An Eddy Closure Based on Potential Vorticity

Todd Ringler
Los Alamos National Laboratory
and
Peter Gent
National Center for Atmospheric Research

Isopycnal Coordinates

$$Thickness: h = -\partial z / \partial \rho$$

$$Eq: \partial h / \partial t + \underline{\nabla}.(h\underline{u}) = 0$$

$$Tracer: \tau$$

$$Eq: \partial(h\tau) / \partial t + \underline{\nabla}.(h\underline{u}\tau) = 0$$

$$When: \tau = q \Rightarrow h\tau = f + \underline{\nabla} \times \underline{u}$$

Now do averaging to produce equations

Thickness :

$$\partial h / \partial t + \underline{\nabla}.(h\underline{U}) = 0$$

Potential.Vorticity :

$$\partial q / \partial t + \underline{U}.\underline{\nabla} q = \underline{\nabla}.(\kappa h \underline{\nabla} q) / h$$

Momentum :

$$\partial \underline{u} / \partial t + (f + \zeta)\underline{k} \times \underline{U} + \underline{\nabla} M + \underline{\nabla} K' = \underline{k} \times (\kappa h \underline{\nabla} q)$$

Note that the momentum equation is for the mean velocity u; not total velocity U.

Conclusions

- PV mixing gives a non-unique momentum eqn because of unknown gradient term.
- In general, this term cannot be chosen to give a conserved mean kinetic energy.
- If the PV Redi coefficient is chosen to be the same as the GM coeff, then this also becomes the horizontal viscosity coeff.
- The viscosity terms do not satisfy two of the usual properties of viscosity closure.

Results using MPAS in a re-entrant channel 2000km x 2000km forced by wind jet 0.1Nm^{-2}

$$REF: 10km;.3.isopycnal.layers;..-\alpha_h \nabla^4 \underline{u}$$

$$STGM: 62.5km$$

$$\partial h/\partial t + \underline{\nabla}(h\underline{U}) = 0;..momentum.eqn.with.\alpha_l$$

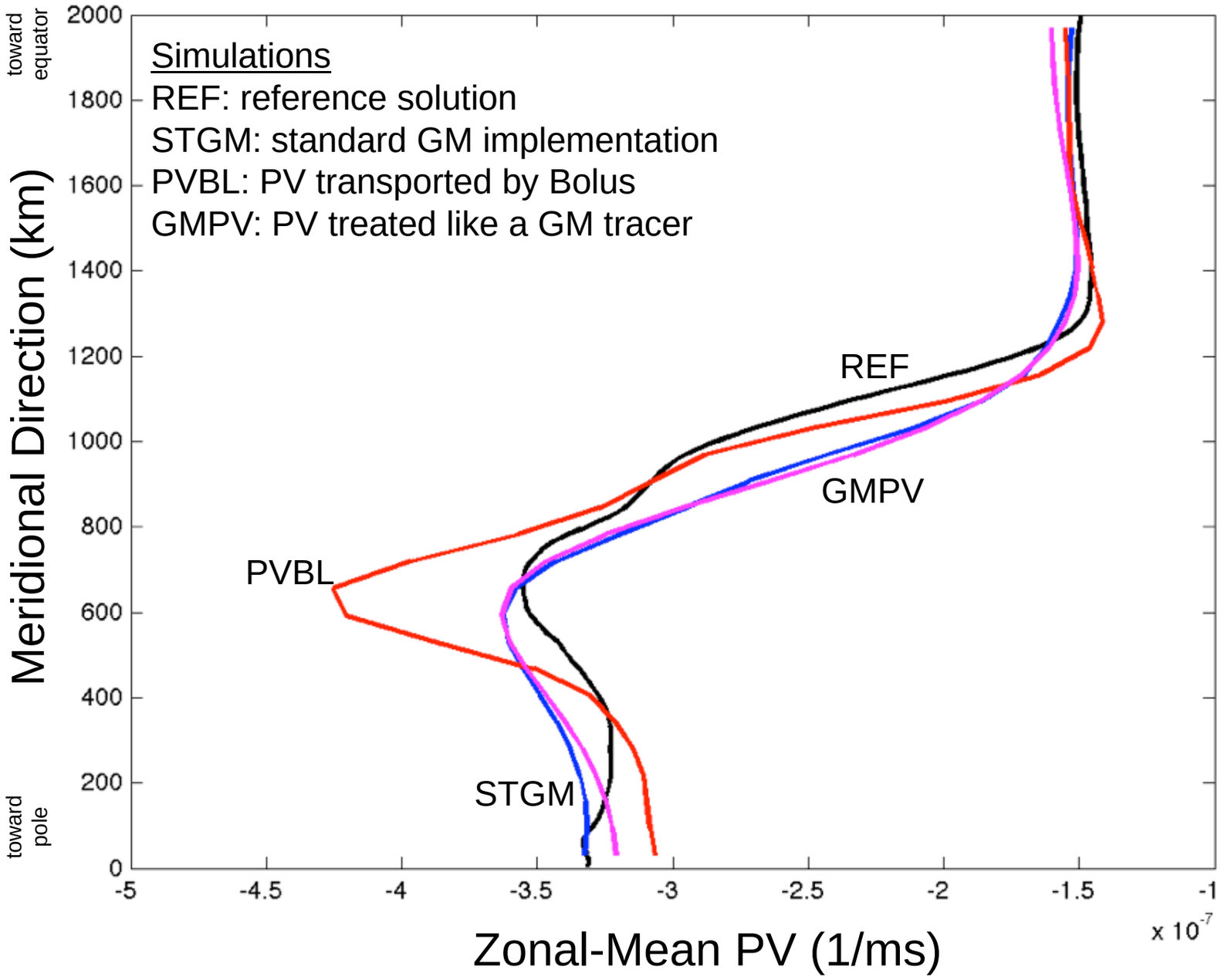
$$PVBL:$$

$$\partial \underline{u}/\partial t + (f+\zeta)\underline{k}\times\underline{U} + \underline{\nabla}M + \underline{\nabla}K' = -\alpha_l \nabla^4 \underline{u}$$

$$GMPV:$$

$$\partial \underline{u}/\partial t + (f+\zeta)\underline{k}\times\underline{U} + \underline{\nabla}M + \underline{\nabla}K' = \underline{k}\times(\kappa h \underline{\nabla} q)$$

Impact of different PV closures on zonal-mean PV



Impact of different PV closures on zonal-mean flow