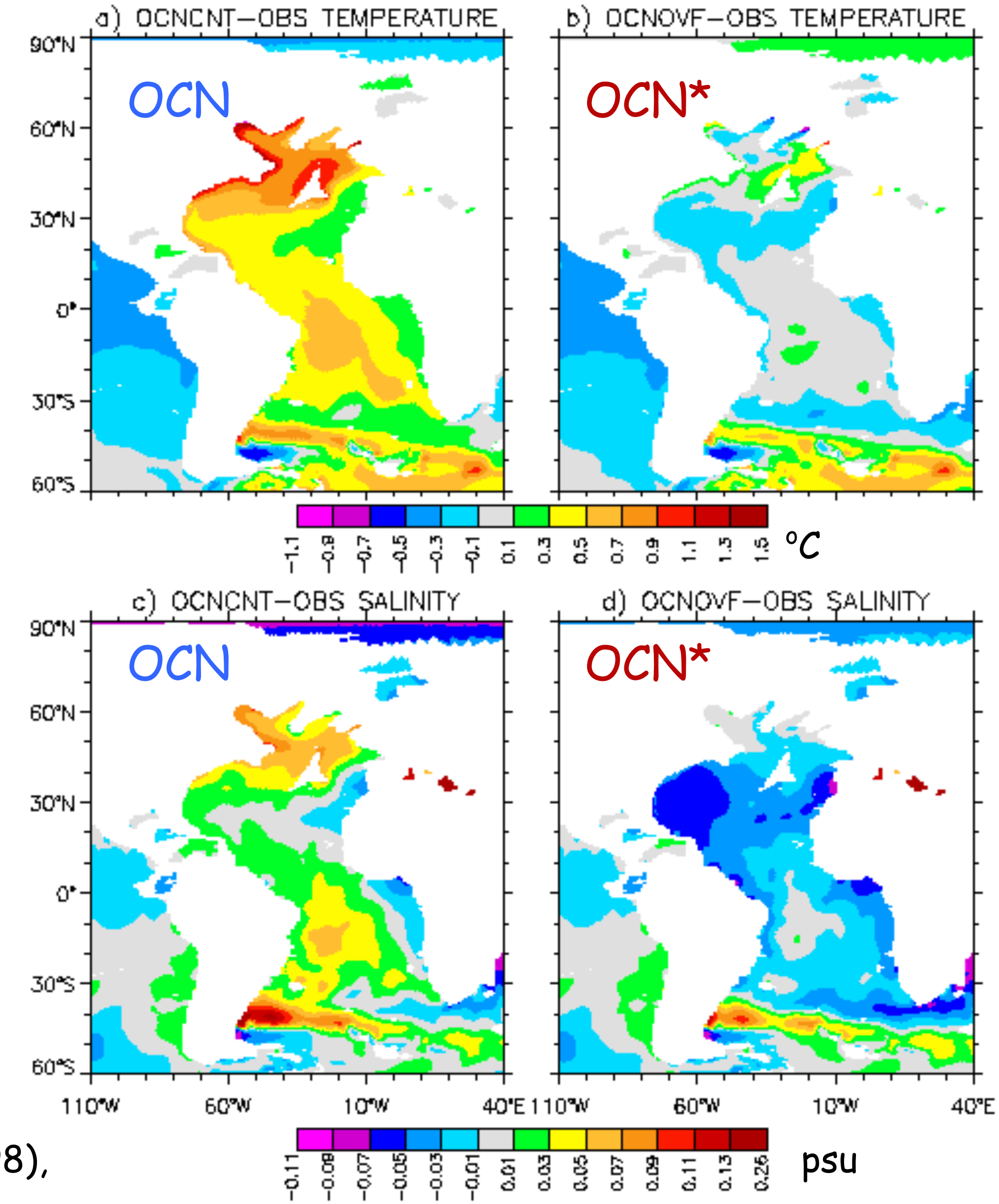
TEMPERATURE AND SALINITY DIFFERENCES FROM OBSERVATIONS AT 2649-m DEPTH

mean= 0.45°C

rms= 0.50°C

mean= -0.04°C

rms= 0.13°C



mean= 0.02 psu

rms= 0.03 psu

mean= -0.03 psu

rms= 0.03 psu

Obs: Levitus et al. (1998), Steele et al. (2001)

DWBC transport

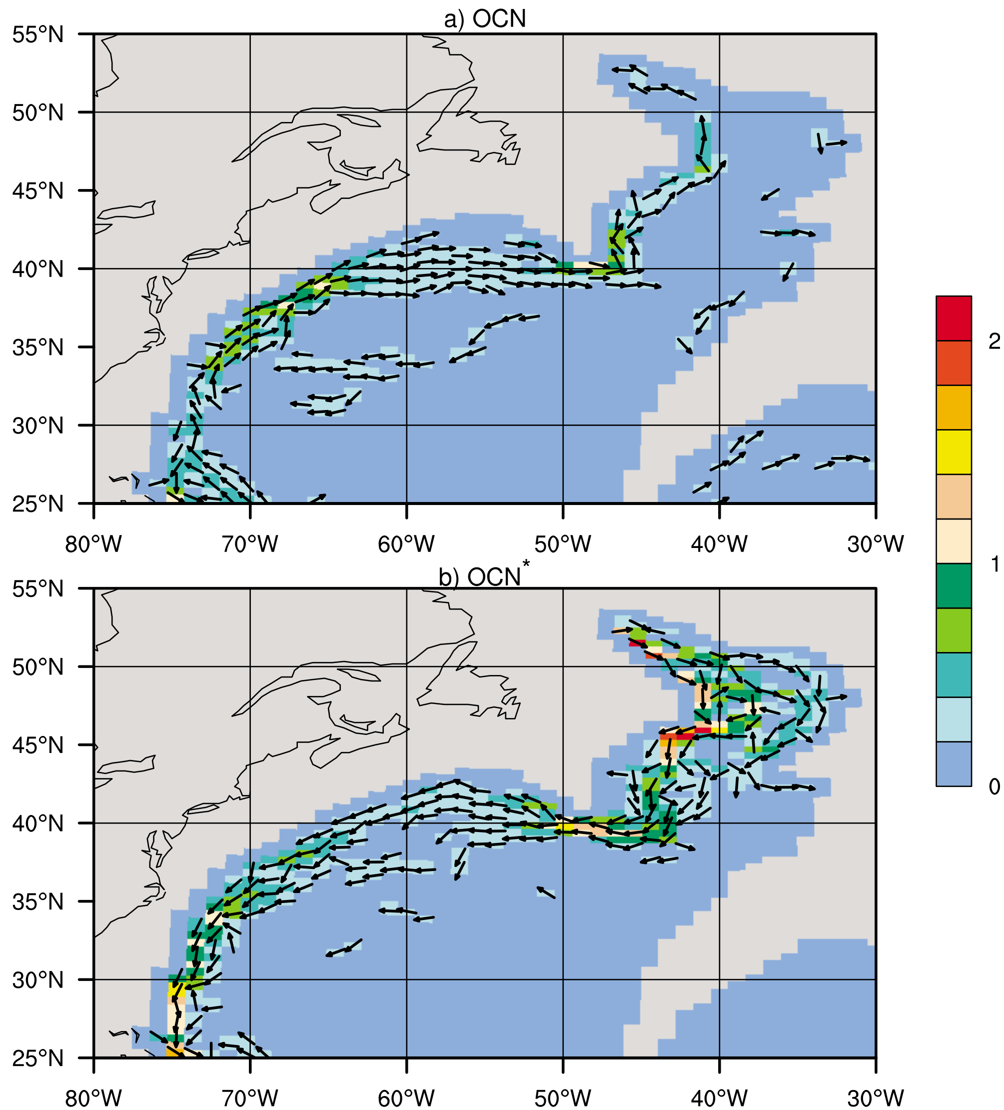


Figure 7. Time mean horizontal velocity at a depth of 3876 m from (a) OCN and (b) OCN*. Arrows and colors give the flow direction and magnitude in cm s^{-1}, respectively. Arrows are plotted only for speeds larger than 0.2 cm s^{-1}.

Table 4. Volume Transports in Sv Across the Sections Indicated in Figure 8[a]

Case	44°W Westward	49.3°W Eastward	49.3°W $\sigma_0 \geq$ 27.74 Eastward	69°W Westward
OCN	5.3	3.5	17.3	0.2
OCN*	10.7	9.3	26.7	2.0
OBS	13.3	14.7	26±5	12.5

[a]Except for the 49.3°W $\sigma_0 \geq 27.74$ column, all transports are for $\sigma_0 \geq$ 27.80 kg m^{-3}. OBS represents observational estimates as follows: the 44°W section corresponds to the Cape Farewell transport from *Dickson and Brown* [1994]; the 49.3°W and 69°W sections are approximations to the 53°N and 70°W lines of *Fischer et al.* [2004] and *Joyce et al.* [2005] lines, respectively. Approximate flow directions are also noted.

Danabasoglu et al. (2010, JGR)

Impacts on AMOC variability

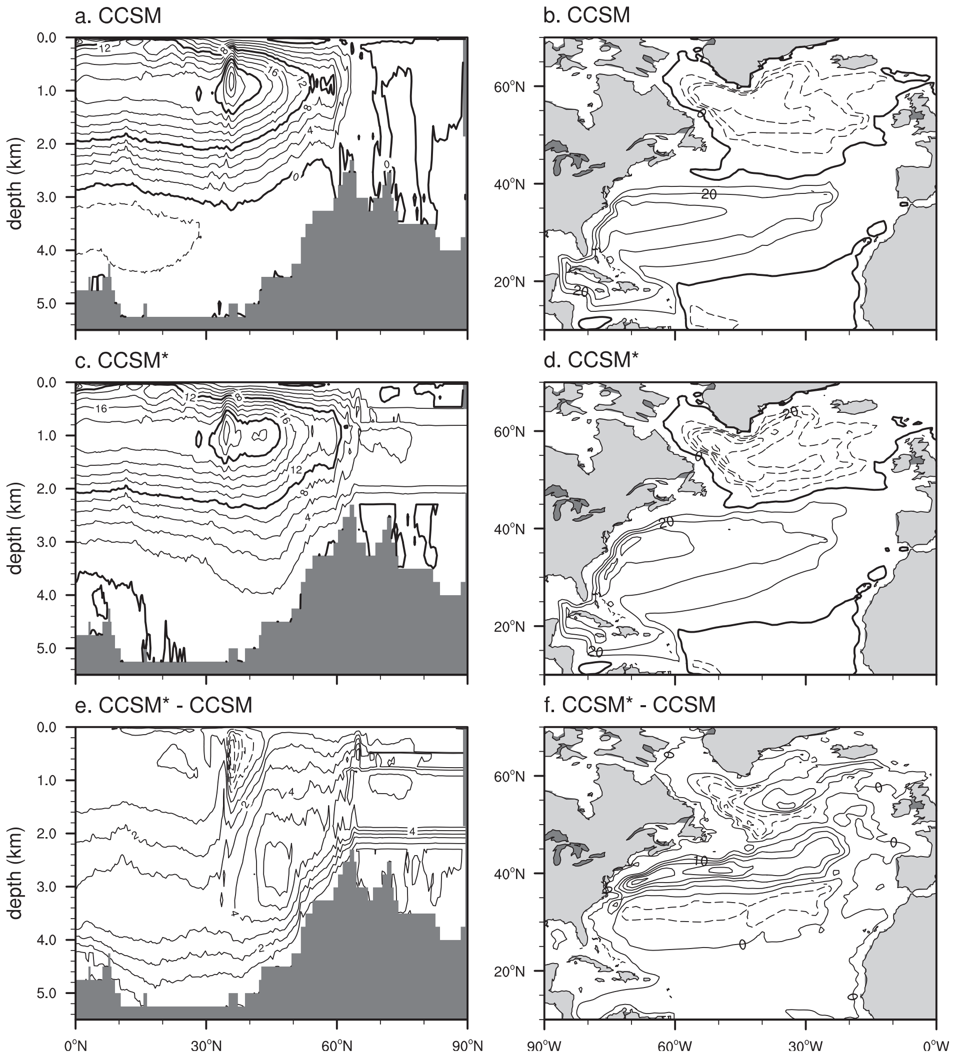


FIG. 2. Time-mean North Atlantic (left) meridional overturning streamfunction and (right) barotropic streamfunction from (a),(b) CCSM, (c),(d) CCSM*, and (e),(f) CCSM* − CCSM. The contour intervals are 2 Sv in (a),(c); 1 Sv in (e); 10 Sv in (b),(d); and 5 Sv in (f). Negative contour levels are dashed.

Here, CCSM and CCSM* are 700-yr long coupled expts without/with overflow param

Yeager & Danabasoglu (2012, J Clim)

Impacts on AMOC variability

Yeager & Danabasoglu (2012, J Clim)

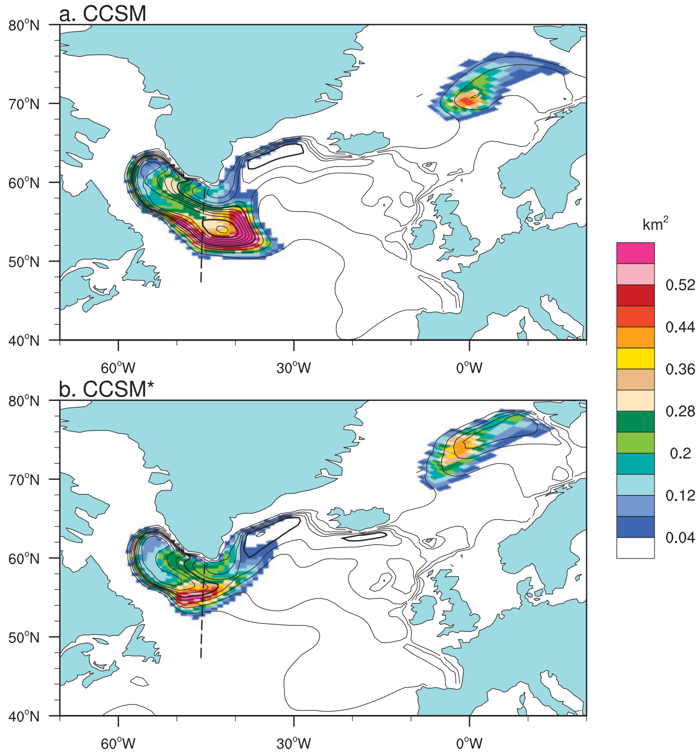


FIG. 4. Total variance (colors, km^2) of detrended maximum March boundary layer depth ($XBLD_{mar}$) time series from (a) CCSM and (b) CCSM*. Black contours show 550-yr time-mean maximum March boundary layer depth contoured at 0.2 km. Thick contours highlight the 1- and 2-km levels. Thick black dashed line shows the meridional section examined in Fig. 5.

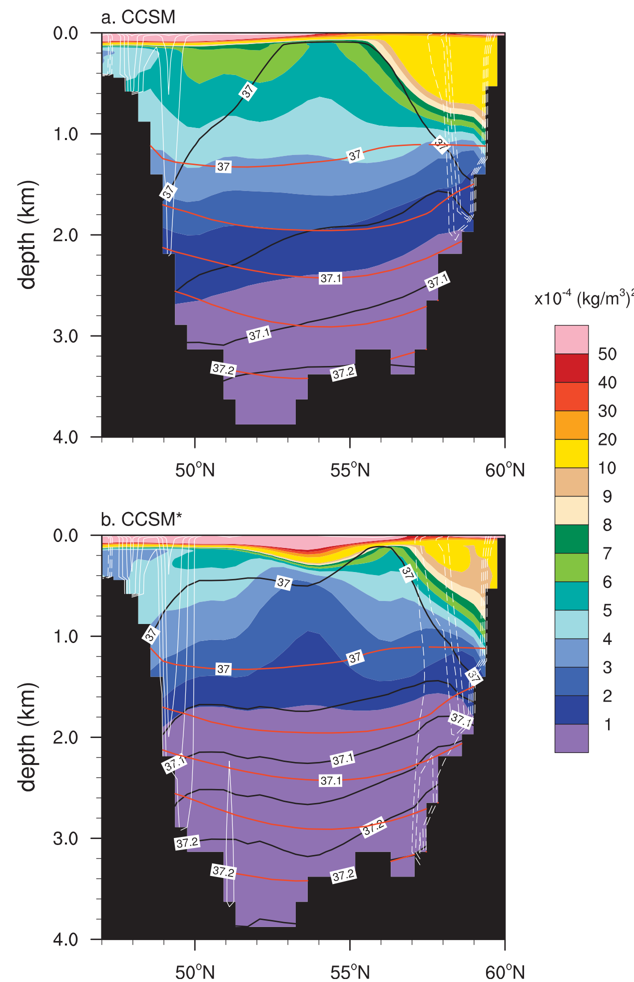


FIG. 5. Total variance (colors, 10^{-4} kg^2 m^{-6}) of detrended annual σ_2 time series from (a) CCSM, and (b) CCSM*. The section shown is along a curvilinear model grid line at approximately 45°W, such that the northern land boundary is the shelf near Cape Farewell Greenland (see dashed line in Fig. 4). Black contour lines show 550-yr mean σ_2 ($\geq$37 kg m^{-3}) contoured at 0.05 kg m^{-3}, and red contour lines show equivalent σ_2 levels from PHC2 climatology. White contour lines are 550-yr mean zonal velocity contoured at ±4, 6, 8, 10 cm s^{-1}, with negative contours dashed (note: positive velocity corresponds to flow out of the Labrador Sea).

Reduced mean/variability in Lab Sea MLD associated with enhanced (more realistic) deep stratification…

Impacts on AMOC variability

Yeager & Danabasoglu (2012, J Clim)

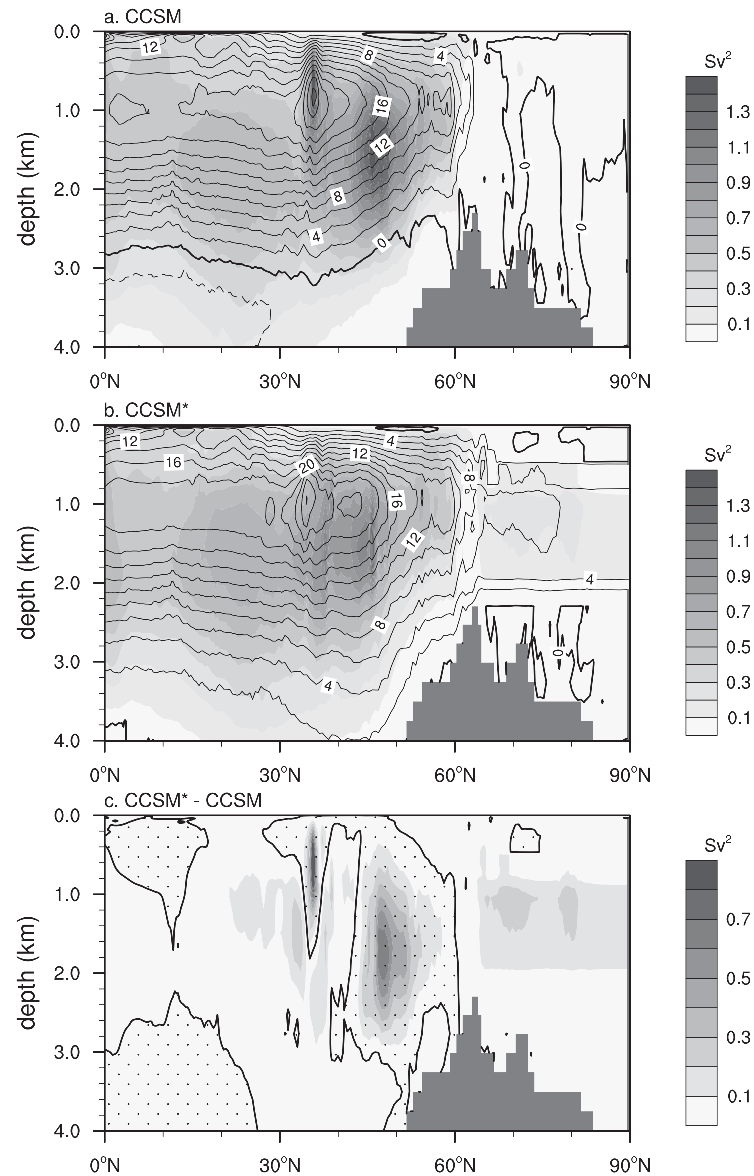


FIG. 8. Total variance (Sv²) of 550-yr detrended annual AMOC time series as a function of latitude and depth from (a) CCSM, (b) CCSM*, and (c) the difference CCSM* − CCSM. Black contours show respective 550-yr mean AMOC streamfunctions (contour interval is 2 Sv) in (a),(b), and the 0 line in (c). Stippling in (c) indicates negative values.

➔ Substantially reduced interannual AMOC variance in 40-60°N latitude range.

The End

Questions?

HORIZONTAL VISCOSITY

Spatially uniform, isotropic, Cartesian, Δ=250km grid for illustration

$$D(U) = A \; U_{xx} + A \; U_{yy}$$
$$D(V) = A \; V_{xx} + A \; V_{yy}$$

Grid Re (Diffuse Noise) $\rightarrow$ $A > 0.5 \; V \, \Delta \; = \; 100{,}000 \; m^2/s$

Resolve WBC (Munk Layers) $\rightarrow$ $A > \beta \; \Delta^3 \; = \; 80{,}000 \; m^2/s$

Diffusive CFL $\rightarrow$ $A < 0.5 \, \Delta^2 / \Delta t \; = 8000{,}000 \; m^2/s$

Realism (EUC, WBC) $\rightarrow$ $A \sim$ physical $= \; 1{,}000 \; m^2/s$

Smagorinsky $\rightarrow$ $A = C \, \Delta^2 \sqrt{\;} \, (\partial_x U)^2 + (\partial_y V)^2 + (\partial_x V + \partial_y U)^2$

ANISOTROPIC HORIZONTAL VISCOSITY

$$\partial_t u + \ldots = \partial_x (A \partial_x u) + \partial_y (B \partial_y u)$$

$$\partial_t v + \ldots = \partial_x (B \partial_x v) + \partial_y (A \partial_y v)$$

Grid Re (Diffuse Noise) $\rightarrow$ Live with the "noise"

Resolve WBC (Munk Layers) $\rightarrow$ $A = B = \beta \ \Delta^3$, only near WBC

elsewhere:

Realism (EUC, WBC) $\rightarrow$ $A = 300\ m^2/s$

$B = 300\ m^2/s$ in the tropics

$= 600\ m^2/s$ polewards of 30^o

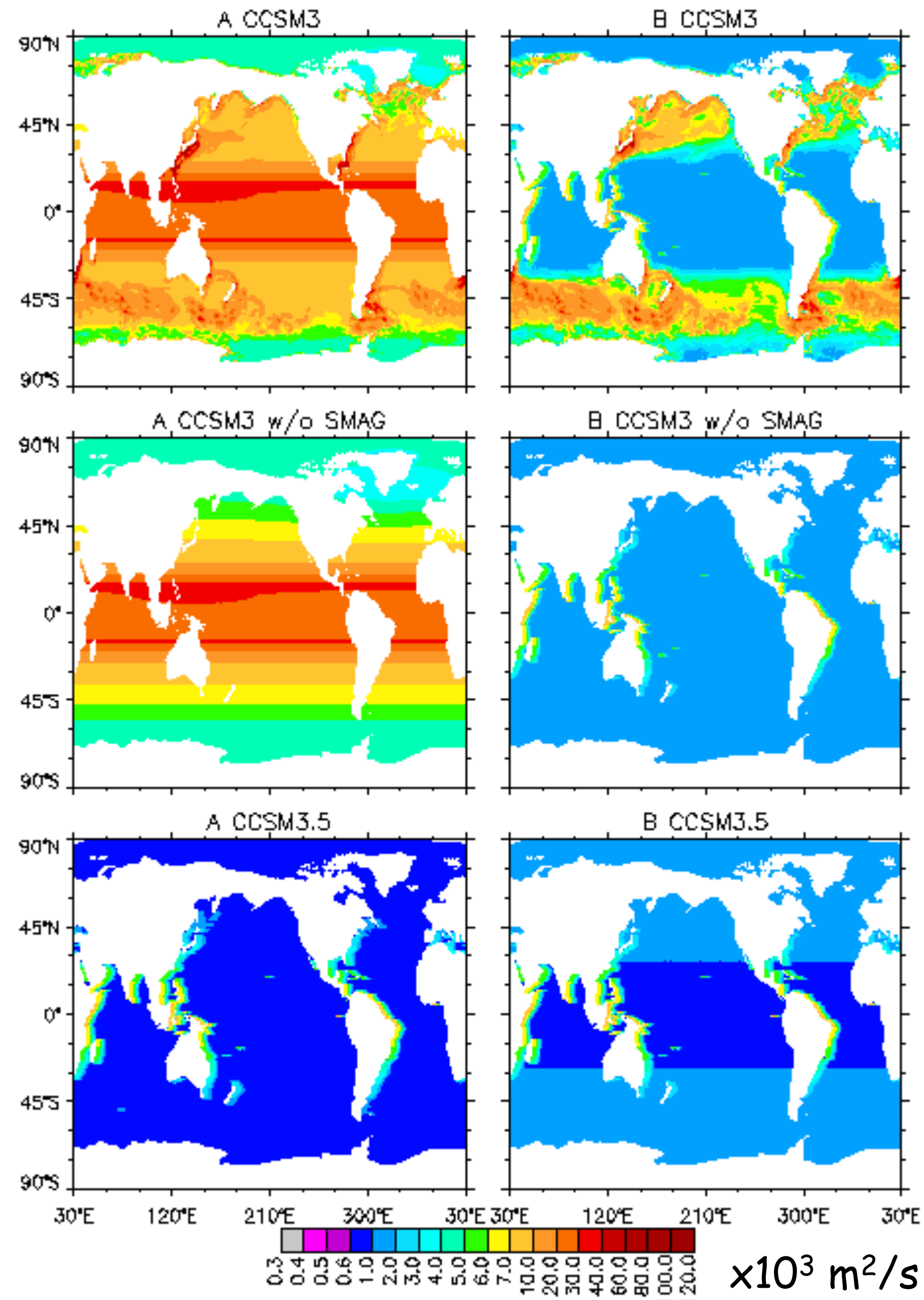
Subject to diffusive CFL, but NO Smagorinsky

ANISOTROPIC HORIZONTAL VISCOSITY at 100-m DEPTH

CCSM4 Ocean :

-Minimally Numerically Viscous

-Maximally Physically Viscous

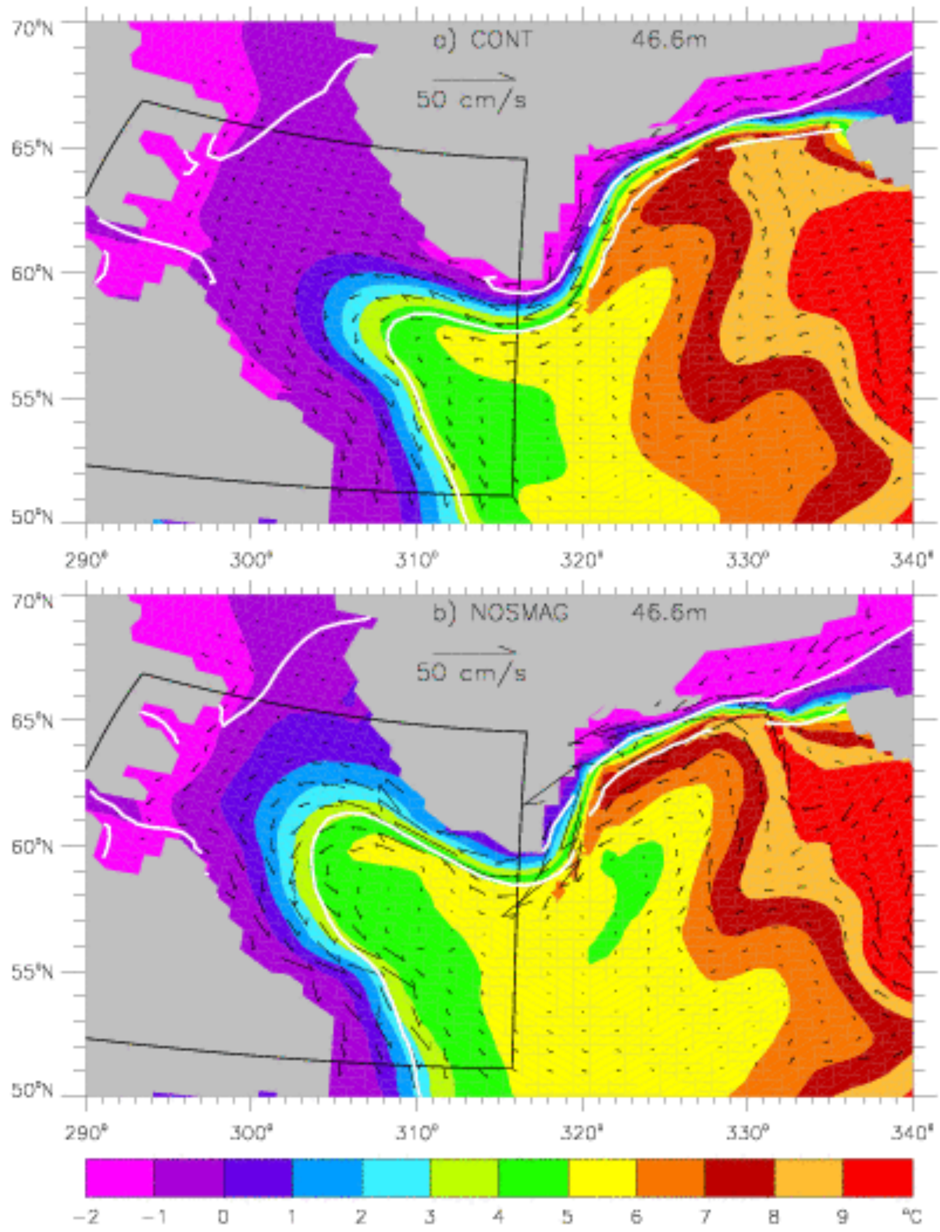


Large et al. (2001, JPO), Jochum et al. (2008, JGR)

IMPACTS ON LABRADOR SEA CIRCULATION AND SEA-ICE

w/ SMAGORINSKY

NO SMAGORINSKY



T (46.6m)

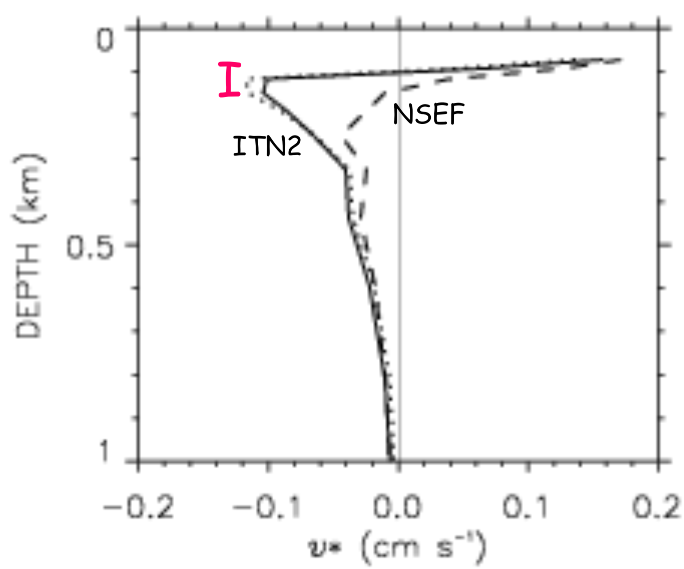
Jochum et al. (2008, JGR)

The eddy-induced meridional velocity is given by

$$v^* = -\frac{\partial\left(A_{ITD} S_y\right)}{\partial z} = -\left(\overset{\mathrm{I}}{\frac{\partial A_{ITD}}{\partial z}} S_y + A_{ITD}\frac{\partial S_y}{\partial z}\right)$$

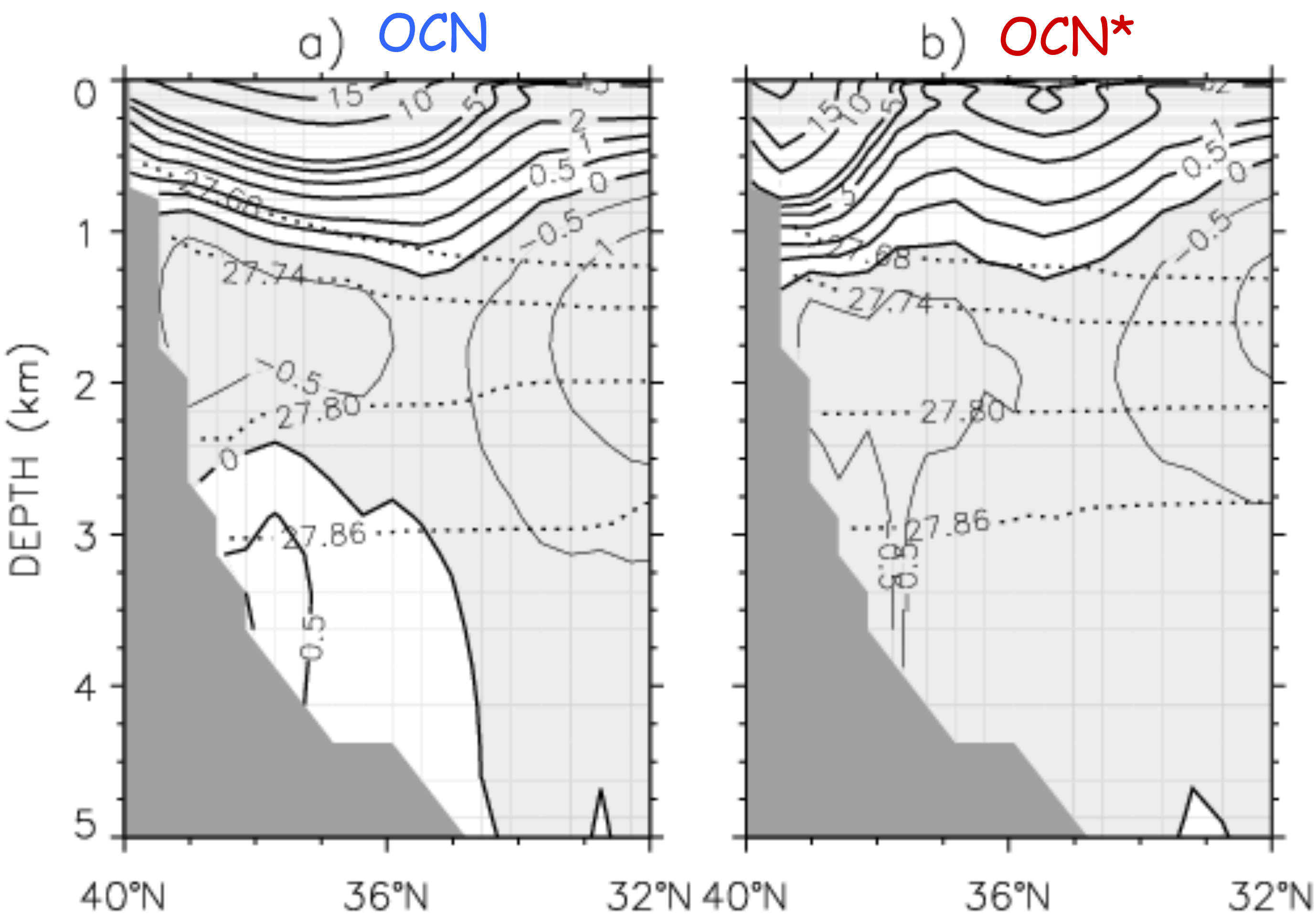
where S_y is the meridional slope of the isopycnal surfaces and z is the vertical coordinate (positive upwards).

Horizontal-mean v* profiles computed between 20°N and 40°N in the North Pacific



Danabasoglu & Marshall (2007, Ocean Modelling)

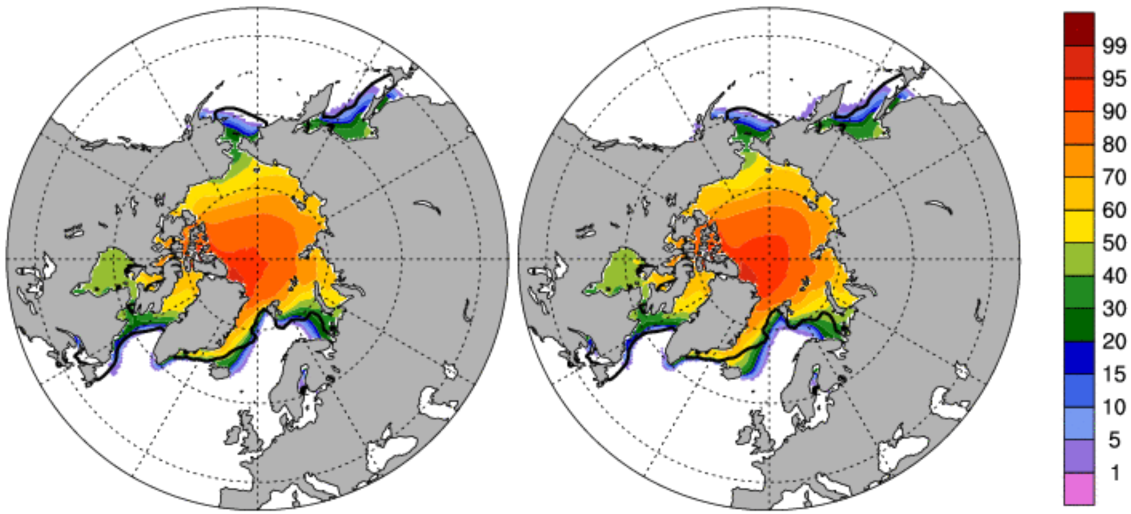
ZONAL VELOCITY ACROSS 69°W IN THE NORTH ATLANTIC



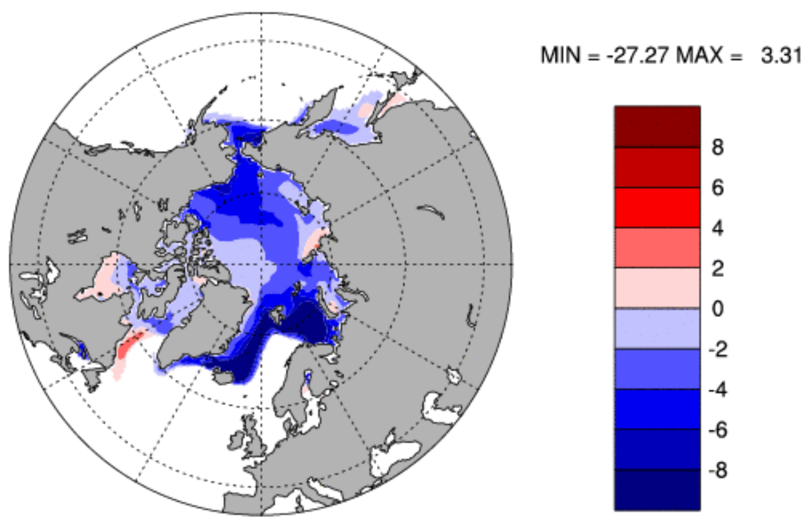
velocity: cm/s, density:kg/m^3

IMPACTS ON SEA-ICE CONCENTRATION

CCSM* CCSM



CCSM* - CCSM



% of grid area