Seasonal evolution of subglacial drainage and ice motion in a glacio-hydrodynamic flow-band model

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Numerical Model

- Ice Dynamics
- Basal Sliding
- Subglacial Hydrology

Examples

- Seasonal transition
- Subglacial flood

Future Applications

- Belcher Glacier
- Russell Glacier

Ice-flow Model (Pimentel et al., *JGR*, 2010)

- **Flow-band:** 2-D flowline model with flow-unit width parameter
- **Higher-order stresses:** 1st-order approximation of the Stokes equation (Blatter, 1995; Pattyn, 2002), includes longitudinal stress gradients

$$\frac{\partial}{\partial x}(2\sigma'_{xx} + \sigma'_{yy}) + \frac{\partial \sigma_{xz}}{\partial z} + F_{\mathrm{lat}} = \rho g \frac{\partial s}{\partial x}$$

- **Lateral Drag:** lateral shear stress parameterization, includes sliding at the side walls and glacier basin shape

$$F_{\mathrm{lat}} = F_{\mathrm{lat}}(x, z, u(x, z))$$

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Basal Sliding

$$\tau_b = C\left(\frac{u_b}{u_b + C^n N^n \Lambda}\right)^{1/n} N, \qquad \Lambda = \frac{\lambda_{max} A}{m_{max}}$$

- The hydrology will be coupled to the ice mechanics by use of a regularized Coulomb friction law (Schoof, 2005; Gagliardini et al.,2007)
- This is a pressure dependent sliding rule utilizing the spatial and temporal variations in basal water pressure from the hydrology model
- Overcomes problem of standard sliding laws that allow arbitrarily large basal shear stresses regardless of effective pressure
- Implemented as a non-linear Robin-type boundary condition which cannot be solved independently but forms part of the solution to the ice-flow problem

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Subglacial Model (Pimentel & Flowers, *Proc. R. Soc. A*, 2010)

A mixed subglacial drainage network which includes dynamic switching between drainage components

- **Distributed**
  - macroporous water sheet
  - low capacity and efficiency
  - characteristic of winter
- **Channelized**
  - ice-walled conduits
  - high capacity and efficiency
  - characteristic of summer
- **Uplift**
  - When large amounts of water impinge on the glacier bed high water pressures are generated and cause flexure of the overlying ice
  - Elastic uplift is parameterized by treating the glacier as a uniform static beam

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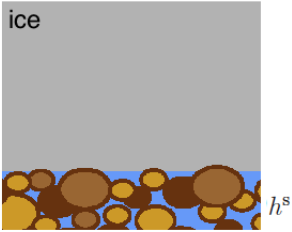
Distributed System



macroporous water sheet
low capacity
low efficiency
typical of winter

Conservation equation:

$$\frac{\partial h^s}{\partial t} + \frac{\partial q^s}{\partial x} = \frac{Q_G + u_b \tau_b}{\rho L} + \dot{b}^s + \phi^{s:c}$$

Water flux:

$$q^s = -\frac{K h^s}{\rho_w g} \frac{\partial \psi^s}{\partial x}$$

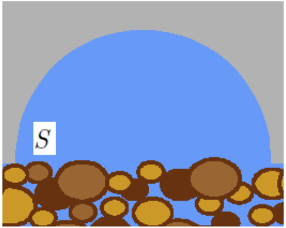
Fluid potential:

$$\psi^s = P_w^s + \rho_w g b$$

Basal water pressure:

$$P_w^s = P_w^s\left(h^s\right)$$

Channelized System



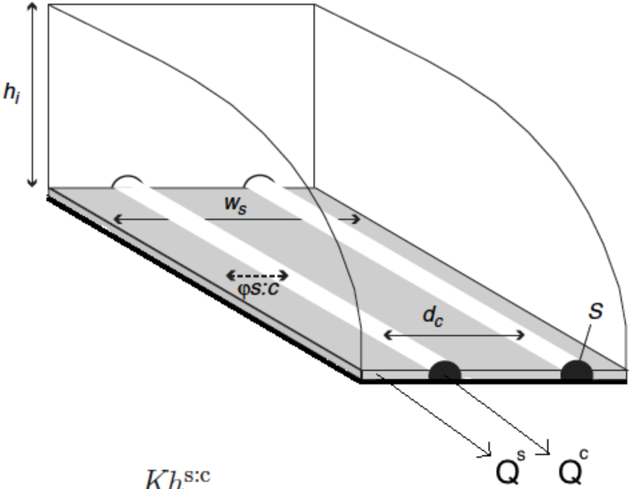
ice-walled conduit
high capacity
high efficiency
typical of summer

Conservation equation:

$$\frac{\partial S}{\partial t} = -\frac{Q^{\mathrm{c}}}{\rho L}\left(\frac{\partial \psi^{\mathrm{c}}}{\partial x} - c_{\mathrm{p}}\,\rho_{\mathrm{w}}\,\Phi\,\frac{\partial P^{\mathrm{c}}_{\mathrm{w}}}{\partial x}\right) - 2AS\left(\frac{P_{\mathrm{i}} - P^{\mathrm{c}}_{\mathrm{w}}}{n}\right)^{n}$$

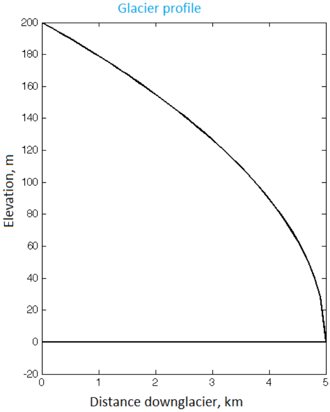
Conduit discharge:

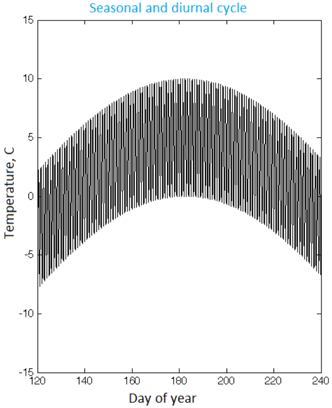
$$Q^{\mathrm{c}} = -\left(\frac{8S^{3}}{P_{\mathrm{wet}}\,\rho_{\mathrm{w}}\,f_{\mathrm{R}}}\right)^{1/2}\frac{\partial \psi^{\mathrm{c}}}{\partial x}\left|\frac{\partial \psi^{\mathrm{c}}}{\partial x}\right|^{-1/2}$$

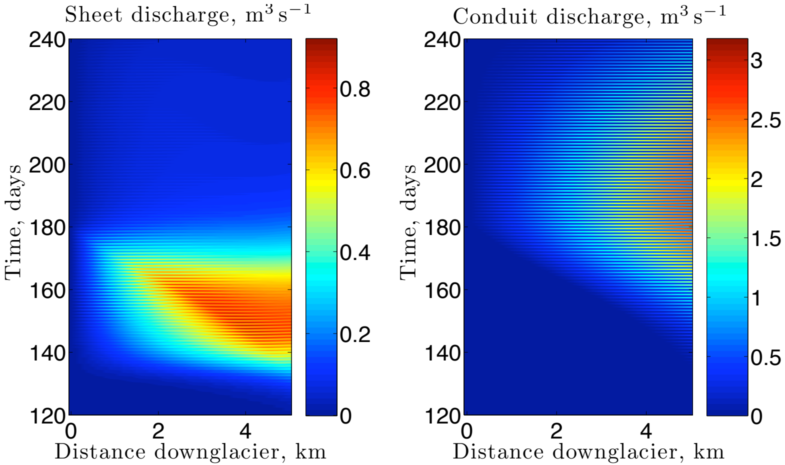


$$\phi^{s:c} = \chi^{s:c} \frac{K h^{s:c}}{\rho_w g d_c^2} (P_w^s - P_w^c)$$

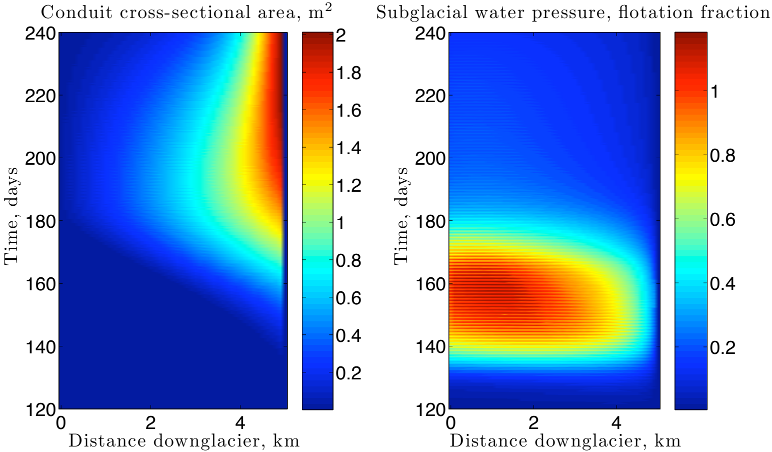
An idealized mountain glacier

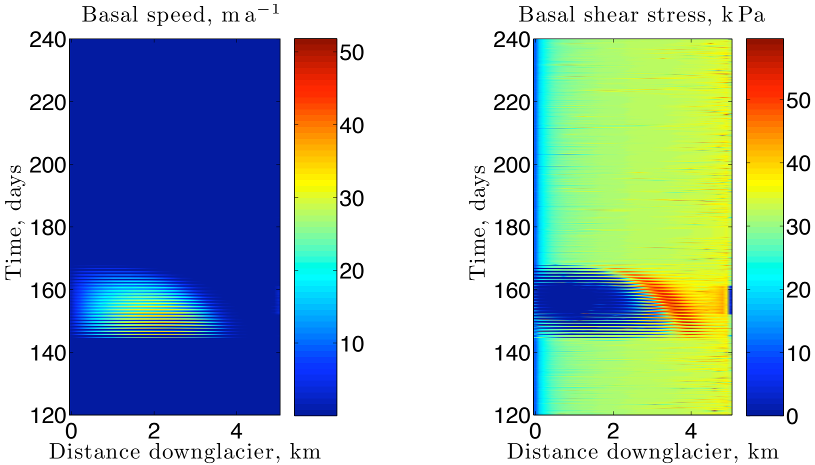




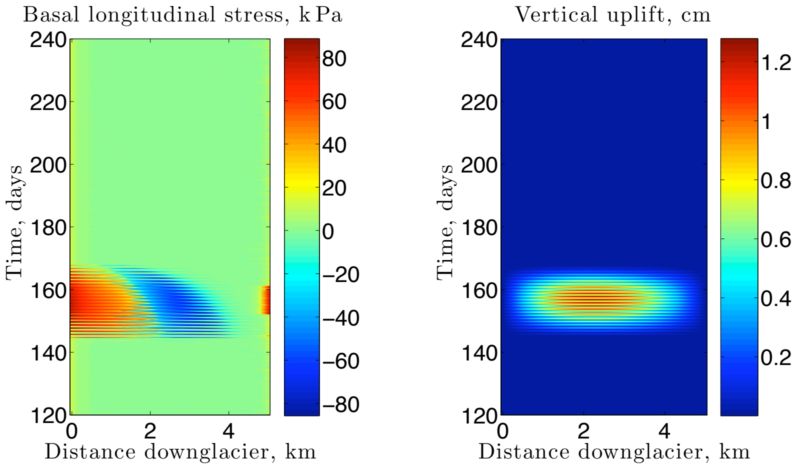


Sheet discharge, m³ s⁻¹
Conduit discharge, m³ s⁻¹
Time, days
Distance downglacier, km
240
220
200
180
160
140
120
0
2
4
0.8
0.6
0.4
0.2
3
2.5
1.5
1
0.5


Conduit cross-sectional area, m^2
Subglacial water pressure, flotation fraction
Time, days
Distance downglacier, km
240
220
200
180
160
140
120
0
2
4
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
Time, days
Distance downglacier, km
1
0.8
0.6
0.4
0.2



Basal speed, m a^{-1}
Time, days
Distance downglacier, km
Basal shear stress, k Pa
Time, days
Distance downglacier, km


Basal longitudinal stress, k Pa
Time, days
Distance downglacier, km
240
220
200
180
160
140
120
0
2
4
80
60
40
20
0
−20
−40
−60
−80
Vertical uplift, cm
Time, days
Distance downglacier, km
240
220
200
180
160
140
120
0
2
4
1.2
1
0.8
0.6
0.4
0.2

Comments

- Model captures seasonal and diurnal cycles as well as the spring-transition
- Such features have been well observed in Alpine glacier systems (e.g. Haut Glacier d'Arolla)
- Increasing evidence of similar behaviour on Arctic and Greenland glaciers (e.g. Bartholomew et al., Nat. Geo., 2010)
- Suggesting a unified treatment of basal processes across a range of scales

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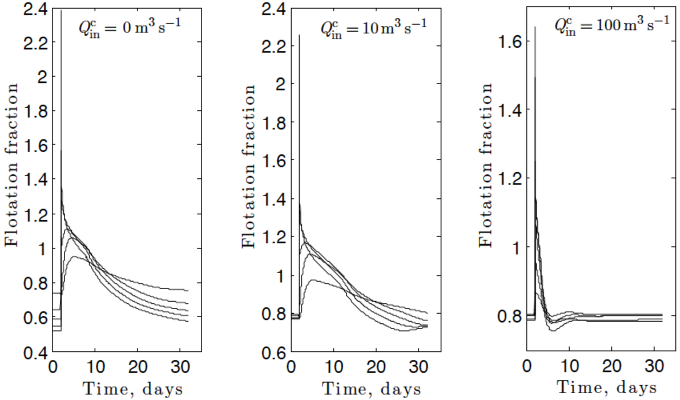
Subglacial Floods

Supraglacial Lake Drainage Event

- Das et al. Science, 2008
- Supraglacial lake of volume 0.044 km^3
- Drains through 980 m of ice in 1.4 h
- 1.2 m of vertical uplift and 0.8 m of horizontal displacement
- Rapid response followed by subsidence and deceleration over 24 hrs

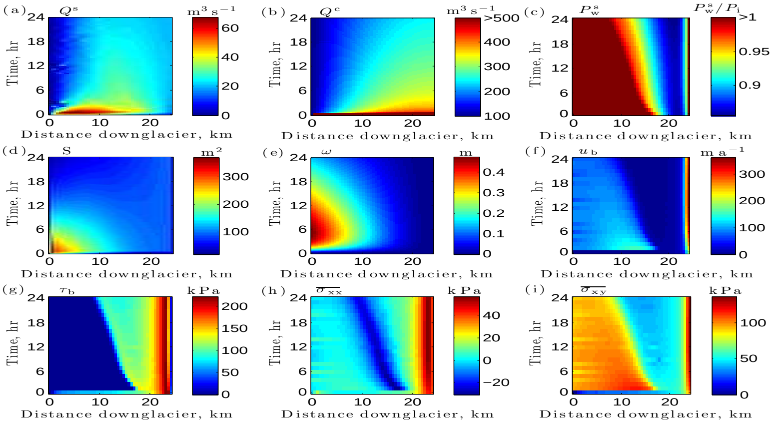
Supraglacial lake on Belcher Glacier, Devon Island Ice Cap. Photo by A. Garner.



No
pre-existent channels

Smaller
pre-existent channels

Larger
pre-existent channels


(a) Q^s m^3 s^{-1}
(b) Q^c m^3 s^{-1}
(c) P_w^s P_w^s/P_i
(d) S m^2
(e) ω m
(f) u_b m a^{-1}
(g) τ_b kPa
(h) $\overline{\sigma_{xx}}$ kPa
(i) $\overline{\sigma_{xy}}$ kPa
Time, hr
Distance downglacier, km

Comments

- Pre-existing channel network needed to dissipate flood response as quickly as observed
- "Regular" seasonal melt as well as lake tapping events condition subglacial system
- Model limitations - multi-directional flow of flood water
- Other processes - horizontal turbulent hydraulic fracture for basal crack propagation (Tsai & Rice, *JGR*, 2010)

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Future Applications

Belcher Glacier, Canadian Arctic

A large, fast-flowing tidewater outlet glacier

Belcher Glacier, Canadian Arctic

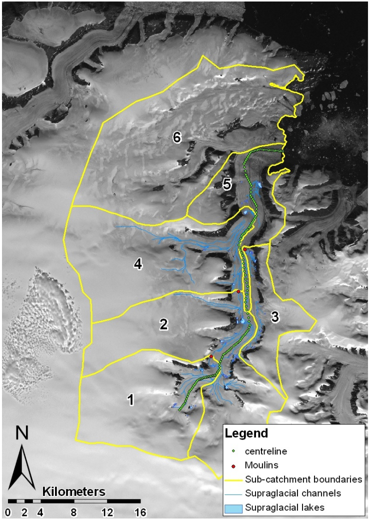
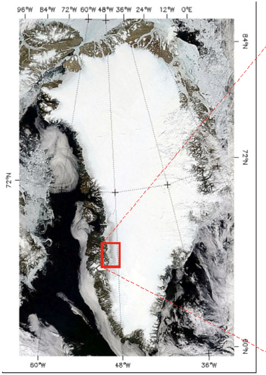
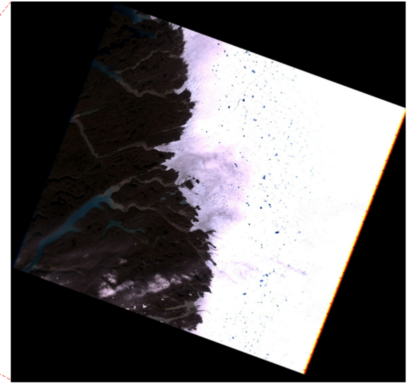


Image from Angus Duncan (University of Alberta)

Russell Glacier, Greenland





MODIS Mosaic

67° N, -50° W

Landsat image
16th June 2010

Image from Andrew Fitzpatrick (University of Aberystwyth)

Russell Glacier, Greenland

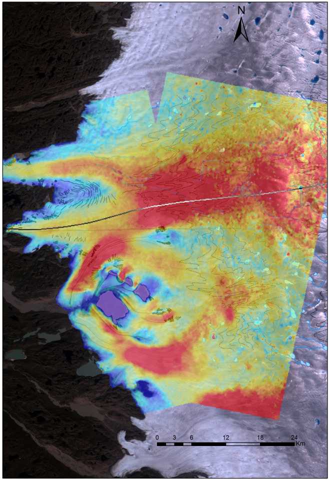
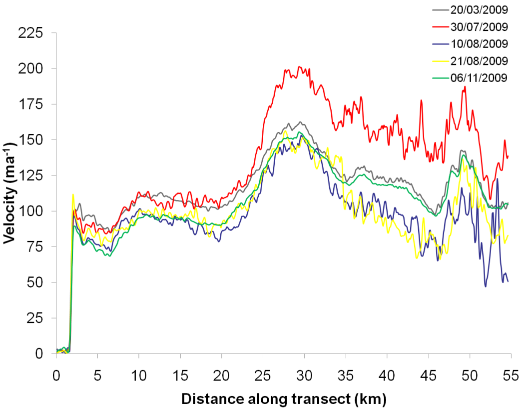


Image from Andrew Fitzpatrick (University of Aberystwyth)

2009 velocities using TerraSAR-X images and speckle tracking algorithms



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Questions?