A simultaneous heat and water transfer model in frozen and thawed soil

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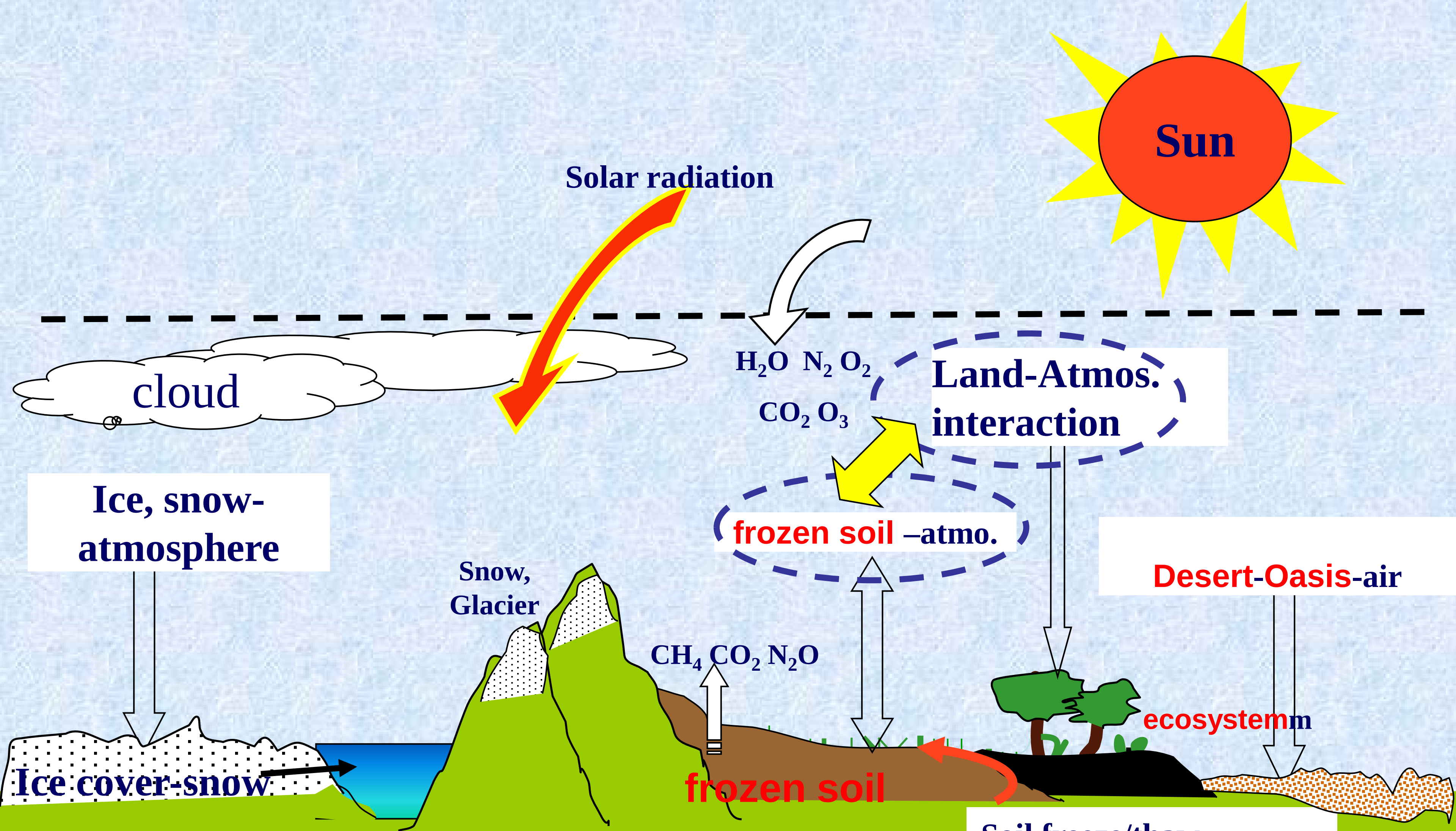
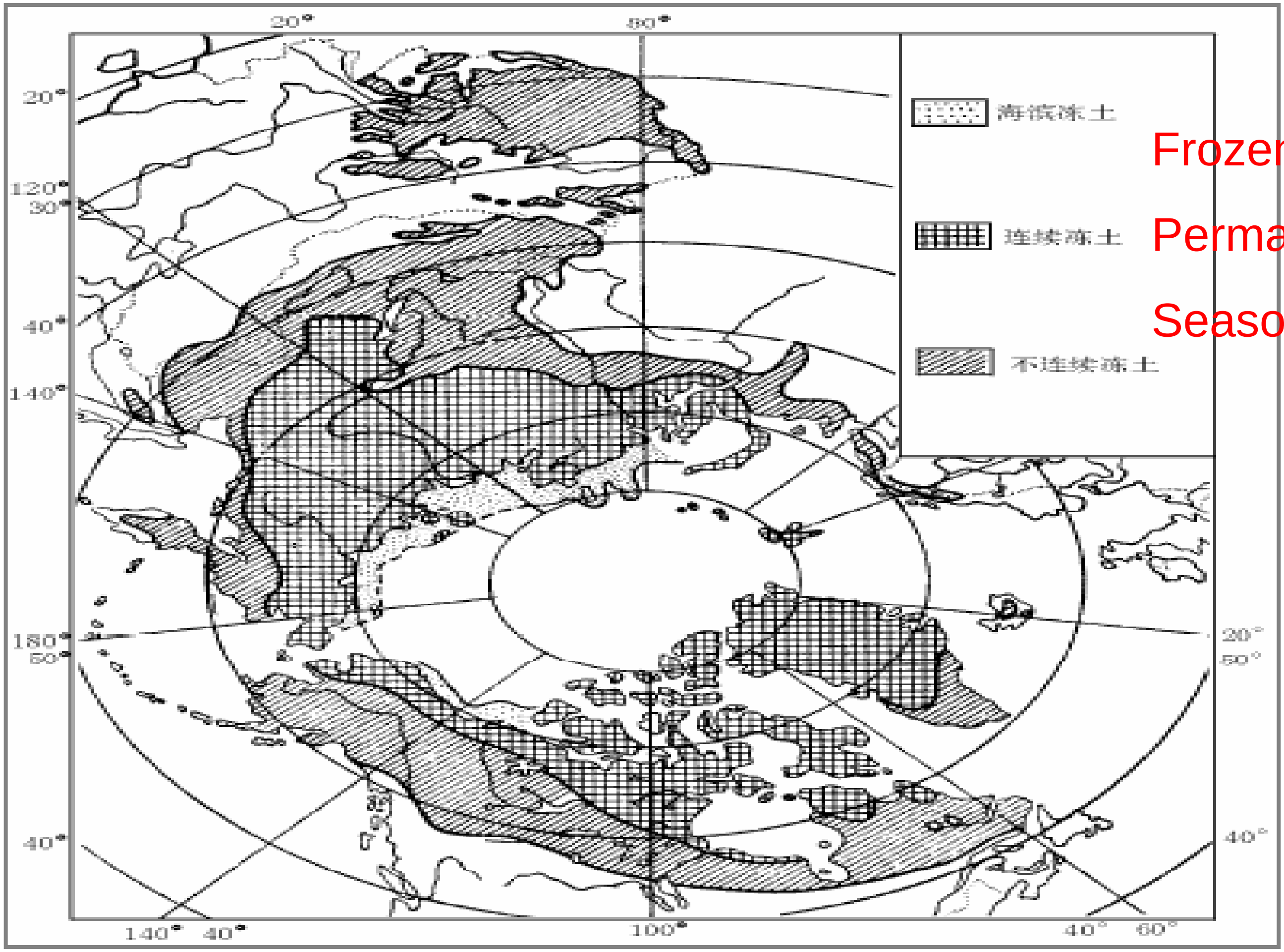
Outline

- Introduction
- A moving boundary problem
- A heat and water transfer model
- Model validation
- Summary and discussion



Soil freeze/thaw processes including change of frost/thaw depths significantly influence energy and water exchanges between land surface and sub-surface, as well as vegetation growth and organic matter decomposition through thermal and hydrological processes

Frozen soil distribution in the North Hemisphere



Frozen by sea

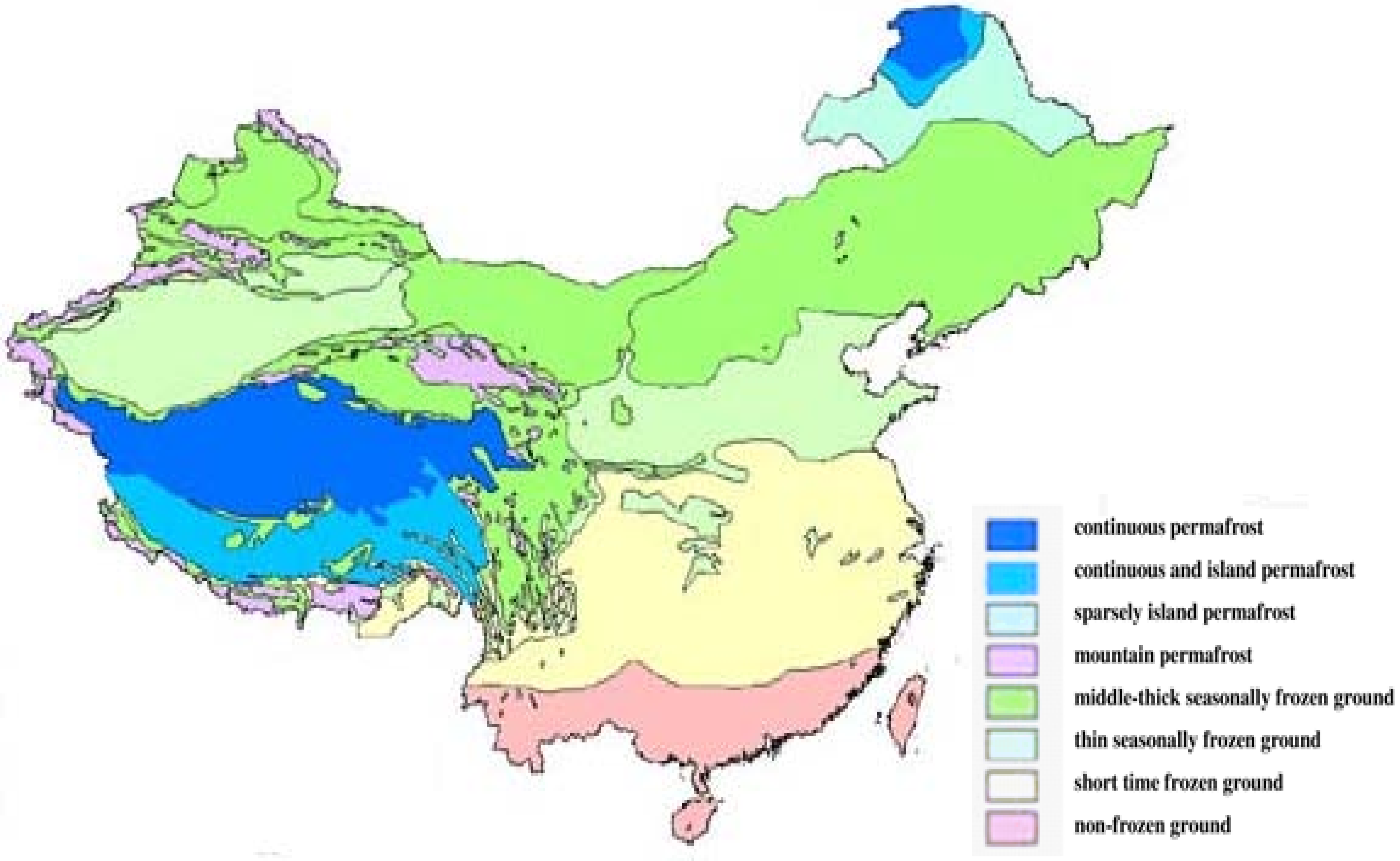
Permafrost

Seasonal

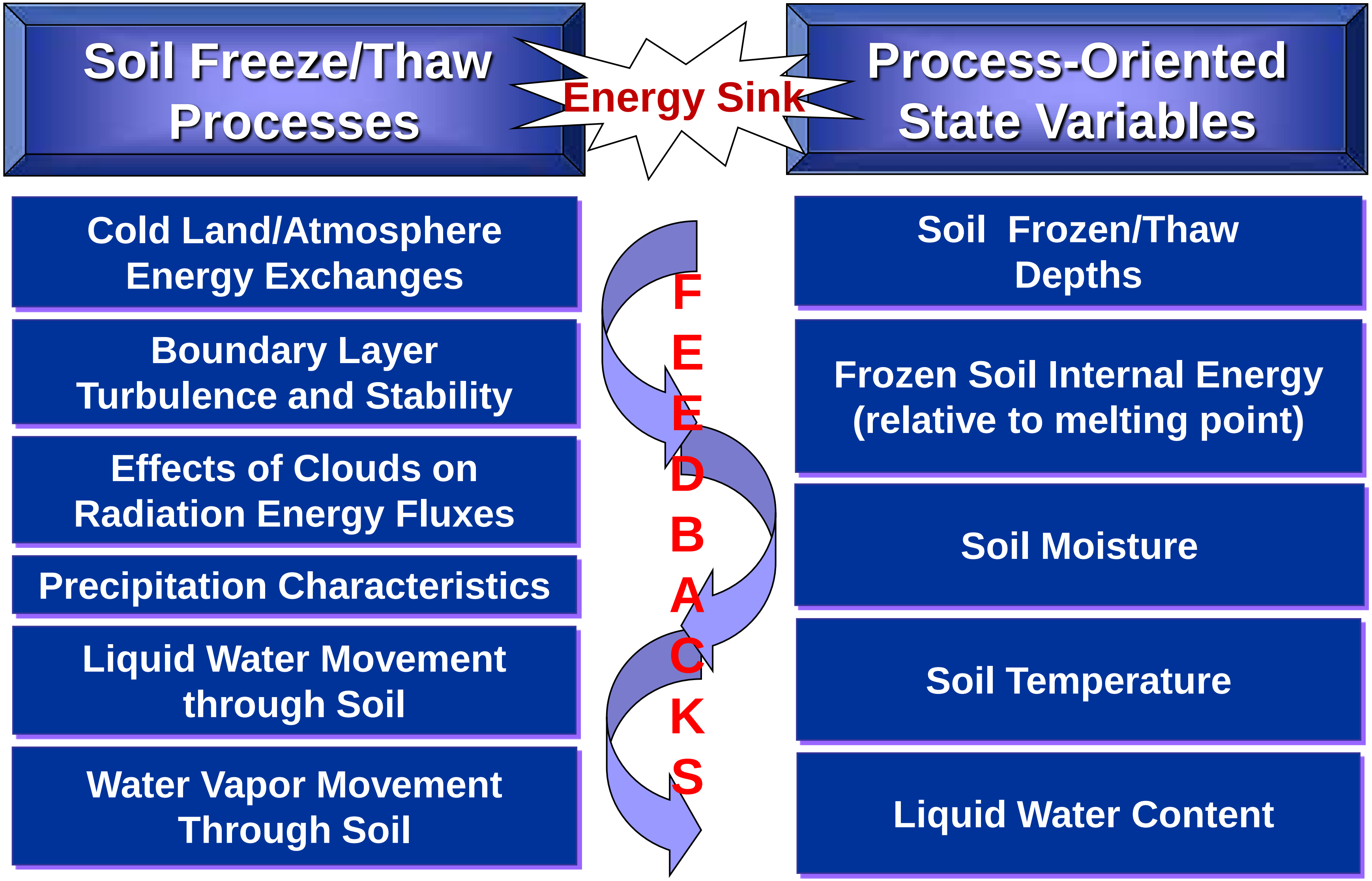
Frozen soil： all kinds of ice-containing frozen soil at 0℃ or below 0℃。

- **Permafrost and seasonally frozen soil account for 24% and 30% of the land area in the north Hemisphere, respectively;**
- **Russia and Canada are the countries where the frozen soil are most widely distributed.**

Distribution of frozen soil types in China



Earth System Responses and Feedbacks To Soil Freeze/Thaw Processes



Soil Freeze/Thaw Processes

Energy Sink

Process-Oriented State Variables

- Cold Land/Atmosphere Energy Exchanges
- Boundary Layer Turbulence and Stability
- Effects of Clouds on Radiation Energy Fluxes
- Precipitation Characteristics
- Liquid Water Movement through Soil
- Water Vapor Movement Through Soil

FEEDBACKS

- Soil Frozen/Thaw Depths
- Frozen Soil Internal Energy (relative to melting point)
- Soil Moisture
- Soil Temperature
- Liquid Water Content

Introduction

- The soil freezing-thawing processes including change of frost/thaw depth, significantly influence energy and water exchanges between land surface and sub-surface;
- Accurate representation of frost and thaw depths and their climate feedback is significant for improving simulations of the hydrological and greenhouse gas exchange processes in cold regions;
- Current land surface models used for climate studies do not represent suitably the dynamics of frost/thaw depths and their feedback to the climate system, which give delayed or rapid freezing/thaw due to the frozen soil parameterization in the models.

The purpose of this work

- A heat and water transfer model with representation of the dynamics of frost/thaw depths and their feedback to the climate system, was developed;
- It treats the soil frost/thaw depths as moving interfaces governed by some Stefan-type moving boundary conditions, and describes the liquid water and solid ice states as well as the positions of the frost/thaw depths;
- An adaptive mesh method for the moving boundary problem is adopted to solve the relevant equations and to determine frost/thaw depths, water content and temperature distribution.

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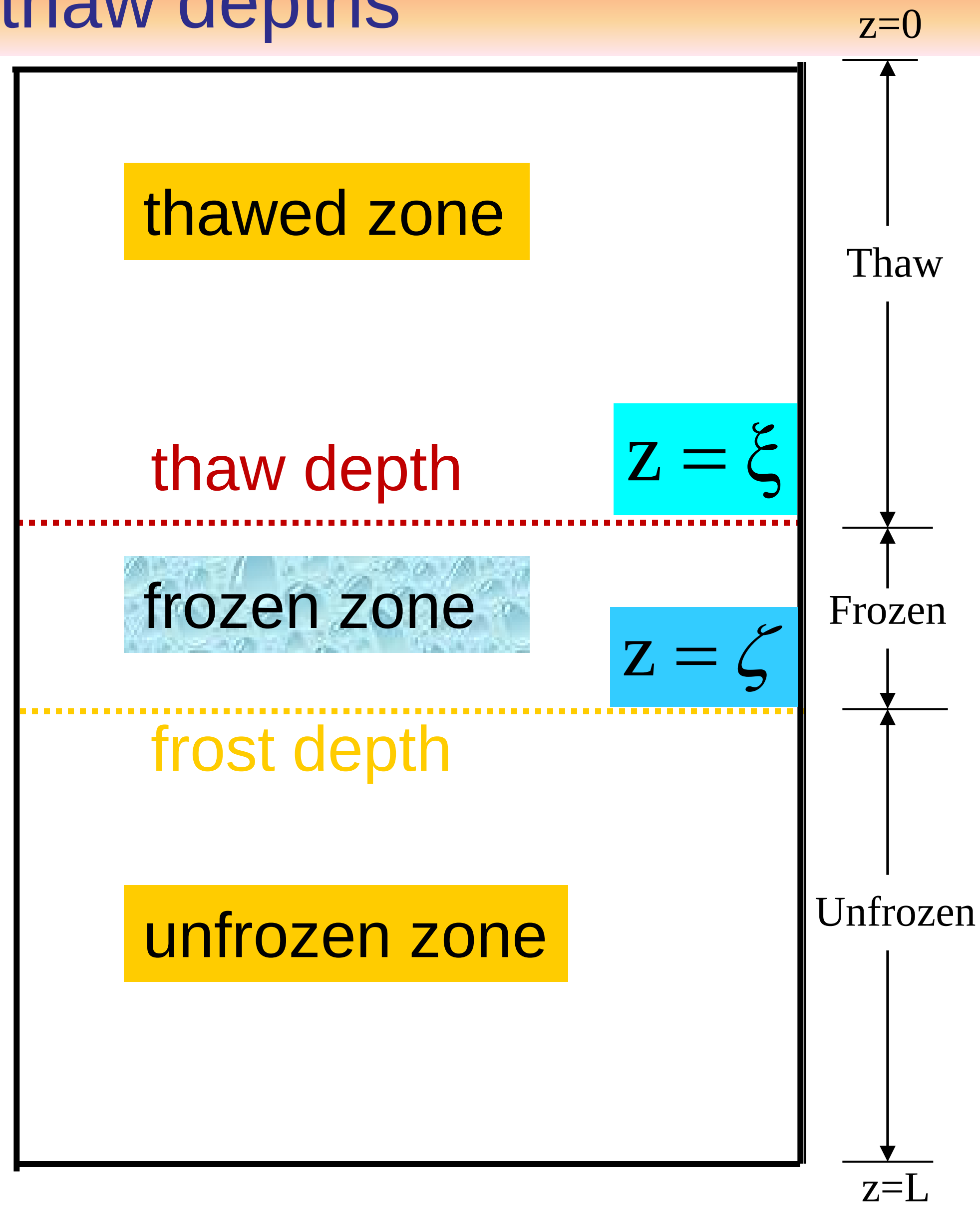
Two moving boundaries: Phase-transition interfaces
Frost and thaw depths

- A thawed zone from ground surface to the first phase-transition interface (namely thaw depth)；
- A frozen zone from the first phase-transition interface to the second phase-transition interface (namely frost depth)；
- An unfrozen zone from frost depth to the bottom of the calculation depth.



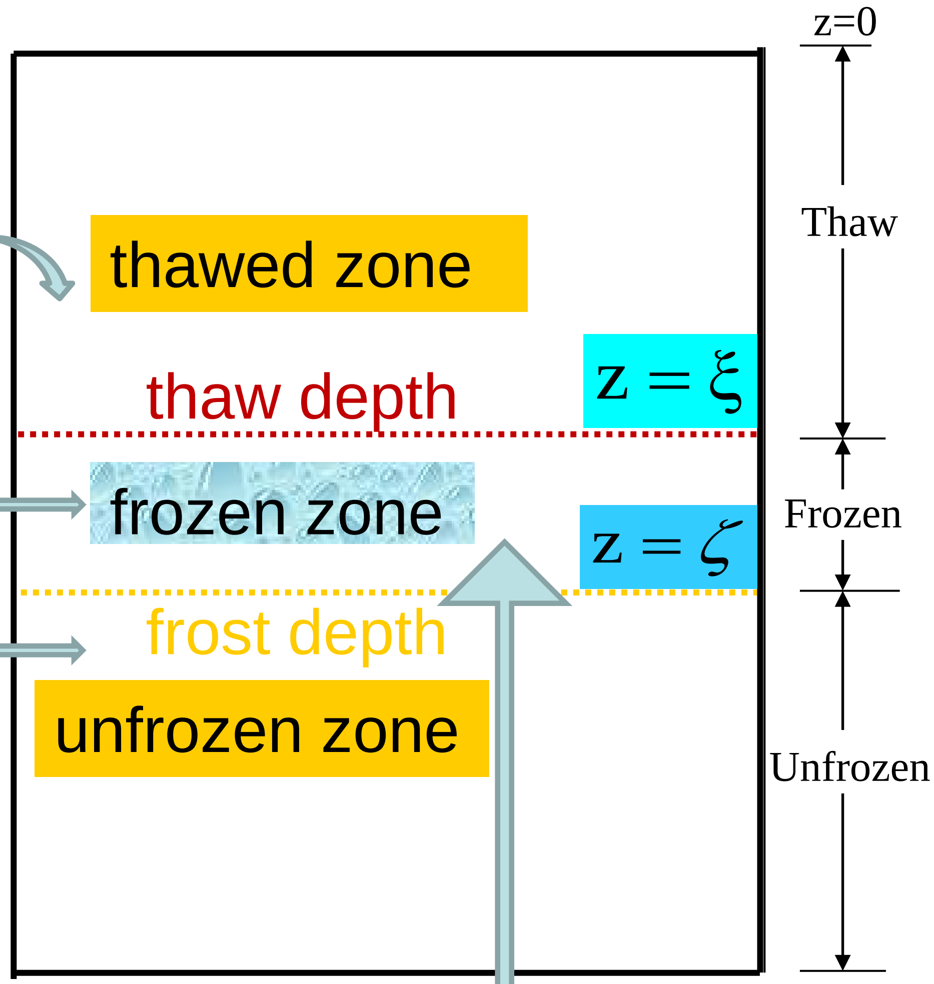
Control equations

Energy balance equations

$$\frac{\partial(c_u T)}{\partial t} = \frac{\partial}{\partial z}\left(\lambda_u \frac{\partial T}{\partial z}\right), 0 < z < \xi,$$

$$\frac{\partial(c_f T)}{\partial t} = \frac{\partial}{\partial z}\left(\lambda_f \frac{\partial T}{\partial z}\right), \xi < z < \zeta,$$

$$\frac{\partial(c_u T)}{\partial t} = \frac{\partial}{\partial z}\left(\lambda_u \frac{\partial T}{\partial z}\right), \zeta < z < L,$$



z=0

Thaw

thawed zone

thaw depth

$z = \xi$

frozen zone

$z = \zeta$

Frozen

frost depth

unfrozen zone

Unfrozen

Mass balance equation

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z}\left(D \frac{\partial \theta}{\partial z}\right) - \frac{\partial K}{\partial z} + \frac{\partial q_v}{\partial z} + S, 0 < z < \xi, \zeta < z < L,$$

A moving boundary problem for heat and water transfer processes

$$\frac{\partial(c_u T)}{\partial t}=\frac{\partial}{\partial z}\left(\lambda_u \frac{\partial T}{\partial z}\right), \quad 0<z<\xi, \zeta<z<L, \qquad \frac{\partial(c_f T)}{\partial t}=\frac{\partial}{\partial z}\left(\lambda_f \frac{\partial T}{\partial z}\right), \quad \xi<z<\zeta,$$

$$\frac{\partial \theta}{\partial t}=\frac{\partial}{\partial z}\left(D \frac{\partial \theta}{\partial z}\right)-\frac{\partial K}{\partial z}+\frac{\partial q_v}{\partial z}+S, 0<z<\xi, \zeta<z<L,$$

$$T(0,t)=f_1(t), t>0, \qquad \theta(0,t)=f_2(t), t>0,$$

$$T(z,0)=g_1(z), 0<z<L, \qquad \frac{\partial T}{\partial z}\Big|_{z=L}=G_g,$$

$$\theta(z,0)=g_2(t), 0<z<L, \qquad \theta(L,t)=\theta_r, t>0,$$

Initial and boundary conditions

$$T\big|_{z=\xi^+}=T\big|_{z=\xi^-}=T_f, \qquad T\big|_{z=\zeta^+}=T\big|_{z=\zeta^-}=T_f,$$

$$\lambda_f \frac{\partial T}{\partial z}\Big|_{z=\xi}-\lambda_u \frac{\partial T}{\partial z}\Big|_{z=\xi}=Q\frac{d\xi}{dt}, \qquad \lambda_f \frac{\partial T}{\partial z}\Big|_{z=\zeta}-\lambda_u \frac{\partial T}{\partial z}\Big|_{z=\zeta}=Q\frac{d\zeta}{dt}.$$

Moving boundary conditions

Xie, Song, Feng, ***Science in China(A***), 2008

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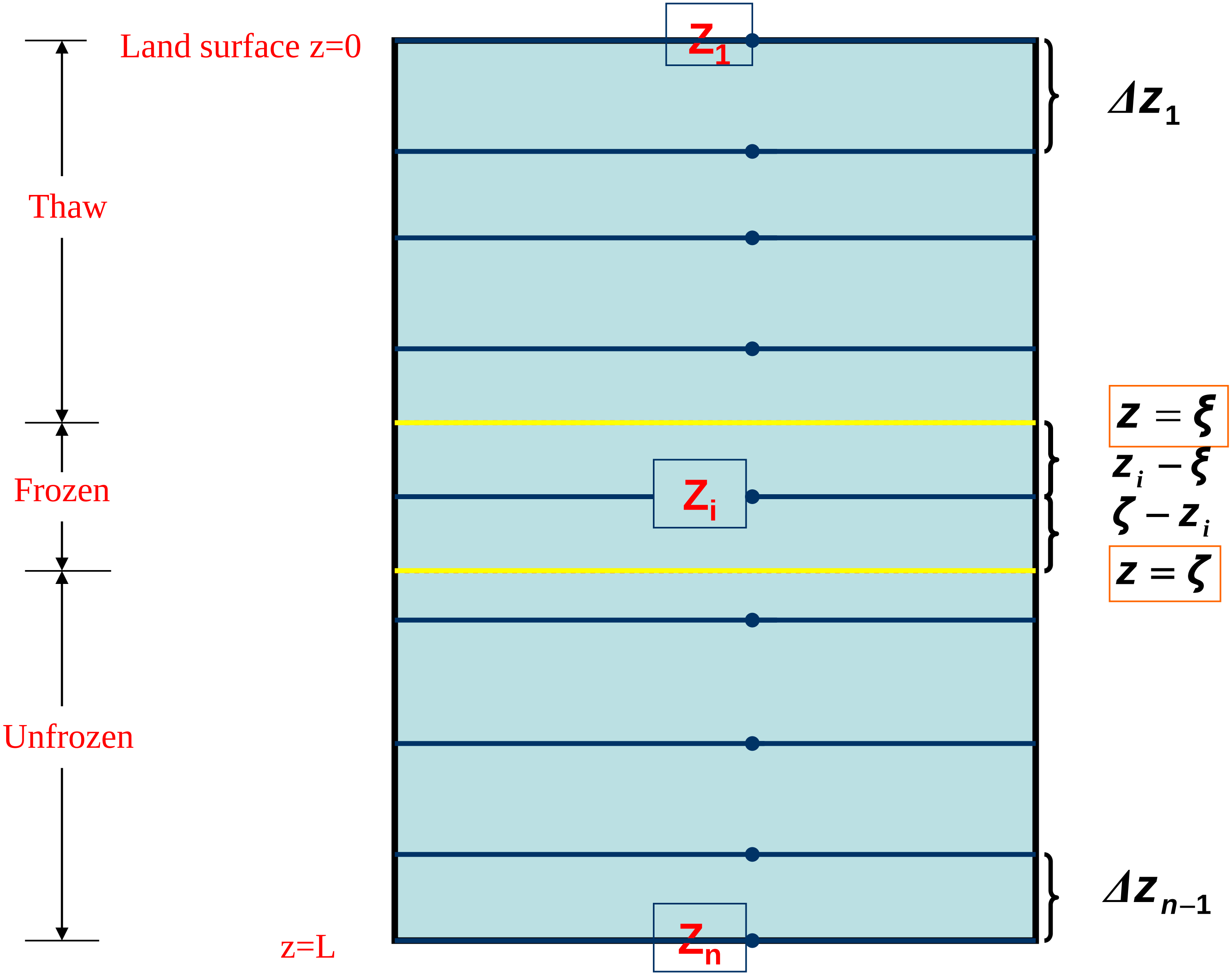
Land surface z=0
Z_1
Δz_1
Thaw
$z = \xi$
Z_i
Frozen
$z_i - \xi$
$\zeta - z_i$
$z = \zeta$
Unfrozen
Δz_{n-1}
z=L
Z_n

$$D^b\theta_j^{k+1} - \frac{\omega}{h_{j+\frac{1}{2}}}\left(D_{j+\frac{1}{2}}^{k+1}\mathrm{E}_f\theta_j^{k+1} - D_{j-\frac{1}{2}}^{k+1}\mathrm{E}_b\theta_j^{k+1}\right)$$

$$-\frac{(1-\omega)}{h_{j+\frac{1}{2}}}\left(D_{j+\frac{1}{2}}^{k}\mathrm{E}_f\theta_j^{k} - D_{j-\frac{1}{2}}^{k}\mathrm{E}_b\theta_j^{k}\right) - \omega\mathrm{E}_cK_j^{k+1} - (1-\omega)\mathrm{E}_cK_j^{k}$$

$$-S_j - \omega\frac{q_{v\,j+1}^{k+1} - q_{v\,j-1}^{k+1}}{h_j + h_{j+1}} - (1-\omega)\frac{q_{v\,j+1}^{1} - q_{v\,j-1}^{k}}{h_j + h_{j+1}} = 0,$$

$$D^b\left(c_{u\,j}^{k+1}T_j^{k+1}\right) - \frac{\omega}{h_{j+\frac{1}{2}}}\left(\lambda_{u\,j+\frac{1}{2}}^{k+1}\mathrm{E}_fT_j^{k+1} - \lambda_{u\,j-\frac{1}{2}}^{k+1}\mathrm{E}_bT_j^{k+1}\right)$$

$$-\frac{(1-\omega)}{h_{j+\frac{1}{2}}}\left(\lambda_{u\,j+\frac{1}{2}}^{k}\mathrm{E}_fT_j^{k} - \lambda_{u\,j-\frac{1}{2}}^{k}\mathrm{E}_bT_j^{k}\right) = 0,$$

$$D^b\left(c_{f\,j}^{k+1}T_j^{k+1}\right) - \frac{\omega}{h_{j+\frac{1}{2}}}\left(\lambda_{f\,j+\frac{1}{2}}^{k+1}\mathrm{E}_fT_j^{k+1} - \lambda_{f\,j-\frac{1}{2}}^{k+1}\mathrm{E}_bT_j^{k+1}\right)$$

$$-\frac{(1-\omega)}{h_{j+\frac{1}{2}}}\left(\lambda_{f\,j+\frac{1}{2}}^{k}\mathrm{E}_fT_j^{k} - \lambda_{f\,j-\frac{1}{2}}^{k}\mathrm{E}_bT_j^{k}\right) = 0,$$

Finite difference scheme

for the control equations

ω=1
the backward Euler scheme

ω=0
the explicit Euler scheme

Algorithm

- Step 1. Choose the initial datum functions;
- Step 2. Compute soil temperature and soil moisture, and Soil frozen/thaw depths $\theta_j^{k+1}, T_j^{k+1}, \xi^{k+1}, \varsigma^{k+1}$ by the Finite difference scheme;
- Step 3. Check whether the stopping criterion is met. If the relative error of two iterates is less than a prescribed tolerance, stop the iteration and set *$k := k + 1$; go back to Step 2;* Otherwise, let the iteration run until the maximum number of iterations is reached, then either stop or set *$k := k + 1$ and go back to Step 2.*
- Step 4 Continue the time integration until $t_{k+1} = t_{fnal}$.

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Numerical Experiment 1

- Purpose: To test the effects of the surface temperature on the simulated frost and thaw depths;
- With the simulated surface temperature

$$f(t) = T_0 + G_t t + A_0 \sin(\omega t)$$

T_0： mean annual ground surface temperature （GST）

G_t： the rising rate of GST

A_0： annual amplitude of GST $A_0 = 13^\circ C$; $\omega = \frac{2\pi}{8760}$

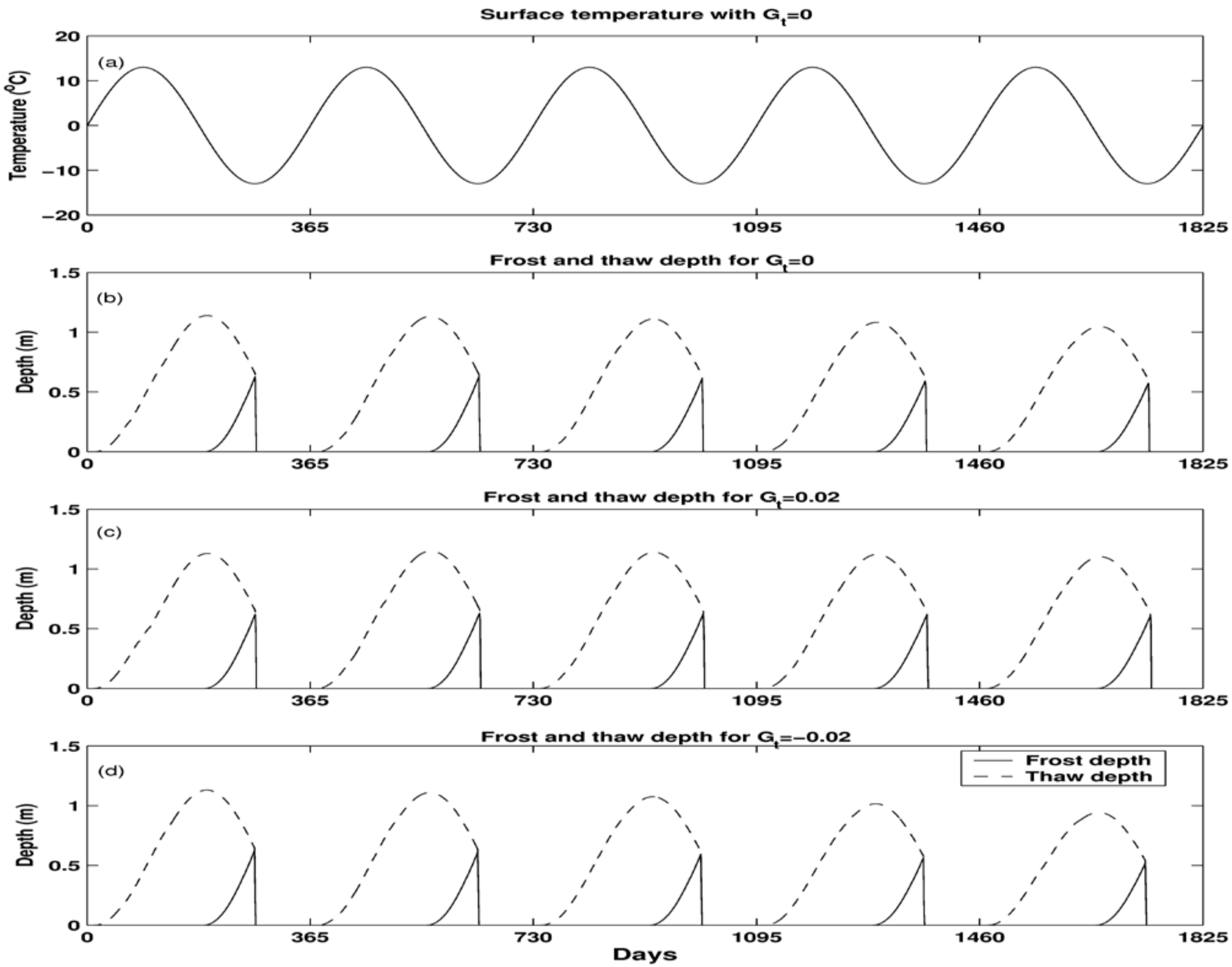
The ground surface temperature and the simulated frost/thaw depths

(a) Soil surface temperature;

(b) frost and thaw depths for G_t=0;

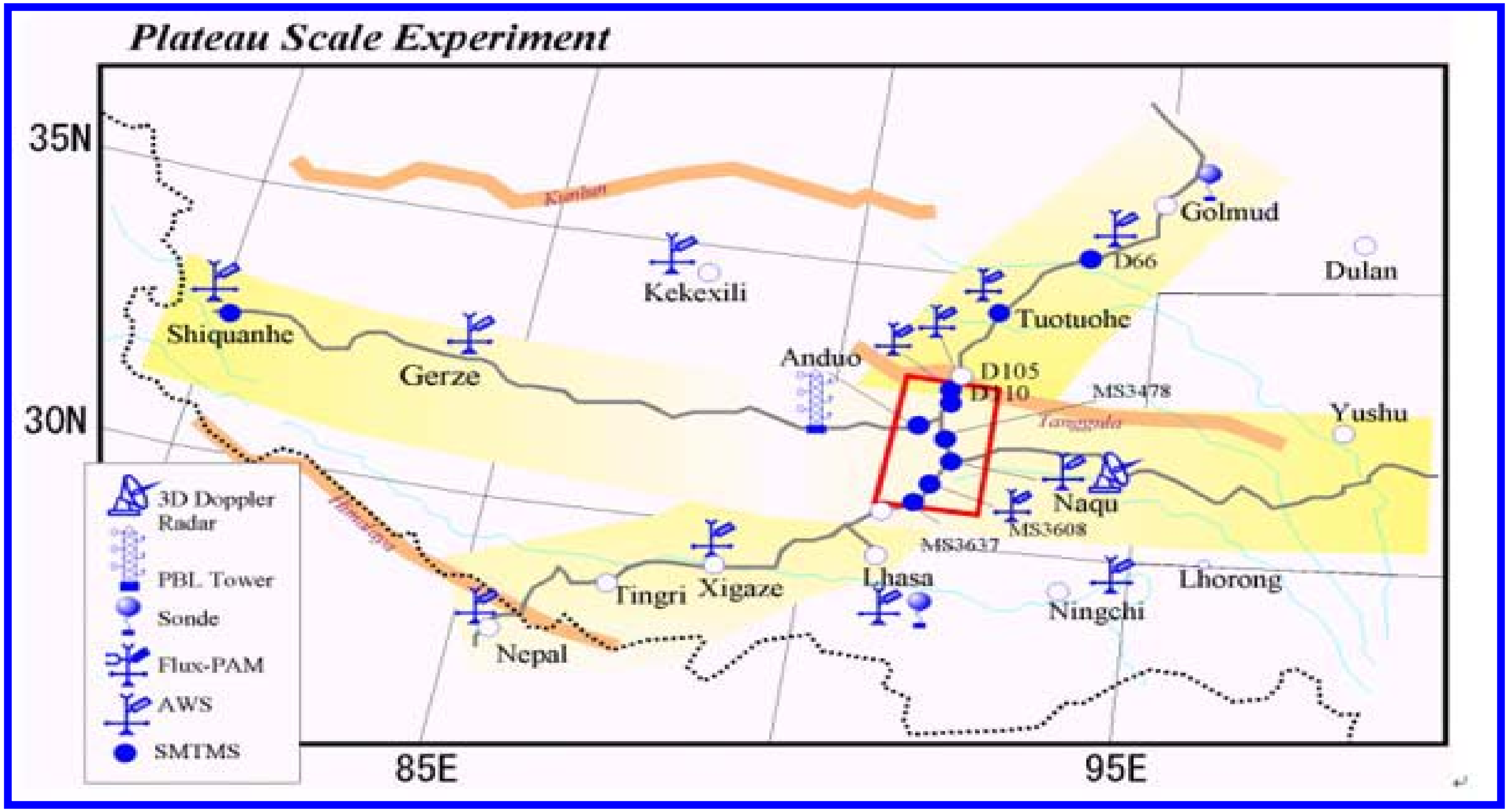
(c) those for G_t=0.02;

(d) those for G_t= -0.02.

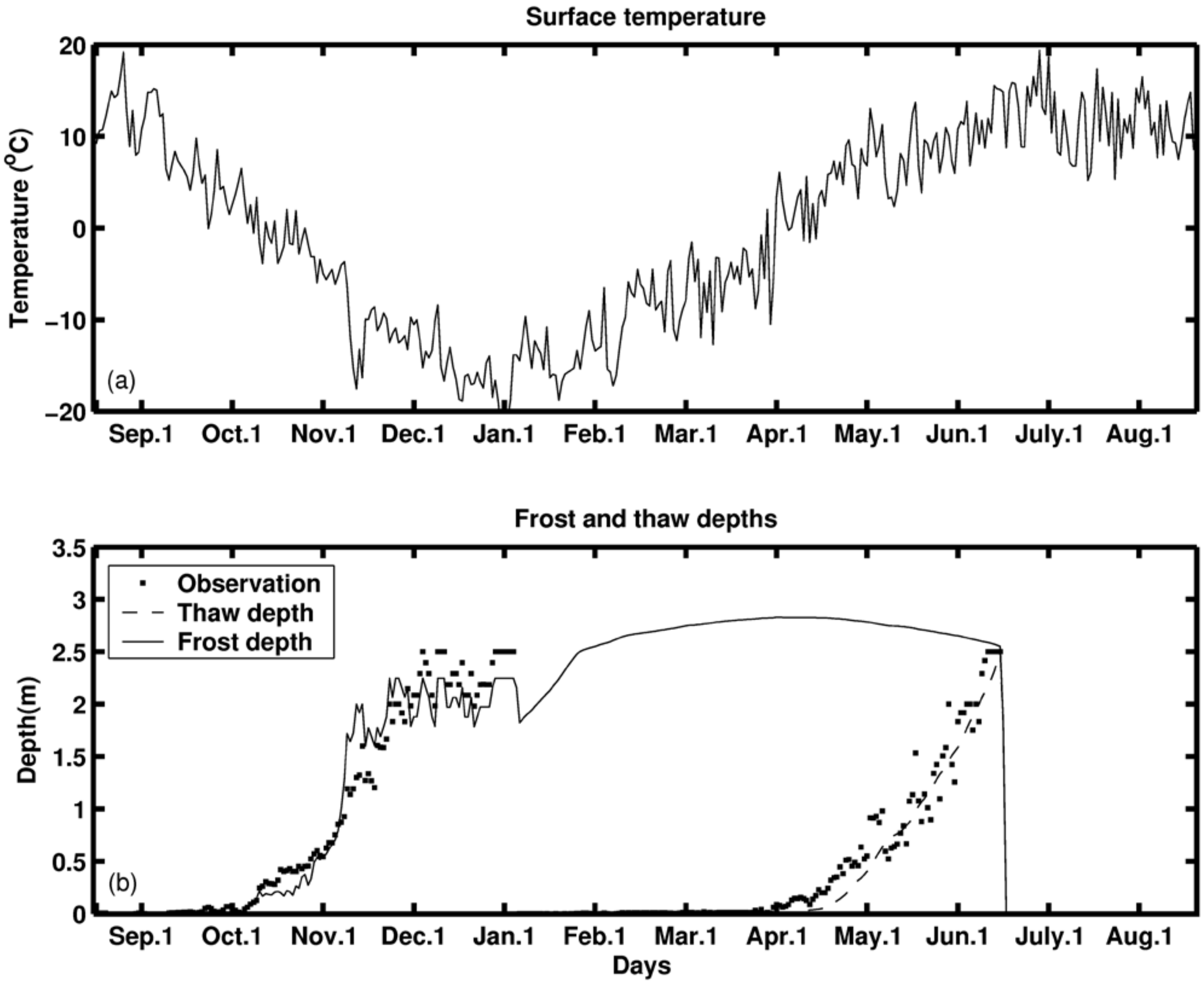


Numerical Experiment 2

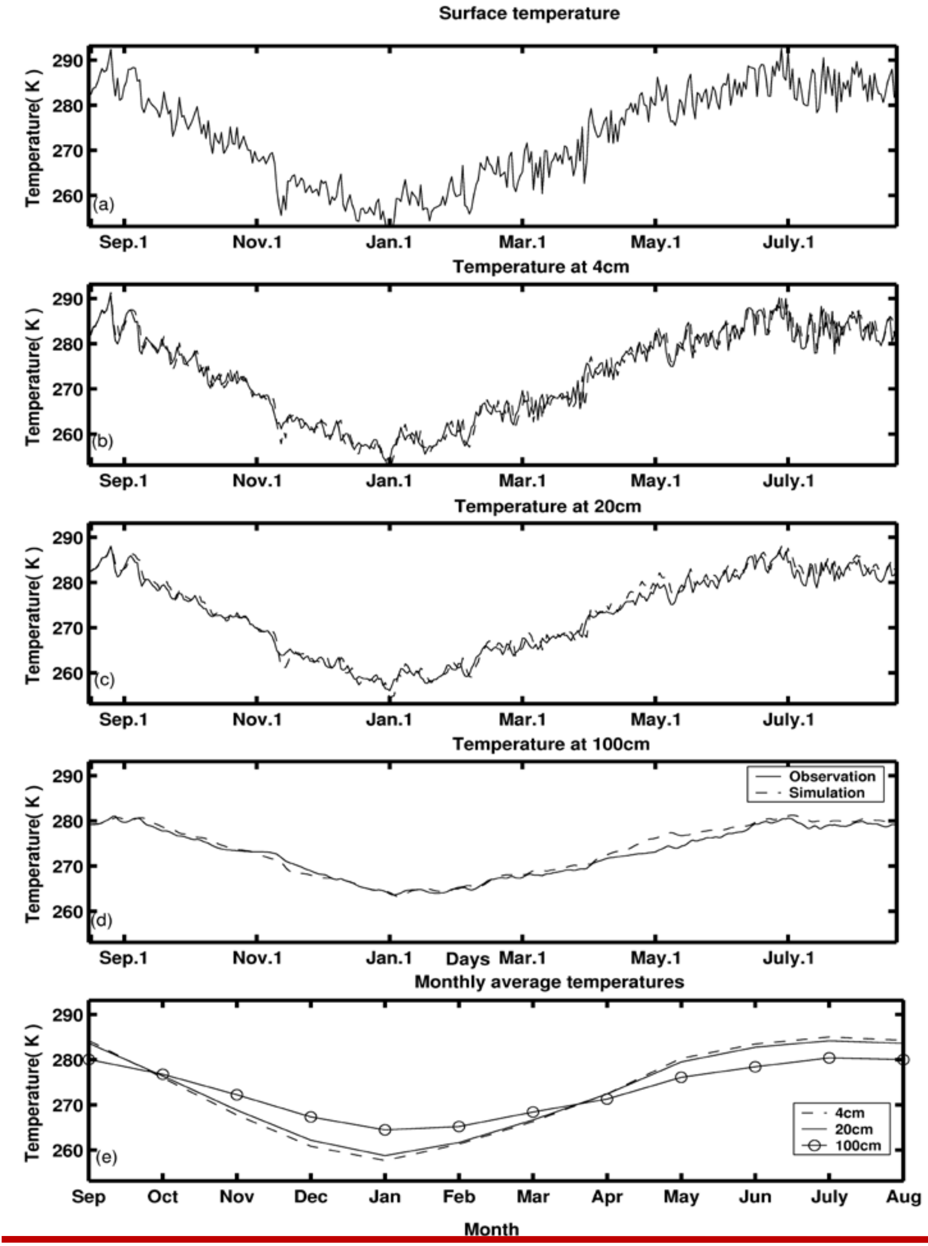
- With the observed surface temperature and soil moisture as the upper boundary conditions
- D66, D110

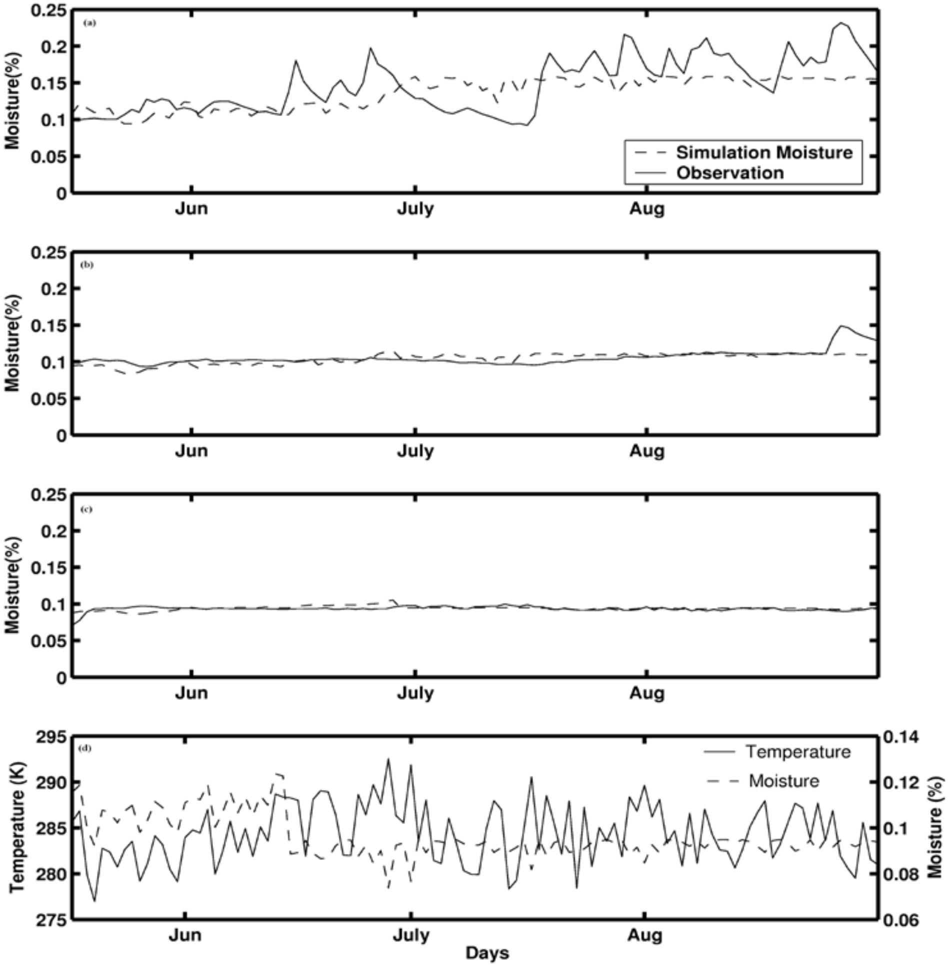


The Simulated frost and thaw depths at D66 station from 1997.8 to 1998 .8

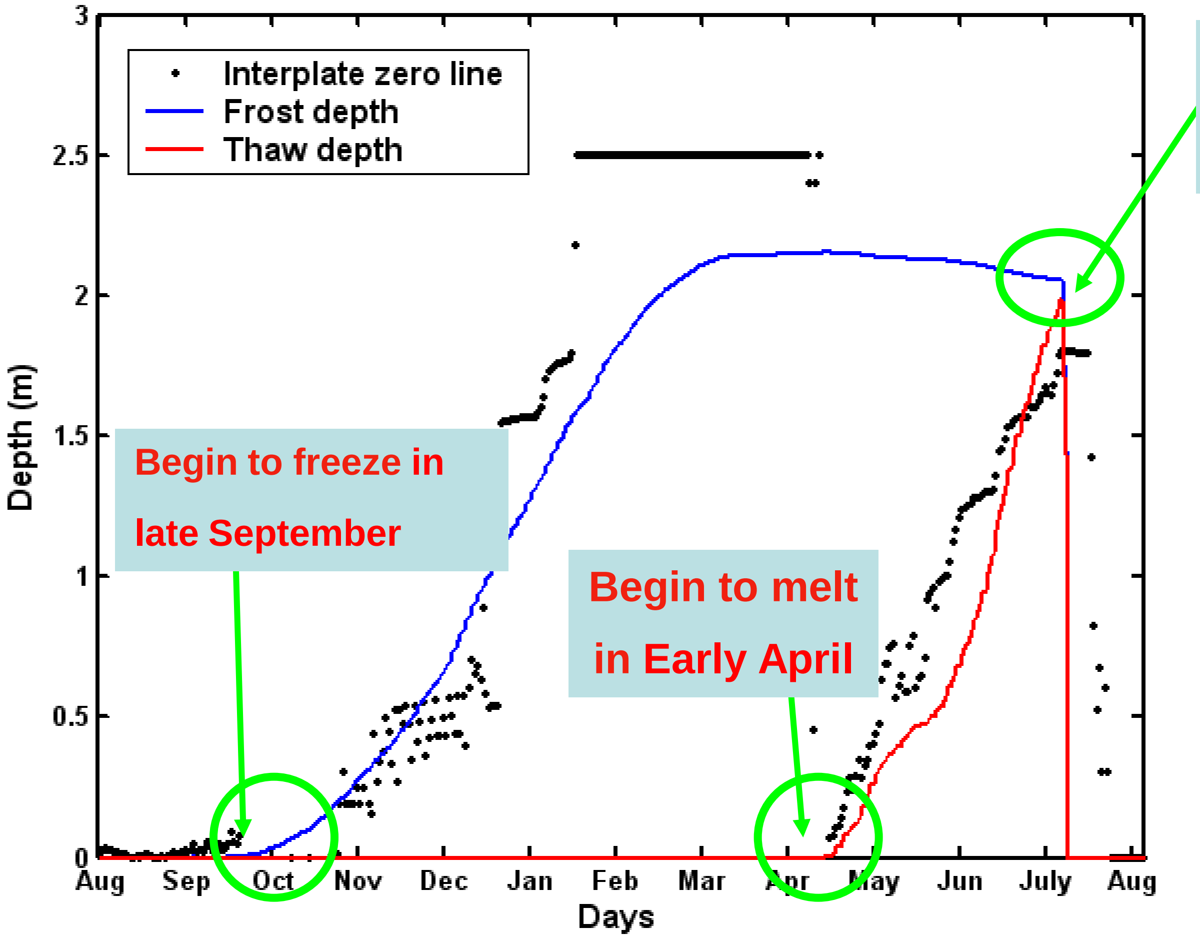


The Simulated soil moisture and temperature at D66 station from 1997.8 to 1998 .8





The Simulated frost and thaw depths at D110 station from 1997.8 to 1998 .8



Finish a freeze-thaw cycle in July

Begin to freeze in late September

Begin to melt in Early April

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Summary

- A new simultaneous heat and water transfer model for simulating the active layer and frost/thaw depths is developed；
- The new model explicitly tracks the freezing-thawing interface and frost/thaw depths ；
- An adaptive mesh method for the moving boundary problem is adopted to solve the relevant equations and to determine frost/thaw depths, water content and temperature distribution.

Future works and discussion

- The dynamical representation of frost/thaw depth was coupled to CLM 3.0, coupling with updated version of the CLM Models and its validation should be done ;
- Soil temperature was calculated by using frost and thaw depths as mesh points, then soil liquid and ice content were adjusted by the variety of phase change;
- Land surface models with dynamical representation of frost/thaw depth can improve the ability to model the soil moisture and temperature.

Thanks for your Attentions！

http://web.lasg.ac.cn/staff/xie/xie.htm

Reference

Zhenghui Xie, Liye Song , Xiaobing Feng, A moving boundary problem derived from heat and water transfer processes in frozen and thawed soils and its numerical simulation, *Science in China(A),* 51(8), 1510-1521, doi:10.1007/s11425-008-0096-x, 2008.