

# Metrics for assessing ice sheet model performance

Jesse Johnson, Douglas Brinkerhoff, Glen Granzow

17 February, 2010

Land Ice Working Group, Paleo Climate Working Group Joint  
Session

## 1 Introduction

- Background
- Challenges

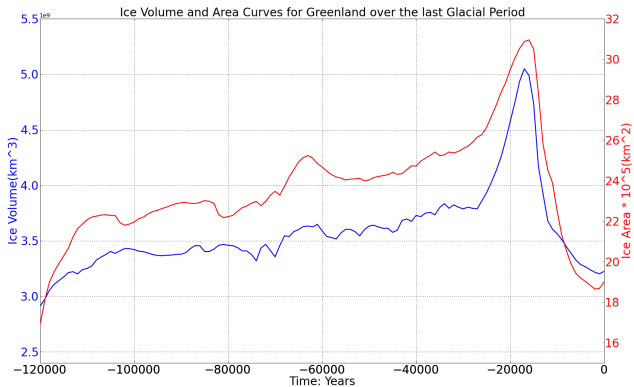
## 2 Distributions

- PDFs and binning
- CDFs and maximum likelihood
- Sensitivity

## 3 Summary

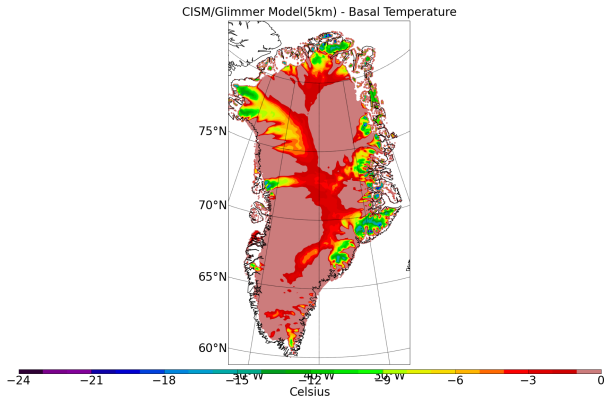
# Model validation

## Volume and Area



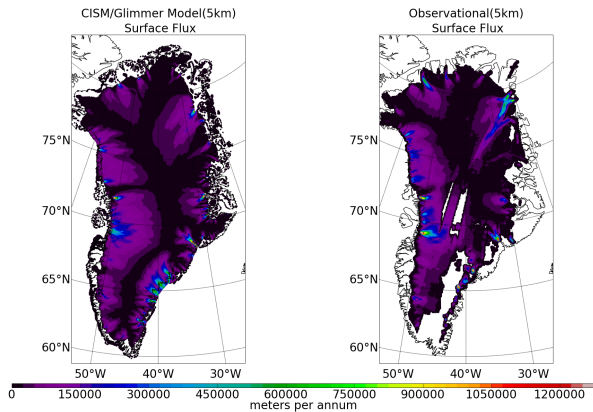
# Model validation

## Basal Temperature



# Model validation

## Surface Velocity



# A brief history of ice sheet modeling

## The Purpose

- ice sheet inception
- glacial-interglacial cycles
- a source for gravity models
- a source of freshwater for ocean models
- locating and dating ice cores

# A brief history of ice sheet modeling

## “Validation”

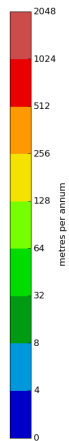
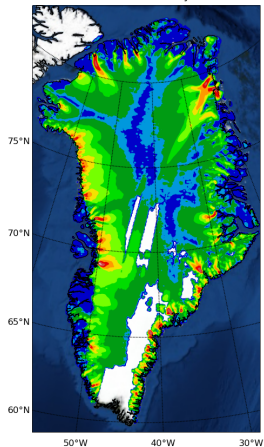
- *“Paucity of suitable test data...”*
- *“geological record is often ambiguous...”*
- *“Gross comparisons of the overall patterns...”*
- *“...look very reasonable”*
- *“...but not full exploited yet”*

Huybrechts. Numerical modelling of polar ice sheets through time. *Glacier Science and Environmental Change*, Chapter 80 (2006) pp. 1-12

# Has the situation changed?

Rich data sources

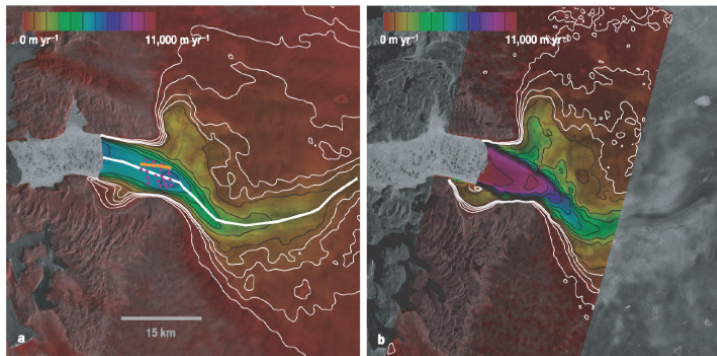
Interferometric Surface Velocity Measurements





# Has the situation changed?

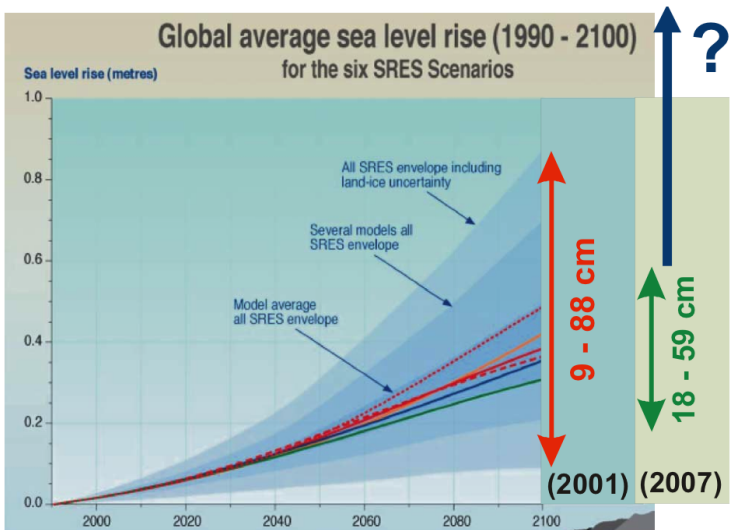
Emphasis on rapid changes



Joughin et al. Large fluctuations in speed on Greenland's Jakobshavn Isbr glacier. *Nature* (2004) vol. 432 pp. 608-610

# Has the situation changed?

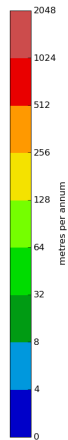
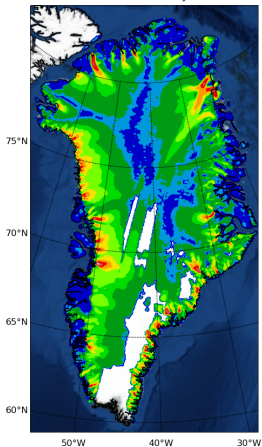
Emphasis on short term sea level rise



# Can data be aggregated in meaningful ways?

Velocity

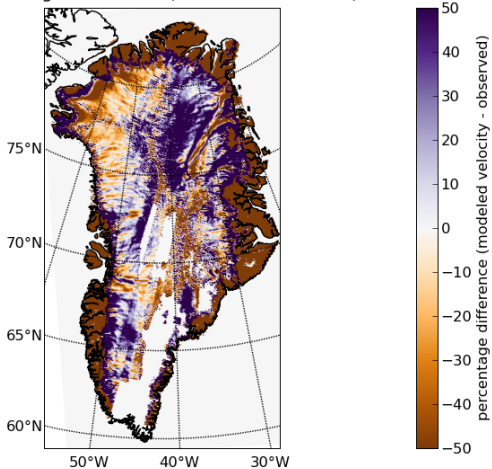
Interferometric Surface Velocity Measurements



# Can data be aggregated in meaningful ways?

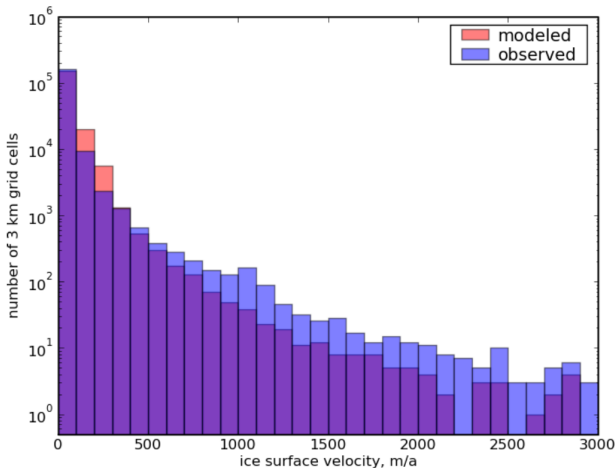
## Velocity Differences

Percentage difference, (modeled - observed)



# Can data be aggregated in meaningful ways?

Distribution, Bueler



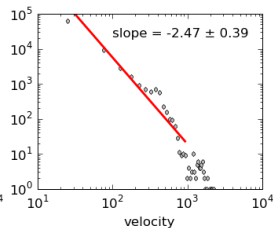
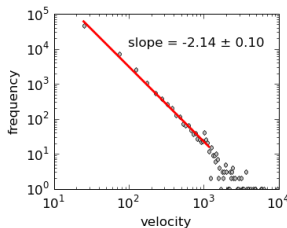
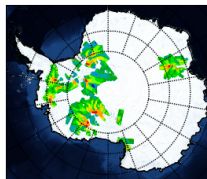
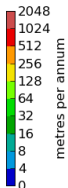
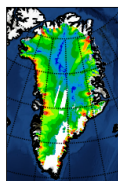
E. Bueler, C. Khroulev, A. Aschwanden, and I. Joughin, Modeled and observed fast flow in the Greenland ice sheet, Presentation at IGS International Symposium on Glaciology in the International Polar Year, Newcastle, UK, July 2009

# Power-law behavior

Simple method

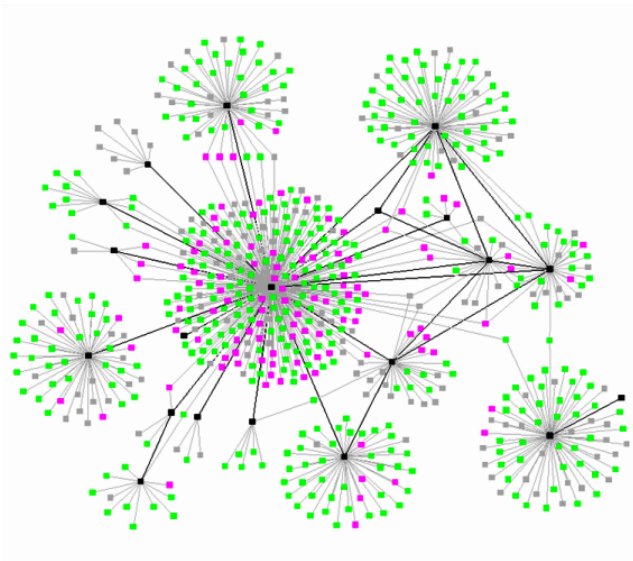
$$N \propto V^{-\alpha}$$

Interferometric Surface Velocity Measurements



# Power-law behavior

Meaning???



# Power-law behavior

Meaning???





# Power-law behavior

Meaning???



# Power-law behavior

## Problems with method

The trouble with binning data:

<b>Method</b>	<b>Value</b>	<b>Error</b>
LS + PDF, Constant width	1.39	$\pm .05$
LS + CDF, Constant width	2.48	$\pm .04$
LS + PDF, Log. width	1.19	$\pm .02$

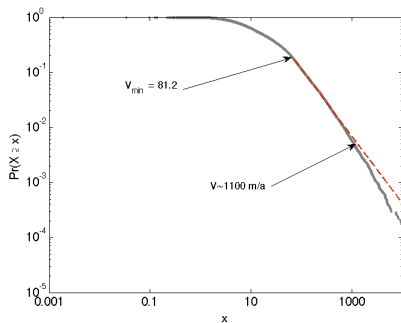
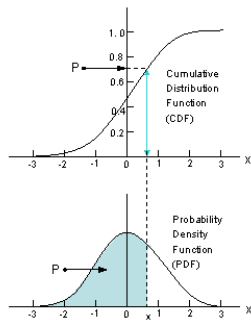
from: Clauset, A., Shalizi, C. R. and Newman, M. E. J. (2007). Power-law distributions in empirical data

# Power law behavior

## Improved method

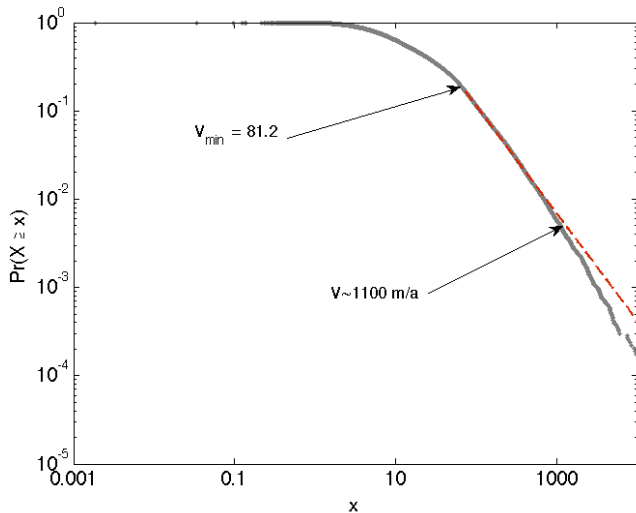
Consider the cumulative distribution function (CDF):

$$P(x > X) = \int_{-\infty}^x p(x) dx$$

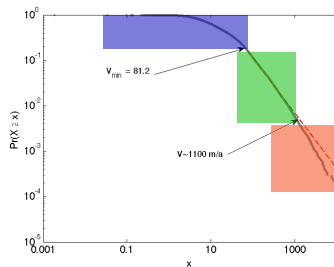
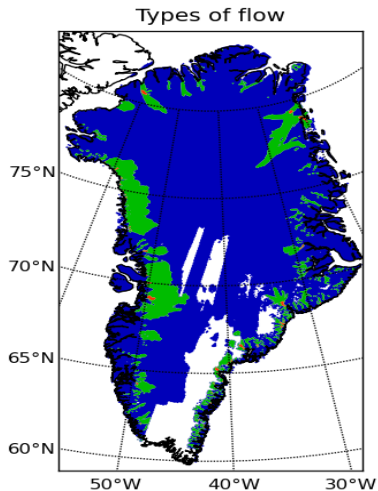


# Power law behavior

CDF of ice velocity data



# Power law behavior



# Comparing the metric to model output

Several models participating in seaRISE

## Results from various models

Model	exponent, $\alpha$	$x_{\min}$	p-value
INSAR Data	$2.41 \pm 0.08$	$81.9 \pm 17$ m/a	.578
Balance Velocity <sup>†</sup>	$2.28 \pm 0.06$	$86.3 \pm 18$ m/a	.158
PISM	$2.70 \pm 0.15$	$62.5 \pm 18$ m/a	.532
CISM (SIA)	$2.91 \pm 0.12$	$94.8$ m/a $\pm 19$ m/a	.926
CISM (HO), isothermal	$1.24 \pm 0.04$	$2.09 \pm 0.15$ m/a	0.0

<sup>†</sup> these are *vertically averaged* velocities.

# Is this metric sensitive to the right thing?

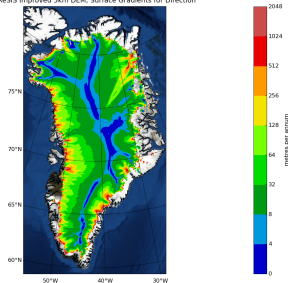
Balance velocity test

## Balance Velocity

$$\frac{\partial H}{\partial t} = -\nabla \cdot \mathbf{u}H + a$$

$$\frac{\partial H}{\partial t} = 0$$

Greenland Balance Velocities (5 ice thickness averaging)  
CRISIS Improved 5km DEM, Surface Gradients for Direction

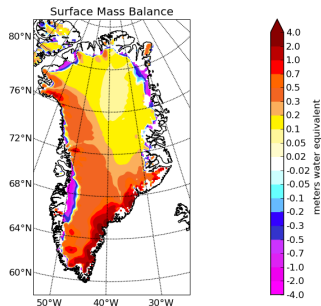


# Is this metric sensitive to the right thing?

Balance velocity test

Accumulation field

$$a = \nabla \cdot \mathbf{u}H$$



Ettema J., M.R. van den Broeke, E. van Meigaard, W.J. van de Berg, J.L. Bamber, J.E. Box, and R.C. Bales (2009), "Higher surface mass balance of the Greenland ice sheet revealed by high-resolution climate modeling", *Geophys. Res. Lett.*, 36, L12501, doi:10.1029/2009GL038110



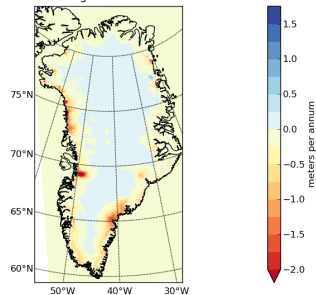
# Is this metric sensitive to the right thing?

Balance velocity test

$\frac{\partial H}{\partial t}$  field

$$a - \frac{\partial H}{\partial t} = \nabla \cdot \mathbf{u}H$$

Rate of change of surface elevation



Csatho 2009, personal communication

# Is this metric sensitive to the right thing?

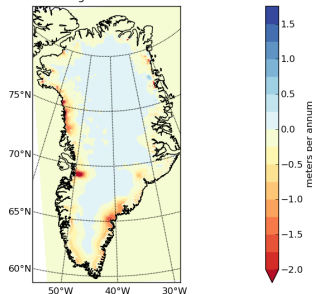
Balance velocity test

Utilizing the  $\frac{\partial H}{\partial t}$  field

$$a - \frac{\partial H}{\partial t} = \nabla \cdot \mathbf{u}H$$

$$a - \gamma \left( \frac{\partial H}{\partial t} \right)_{\text{Dyn}} - (1 - \gamma) \left( \frac{\partial H}{\partial t} \right)_{\text{SMB}} = \nabla \cdot \mathbf{u}H$$

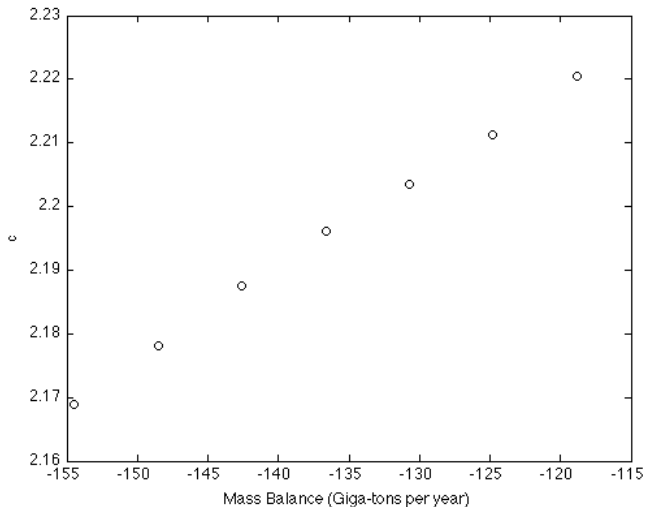
Rate of change of surface elevation



Csatho 2009, personal communication

# Sensitivity of $\alpha$ to mass balance

Using balance velocity calculation



# Summary

The point of this talk

Distributions of observation and model output may provide a useful metric for evaluating ISM output because:

- They provide a simple aggregation of a large amount of data
- Power law distributions have mature statistical tools for evaluation of data
- These metrics appear to be sensitive to changes in mass balance

# Extra Slide

## Benford's Law

