Lagrangian Particle Models for Ice Sheet and Ice Shelf Dynamics

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Main objectives:

- Develop Smoothed Particle Hydrodynamics (SPH) model for coupled ice sheet and ice shelf dynamics;
- Develop Discrete Element Methods (DEM) model for ice shelf fracturing;
- Use the models to investigate assumptions in simplified but computationally more efficient grid-based ice sheet models for different types of ice sheets and glaciers.

Motivation: Why to use Lagrangian Models

- Grid-based solutions of fully 3D free-surface problems are very complex and are rarely sought in ice sheet models
- Most of ice sheet models use (quasi) two-dimensional First Order Shallow Ice Approximations and Shallow Shelf Approximations.
- Under certain conditions these approximations may lead to significant errors. **Examples**: large ice sheet aspect ratio and/or large bedrock slope; tidewater glaciers; ice streams; surge dynamics; the dynamics of flow across the grounding line; and the dynamics in the vicinity of ice sheet divides [Marshall, 2005].
- Errors in coupling of different models
- Lagrangian particle methods are very efficient for free-surface problems and for problems involving large material deformation and fracturing.

SPH model for coupled ice sheet and ice shelf dynamics: Governing equations in SPH model

3D momentum conservation and continuity equations:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathrm{v}) = 0 \; ; \qquad \rho\left(\frac{\partial \mathrm{v}}{\partial t} + \mathrm{v} \cdot \nabla \mathrm{v}\right) = -\nabla p + \nabla \cdot \mathbf{T} + \mathrm{g}$$

Non-Newtonian constitutive equation [1]:

$$\dot{\varepsilon}_{ij} = A \frac{\tau^{n-1} \tau'_{ij}}{d^{m}} \exp\left(-\frac{Q+PV}{RT}\right) \qquad \tau = \sqrt{\frac{1}{2} \tau'_{ij} \tau'_{ij}}$$

$\dot{\varepsilon}_{ij}$: Strain rate tensor

d : Grain size

m : Grain size exponent

A : Material parameter

V : Activation volume for creep

R : Gas constant

$\tau'_{ij} = T_{ij} - \frac{T_{kk}}{3} \delta_{ij}$: Stress deviator tensor

Q : Activation energy for creep

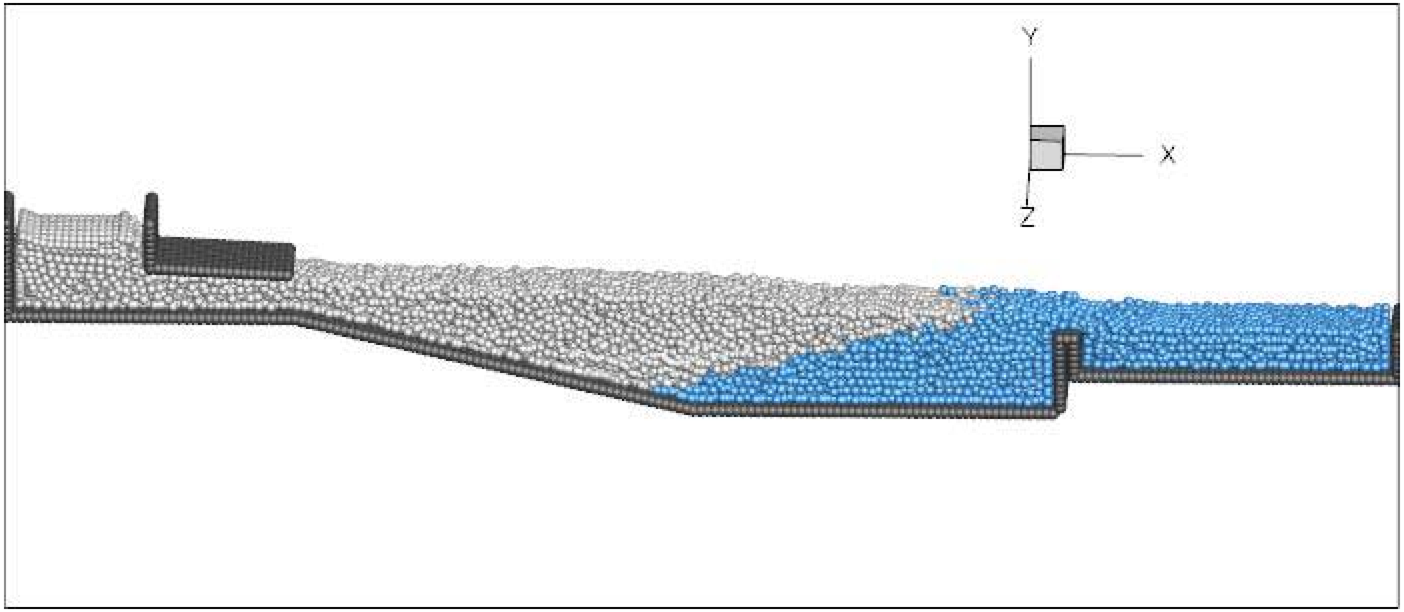
n : Stress exponent

P : Hydrostatic pressure

T : Absolute temperature

[1] Goldsby, D. L. and Kohlstedt, D. L., *Journal of Geophysical Research* (106), 11017-11030, 2001

SPH coupled ice sheet and ice shelf model



•Ice, water and bedrock are discretized with particles,

•Positions of the particles are used as node points to discretize governing equations.

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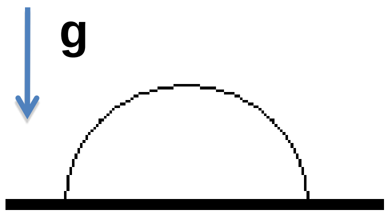
2D model verification

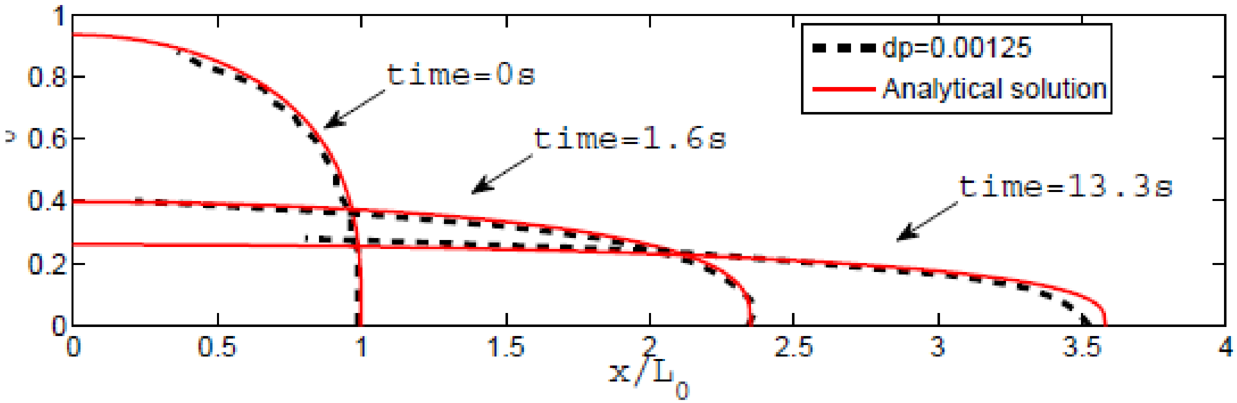
Case II: (gravity current propagation)

$$\frac{\partial H}{\partial t} - \frac{1}{3}\frac{g}{\nu}\frac{\partial}{\partial x}(H^3\frac{\partial H}{\partial x}) = 0.$$

$$H(x,t) = \eta_L^{\frac{2}{3}}(3q^2\nu/g)^{\frac{1}{5}}t^{-\frac{1}{5}}\phi(\eta/\eta_L),$$

$$\eta = (\frac{1}{3}gq^3/\nu)^{\frac{1}{5}}xt^{-\frac{1}{5}},$$





Bueller et al, 2005, Exact solutions and verification of numerical models for isothermal ice sheets, J. Glaciology.

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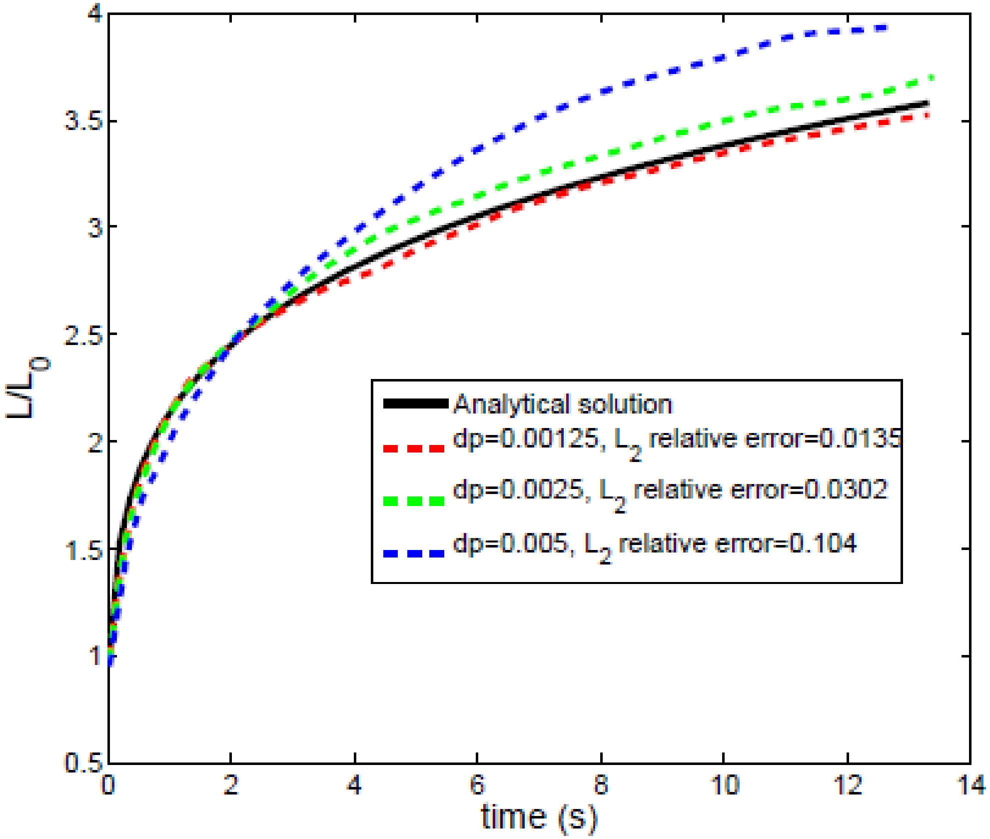
2D model verification

Case II: (gravity current propagation)



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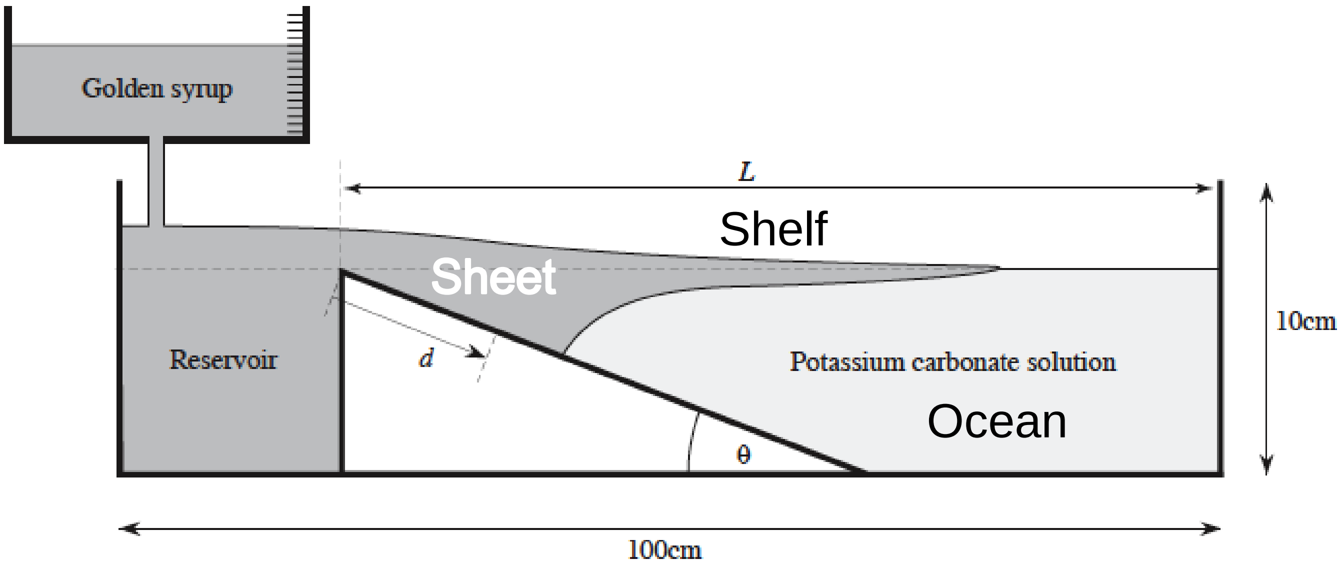
Comparison with Laboratory Experiment



Golden syrup is supplied at a constant rate to the reservoir and spills onto a rigid slope and into a dense solution of potassium carbonate (Rosalyn et al., JFM, 2010) .

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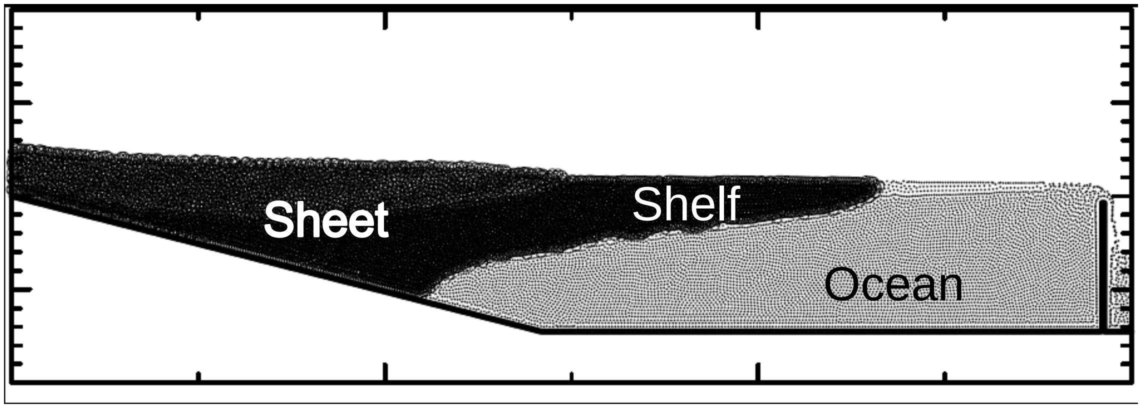
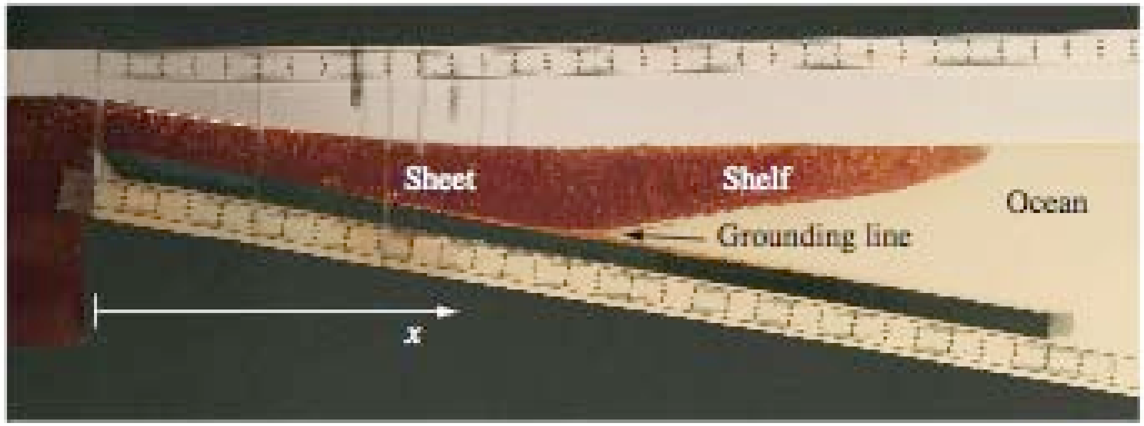
Coupled 2D Smoothed Particle Hydrodynamics model for ice sheets and ice shelves.



The model allows for a seamless coupling between ice sheet and ice shelf computational domains.

Laboratory experiment with viscous syrup



(Rosalyn et al., JFM, 2010)

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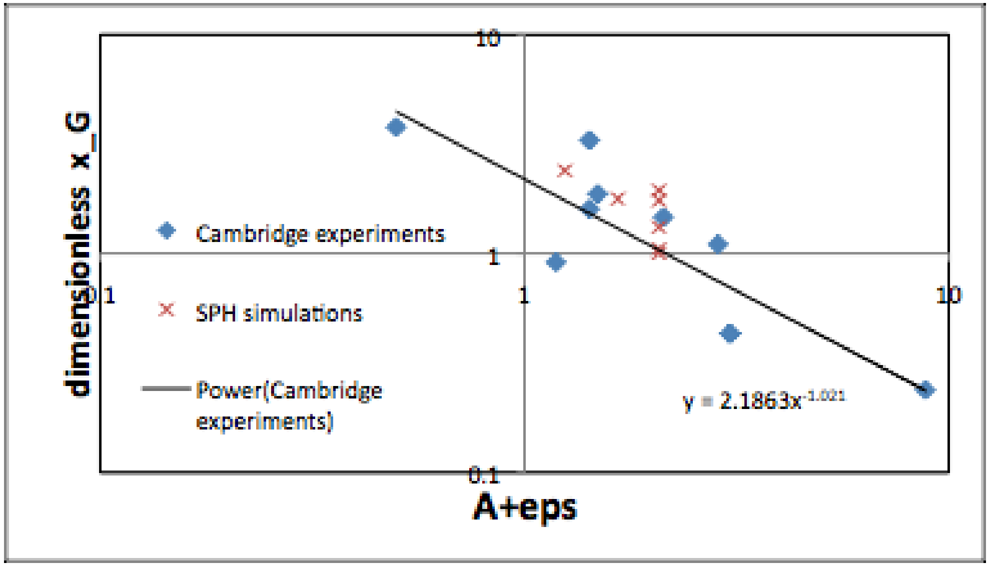
Comparison of experimental and numerical results



$$A = 2\alpha\left(\frac{g'}{g}\right)^{-1/2}$$

$$\varepsilon = 2\alpha\left(\frac{g'}{g}\right)^{1/2}\frac{\rho_w}{\rho}$$

$$g' = g\left(\rho - \rho_w\right)/\rho_w$$

Dimensionless
position of grounding line

$$\hat{x}_G = \frac{x_G}{2BC^4}$$

$$B = \left(6q_0\nu / g\right)^{1/3} \qquad C = \left(g / g'\right)^{1/6}$$

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Next Steps

- Develop highly scalable 3D SPH model (based on LAMMPS code)
- Validate the 3D model
- Compare model with ice sheets community models
- Use the model to simulate meso-scale ice sheets
- Publications

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Modeling Ice Shelf Fracturing Subject to Ice-water Interaction Using an DEM Method

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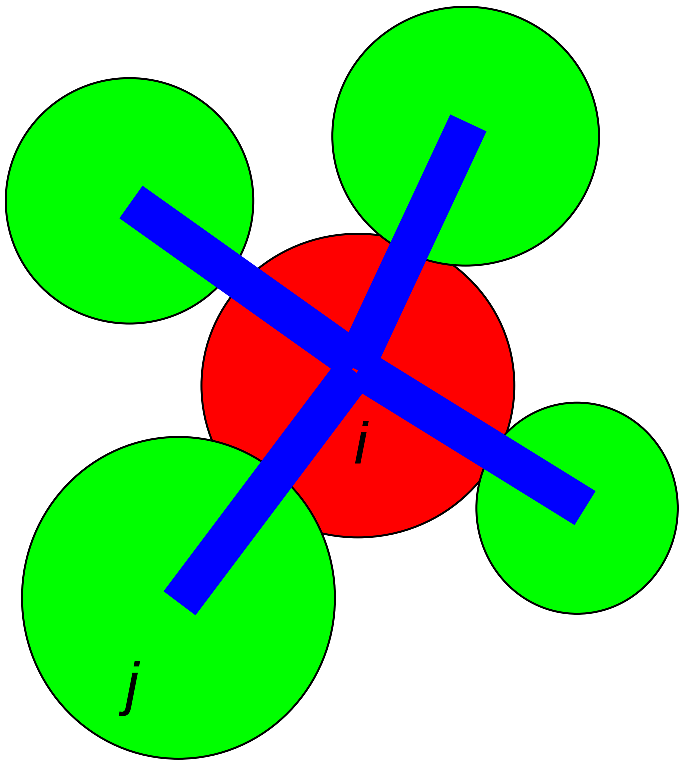
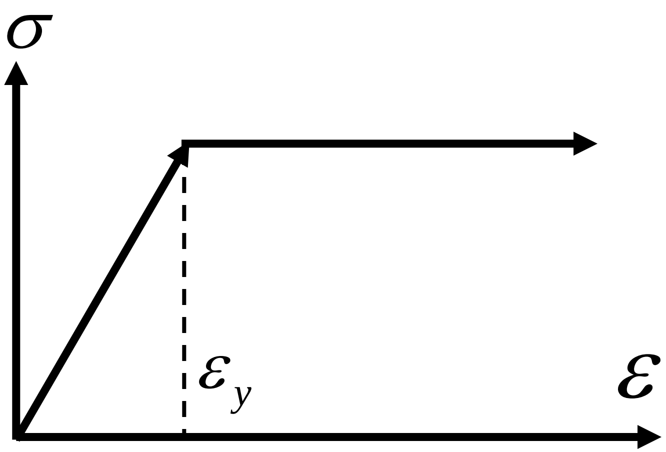
Objective

- Develop a robust particle model suitable for modeling the dynamic response of ice shelf subject to wave waves
- Investigate the mechanical response of ice shelf when subject to ice-water interaction
- Investigate the fracturing process of ice shelf when subject to ice-water interaction

Approach: Discrete Element Model (DEM)



- Particles *i, j* can interact through bond.
- Bond can be broken and removed if shear stress and/or tensile stress in bond exceeds a threshold.
- Repulsive force between grains if *i* and *j* are in contact after the bond is broken



- Depend on the ice material model (deformation rate and temperature), standard DEM can be extended to incorporate the plastic deformation if the individual bond deformation exceeds elastic limit: i.e. a simple ideal plasticity model

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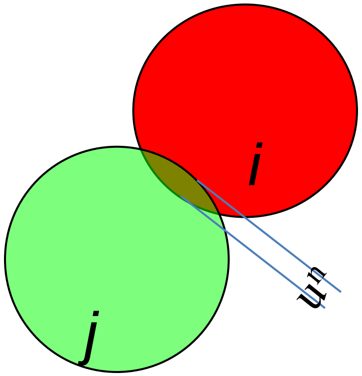
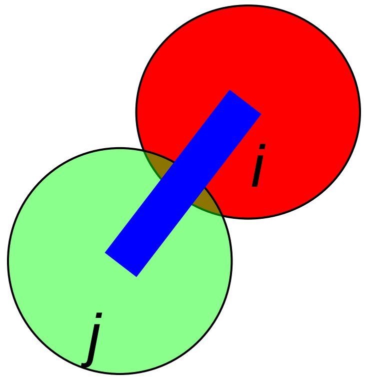
DEM main equations:



Grain force



Bond force

Grain Normal Force: $f_G^n = K^n u^n$

Grain Shear Force: $\Delta f_G^s = -K^s \Delta u^s$

K^n : contact normal stiffness
K^s : contact shear stiffness
k^n : bond normal stiffness
k^s : bond shear stiffness
A : bond area
I: moment of inertial
J: polar moment of inertia

Bond Normal Force: $f_B^n = k^n A u^n$

Bond Shear Force: $\Delta f_B^s = -k^s A \Delta u^s$

Bond Bending Moment: $\Delta M_B^s = -k^s I \Delta \theta^s$

Bond Twisting Moment: $\Delta M_B^n = -k^n J \Delta \theta^n$

$$\sigma_B = -\frac{f_B^n}{A} + \frac{|M_B^s| R}{I} < \sigma_b^{MAX} \qquad \tau_B = \frac{|f_B^s|}{A} + \frac{|M_B^n| R}{J} < \tau_B^{MAX}$$

$$m d\mathbf{V}_i / dt = \mathbf{f}_i = \sum_{j \neq i} \mathbf{f}_{ij}$$

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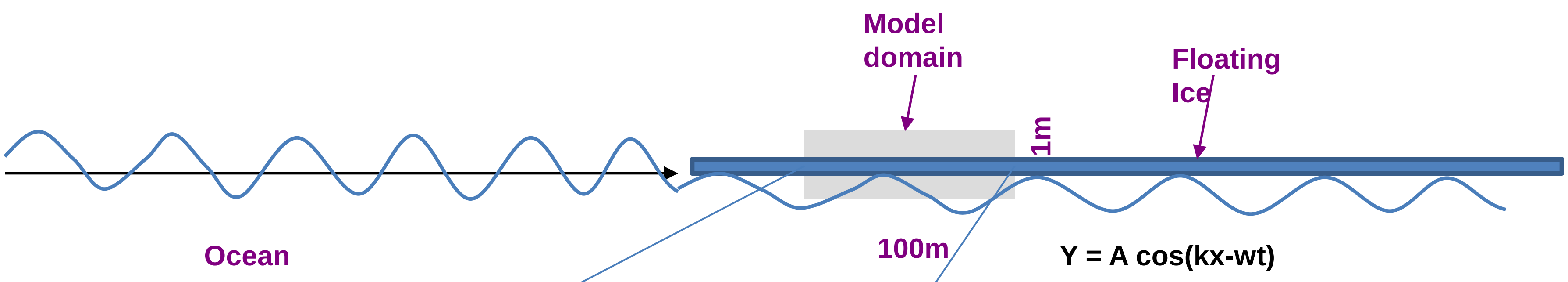
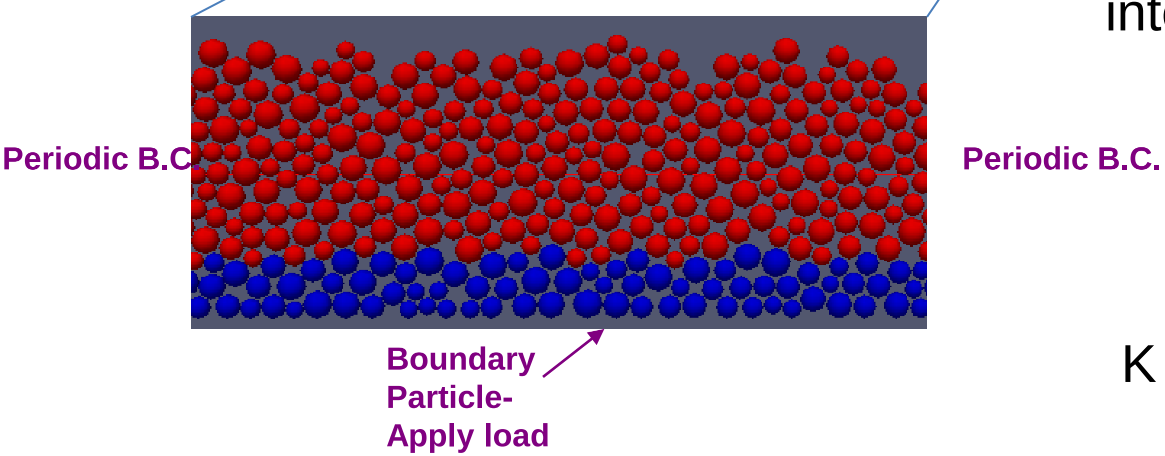
Conceptual and Preliminary Models





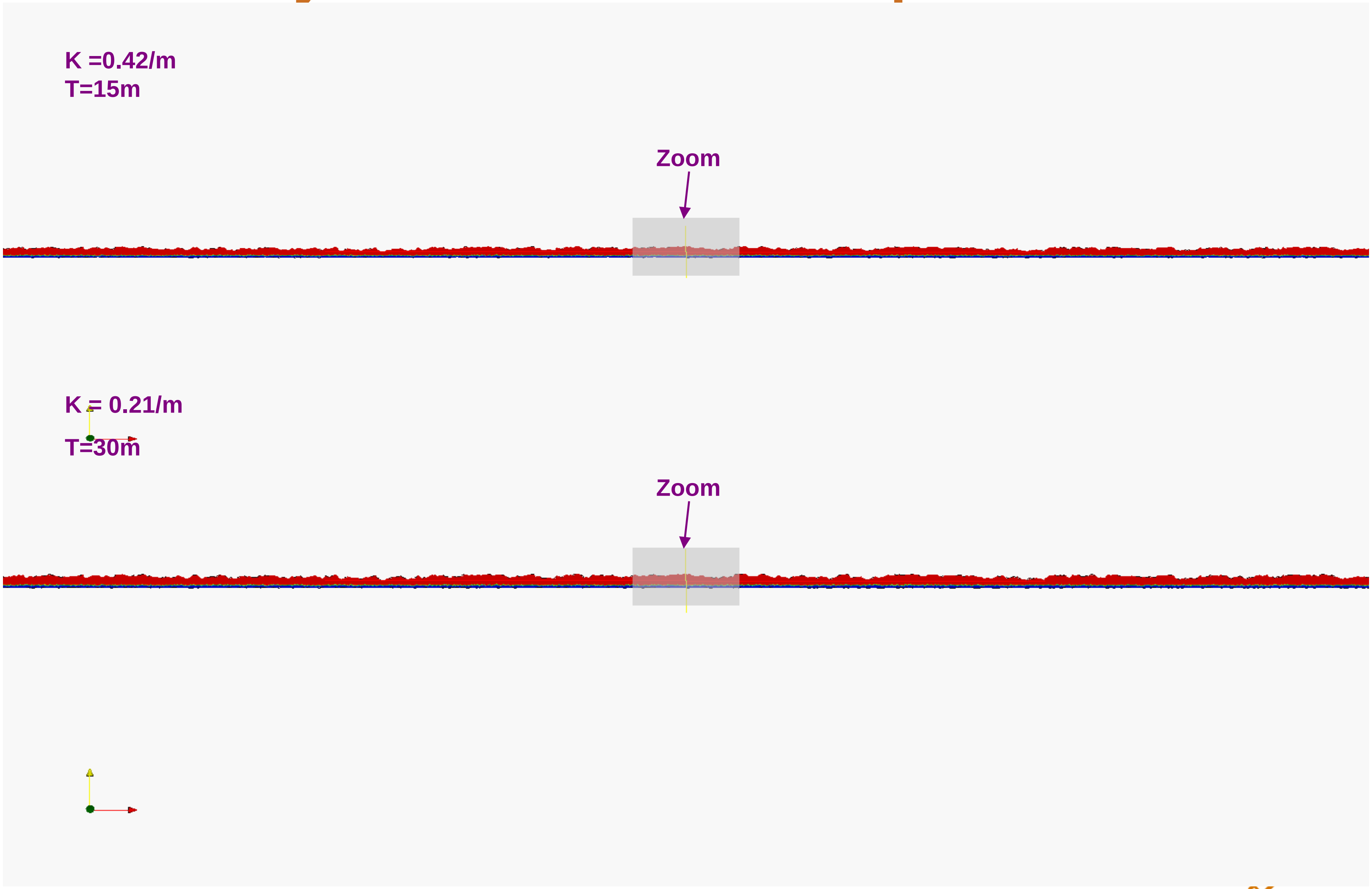
Parameters are selected to represent the typical interactions between ice and water wave

A=1.0m

W = 1/s

K = 0.42/m and 0.21/m

Preliminary Results: overall response



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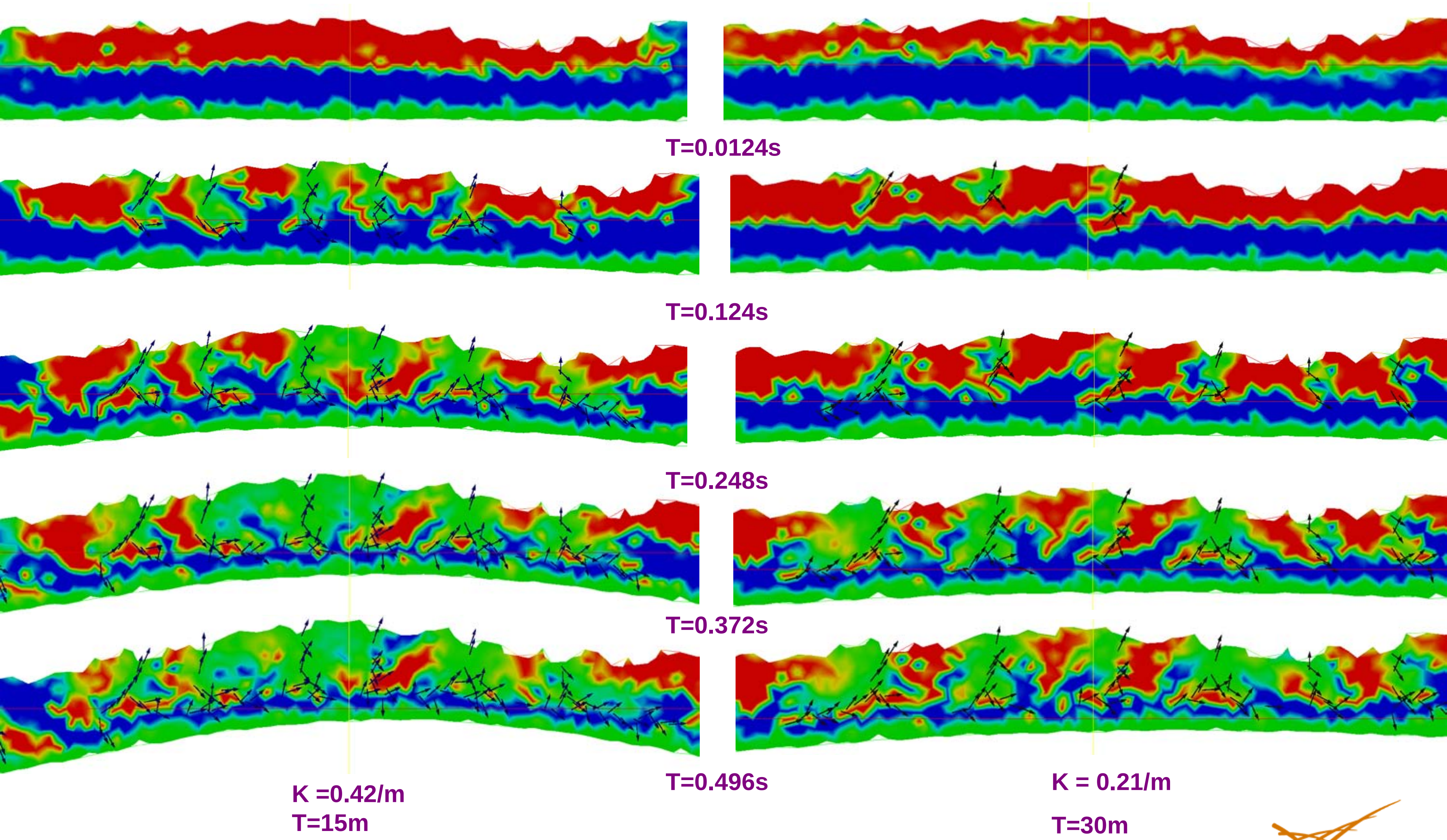
Preliminary Results: Micro-fractures



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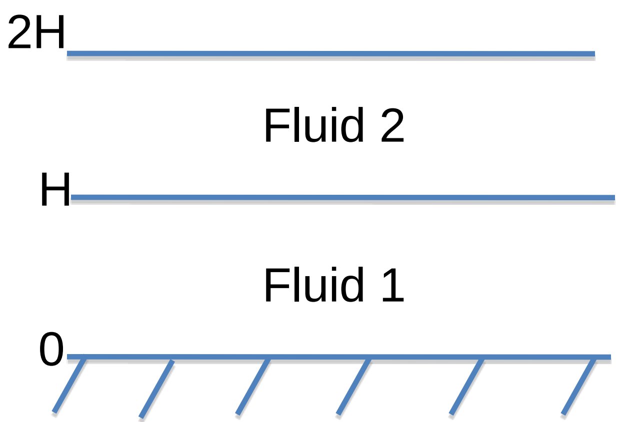
Summary and Impact

- Particle models avoid shallow ice and shallow shelf approximations
- The SPH model can improve the predictive ability of numerical ice sheet simulations.

2D model validation

Case I: (free-surface plane shear flow)

$$
\begin{cases}
\frac{\partial u}{\partial t} = \nu_1 \frac{\partial^2 u}{\partial y^2}, \quad \frac{\partial u}{\partial t} = \nu_2 \frac{\partial^2 u}{\partial y^2}, \\
u(t,y) = u_0 & \text{at} \quad 0 \le y \le 2H \quad t = 0, \\
u(t,y) = 0 & \text{at} \quad y = 0 \quad t > 0, \\
\frac{\partial u}{\partial y} = 0 & \text{at} \quad y = 2H \quad t > 0, \\
\mu_1 \frac{\partial u}{\partial y} = \mu_2 \frac{\partial u}{\partial y} & \text{at} \quad y = H \quad t > 0.
\end{cases}
$$



Fluid	$\nu(m^2/s)$	$\rho(kg/m^3)$
1	1.0	1500
2	10.0	1300

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2D model verification

Case I: (Free-surface plane shear flow)

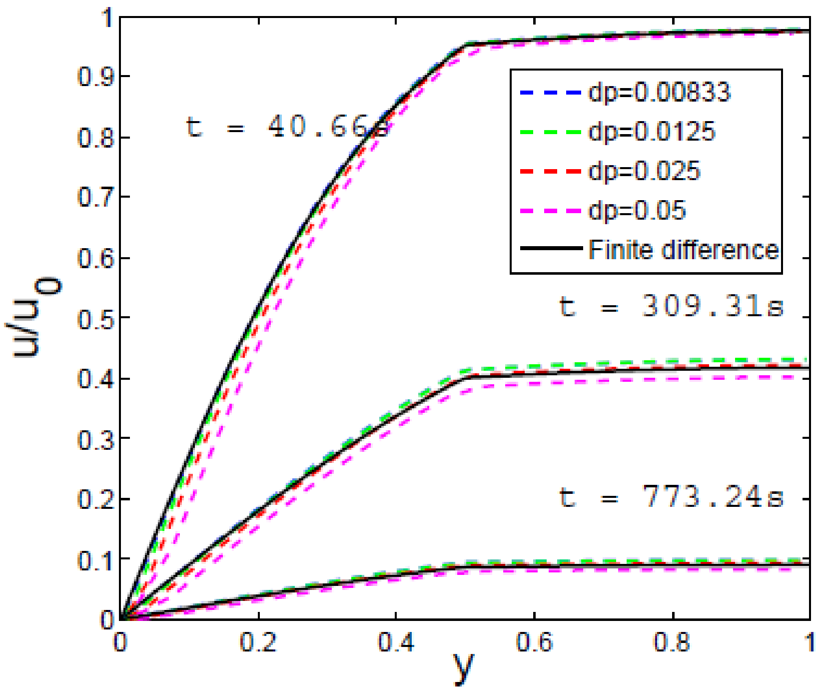

Fig. 1. The velocity profiles of the flow at different times.

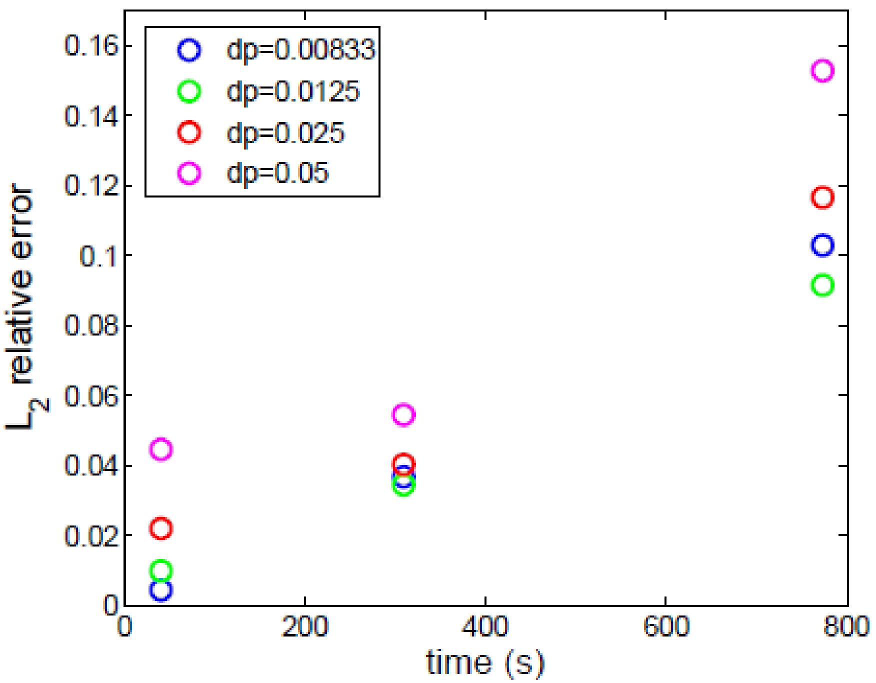

Fig. 2. The L_2 relative error of the velocity.

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