Towards a 2D, Computationally Light, Single-Head Ice Sheet Hydrology Model

William COLGAN[1], Harihar RAJARAM[2], Robert ANDERSON[3], Thomas PHILLIPS[1], Ian JOUGHIN[4], Jay ZWALLY[5], and Konrad STEFFEN[1]

[1]Cooperative Institute for Research in Environmental Sciences, University of Colorado
[2]Department of Civil, Environmental and Architectural Engineering, University of Colorado
[3]Institute of Arctic and Alpine Research, University of Colorado
[4]Applied Physics Laboratory, University of Washington
[5]Goddard Space Flight Center, National Aeronautics and Space Administration

CESM Land Ice Working Group Winter Meeting

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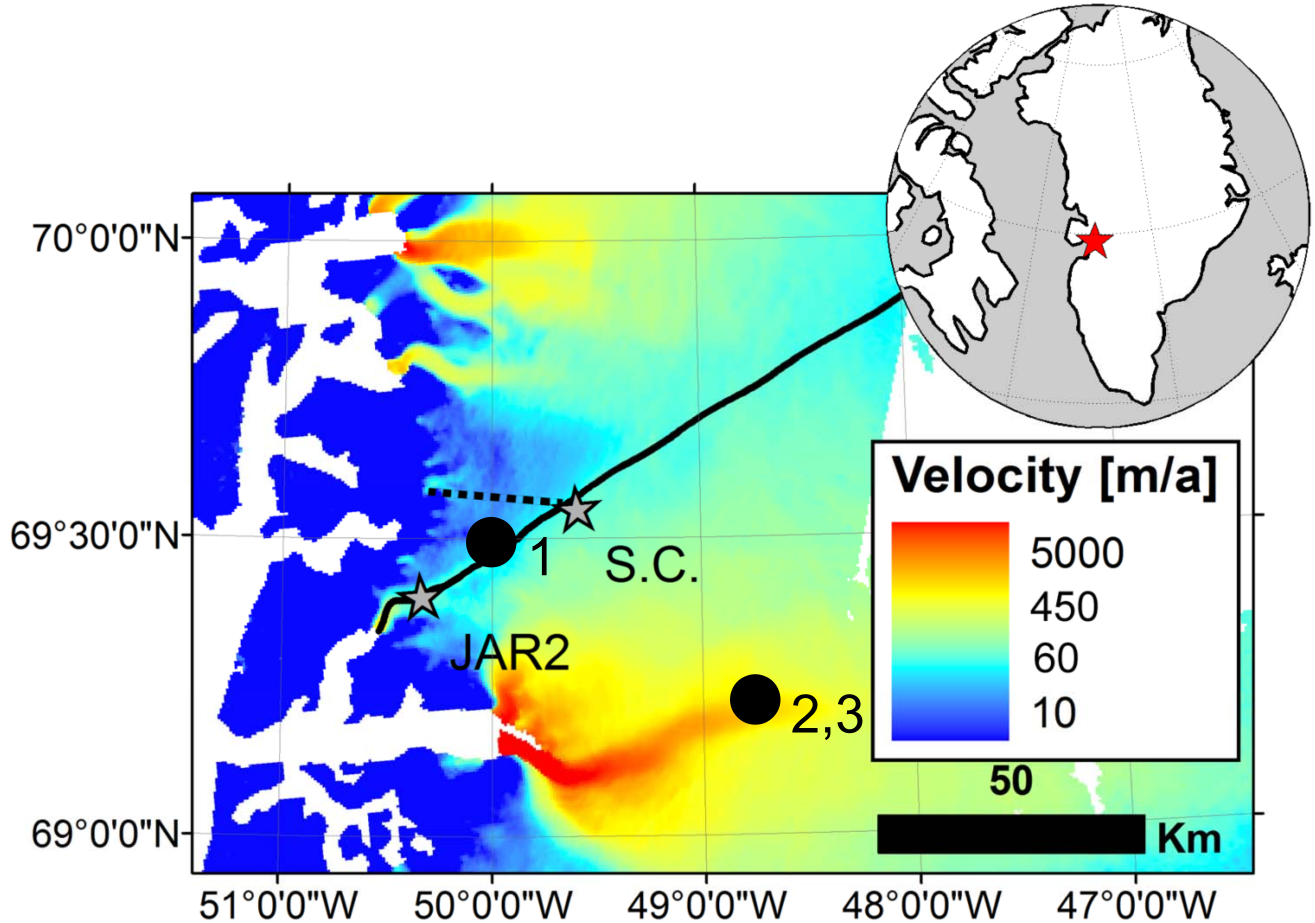
Study Site and Data: Sermeq Avannarleq Flowline

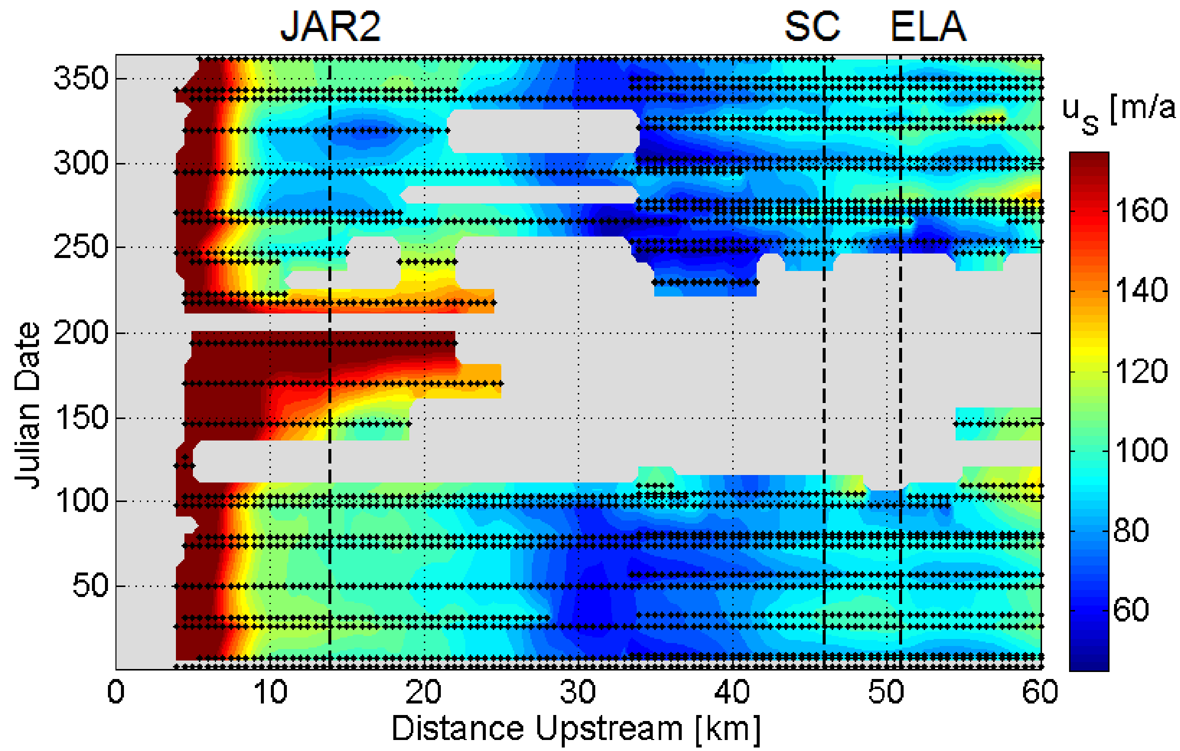


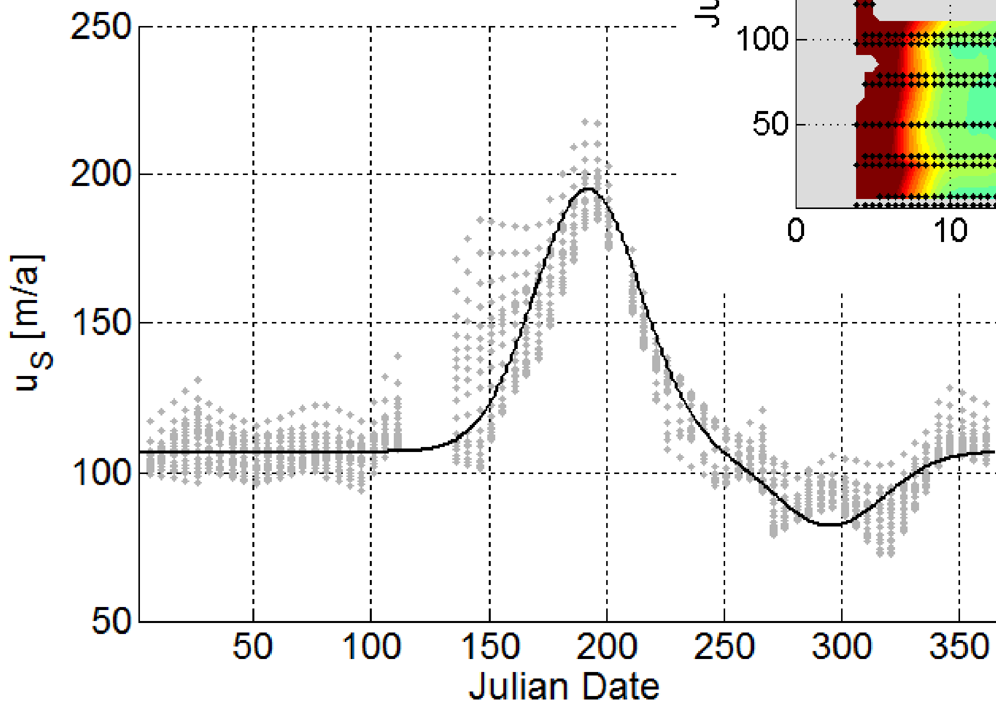
1. *Thomsen and Olsen* (1991)
2. *Iken et al*. (1993)
3. *Lüthi et al.* (2002)

(*Joughin et al*., 2010)

Empirical Motivation: InSAR 2005/2006 Velocities

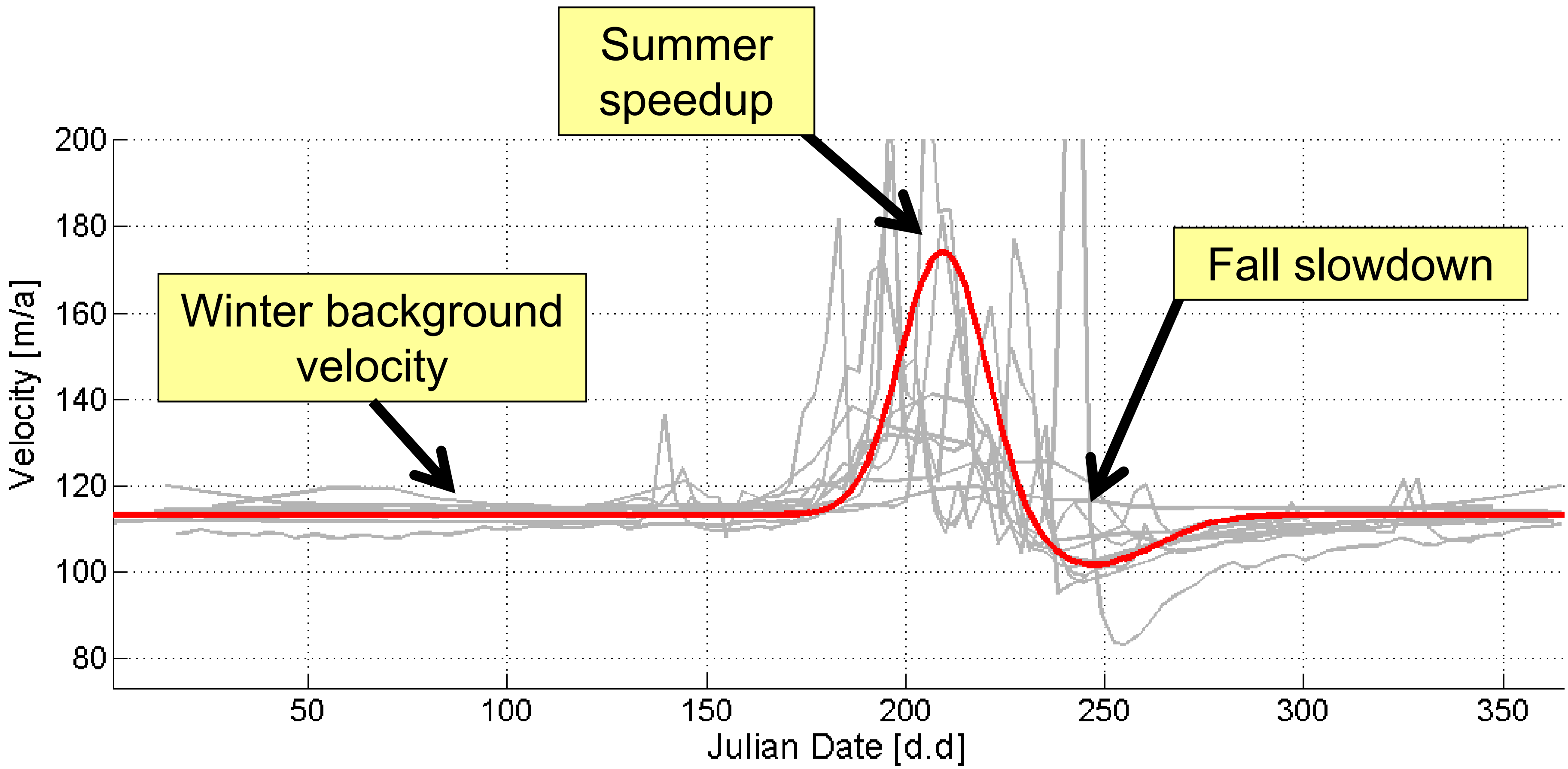
Pursuing a hydrology model which can be used to explain velocity variations…





(*Joughin et al.*, 2010)

Empirical Motivation: GPS 1996/2008 Velocities



(*Jay Zwally*, per. comm.)

Theoretical Motivation: "Alpine" Sliding Model

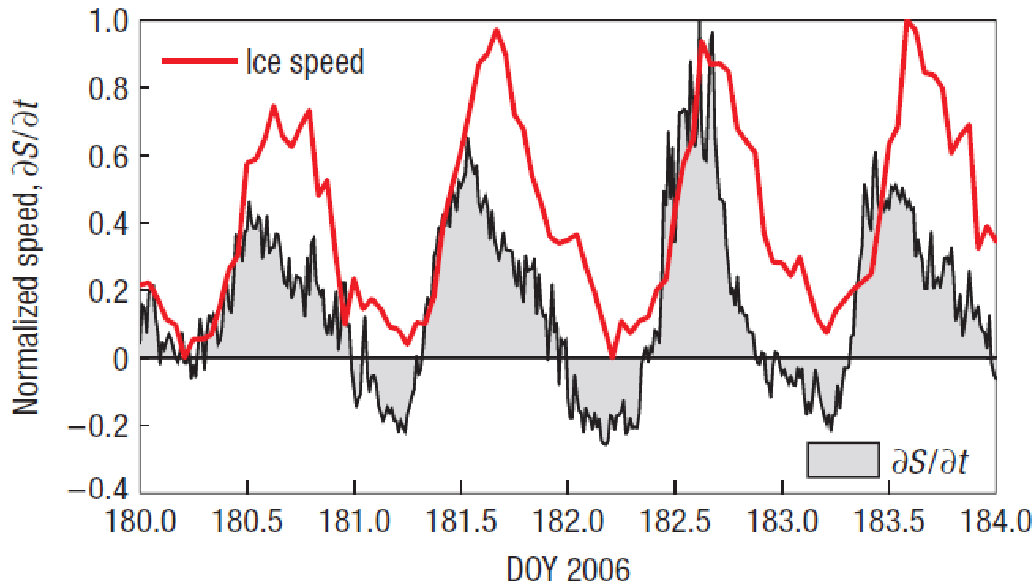
Variations in sliding velocity are due to changes in water storage over time (i.e. "*dS/dt*")…

…where output rate is dominated by conduit efficiency.

$$\frac{dS}{dt} = inputs - outputs$$

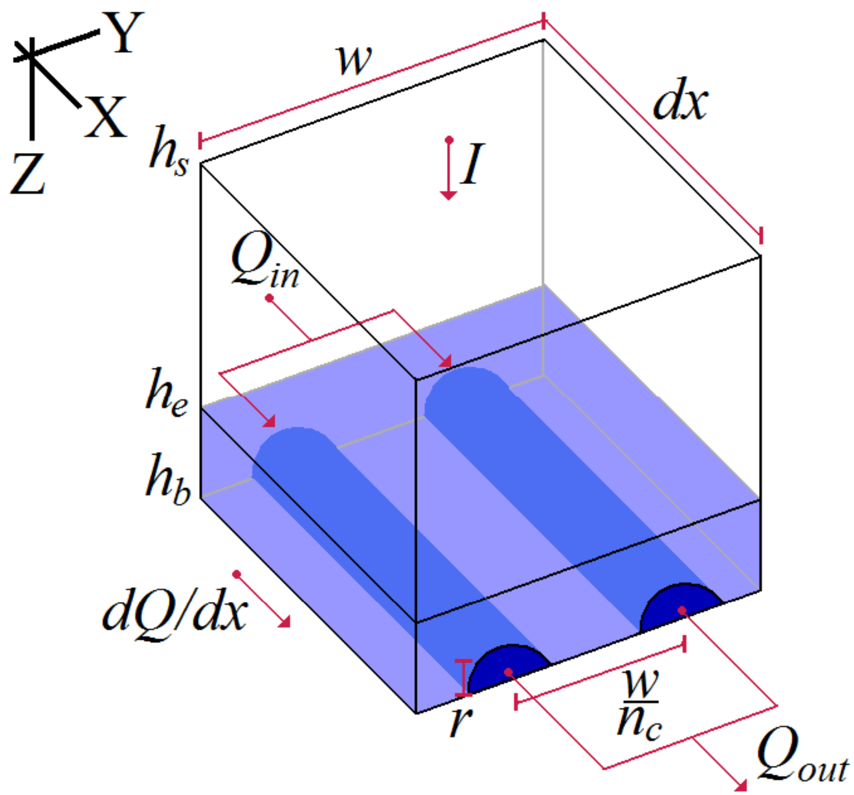
$$+VE\,\frac{dS}{dt} = sliding$$

$$-VE\,\frac{dS}{dt} = no\,sliding$$



(*Bartholomaus et al.*, 2007)

Single-Head Hydrology Model (1D): Overview



Water Storage:

1. Ice aquifer (S_i)
2. Conduits (S_c)

$$S = S_i + S_c$$

Water Mass Conservation:

1. External input (Iw)
2. Internal melt (m/ρ_w)
3. Horizontal divergence (dQ/dx)
4. Change in conduit storage (dS_c/dt)

$$\frac{dS}{dt} = \varphi w \frac{\partial h_e}{\partial t} = Iw + \frac{\dot{m}}{\rho_w} - \frac{\partial Q}{\partial x} \boxed{- \frac{\partial S_c}{\partial t}}$$

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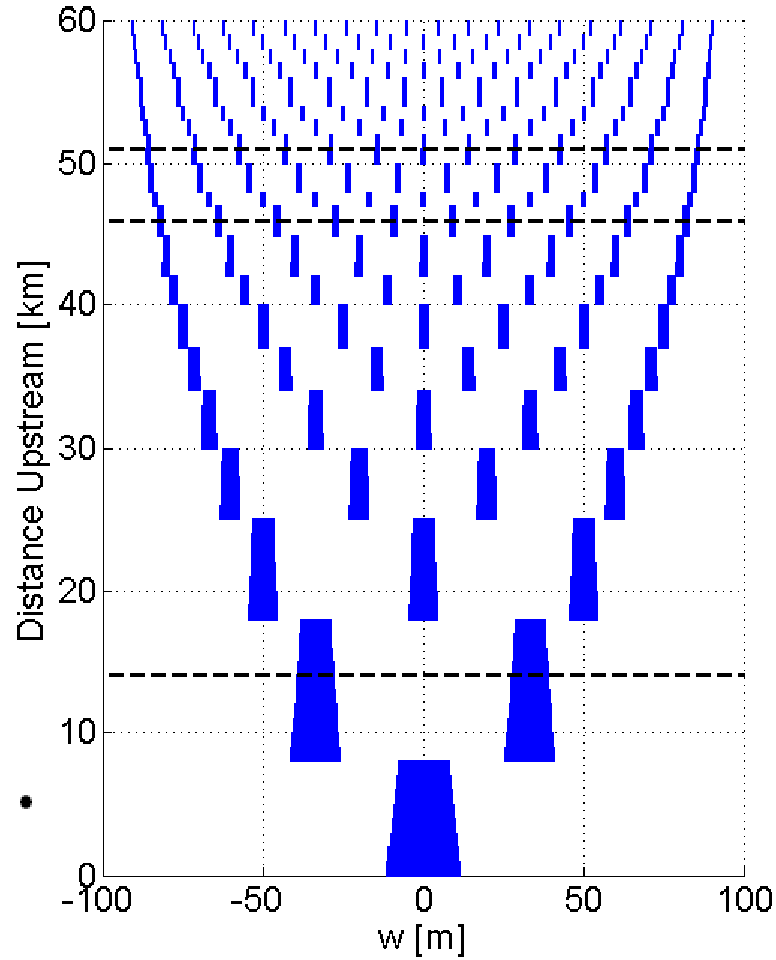
Single-Head Hydrology Model (1D): Conduits

Conduit geometry:

$$S_c = \frac{\pi r^2}{2}(n_c w)$$

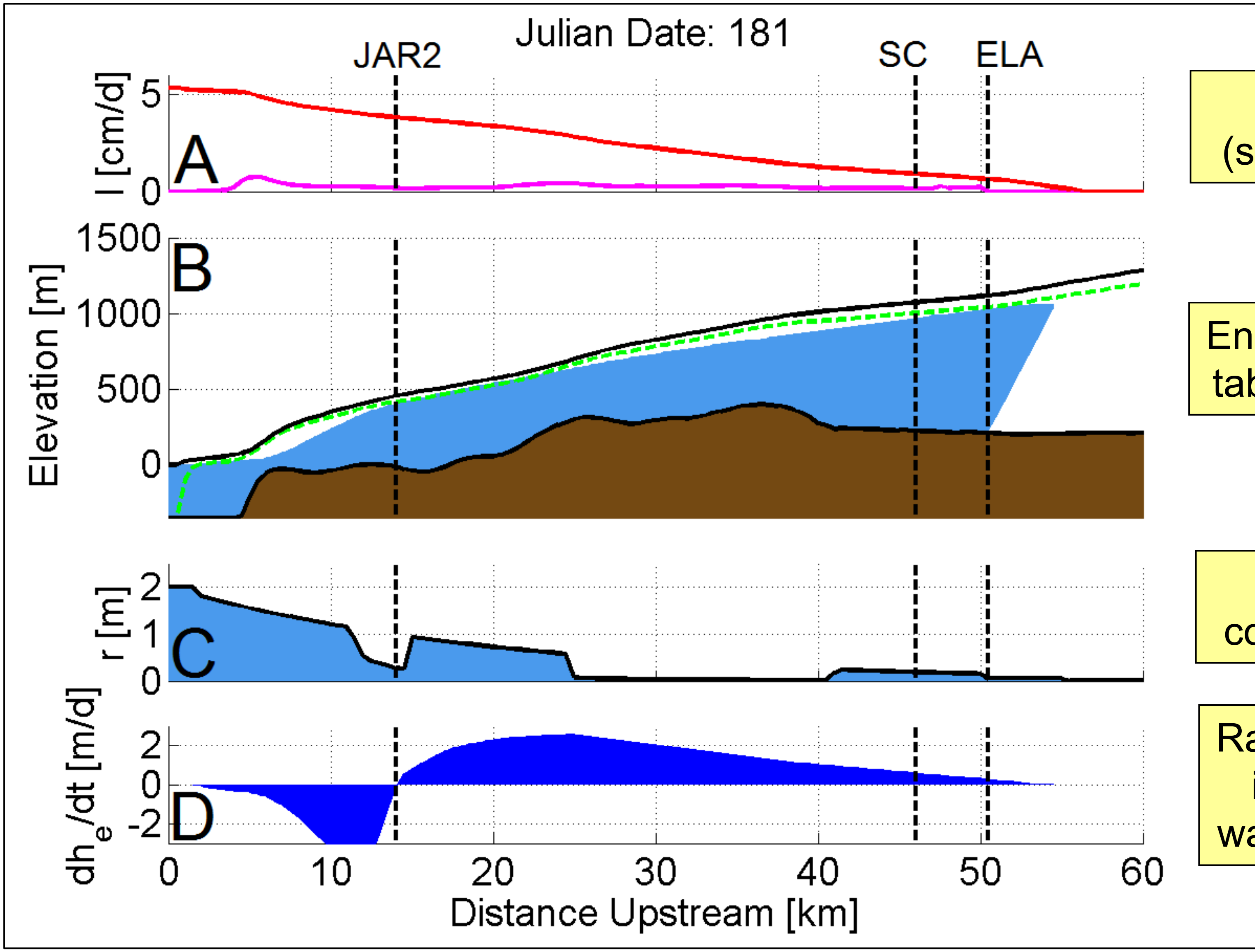
Conduit Mass Conservation:

1. Internal Melt (m/ρ_w) +VE
2. Deformation (…) –/+ VE



$$\frac{dS_c}{dt} = \frac{\dot{m}}{\rho_i} - 2A\left(\frac{\pi r^2}{2}\right)\cdot(n_c w)\cdot \cdots \left(\frac{|P_i - P_w|}{n}\right)^n \cdot \operatorname{sign}(P_i - P_w)$$

1D Results: Animation



Water input (surface & bed)

Englacial water table elevation

Subglacial conduit radius

Rate of change in englacial water elevation

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1D Results: Animation

VIEW
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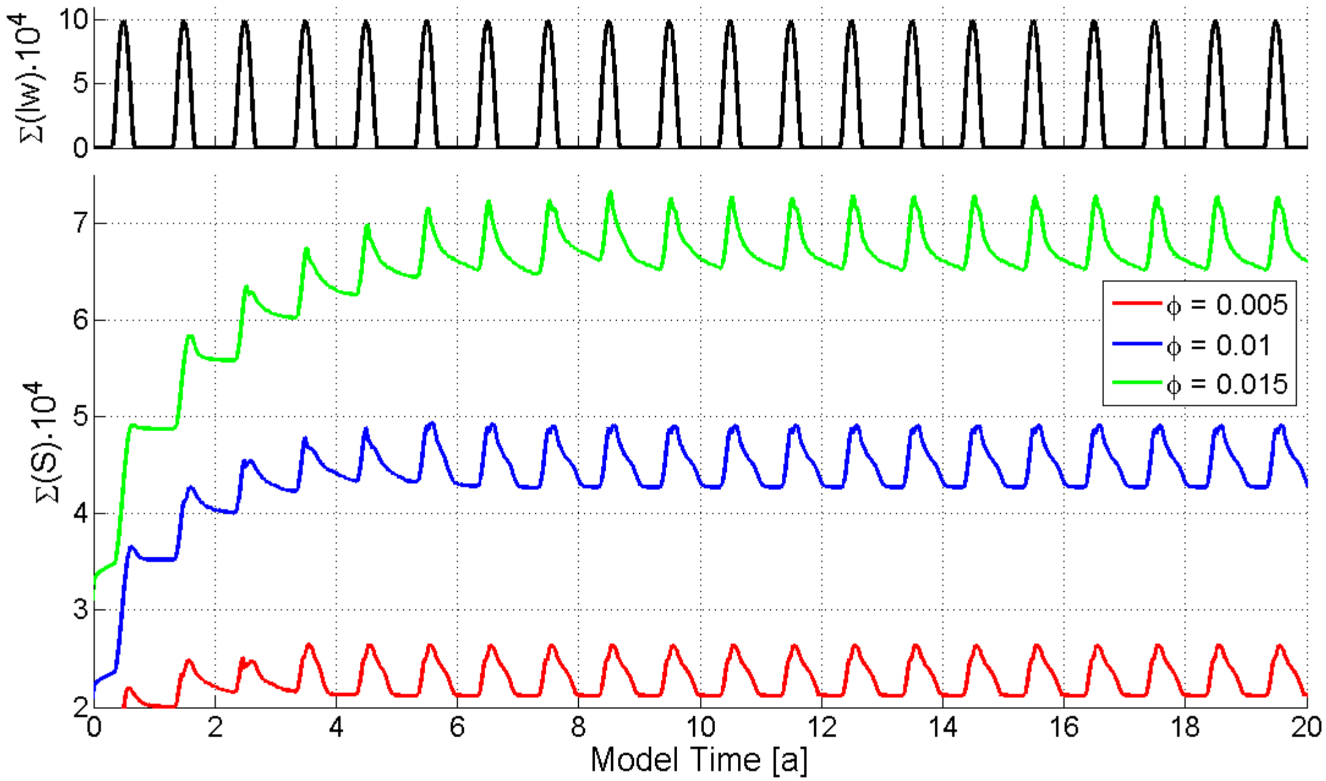
1D Results: Stability and Residence Time

Spin-up to stable state in < 10 years…



$$t_{res} = \frac{\Sigma S}{\Sigma(Iw)}$$

Mean residence time (t_{res}) varies between 1.1 and 3.3 years (depending on bulk ice porosity; ϕ)

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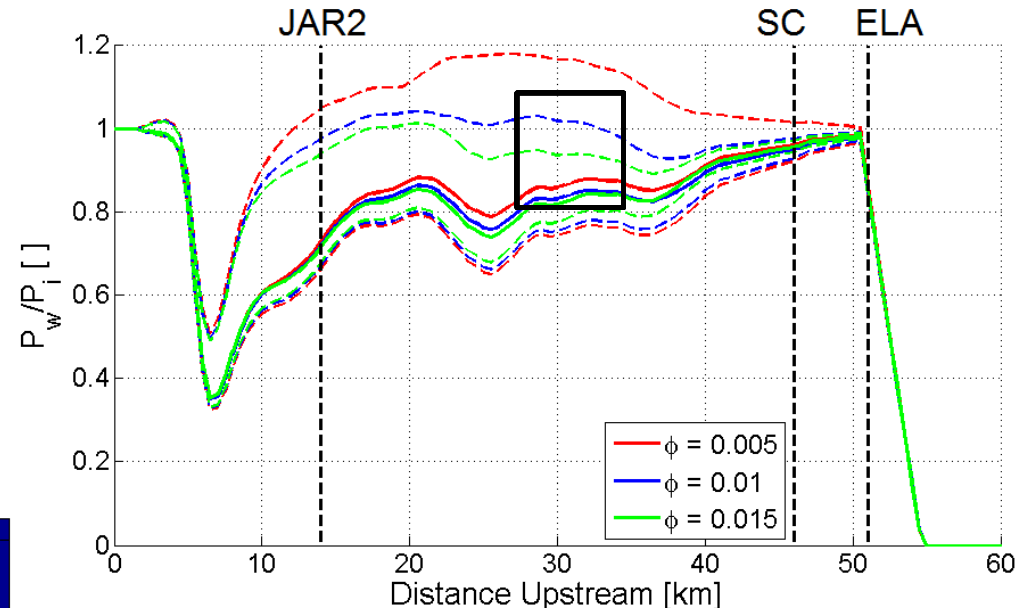
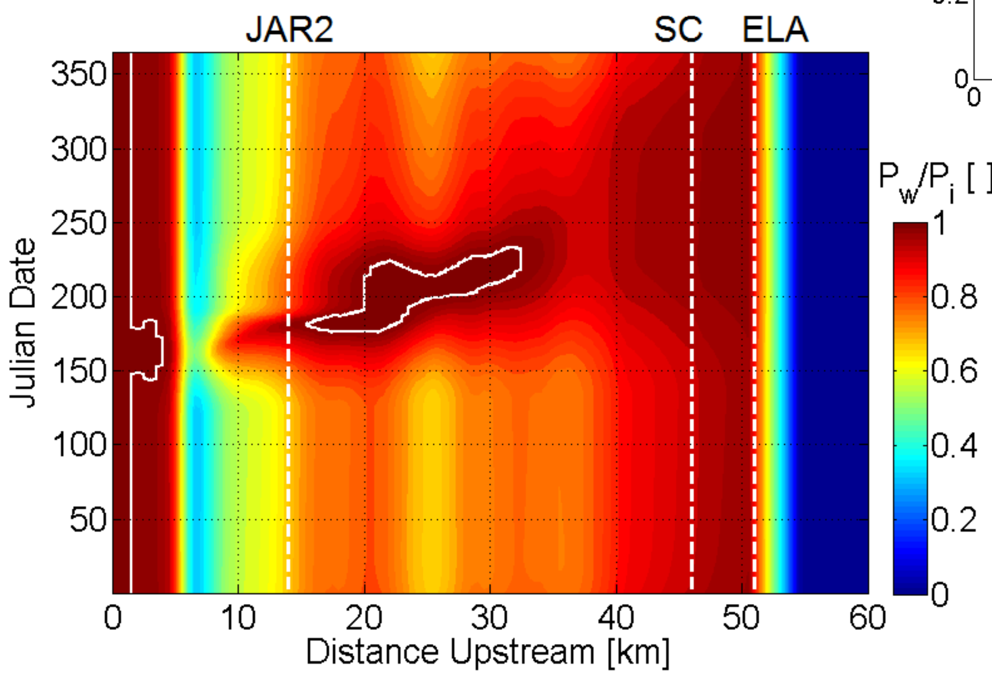
1D Results: Flotation Fraction (P_w/P_i)

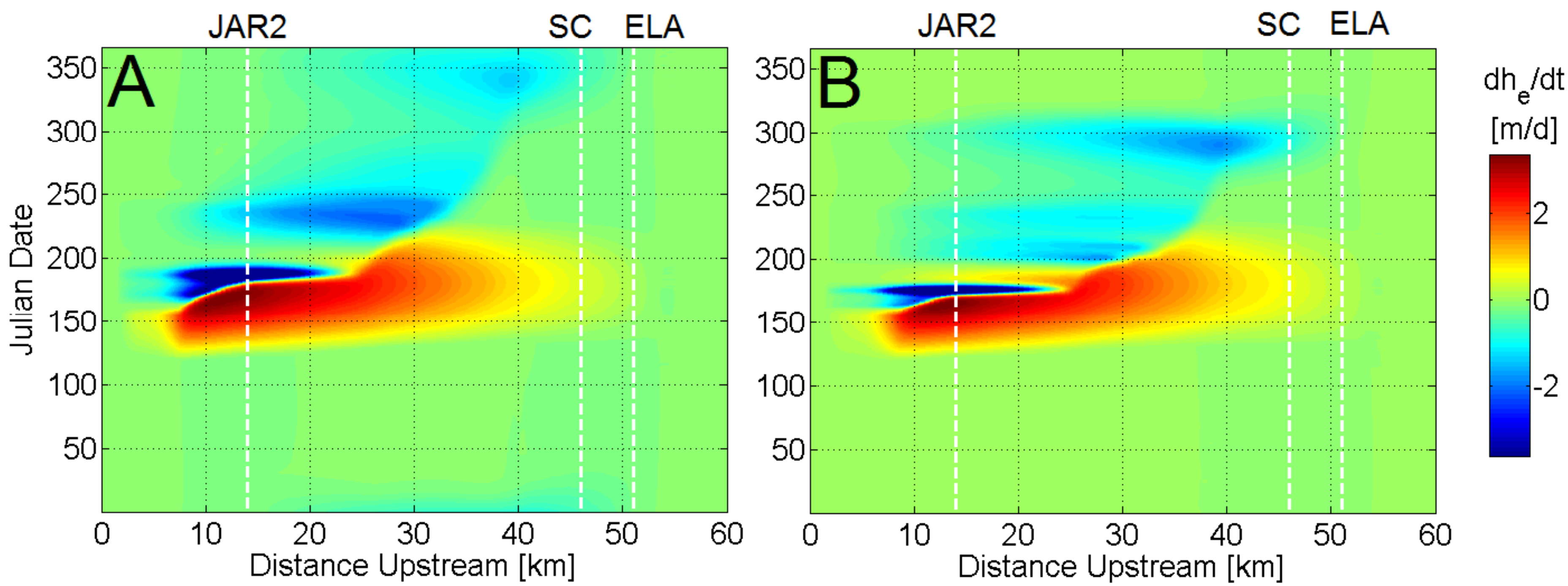


Entire flowline annually oscillates close to flotation (P_w/P_i = 1)… consistent with *in situ* observations



Annual mean and minimum are not sensitive to choice of bulk ice porosity (ϕ)… but annual maximum is.

1D Results: Changes in Water Storage Over Time (*dS/dt*)



$$\frac{dS}{dt} = w\phi \frac{dh_e}{dt}$$

Support for summer speedup during +VE *dS/dt* and fall slowdown during –VE *dS/dt*… (…with strong 1D artifacts).

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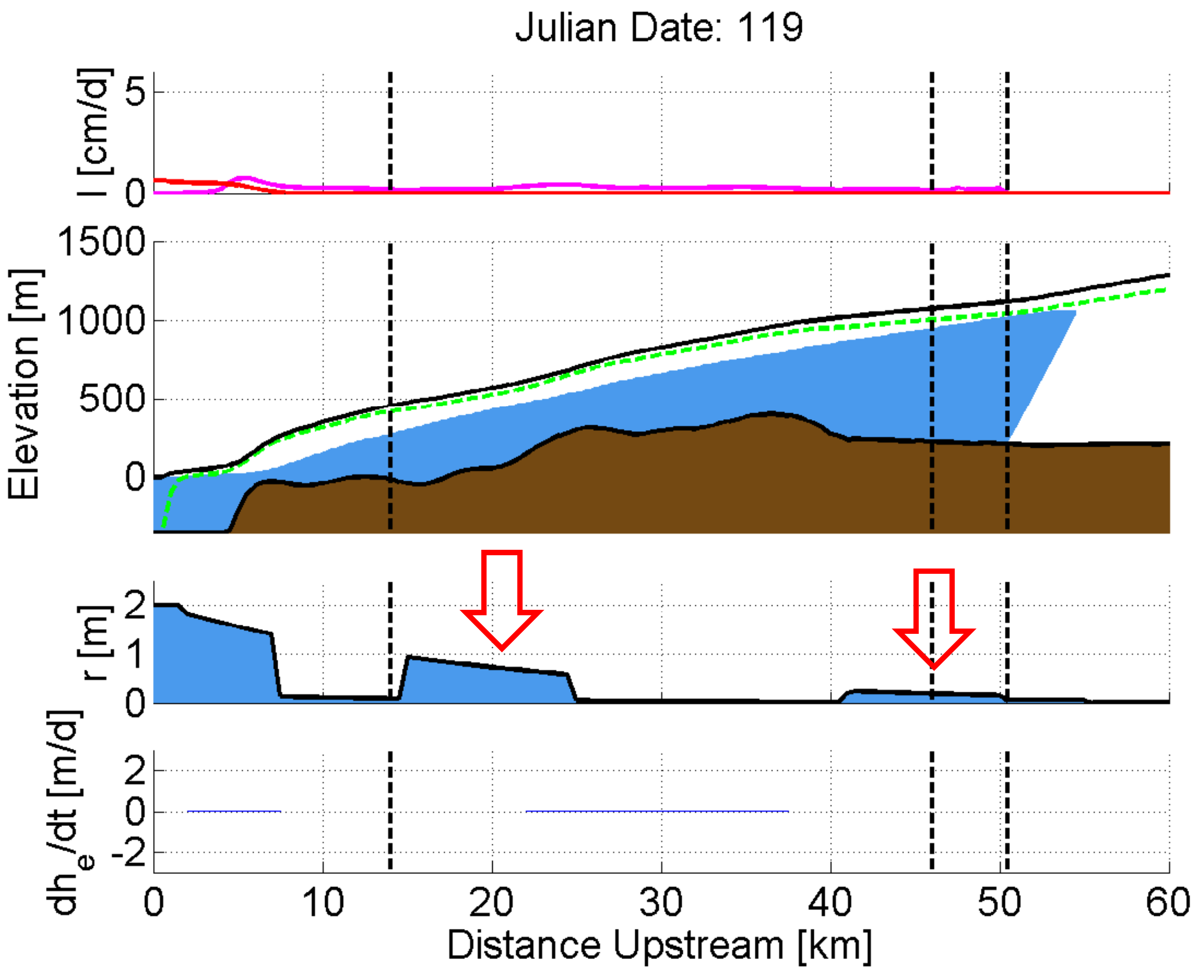
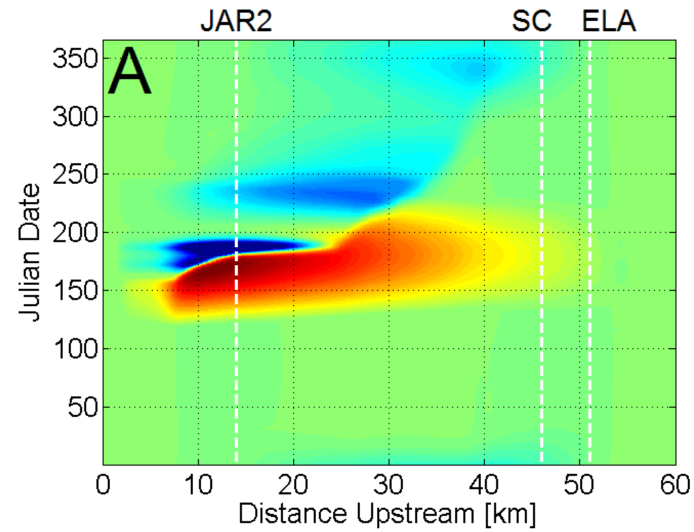
Aside: “Perennial” Conduits





Portions of the conduit system may "overwinter“ to be "reactivated" the following melt season. “New" system may not have to migrate upglacier from the terminus each melt season

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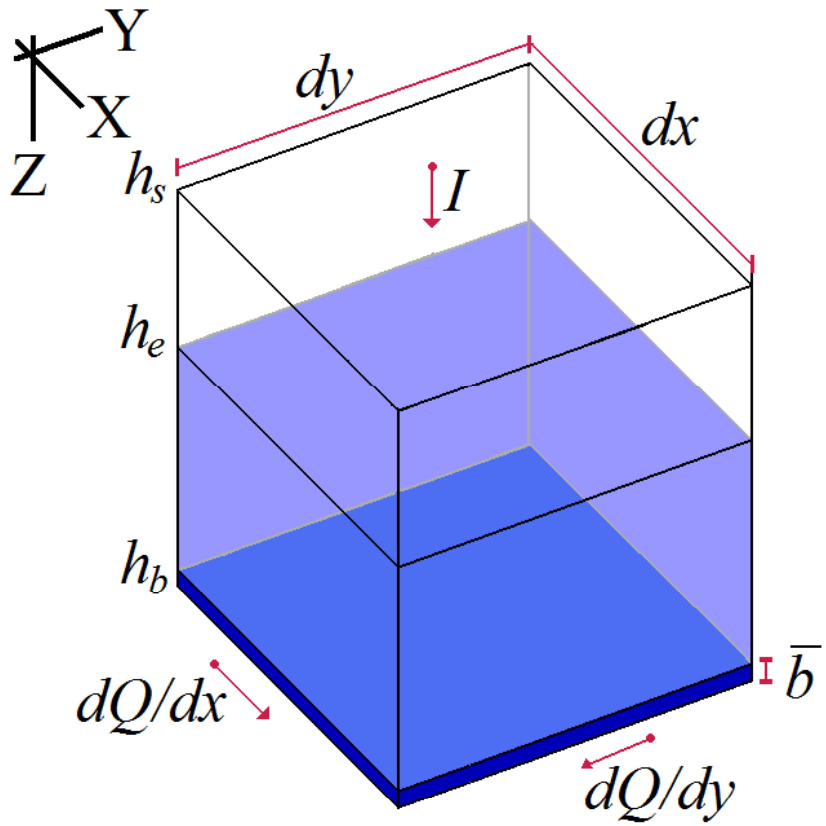
Single-Head Hydrology Model (2D): Overview

Water Mass Conservation:

$$\varphi \cdot \partial x \cdot \partial y \cdot \frac{\partial h_e}{\partial t} = I \cdot \partial x \cdot \partial y + \frac{\dot{m}}{\rho_w} - \frac{\partial Q}{\partial x} - \frac{\partial Q}{\partial y} - \frac{\partial \bar{b}}{\partial t}$$



Sheet-type flow: $\bar{b}$ becomes the mean "gap" width at the ice-bed interface… higher order conduit geometry not required.

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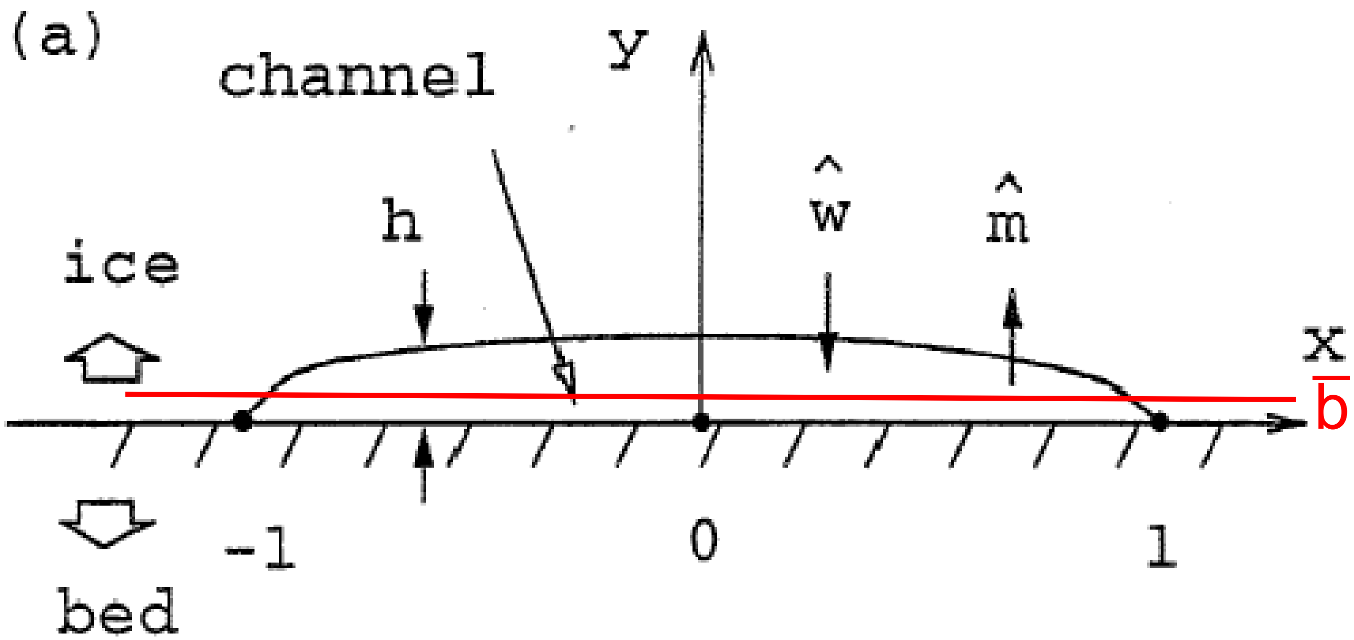
Single-Head Hydrology Model (2D): Overview

Plan: Implement transient equations that describe flow in the center of a *wide* conduit to approximate “gap” flow…



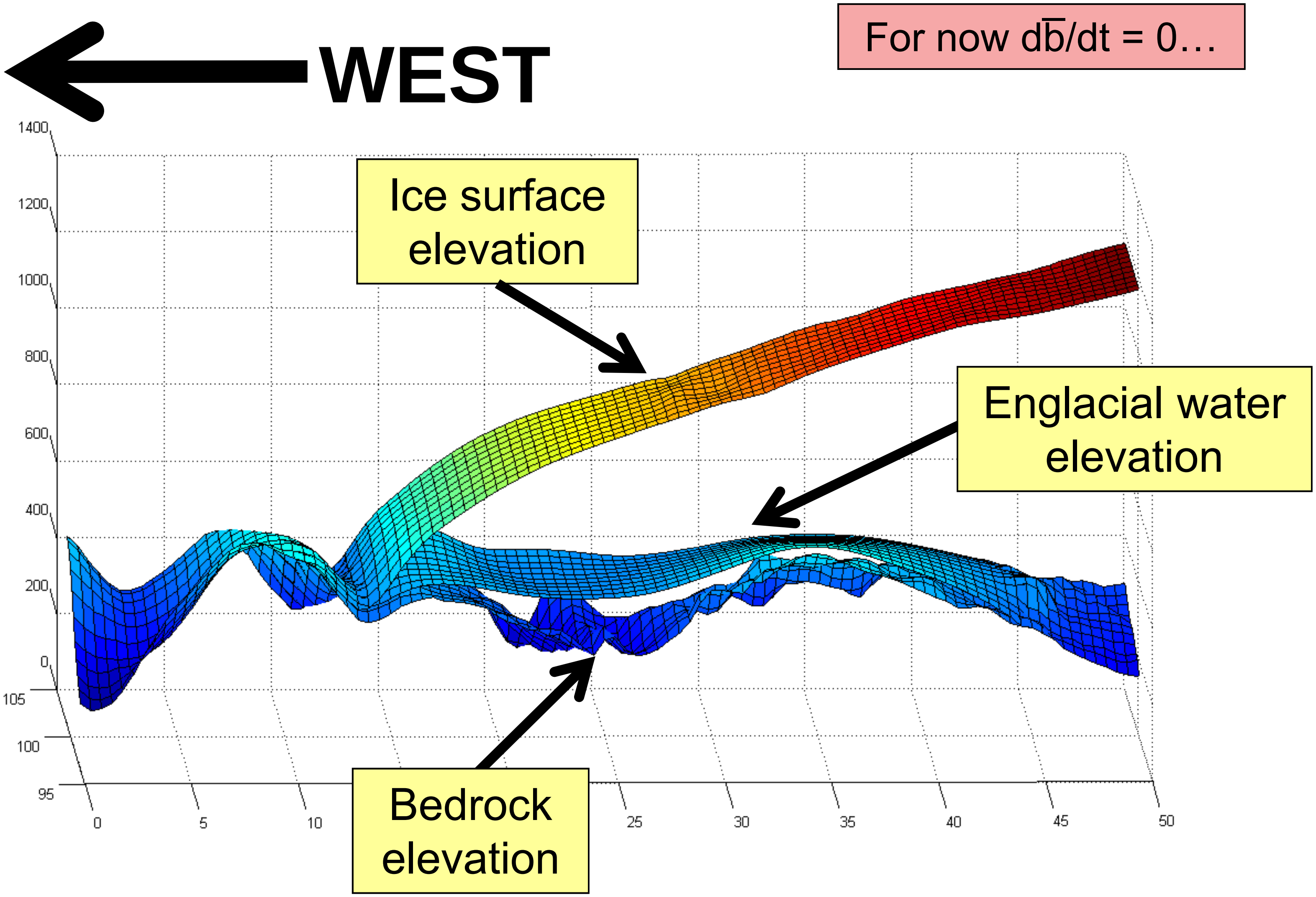
$$\frac{\partial h}{\partial t} = \frac{\hat{m}}{\rho_i} - \hat{w}, \qquad \hat{w} = \frac{N_c}{2\eta_i}\sqrt{l^2 - x^2}.$$

← (one integration too far)

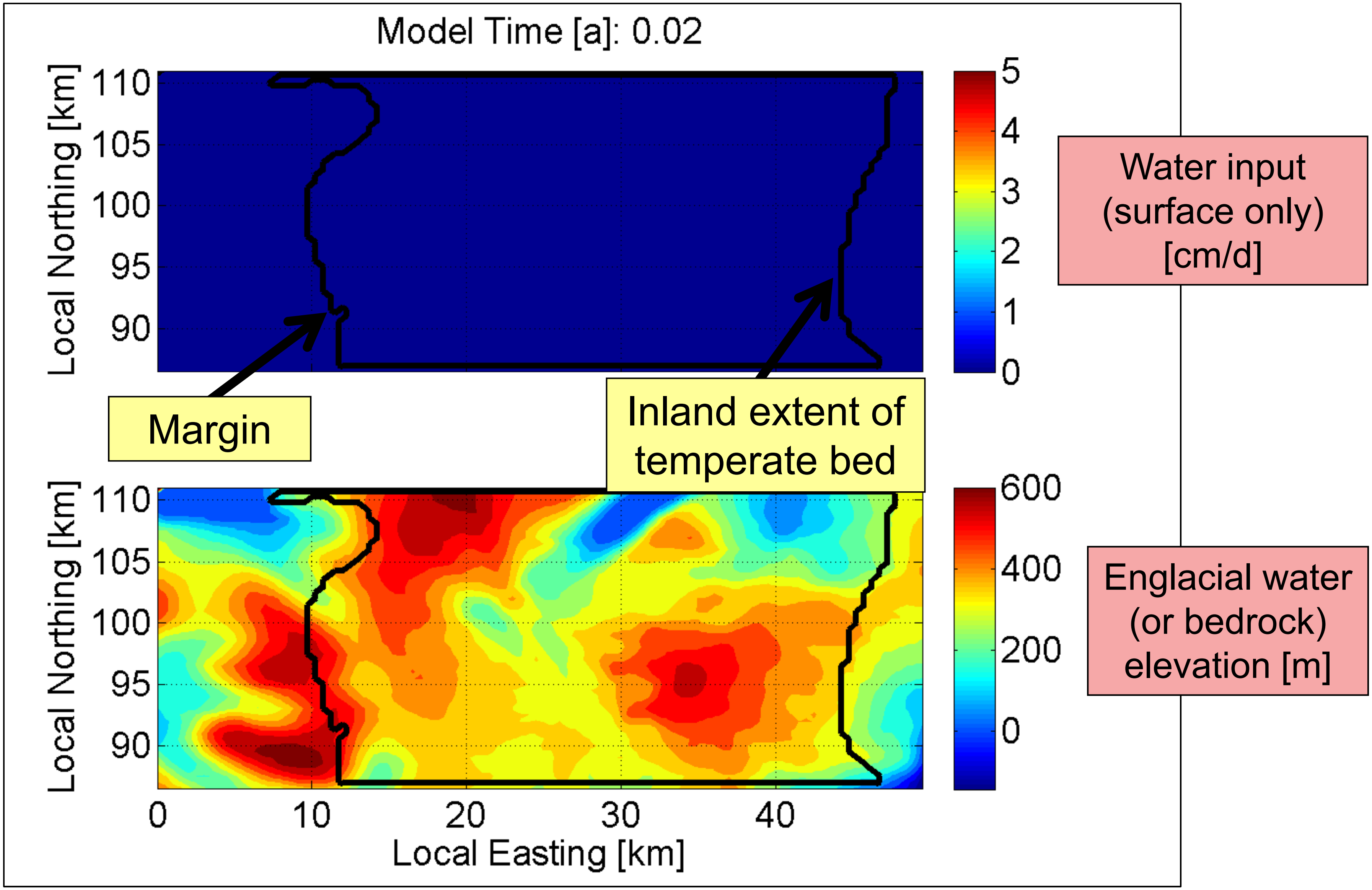
(*Ng*, 1999)

2D Sample Region Output: Overview

For now $d\bar{b}/dt = 0$…



2D Sample Region Output: Animation



2D Sample Region Output: Animation

VIEW
ANIMATION

Computational Efficiency (Single Processor)

	1D (Flowline)	**2D (Test Region)**
Numerical Method	ode15s	1st iteration Piccard
Transients (/node)	h_E and S_c	h_E and $\overline{b}$
Unknowns (/node)	47	46 (est.)
Constants	~ 50	~ 50
Nodes	120	5000
dt	1 d	6 hr
Processor Time (/dt)	0.2 s (≈ 1.1 min / a)	10 s (≈ 4.1 hr / a)
Processor Time (/dt/node)	0.0017 s	0.0020 s

A 1 by 1 km application to the entire Greenland Ice Sheet would require ~ 2,000,000 nodes… in the realm of feasibility with a quad- or eight core unit.

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