A consideration of two-time-level schemes

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Motivation and Purpose

The Arbitrary Lagrangian Eulerian approach employed in HYPOP will greatly benefit from a two-time level (n to n+1) scheme for time integration.

POP would also benefit from such a scheme, particularly for issues related to conservation and coupler communication.

The purpose is not to conduct original research (i.e. we not aiming to create a new time stepping scheme), but rather to survey existing methods, implement some subset of these methods and move on.

As it turned out, the effort is much more applicable to HYPOP than POP. (It might be that the implementation of a less dissipative Robert filter by Mat Maltrud will be applicable to POP.)

Requirements

1. The scheme should advance model state at time level n to time level n+1 based on model state at only time level n.

2. The scheme should support state-of-the-art, high-order transport schemes.

3. Respects all of the requirements that go along with century-scale (and longer) integration, including conservation of mass and tracer substance.

4. Stable integration in the presence of massless layers.

5. Explicit sub-cycling of barotropic mode in order to remove the global reductions that are required when using the implicit barotropic solver.

6. The ability to "turn-off" the splitting and run with an explicit time step for the whole system. This allows the straightforward implementation of implicit time-stepping.

7. (More a goal than a requirement): Do not require ad hoc dissipation to keep the barotropic-baroclinic splitting stable.

Scope

Here are the set of papers on which this effort is built from:

Bleck and Smith. A wind-driven isopycnic coordinate model of the north and equatorial Atlantic Ocean. 1. Model …. J. Geophys. Res. (1990)

Higdon and de Szoeke. Barotropic-baroclinic time splitting for ocean circulation modeling. Journal of Computational Physics (1997)

Hallberg. Stable split time stepping schemes for large-scale ocean modeling. Journal of Computational Physics (1997) vol. 135 (1) pp. 54-65

Higdon. A Two-Level Time-Stepping Method for Layered Ocean Circulation Models. Journal of Computational Physics (2002) vol. 177 (1) pp. 59-94

Wicker and Skamarock. Time-Splitting Methods for Elastic Models Using Forward Time Schemes. Monthly Weather Review (2002)

Higdon. A two-level time-stepping method for layered ocean circulation models: further development and testing. Journal of Computational Physics (2005) vol. 206 (2) pp. 463-504

Shchepetkin and McWilliams. The regional oceanic modeling system (ROMS): a split-explicit, free-surface, topography-following- …. Ocean Modelling (2005).

Issues: Barotropic-Baroclinic Splitting

One of the core issues in this process is the assumption of splitting, i.e. what is considered barotropic and what is considered baroclinic. We have adopted the splitting of Bleck which was used as the basis of Higdon and Hallberg.

$$p_b = (1+\eta)\Pi$$

- p_b: true bottom pressure
- η: non-dimensional splitting factor
- Π: reference bottom pressure

$$p_b = \sum_{r=1}^{r=R} \Delta p_r$$

← bottom pressure is sum of layer pressures

$$\Delta p_r = \Delta q_r \left(1+\eta\right)$$

← barotropic mode distributed uniformly with depth

Issues: Consistency and Conservation

When implementing this type of barotropic-baroclinic splitting, issues of consistency and conservation come to the fore.

We have TWO estimates of mass at the end of each time step.

$$\frac{\partial}{\partial t}\sum_{r=1}^{R}\Delta p_r + \sum_{r=1}^{R}\left[\nabla\cdot(\Delta p_r \mathbf{u})\right] = 0 \longleftarrow$$

as computed with the high-order baroclinic transport scheme at the long time step.

$$\frac{\partial}{\partial t}p_s + \nabla\cdot(p_s\mathbf{u}) = 0 \longleftarrow$$

as computed with the center-in-space barotropic transport scheme and summed over the many short time steps.

This inconsistency must be removed at the end of each time step. The only reasonable way to do this is to compute the differences in depth-integrated mass flux between the two estimates and use this difference to correct one of the estimates (baroclinic) to match the other estimate (barotropic). We have developed a way to do this even in the presence of massless layers.

Issues: Horizontal Pressure Gradient Force

In addition to the mass field, the pressure gradient must be decomposed into a barotropic component that will be allowed to vary during the barotropic sub-cycling and baroclinic component that is essentially fixed during the entire time step.

There are no less than three approaches to decomposing the pressure gradient force into it barotropic and baroclinic components.

1. Hallberg: At each time step during the sub-cycling, project barotropic component back onto the entire 3D mass field; compute 3D pressure gradient force; average 3D pressure gradient force to find barotropic component. Finding: very robust, relatively expensive.

2. Higdon: Linearize the pressure gradient force based on the depth-average density. I am finding satisfactory results with this approach. This formulation will have to be generalized when not using a pure isopycnal coordinate.

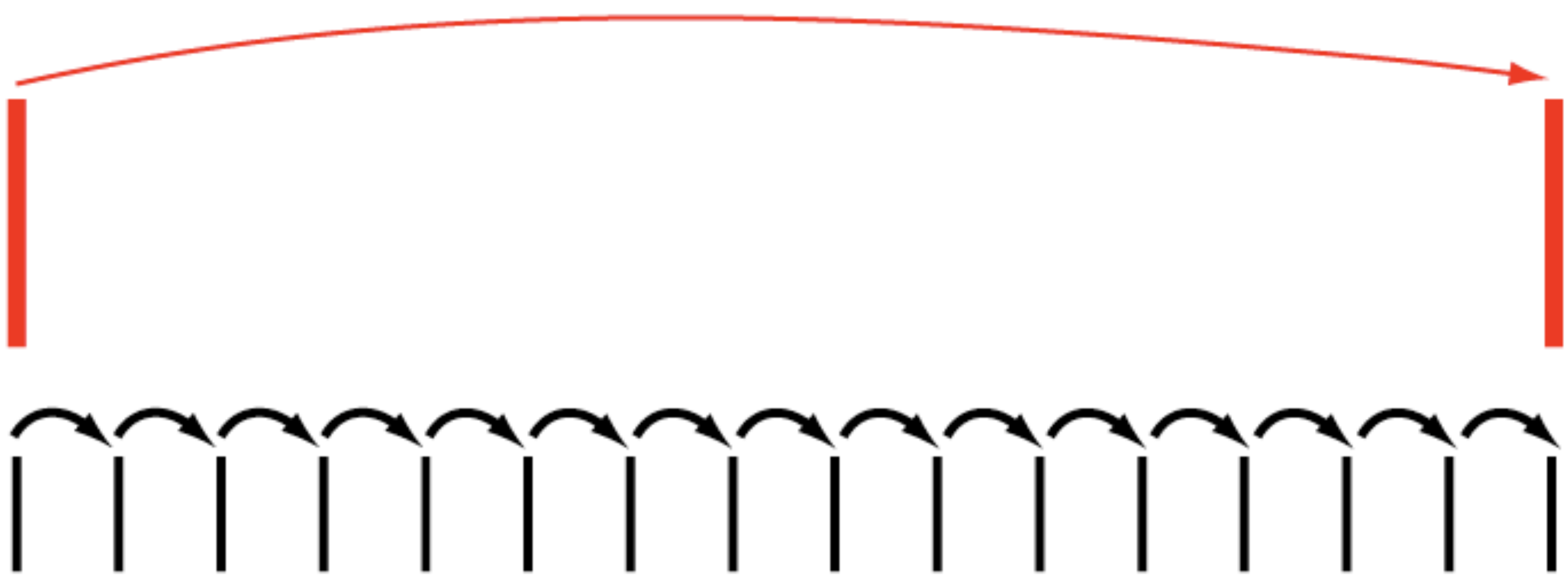
3. Shchepetkin: Extends the linearization of the pressure gradient force beyond accounting for the depth-averaged density to the next term in the expansion (termed the dynamic density).

Issues: Stability in coupling barotropic-baroclinic modes.

Insuring the stability of the barotropic-baroclinic coupling is a tricky business and (at least in my view) is not entirely sorted out.

The issues include the aliasing of time scales resolved by the barotropic system into baroclinic time scales and the consistency between the states on which the barotropic and baroclinic systems evolve.

One way to significantly enhance the stability of the barotropic-baroclinic coupling is to integrate over each time sequence twice to allow the coupling to be centered on the baroclinic time step. Since the barotropic time stepping can be forward-backward with a CFL of approximately one and since the tracer updates are only need at the end of the time step, this is not as computationally demanding as it might appear.



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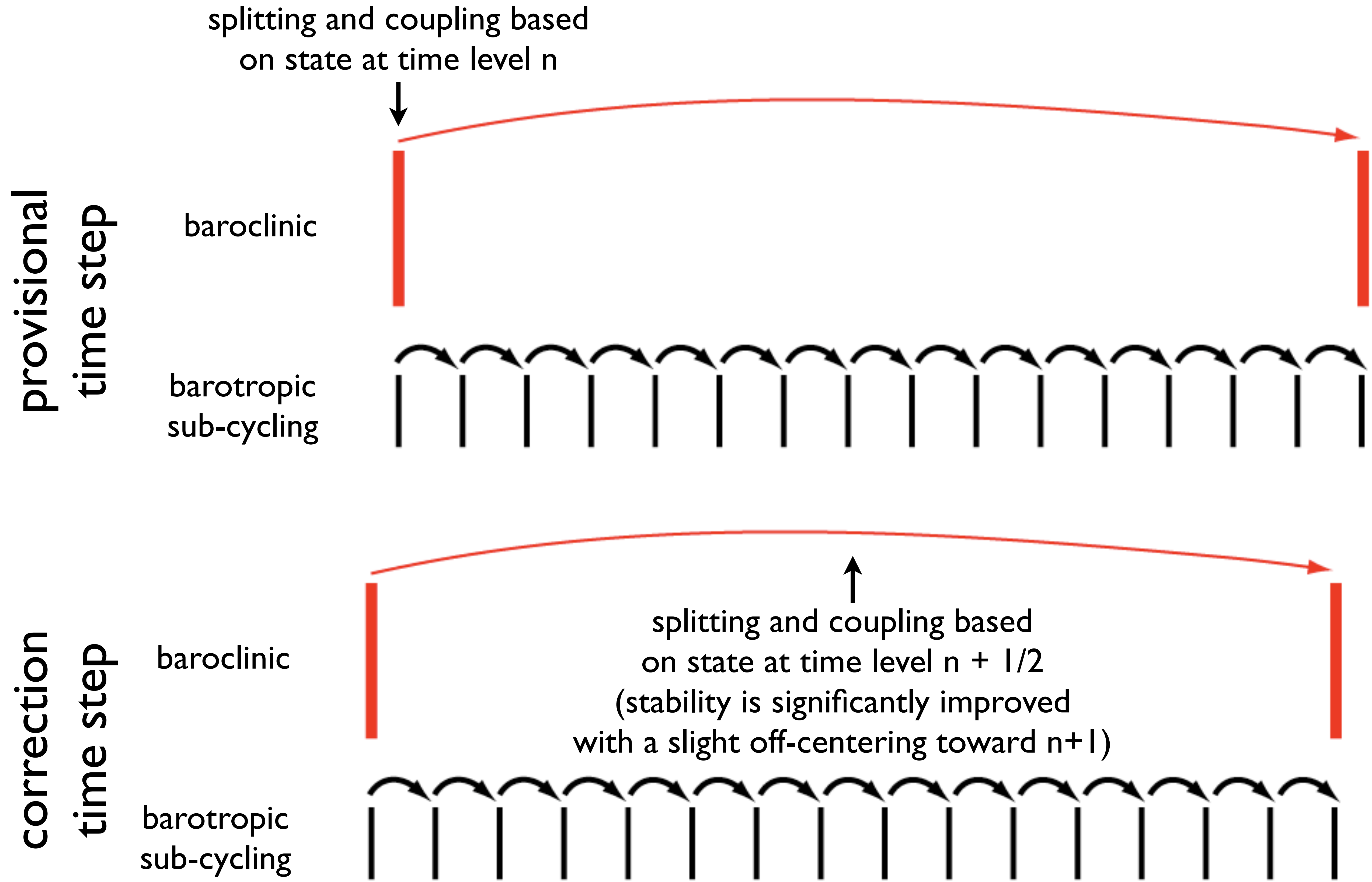


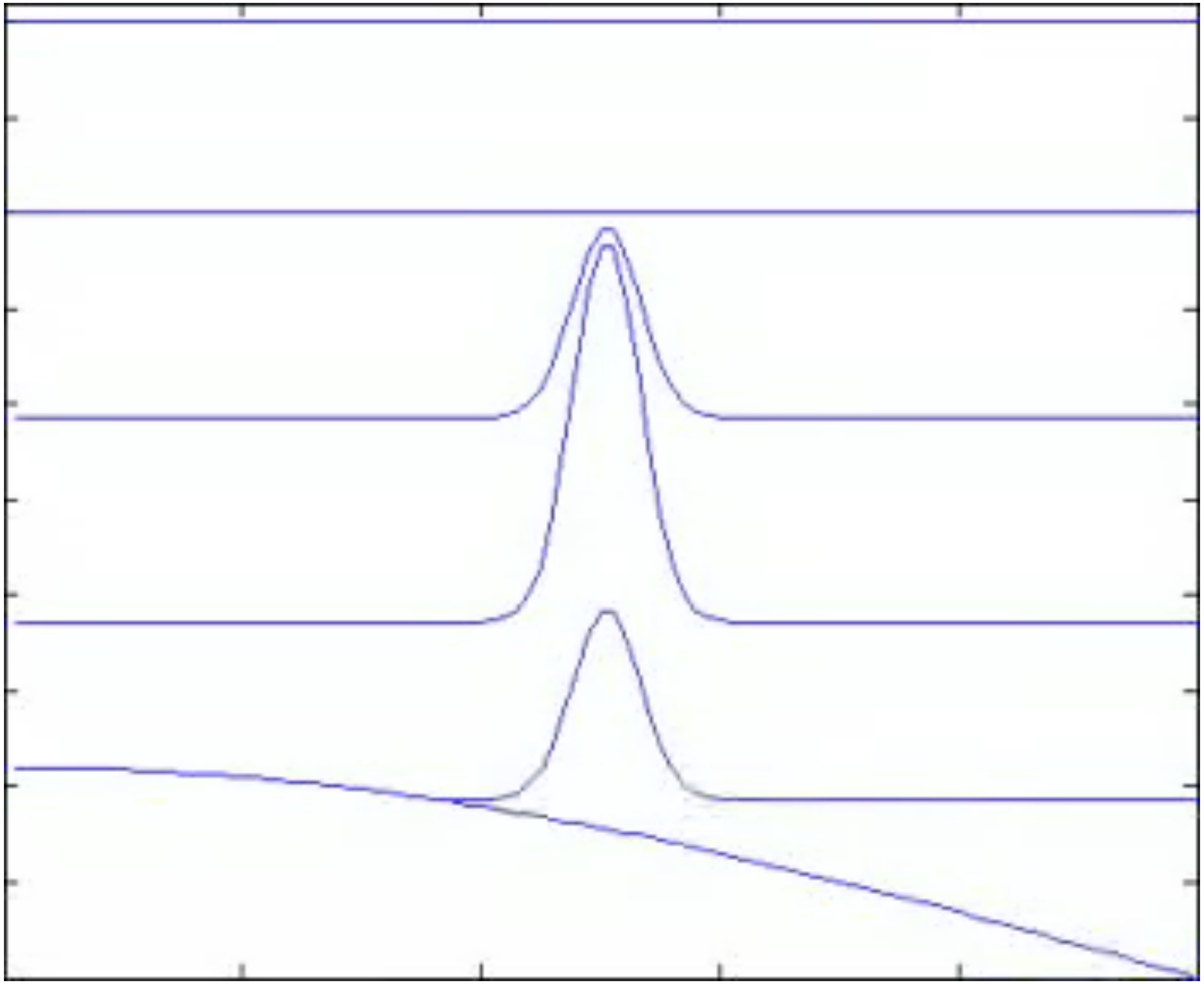
Implemented Scheme

(essentially Higdon 2005)

1. Split modes based on time level N state. Linearize barotropic pressure gradient force about time level N state.
2. Time step the baroclinic velocity to N+1 based on time level N state. Compute forcing G(N) that eliminates barotropic velocity component. Store provisional mid-time velocity (N+1/2).
3. Sub-cycle barotropic system. Hold baroclinic forcing (i.e. G(N)) constant during sub-cycling. Save time mean barotropic flux (F) and store time mean barotropic velocity field.
4. Time step FULL layer pressure thickness based on time level N layer thicknesses and N+1/2 total velocity field. Use F to guarantee that full layer pressure thicknesses sum identically to barotropic pressure.
5. Recompute splitting based on N+1/2 state. Linearize barotropic pressure gradient force about N+1/2 state.
6. Repeat Step #2 based on N+1/2 state. Recompute G(N+1/2).
7. Repeat Step #3. Use new G(N+1/2). Save new F and time mean barotropic velocity.
8. Repeat Step #4 based on new N+1/2 total velocity and new F.
9. Do tracer transport (based on mass fluxes computed above).
10. Compute EOS, vertical diffusion, vertical remapping (transport), GM etc.

Results

Five Layers
5000 m depth
dx = 50 km
2 m SSH perturbation
baroclinic time step : 3600 s
barotropic time step : 150 s
density = 1000 kg/m3 top
density = 1025 kg/m3 bottom



http://public.lanl.gov/ringler/movies/2009/ts_01.mov

Summary of proposed two-time level scheme

1. Stabilize mode coupling with predictor-corrector approach.

2. Compute forcing terms in momentum based on N+1/2 state.

3. Compute mass fluxes based on an inexpensive, positive definite transport algorithm.

4. Compute tracer fluxes based on a high-order, incremental remapping approach.

Proposed Implementation Pathway

The plan is to implement this scheme into a full 3D, eddy-resolving dynamical core with topography.

Test with and without splitting to measure the impact of the splitting on overall solution.

Test with high-order transport to determine the impact of high-order transport and flux-correction on the overall simulation.

Implement into production model.