verification, validation, and basal strength in models for the present state of ice sheets

Ed Bueler[1]
Constantine Khroulev[2]
Andy Aschwanden[3]

[1] Dept of Mathematics and Statistics, University of Alaska Fairbanks

[2] Geophysical Institute, University of Alaska Fairbanks

[3] Arctic Region Supercomputing Center, University of Alaska Fairbanks

17 February 2010 CCSM LIWG supported by NASA grant NNX09AJ38C

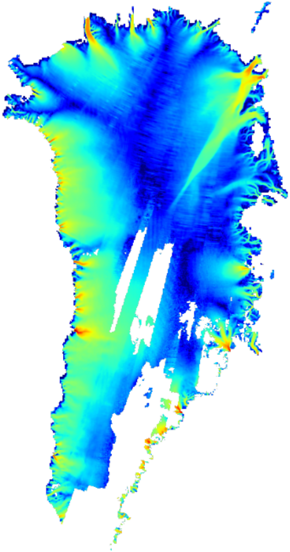
Outline

recent Greenland work

evaluating spun-up present-day states (is this model validation?)

subspace of basal strength parameters: the weak heart of ice dynamics modeling

present day surface velocity



- ⟵ nice map from Ian Joughin and Ben Smith at UW!
- RADARSAT inSAR and speckle-tracking
- average of four winter maps (2000,2006–2008)
- 86% area coverage (on 3 km PISM model grid)

PISM in one slide

- PISM = Parallel Ice Sheet Model `www.pism-docs.org`
- physical model relevant to this talk:
  - mass continuity
  - polythermal conservation of energy
  - shallow hybrid stress balance:
    - shallow ice approximation (SIA)
    - $+$ dragging-or-floating shallow shelf approximation (SSA) *as a sliding law for SIA*
- grid rectangular in horizontal & unequal (fine base) in vertical
- parallelism:
  - fields are PETSc Vecs with DA grid/topology
  - = MPI, but *not* by-hand
  - SSA stress balance:
    - by-hand "outer" viscosity iteration
    - $+$ direct call to KSP for linear solve (= pc gmres)
  - Jed Brown

PISM experiment: identify three parameters

- enhancement factor e in flow law:

$$D = e\,A|\tau|^{n-1}\tau$$

- exponent q in power law till, written as pseudo-plastic:

$$\vec{\tau}_b = -\tau_c \frac{\vec{u}_b}{|\vec{u}_b|^{(1-q)} u_0^q}$$

- allowed maximum α for basal water pressure, as fraction of overburden pressure $p_{\text{over}} = \rho g H$:

$$p_w = \alpha \frac{W}{W_0} p_{\text{over}} \quad \xrightarrow[\text{by Mohr-Coulomb}]{\text{giving yield stress}} \quad \tau_c = (\tan\phi)(p_{\text{over}} - p_w)$$

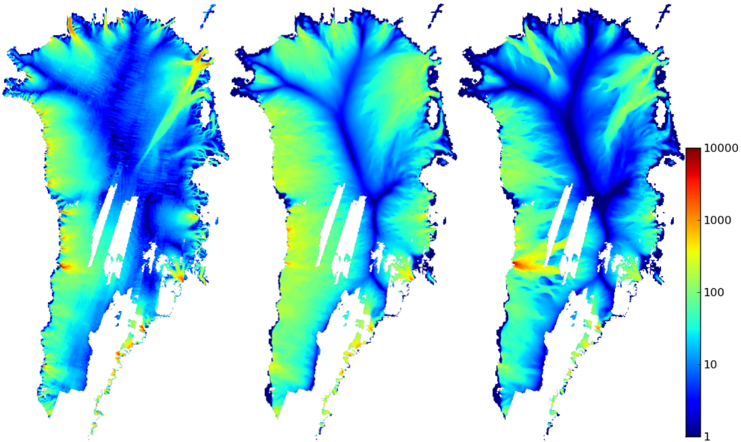
PISM experiment, cont.: parameter study

- these cases

$$e = 1, 3, 5$$
$$q = 0.1, 0.25, 0.5$$
$$\alpha = 0.95, 0.98, 0.99$$

- not doing inverse modeling . . . only *three* global, scalar parameters
- run on 3 km grid for 100 model years, from present thickness
- “spin-ups” also used these parameters (i.e. lots of spinups)
- *steady climate*
- on next slide: compare model snapshot to present-day observed

present-day surface velocity: observed and modeled



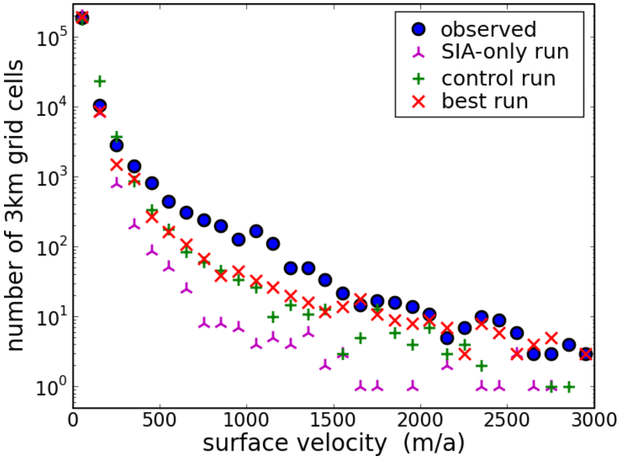
observed

model
$e = 3$ and power law sliding ($u_b \sim \tau_b^4$) and modest allowed basal water pressure

model
$e = 1$ and nearly-plastic sliding and high allowed basal water pressure

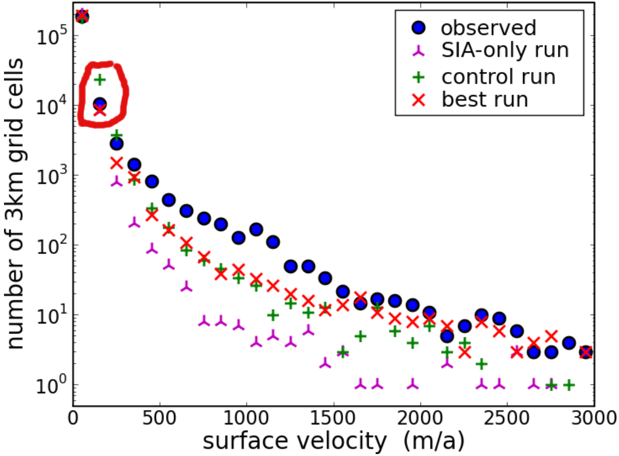
Ed: pause for eyeball norm!

comparison



- prev slide had 3 of 4: observed ● and model run + and model run ×
- note log scale on y-axis

comparison



- prev slide had 3 of 4: observed ● and model run + and model run ×
- note log scale on y-axis

spin-up: we all do it . . .

- present day measurements not sufficient for initial values
- so we do modeling “before getting to” our proposed initial state
- . . . I’ll call that spin-up even if it is *inverse modeling*
- (and I am not trying to be precise about this language; I’m trying to avoid a false dichotomy)

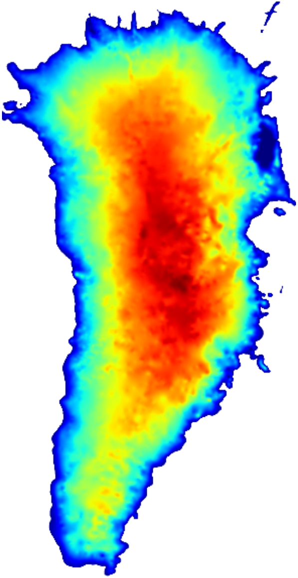
spin-up: we all do it, cont.

- by what standard do we choose among spinup procedures?
- propose a metric or norm on modeled present state m_{PS} with several $\Delta f = f_{\text{observed}} - f_{m_{PS}}$:

$$J[m_{PS}] = c_0 \left\|\Delta H\right\|^2 + c_1 \left\|\Delta u_s\right\|^2 + c_2 \left\|\Delta T_{\text{cores}}\right\|^2 + c_3 \left\|\Delta A_{\text{isochrone}}\right\|^2 + c_4 \left\|\Delta \frac{db}{dt}\right\|^2$$

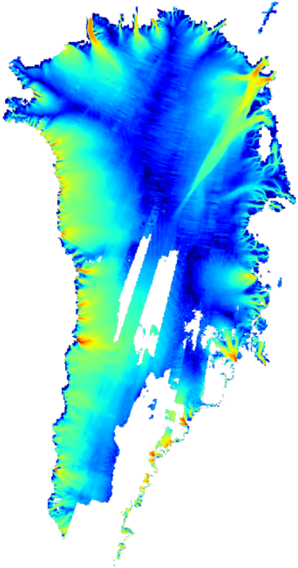
- my understanding of the paradigm:
  1. drop the terms which you have "already inverse-modeled"
  2. assign coefficients c_0, c_1, c_2, c_3, c_4
  3. choose the spinup that minimizes $J[\cdot]$

Greenland ice thickness



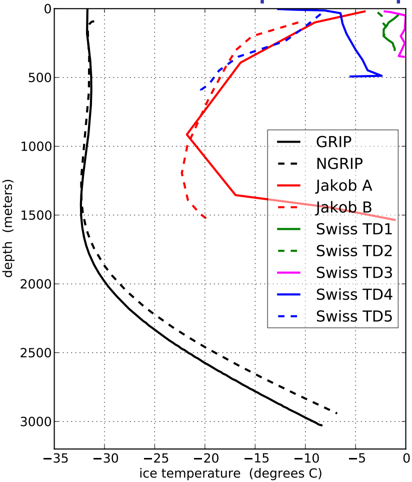
$$J[m_{PS}] = c_0 \left\| \Delta H \right\|^2 + c_1 \left\| \Delta u_s \right\|^2 + c_2 \left\| \Delta T_{\text{cores}} \right\|^2 + c_3 \left\| \Delta A_{\text{isochrone}} \right\|^2 + c_4 \left\| \Delta \frac{db}{dt} \right\|^2$$

Greenland surface velocity

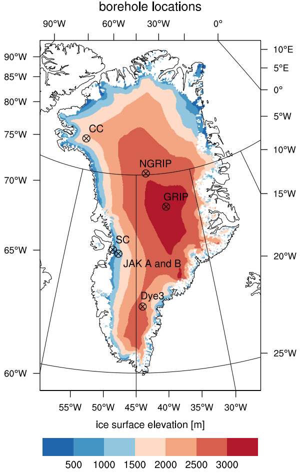


$$
\begin{aligned}
J[m_{PS}] = {} & c_0 \left\|\Delta H\right\|^2 + c_1 \left\|\Delta u_s\right\|^2 \\
& + c_2 \left\|\Delta T_{\text{cores}}\right\|^2 \\
& + c_3 \left\|\Delta A_{\text{isochrone}}\right\|^2 \\
& + c_4 \left\|\Delta \frac{db}{dt}\right\|^2
\end{aligned}
$$

measured temps at depth in Greenland holes/cores

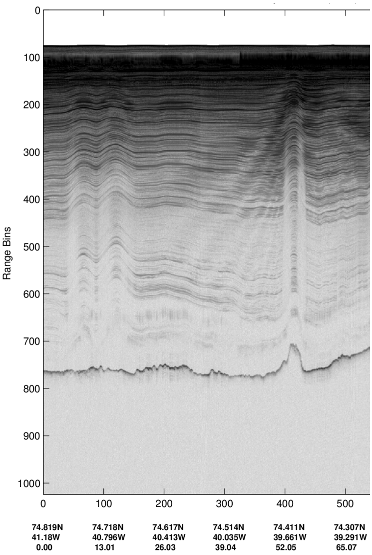


- sparse
- biased?



$$J[m_{PS}] = c_0 \|\Delta H\|^2 + c_1 \|\Delta u_s\|^2 + c_2 \|\Delta T_{\text{cores}}\|^2 + c_3 \|\Delta A_{\text{isochrone}}\|^2 + c_4 \left\|\Delta \frac{db}{dt}\right\|^2$$

radar isochrones in Greenland



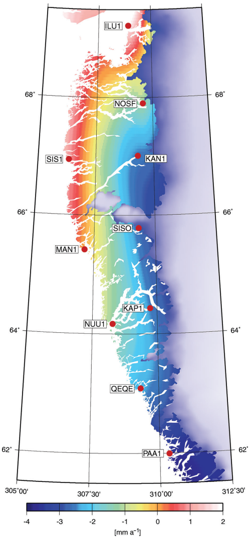
- ← CReSIS 2002 flightline, NE Greenland
- $A =$ (ice age) solves

$$\partial_t A + u\partial_x A + v\partial_y A + w\partial_z A = 1$$

- isochrone = (level surface of A)
- best if isochrones are *dated* (but still provide $\partial_{x,y,z} A$ if not)

$$J[m_{PS}] = c_0 \left\|\Delta H\right\|^2 + c_1 \left\|\Delta u_s\right\|^2 + c_2 \left\|\Delta T_{\text{cores}}\right\|^2 + c_3 \left\|\Delta A_{\text{isochrone}}\right\|^2 + c_4 \left\|\Delta \frac{db}{dt}\right\|^2$$

bedrock uplift rate in SW Greenland



- from Dietrich, Rülke, & Scheinert (2005)

$$J[m_{PS}] = c_0 \left\|\Delta H\right\|^2 + c_1 \left\|\Delta u_s\right\|^2 + c_2 \left\|\Delta T_{\text{cores}}\right\|^2 + c_3 \left\|\Delta A_{\text{isochrone}}\right\|^2 + c_4 \left\|\Delta \frac{db}{dt}\right\|^2$$

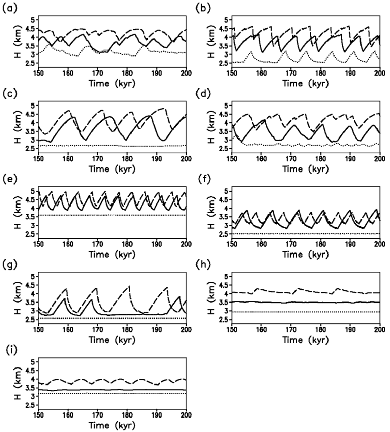
# of course I have no idea how to

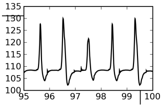
- . . . assign values to c_0, c_1, c_2, c_3, c_4

dancing around the issue

- do the terms above, in the metric, strongly-control the critical model parameter subspace?
- here's what we believe is the critical subspace:
  - something like a sliding law exponent q
  - something like a basal water pressure limit α
  - controls on thickness of near-basal temperate layer
  - controls on near-basal anisotropy
- in fact, are the parameters controlling time-dependent basal strength constrained by existing/available observations?
- the proposed metric is probably dancing around this issue

warning for SeaRISE and ice2sea





- *as a community*, we do not have a clue about the time-dependence of basal sliding:
    - ← **top**: average thickness over sediment area in HEINO intercomparison
    - ← **bottom**: average speed over one ice stream from Bueler & Brown (2009)

goal is more than just a good present-day state

- it is not just that we want to get the present state right
- we want to know that the model went through past states in a reasonable way
- ...so the model might have predictive capability in time
- I'd like to learn more about
  - inverse modeling *in time*?
  - uncertainty quantification?

role of verification in PISM

- yes, PISM has software unit tests
- . . . but we are also dragging around exact solutions and running them as verification before almost every commit
- Q: is this good or sustainable?
- I feel the need for exact solutions to *significant coupled subsystems*
- for such exact solutions the numerical error is the most sensitive and comprehensive "unit test" for that subsystem

www.nakedpastor.com

exact solutions: also dancing around the issue

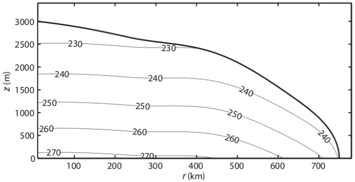


Fig. 2. Thickness and temperature field (K) in test **G**, evaluated at $t = 500$ years.

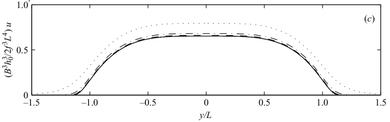


FIGURE 3. Numerical solutions of the depth-integrated ice flow problem compared with analytical solutions and solutions of the full Stokes equations. Each panel shows a numerical

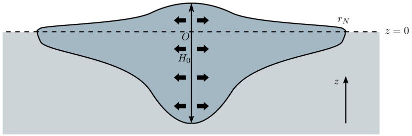


Figure 4: Cross-section of an axisymmetric shelf formed by a vertical line source, which is a point source in the horizontal plane.

Bueler et al. (2007); thermocoupled, non-sliding SIA; test G in PISM

Schoof (2006); SSA with pre-determined yield stress; test I in PISM

Sam Pegler (2009); spreading ice shelf with vertical-line source; not yet in PISM

wanted: the *right* exact solution

- anyone out there have an exact formula for a time-dependent grounded, sliding flow with SSA-type longitudinal stress?
- . . . it never hurts to ask . . .
- MISMIP is in the right direction, but I'd rather have it exact-and-manufactured than asymptotically-matched-and-not-quite-a-solution-to-the-PDEs-I'm-solving