Progress on time stepping method in MPAS

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Background

- **Currently fourth order Runge Kutta in MPAS**
- **Barotropic-baroclinic splitting that separates fast and slow dynamics**
  - Two time level, predictor corrector scheme
  - Allows larger timestep
  - Barotropic terms (2-D) are subcycled rather than treated implicitly
  - Extension to work of R.L. Higdon (JCP 1997 and subsequent)
- **Prototype / MPAS implementation**
- **How barotropic-baroclinic splitting can aid implicit time integration methods**

Background

- Assume discretization in layers $r = 1, \ldots, R$
- Model equations

$$\frac{\partial \mathbf{u}_r}{\partial t} + (\mathbf{u}_r \cdot \nabla)\,\mathbf{u}_r + f\mathbf{u}_r^{\perp} = -\nabla M_r + \frac{g\Delta\tau_r}{\Delta p_r} + \frac{1}{\Delta p_r}\nabla\cdot(A_H \Delta p_r \nabla \mathbf{u}_r)$$

$$\frac{\partial \Delta p_r}{\partial t} + \nabla\cdot(\mathbf{u}_r \Delta p_r) = 0$$

$$M_{r+1} - M_r = p_r\,(\alpha_{r+1} - \alpha_r)$$

Background

- **Introduce 2-D barotropic velocity**

$$\bar{\mathbf{u}}(x, y, t) = \sum_{r=1}^{R} \frac{\Delta p_r}{p_b} \mathbf{u}_r$$

- **3-D Baroclinic velocity**

$$\mathbf{u}'_r(x, y, t) = \mathbf{u}_r - \bar{\mathbf{u}}, \quad \text{with} \quad \sum_{r=1}^{R} \frac{\Delta p_r}{p_b} \mathbf{u}'_r = 0$$

- **Pressure treatment**

$$\Delta p_r(x, y, t) = (1 + \eta(x, y, t))\, \Delta p'_r(x, y, t)$$

Background

- **Baroclinic equations (3-D)**

$$\frac{\partial \mathbf{u}_r'}{\partial t} + (\mathbf{u}_r \cdot \nabla)\,\mathbf{u}_r' + f\mathbf{u}_r'^{\perp} = -\left(\nabla M_r - \overline{\nabla M_r}\right) - G$$

$$\frac{\partial \Delta p_r'}{\partial t} + \nabla \cdot (\mathbf{u}_r \Delta p_r') = \frac{\Delta p_r'}{p_b'} \nabla \cdot (p_b' \bar{\mathbf{u}})$$

- **Barotropic equations (2-D)**

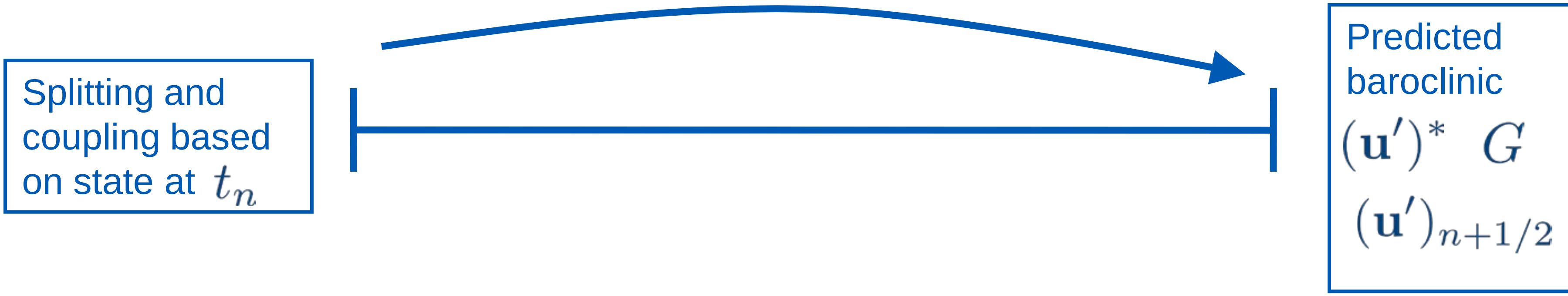
$$\frac{\partial \bar{\mathbf{u}}}{\partial t} + (\bar{\mathbf{u}} \cdot \nabla)\,\bar{\mathbf{u}} + f\bar{\mathbf{u}}^{\perp} = -\overline{\nabla M_r} + G$$

$$\frac{\partial (\eta p_b')}{\partial t} + \nabla \cdot (\bar{\mathbf{u}} p_b') = 0$$

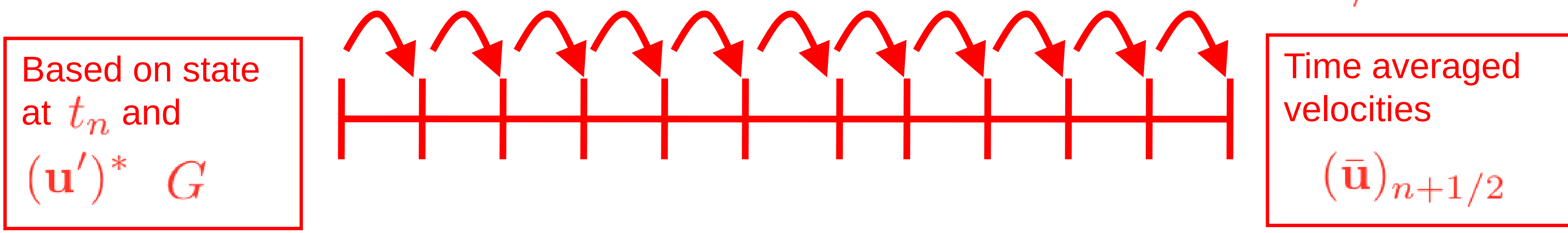
- **The residual term G consists of nonlinear terms and ensures the vertical average of the baroclinic velocity is zero**

Algorithm: Time level 1 (prediction step)

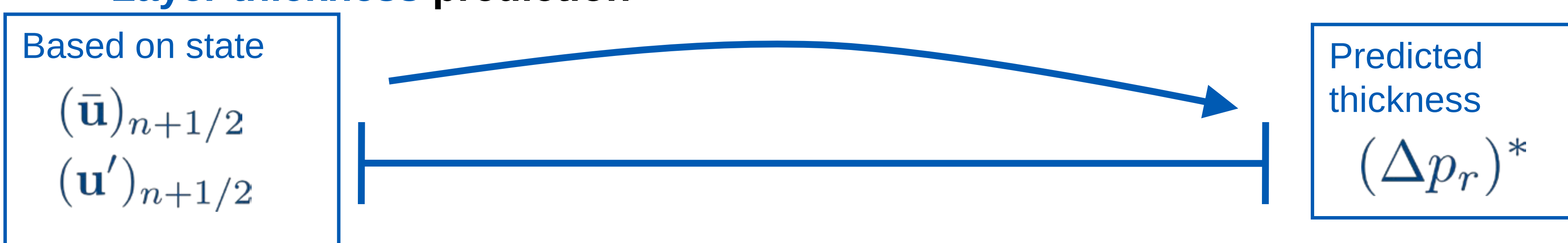
- **Baroclinic system (3-D) prediction explicit with long timestep** $\Delta t = t_{n+1} - t_n$



- **Barotropic system (2-D) prediction explicitly subcycled** $\delta t = \Delta t / m$



- **Layer thickness prediction**



Algorithm: Time level 2 (correction step)

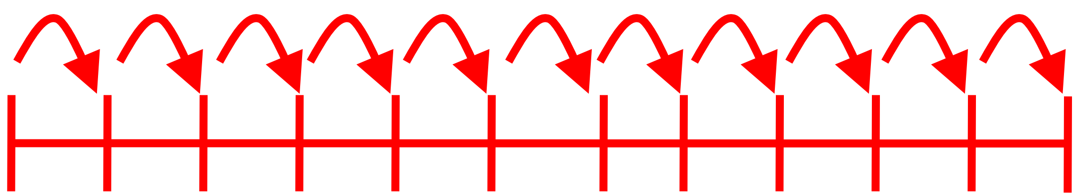
- **Baroclinic system (3-D) correction explicit with long timestep** $\Delta t = t_{n+1} - t_n$

Based on state
$(\bar{\mathbf{u}})_{1/2}\ (\mathbf{u}')_{1/2}$

Final baroclinic
$(\mathbf{u}')_{n+1}\quad G$
$(\mathbf{u}')_{n+1/2}$

- **Barotropic system (2-D) correction explicitly subcycled** $\delta t = \Delta t / m$

Based on state
$(\mathbf{u}')_{n+1}\quad G$



Final barotropic
$(\bar{\mathbf{u}})_{n+1}$
$(\bar{\mathbf{u}})_{n+1/2}$

- **Layer thickness correction**

Based on state
$(\bar{\mathbf{u}})_{n+1/2}$
$(\mathbf{u}')_{n+1/2}$

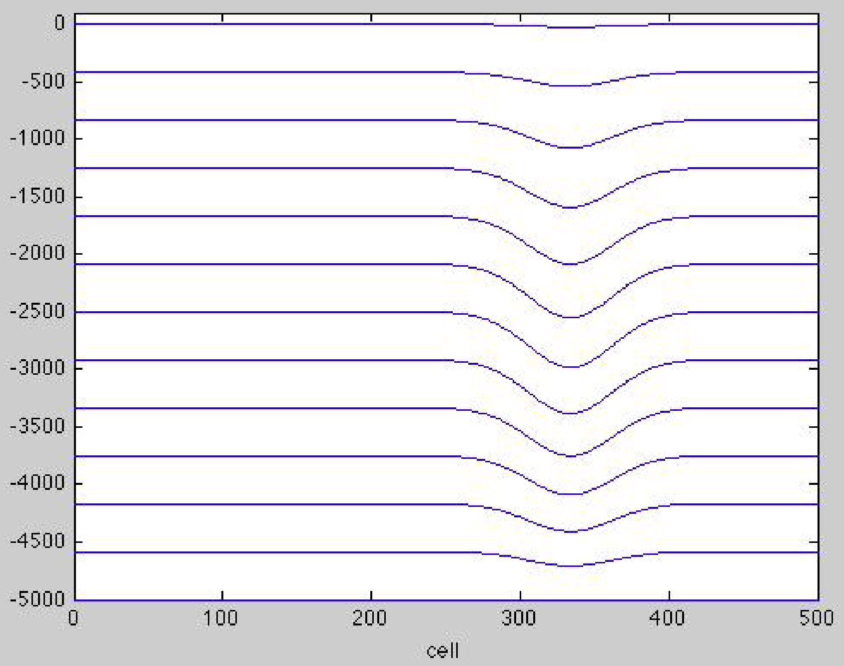
Final thickness
$(\Delta p_r)_{n+1}$

Prototype implementation

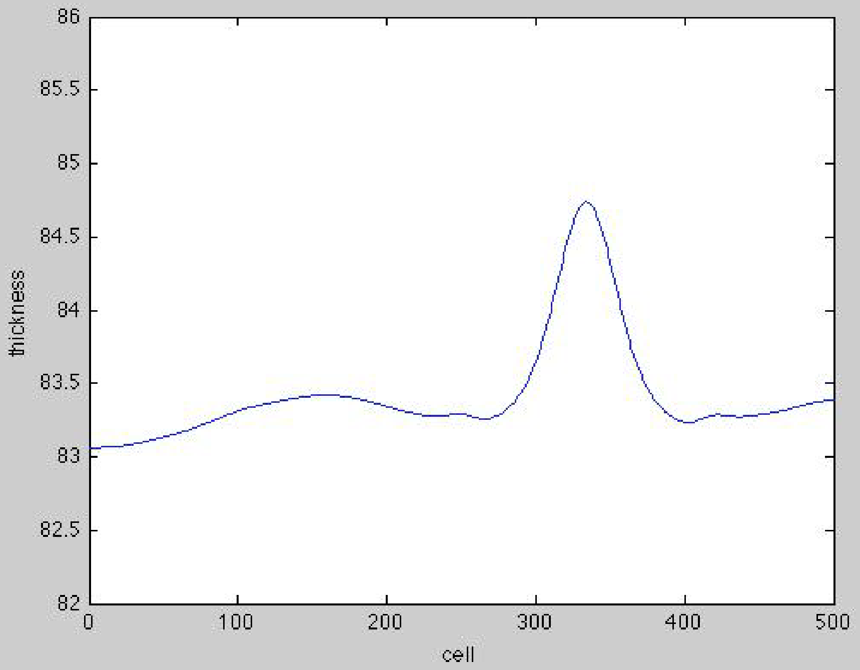
- **2-D, infinite channel, 5000 m depth, 5000 km width**
- **C-grid,** $\Delta x = 1.0 \times 10^4$**,** $n_x = 500$**,** $R = 60$
- **Non-uniform density, large amplitude thickness perturbations and SSH perturbation**
- **Coriolis via fixed point iteration**
- **Reference solution – Fully coupled fourth order Runge Kutta** $\Delta t = 1.0$
- **Barotropic CFL 30 sec; Baroclinic CFL 600 sec**
- **Stable integration to 10 days requires**
  - 229.77 sec wall clock for split (600,20)
  - 815.68 sec wall clock for Runge Kutta (30)
- **Integration to 1 day with various stepsizes**
- **Full implementation of barotropic-baroclinic splitting method to be added to MPAS**

Prototype implementation – 10 day simulation

Layer depth

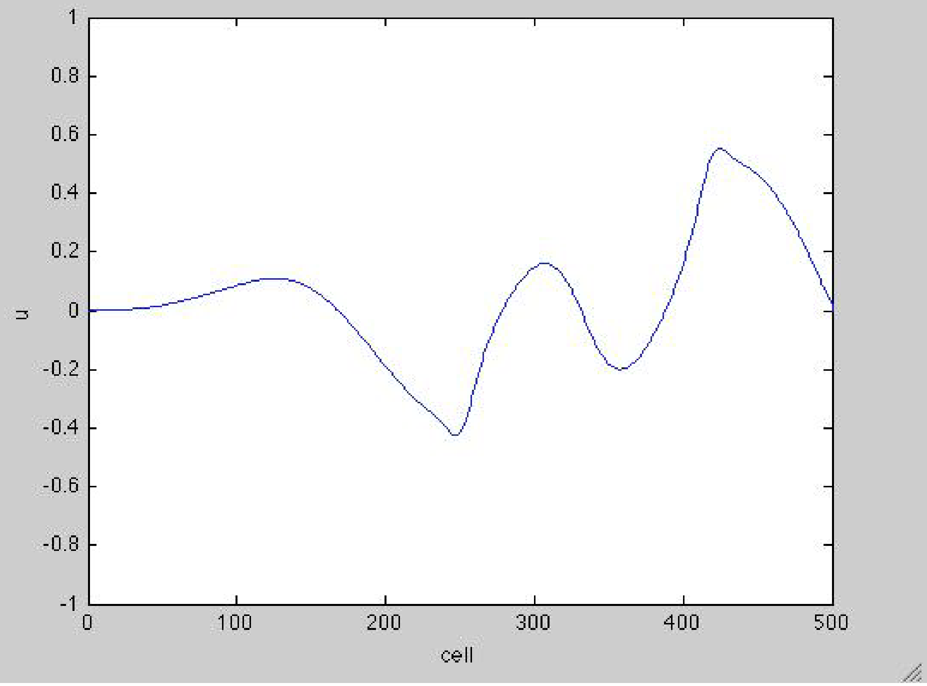


Layer 30 thickness

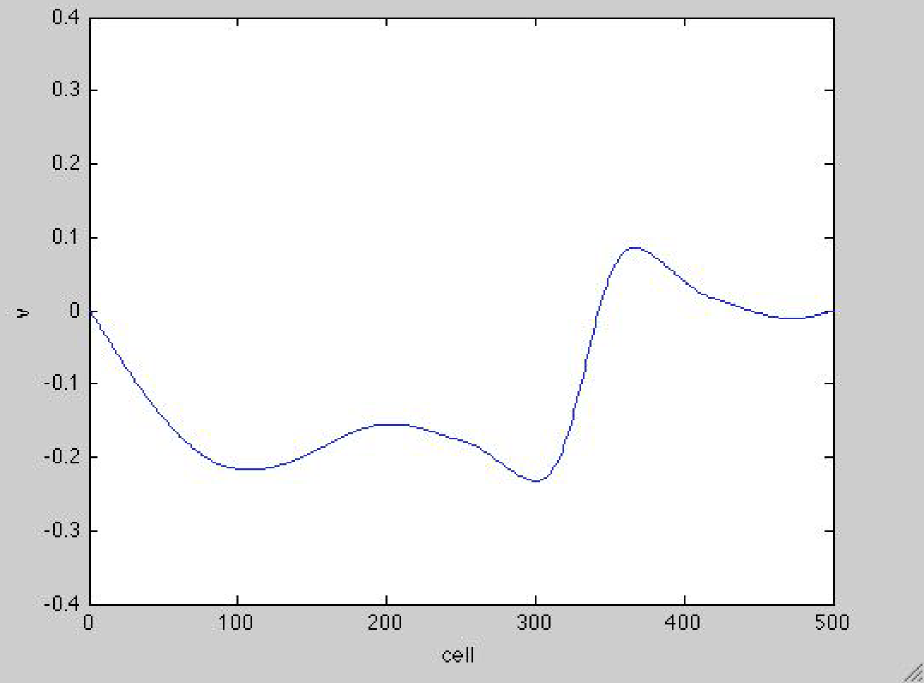


Prototype implementation – 10 day simulation

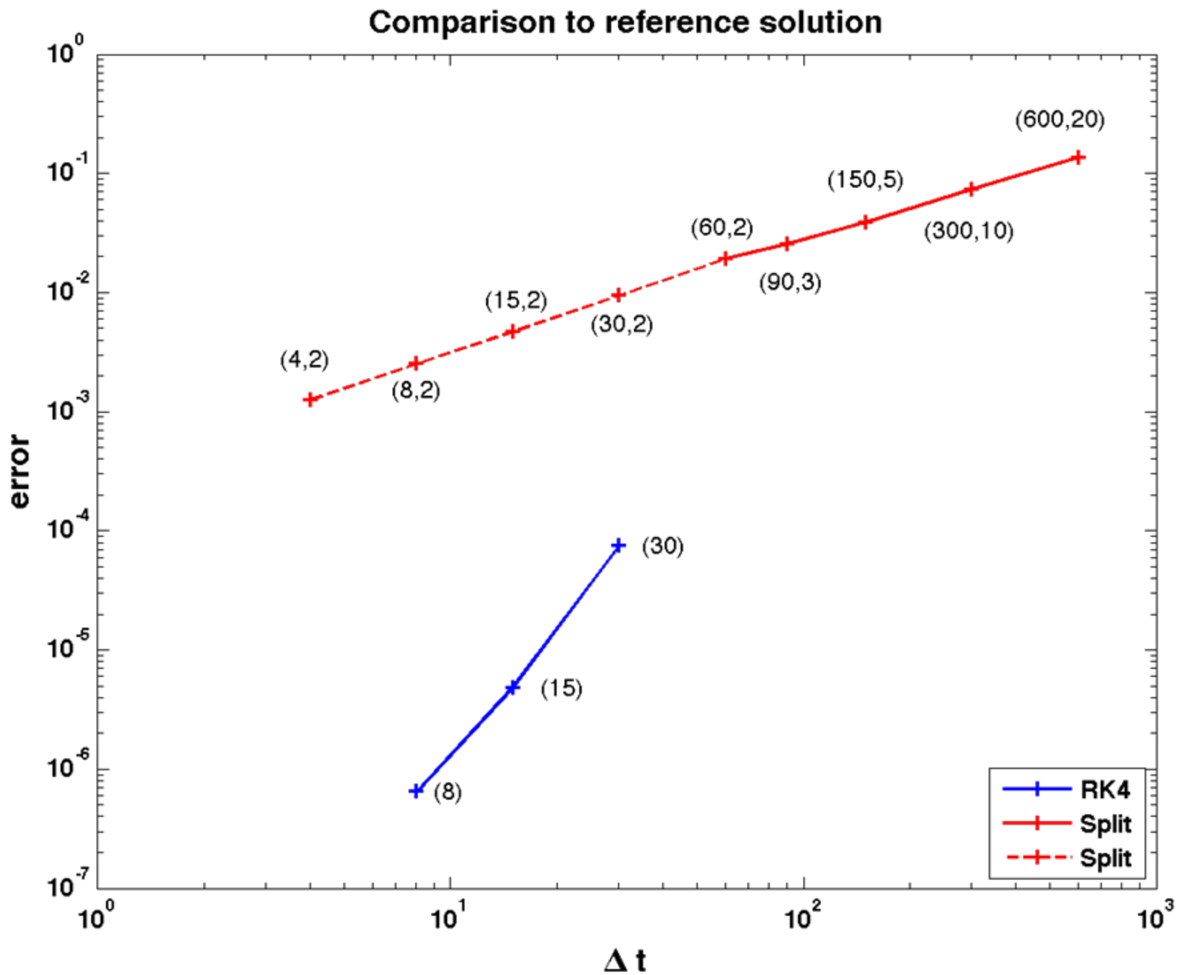
Layer 30 u-velocity



Layer 30 v-velocity

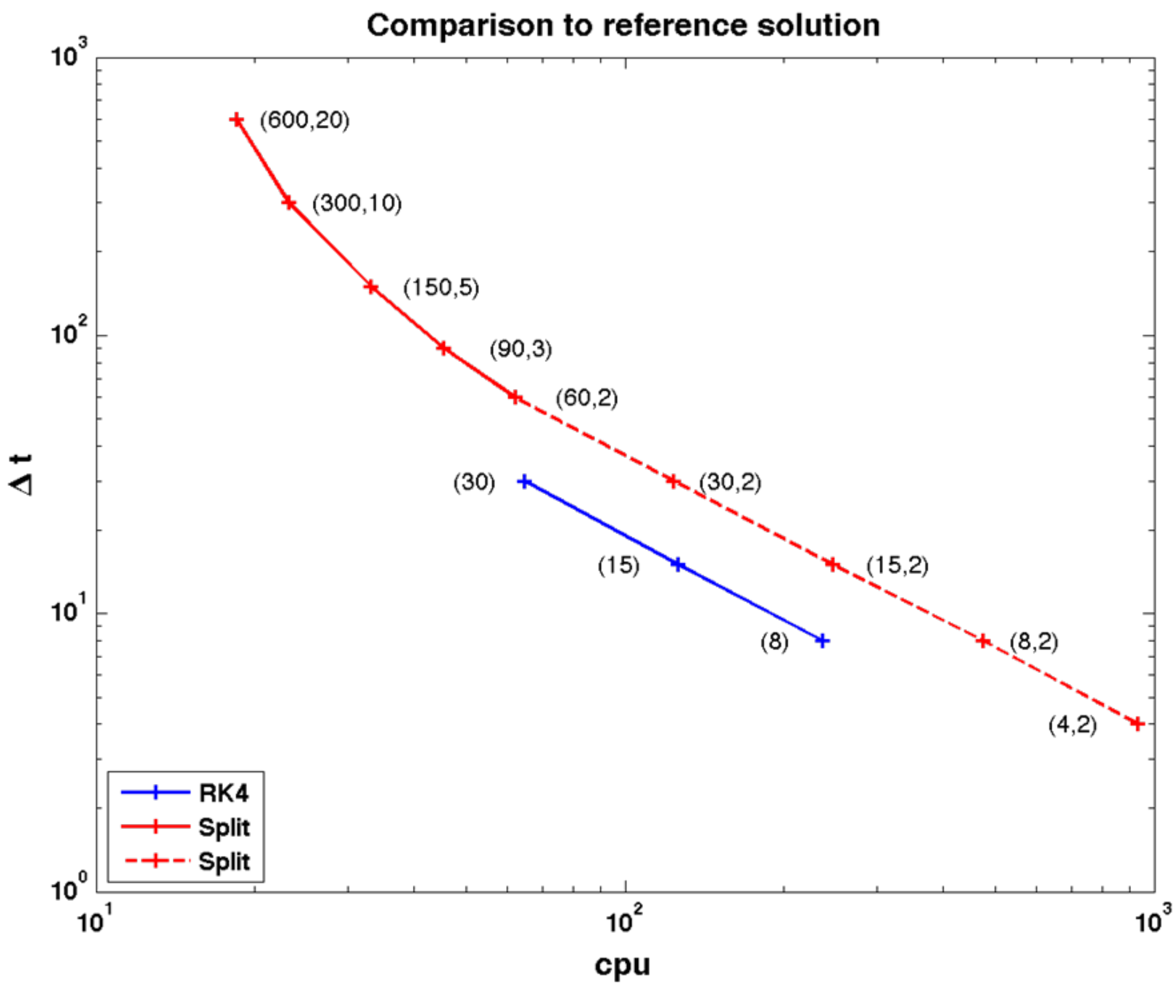


Comparison to reference solution – total error vs baroclinic timestep



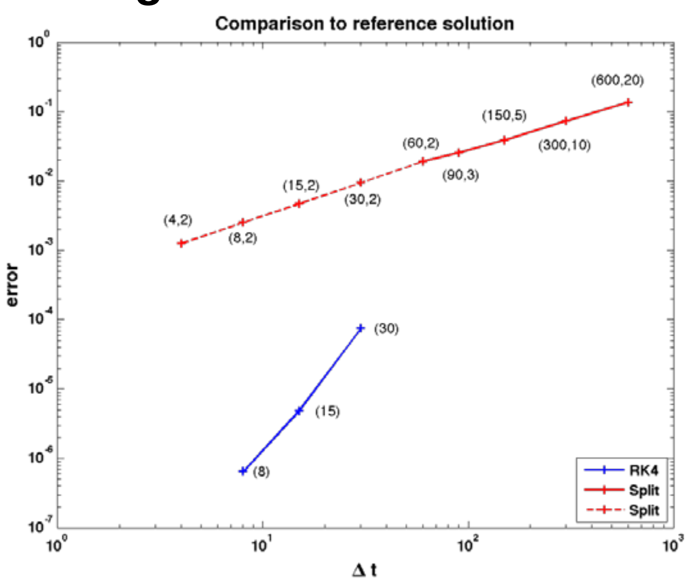
- Split $(\Delta t, m)$; fixed barotropic stepsize δt, # subcycles/step m
- Fourth order Runge Kutta (Δt)

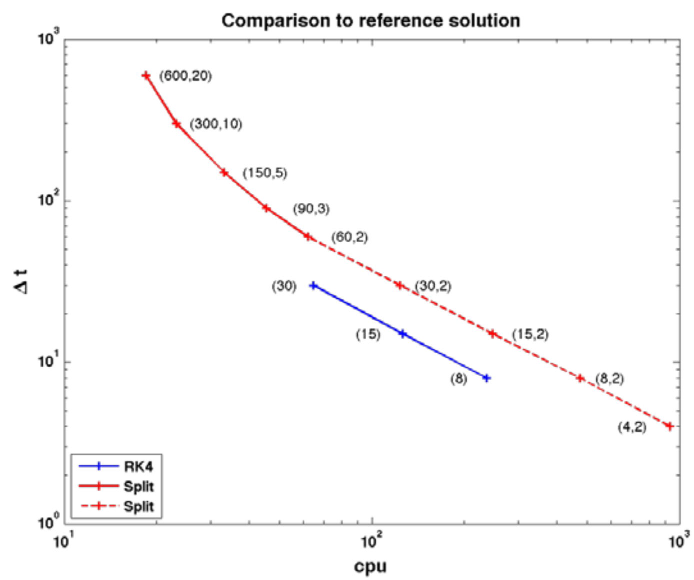
Comparison to reference solution – timestep vs cost



Comparison to reference solution - observations

- **Splitting method has slower convergence**
  - In this example, splitting is also less accurate
- **Convergence is independent of number of subcycles**
- **Splitting stable for larger timesteps and for long time integration**
- **Splitting allows larger timesteps with smaller computational costs**
  - In this example, timestep is 20 times larger and cost is 3.5 times less
- **Given its efficiency, may be a good preconditioner for implicit time integration**





How barotropic-baroclinic splitting can aid implicit time integration methods

- **Jacobian-free Newton-Krylov method for solution of fully coupled system**
  - No need to store Jacobian
  - Only need to form residual vector
  - Requires implementation of a preconditioning operator
  - Eliminates errors due to splitting
- **Operator split methods have been proven to be good physics based preconditioners for fully coupled multiple time scale problems (Mousseau JCP 2003)**
- **Leverage existing software: Trilinos**
- **Implemented appropriately, the barotropic-baroclinic splitting can be used as a solver or preconditioner**

Summary

- **Described a two level barotropic-baroclinic splitting where the barotropic terms are subcycled explicitly**
- **Prototype implementation of barotropic-baroclinic splitting yields stable, efficient time integration with larger timesteps and less compute time than fourth order Runge Kutta**
- **Full implementation of barotropic-baroclinic splitting method to be added to MPAS**
- **Implemented appropriately, the barotropic-baroclinic splitting can be used as a solver or preconditioner for an implicit time integration**

References

- R. Bleck, L. T. Smith, A wind-driven isopycnic coordinate model of the north and equatorial atlantic ocean. 1. Model development and supporting experiments, Journal of Geophysical Research (95) (1990) 3273-3285.
- R. L. Higdon, A two-level time-stepping method for layered ocean circulation models: further development and testing, J. Comput. Phys. 206 (2) (2005) 463-504. doi:http://dx.doi.org/10.1016/j.jcp.2004.12.011.
- R. L. Higdon, Implementation of a barotropic-baroclinic time splitting for isopycnic coordinate ocean modeling, J. Comput. Phys. 148 (2) (1999) 579-604. doi:http://dx.doi.org/10.1006/jcph.1998.6130.
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