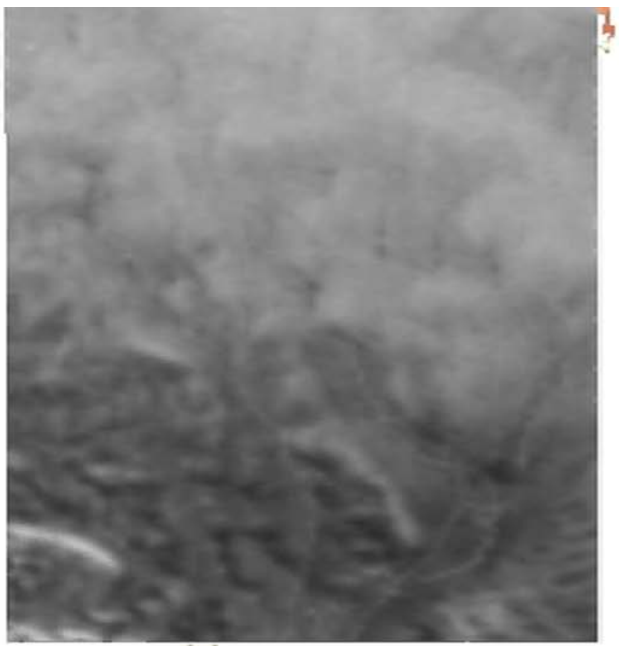
Engineering Robust Ice Sheet Models

A discussion of the members of the Land Ice Working Group

Requirements of Ice Sheet Models

- Tolerant of irregular geometries
- Work with incomplete knowledge of all fields
- 200-200,000 years of modeled dynamics
  - Paleo-climate
  - Dynamic forcing
- **Can not halt**, indeed should
  - Converge to desired tolerance at every time step
  - Be second order accurate
- This is what we call ‘robust’

Rough geometry and missing data



Formal definitions for ISM robustness: “wat”

From the urban dictionary:

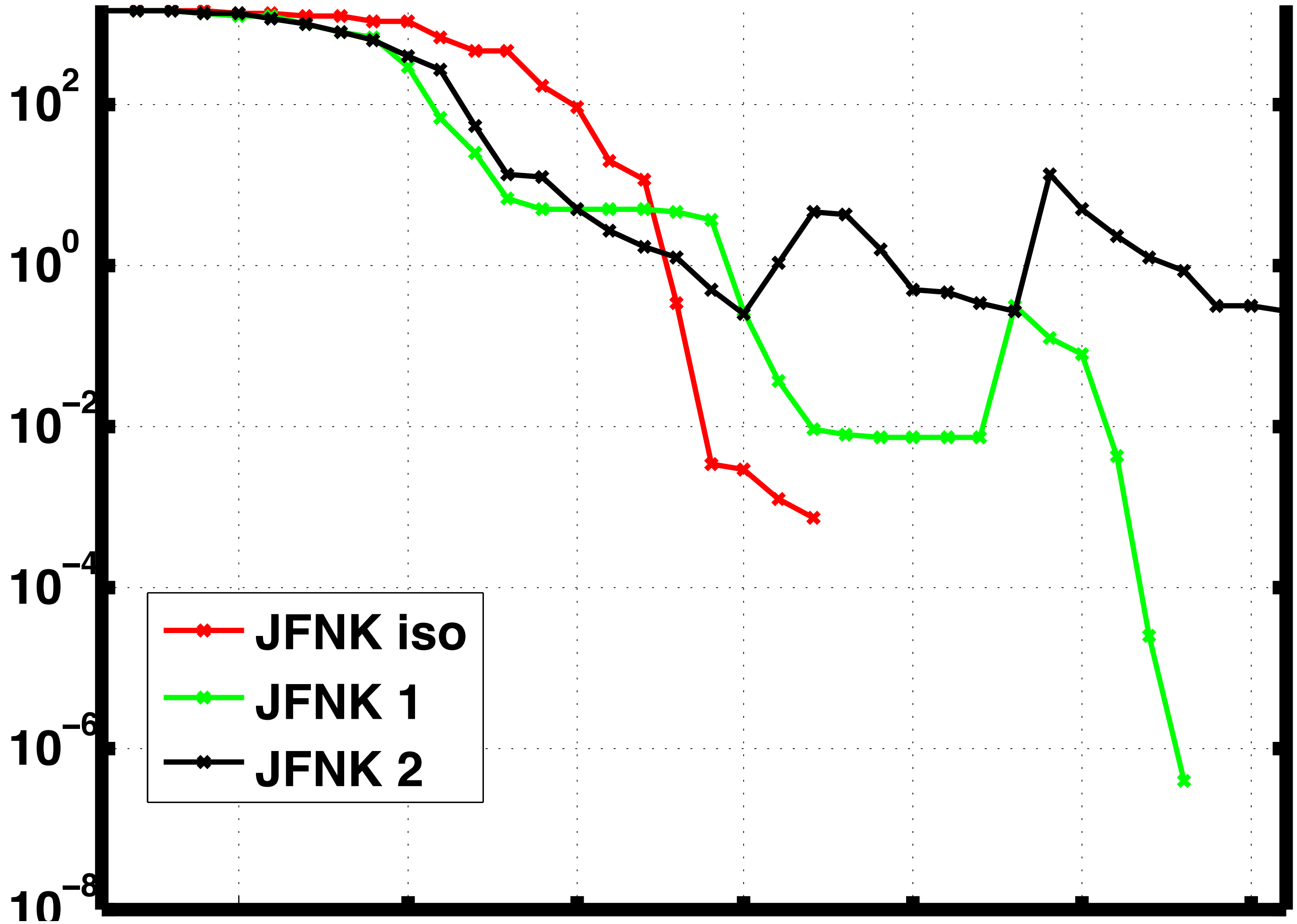
wat - the only proper response to something that makes absolutely no sense.

Something like this:

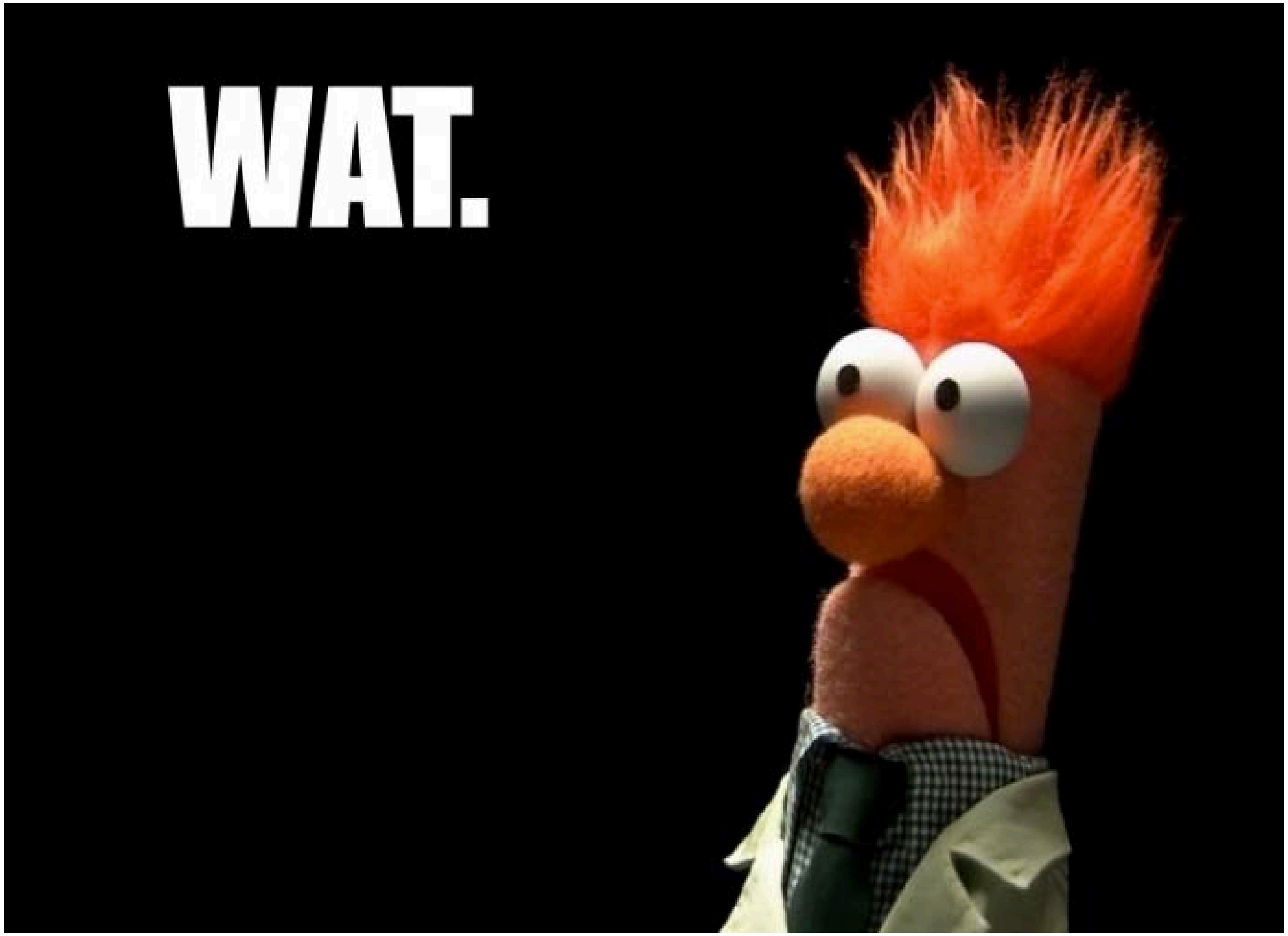
Maximum temperature iterations: 1
  * FATAL ERROR :
(/glade/home/gailg/tg_compset_yrstep/models/glc/cism/source_glimmer-cism/glide_thck.F90:517) SLAP
 solution error at time: 1.000000000 . Data dumped to slap_dump.txt
 ****
******
 Finished logging at 2012-01-27 12:26:28.807
 ****
******

Gets a resounding:

Things like this black line,

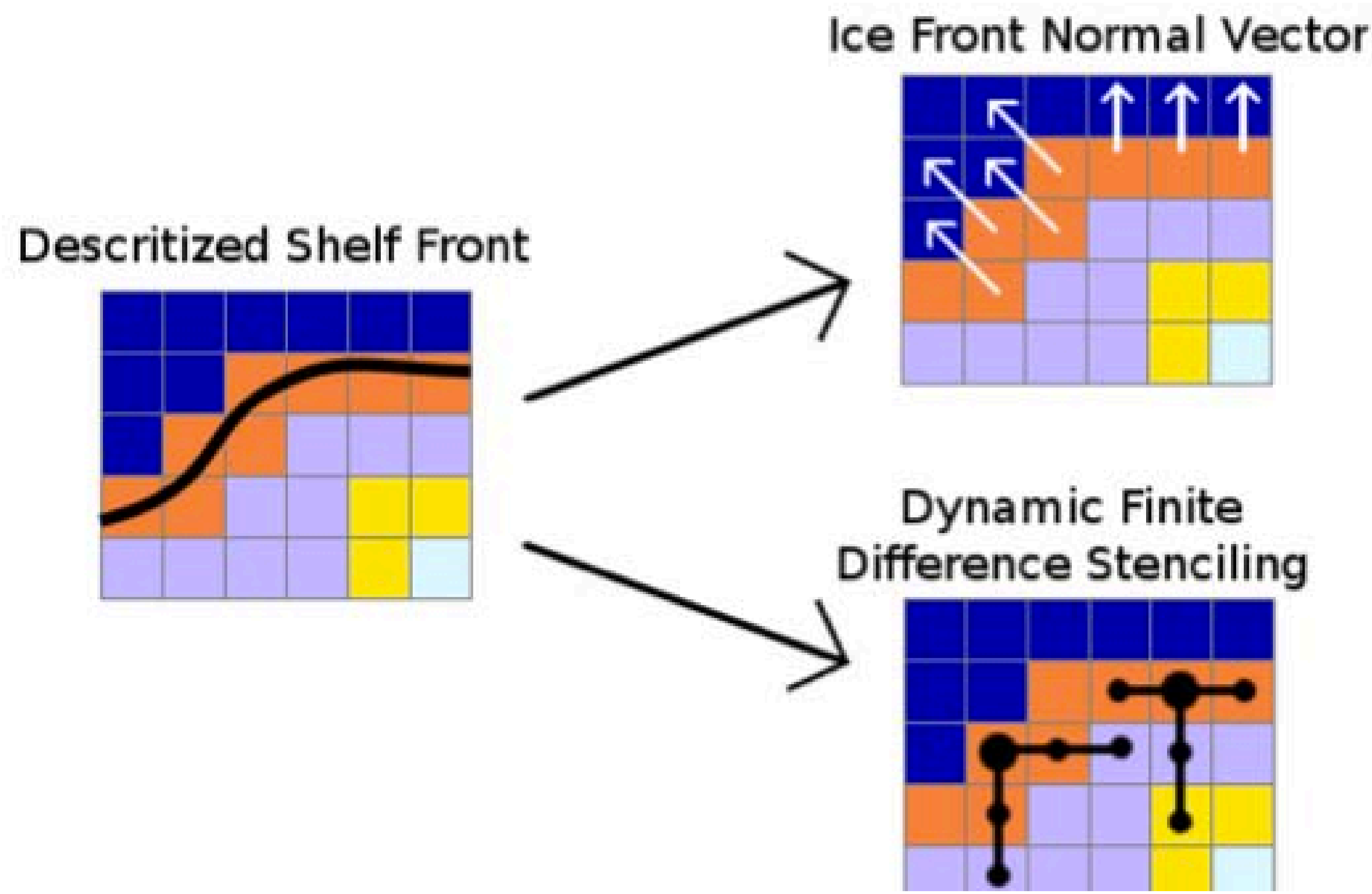


Also, a



Let’s talk about other sources of “wat”. Let’s talk about momentum.

Momentum: Where is the boundary? At the ice edge?



ML preconditioner

- Basal boundary condition:

$$\eta \frac{\partial u}{\partial z} = -\beta^2 u \qquad \text{for } z = b(x)$$

Central Implementation

$$\begin{pmatrix} -\frac{\eta}{2h} & \beta^2 & \frac{\eta}{2h} & 0 & 0 & 0 \\ \frac{\eta}{h^2} & -\frac{2\eta}{h^2} & \frac{\eta}{h^2} & 0 & 0 & 0 \\ 0 & \frac{\eta}{h^2} & -\frac{2\eta}{h^2} & \frac{\eta}{h^2} & 0 & 0 \\ 0 & 0 & \frac{\eta}{h^2} & -\frac{2\eta}{h^2} & \frac{\eta}{h^2} & 0 \\ 0 & 0 & 0 & -\frac{\eta}{2h} & 0 & \frac{\eta}{2h} \end{pmatrix}$$

One-Sided Difference Implementation

$$\begin{pmatrix} \beta^2 - \frac{\eta}{h} & \frac{\eta}{h} & 0 & 0 \\ \frac{\eta}{h^2} & -\frac{2\eta}{h^2} & \frac{\eta}{h^2} & 0 \\ 0 & \frac{\eta}{h^2} & -\frac{2\eta}{h^2} & \frac{\eta}{h^2} \\ 0 & 0 & -\frac{\eta}{h} & \frac{\eta}{h} \end{pmatrix}$$

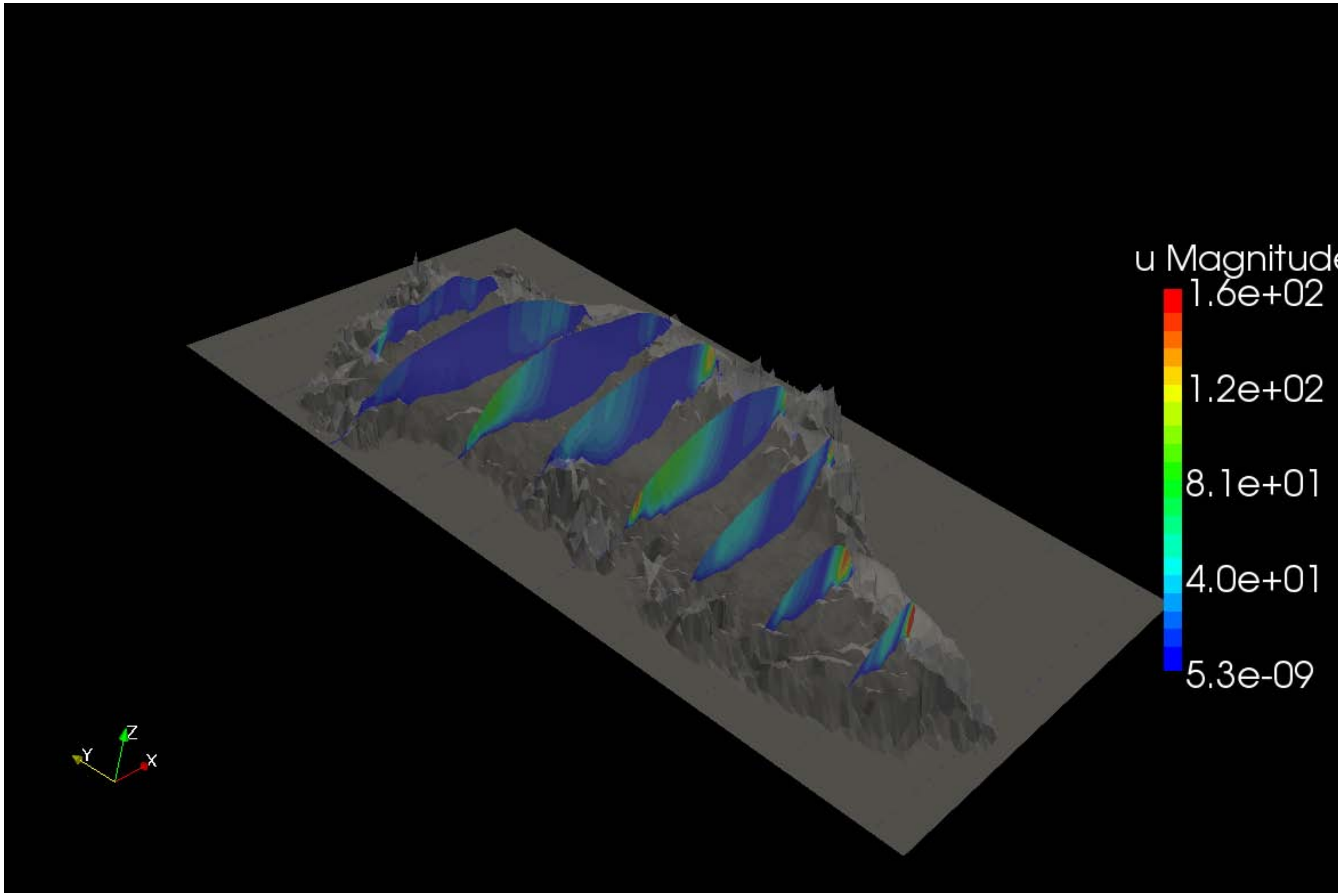
- Ifpack preconditioner optimal for problems with vertical coupling.
- ML preconditioner expected to be better alternative than ILU preconditioner for case with:
  - **Basal sliding (horizontal shear and coupling among horizontal cells).**
  - **Very large problems run on many processors (ILU may not scale well).**

ML preconditioner

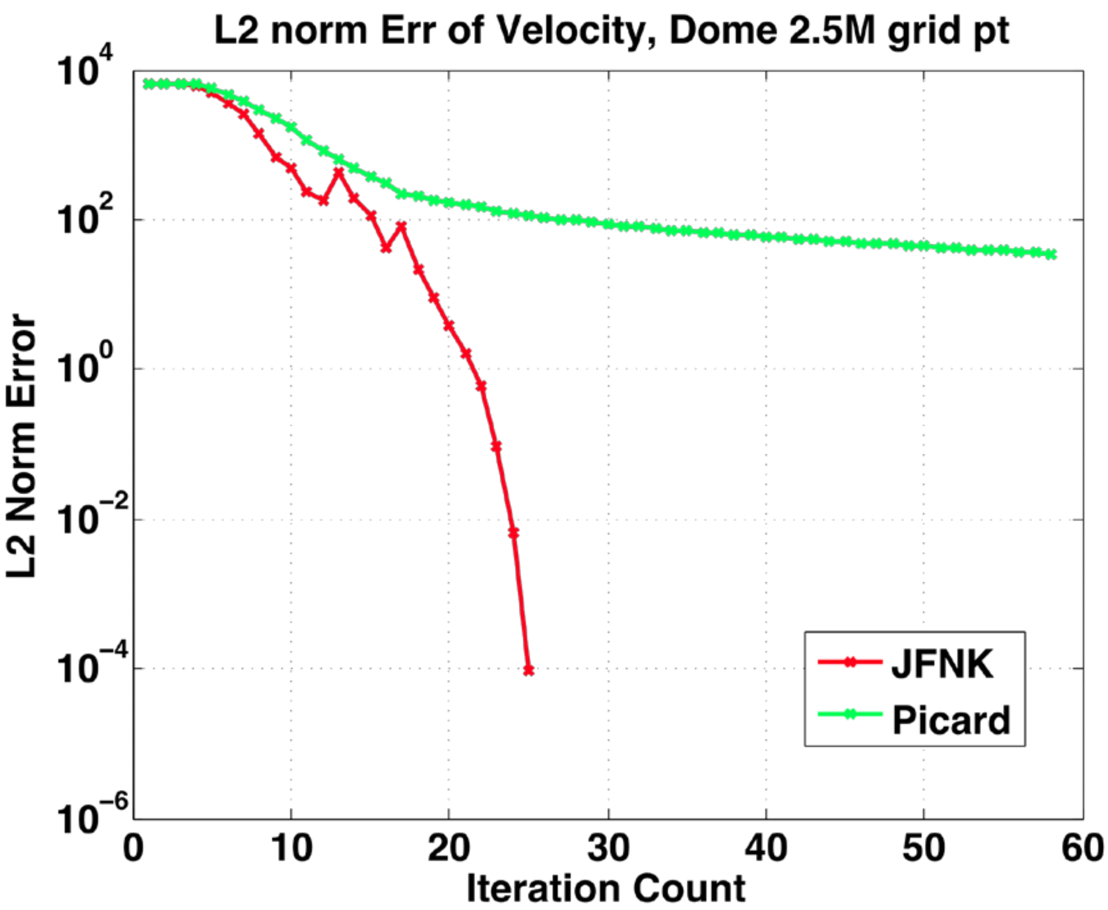
		Ifpack (1 overlap, 1 level-of-fill)	ML
old (central diff) BC	$\|\|F\|\|$	$9.334e-5$	$4.158e2$ **(FAILED)**
	# iter nonlinear solver	14	100 **(FAILED)**
	utime (s)	27,593	921,431
new (one-sided diff) BC	$\|\|F\|\|$	$3.817e-5$	$3.862e-5$
	# iter nonlinear solver	10	10
	utime (s)	45,402	39,638

- **Behavior of preconditioners is as expected (10 km Greenland problem on 512 processors):**
  - **Central difference BC implementation: linear solver with ILU preconditioner converges but linear solver with ML preconditioner fails to converge.**
  - **One-sided central difference BC implementation: linear solver converges with both ILU and ML preconditioners.**
  - **ML preconditioner can yield shorter total solve time.**

Or this boundary?

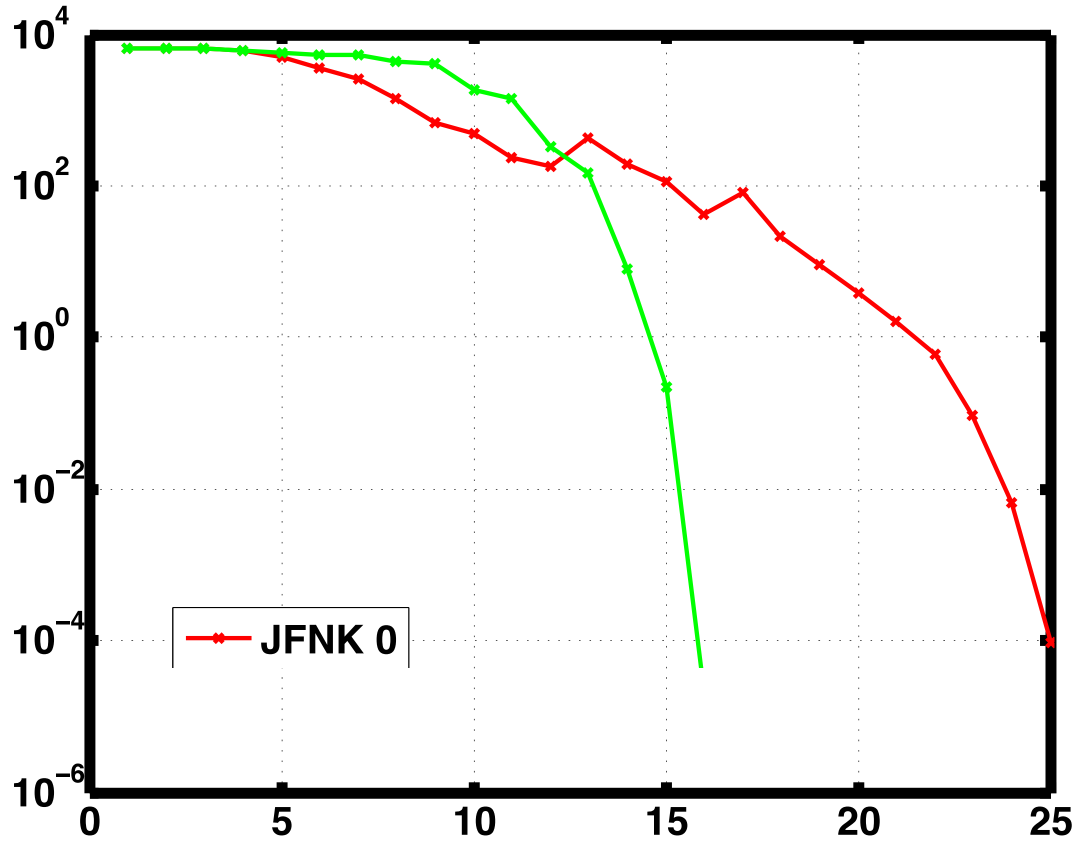


The non-linear solver matters



Note: Picard eventually blows up in about 50+ more iterations,
Regardless of precon settings

As does the pre-conditioner



JFNK 1 behavior identical for 420 or 1600 processors.

The importance of a pre-conditioner

Additive Schwarz Method

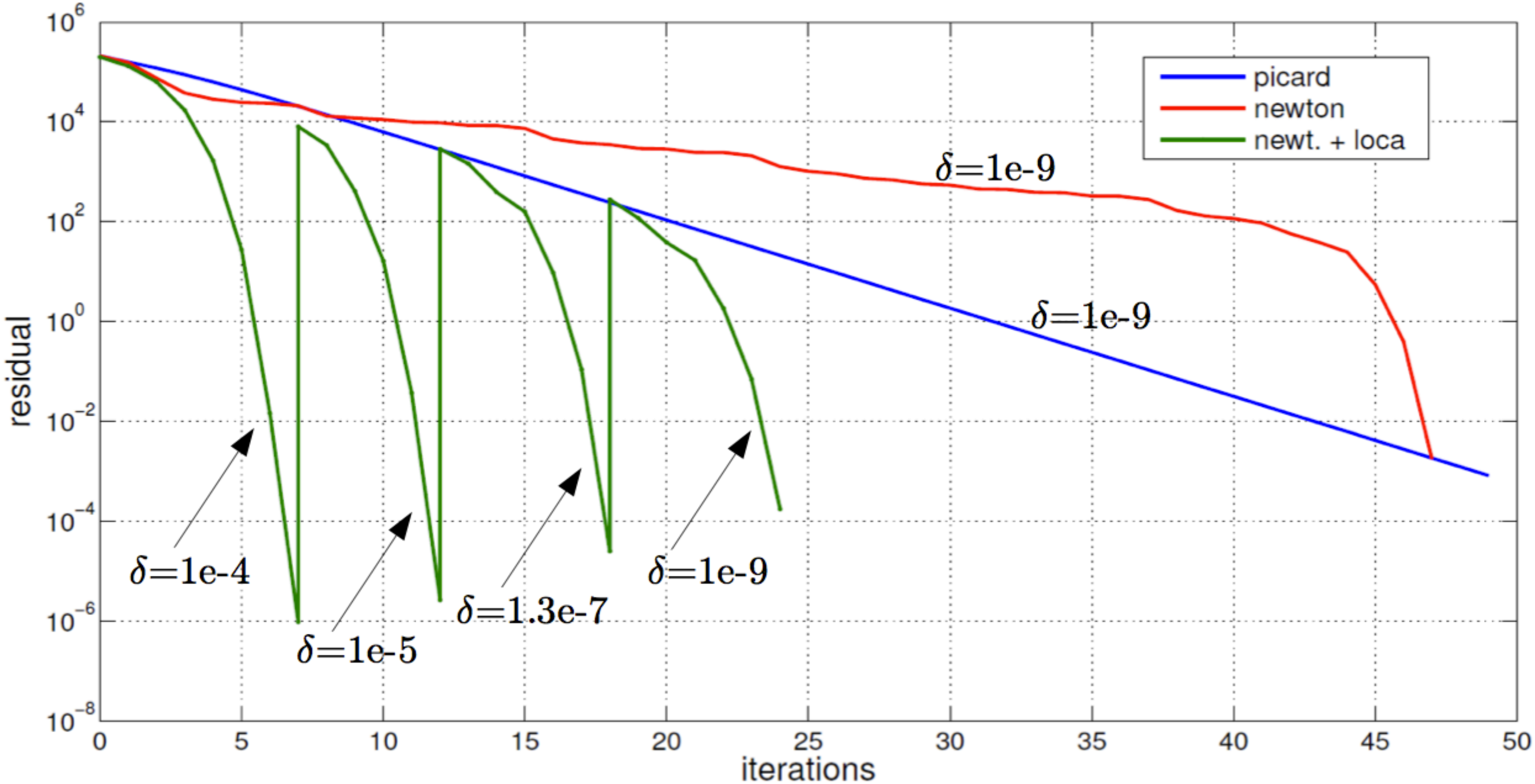
$$P^{-1} = R_0^T A_0^{-1} R_0 + \sum_{i=1}^{N} R_i^T A_i^{-1} R_i$$

- A_0 : coarse matrix (restriction to the coarse space)
- A_i : local matrix (restriction to extended subdomain Ω_i')
- R_0 : restriction to coarse space
- R_i : restriction to extended subdomain Ω_i'

wat

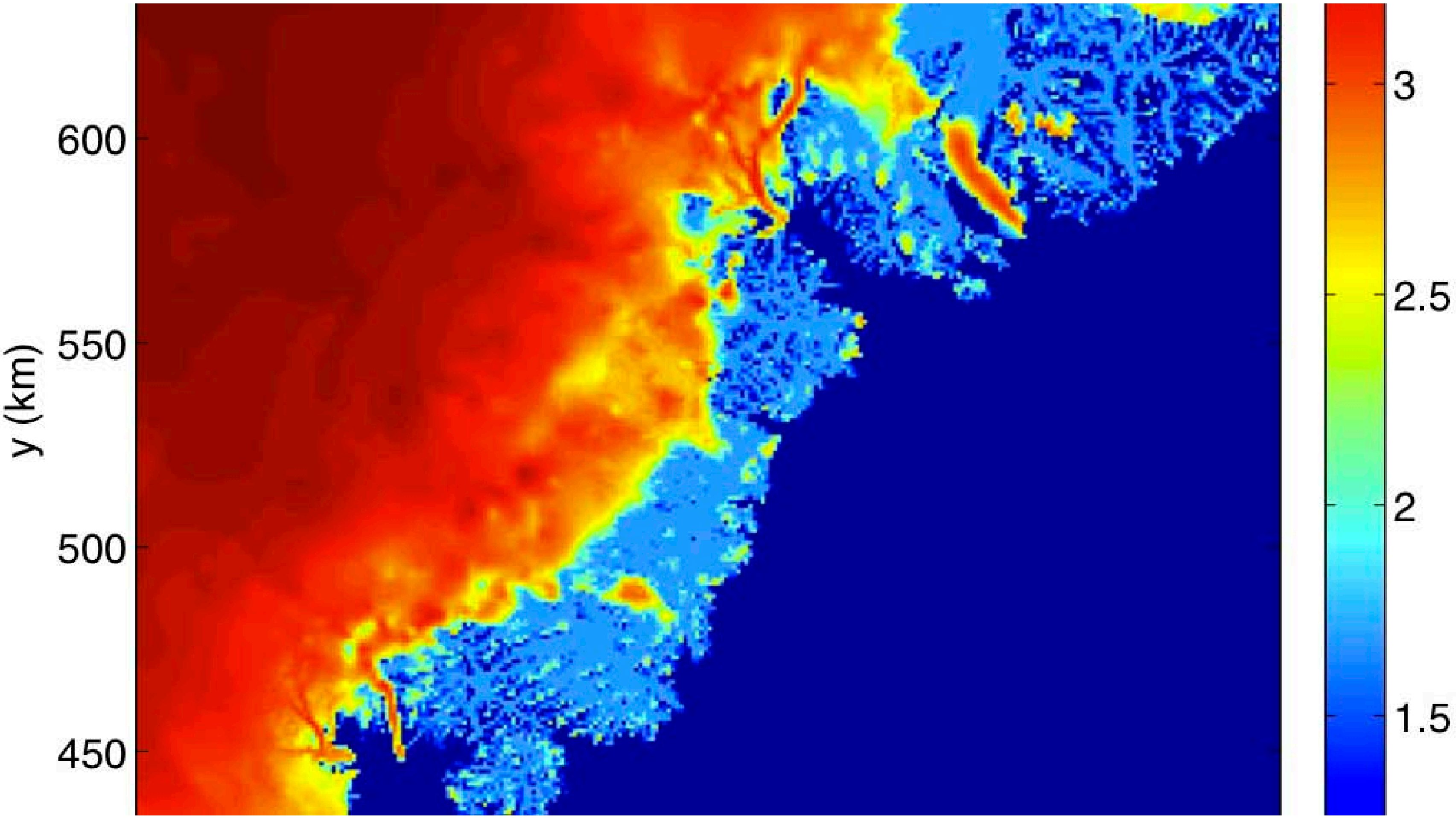
Continuation parameter

$$\dot{\varepsilon}_e^{-\left(1-\frac{1}{n}\right)} \approx \left(\sqrt{\dot{\varepsilon}_e^2+\delta^2}\right)^{-\left(1-\frac{1}{n}\right)}$$

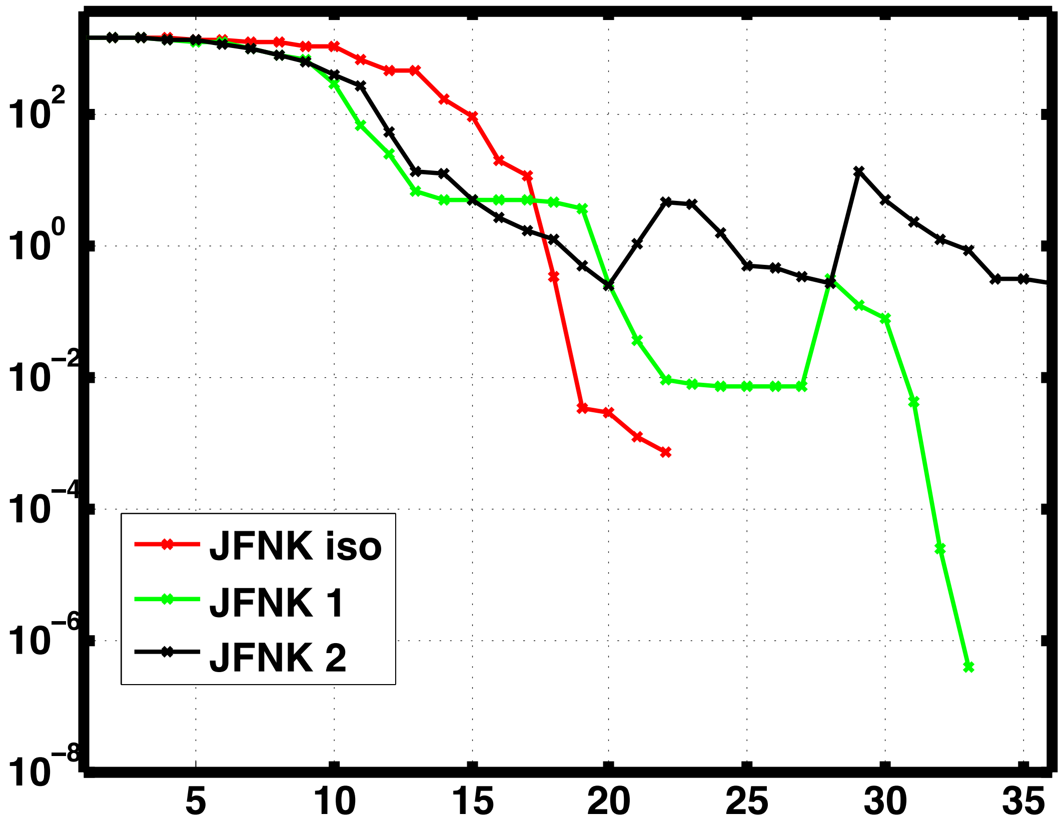


The parameter δ is decreased by LOCA from 1e-4 to 1e-9

Geometry matters

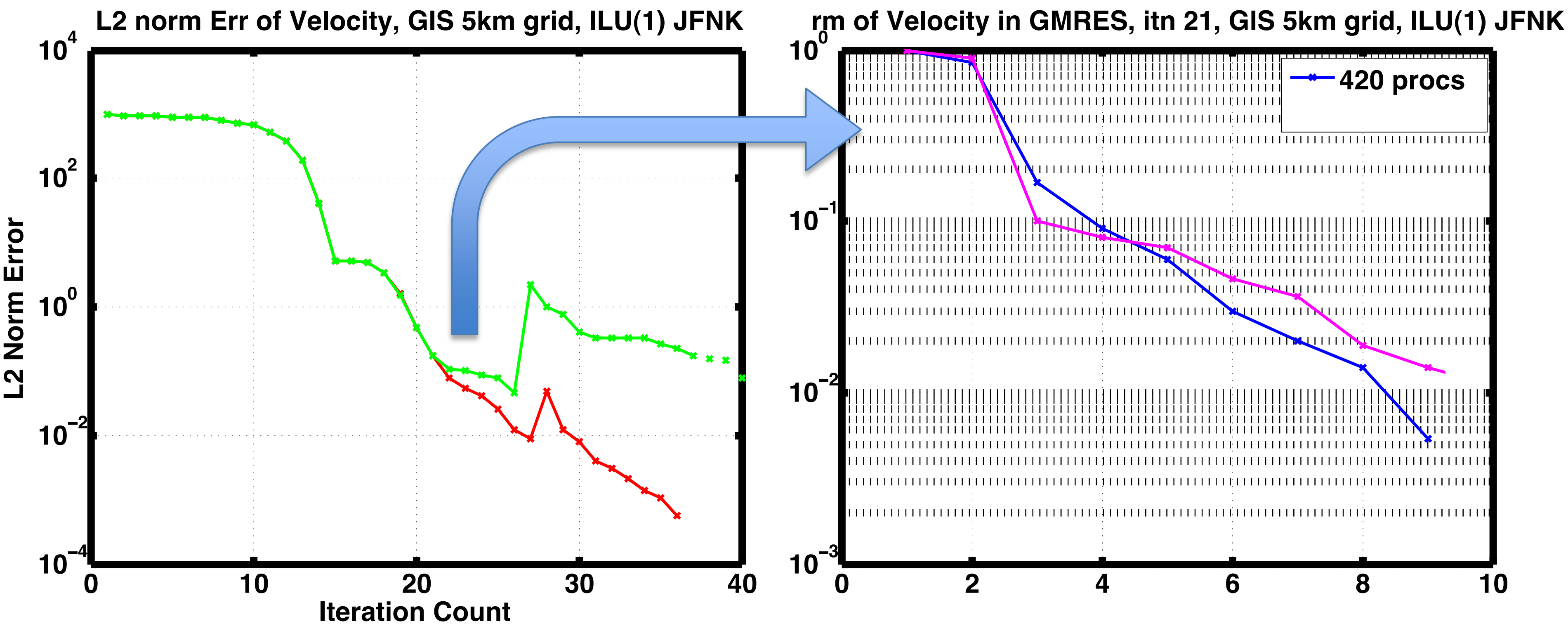


Solver stats for global 5km resolution GIS



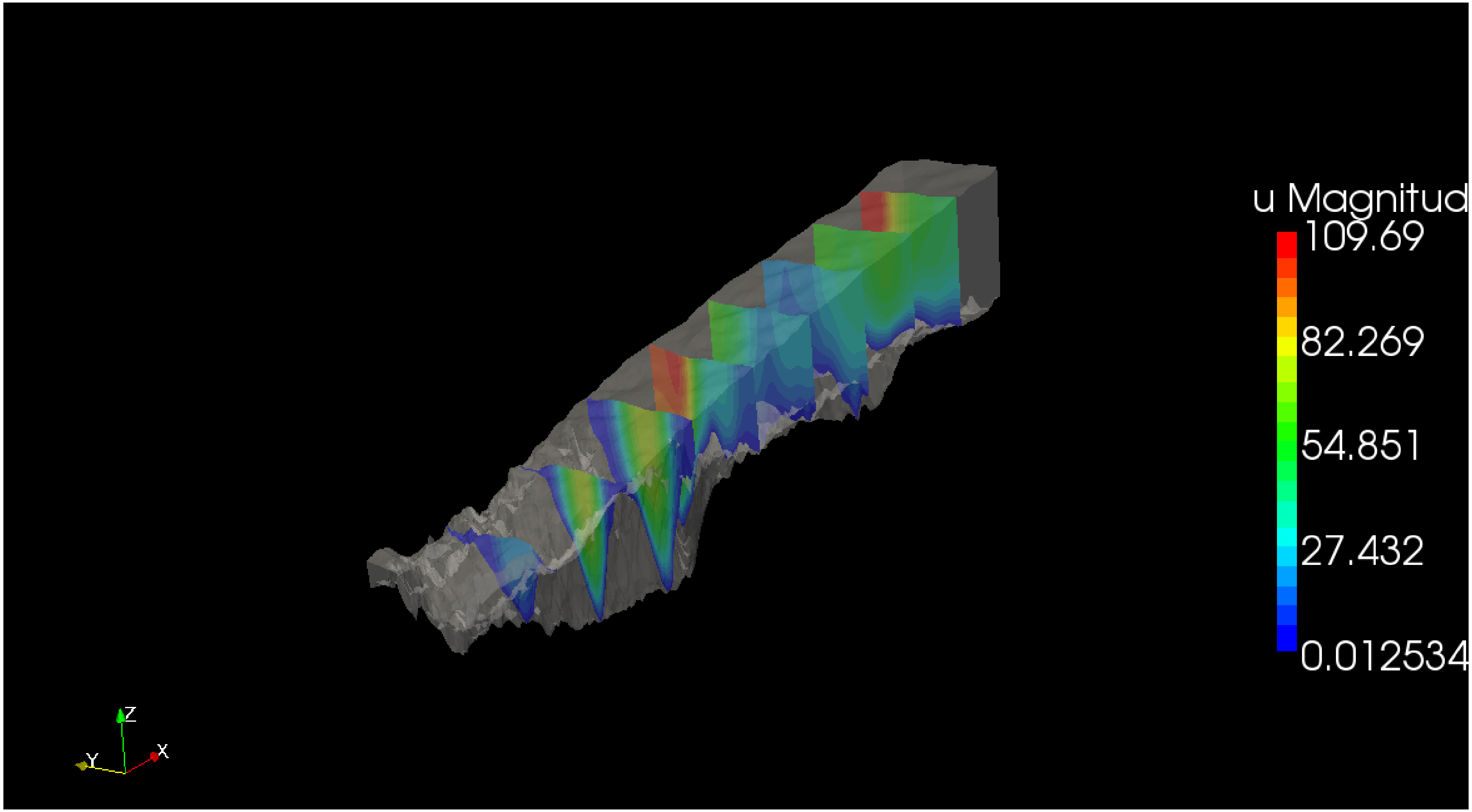
JFNK ‘iso’ is isothermal flow with Bamber et al (2000) dataset, JFNK “1” uses the 1 km res. Greenland Ice2Sea dataset.

Another sensitivity to JFNK convergence is processor count effect on ILU preconditioner



JFNK with isothermal flow law settings.
This sensitivty illustrates need for scalable preconditioning.

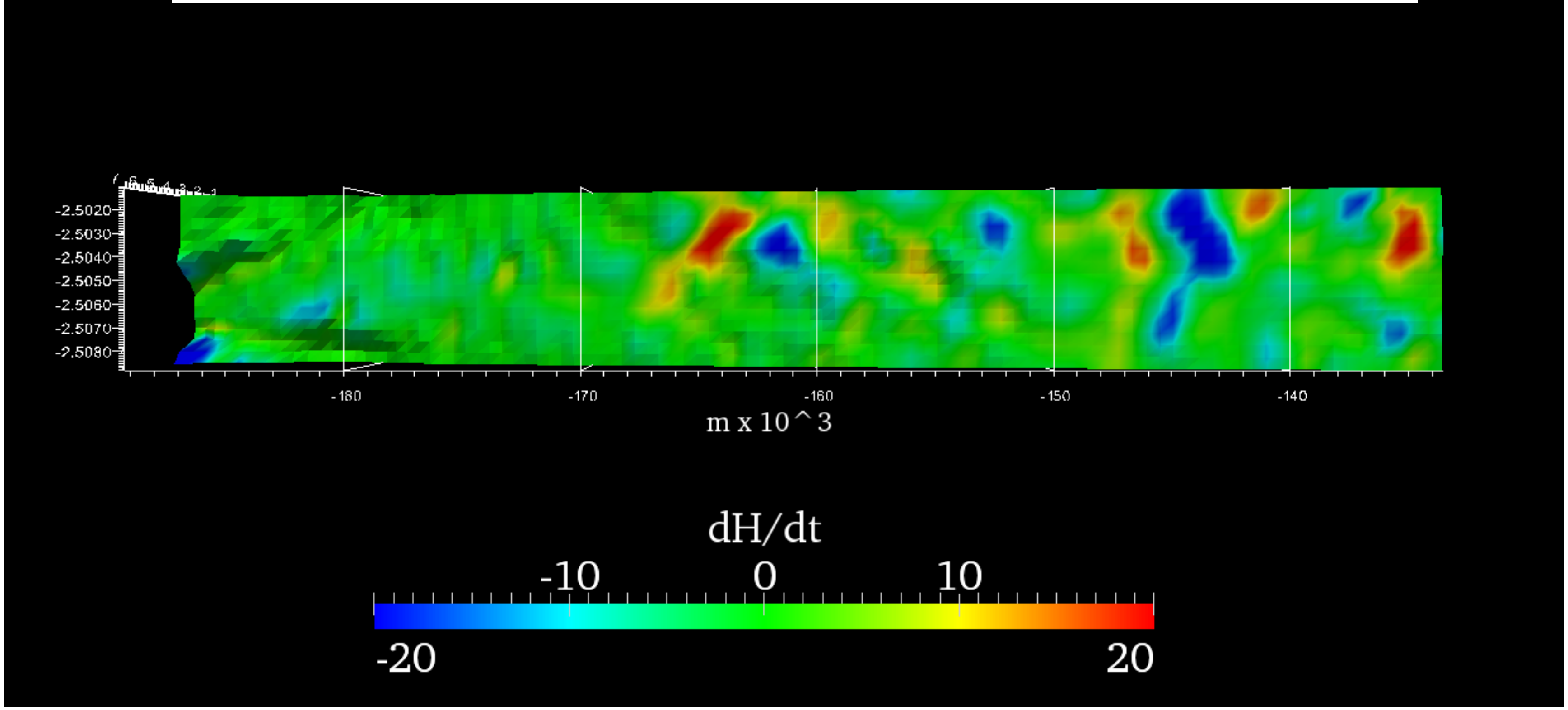
Ice Sheet Initialization



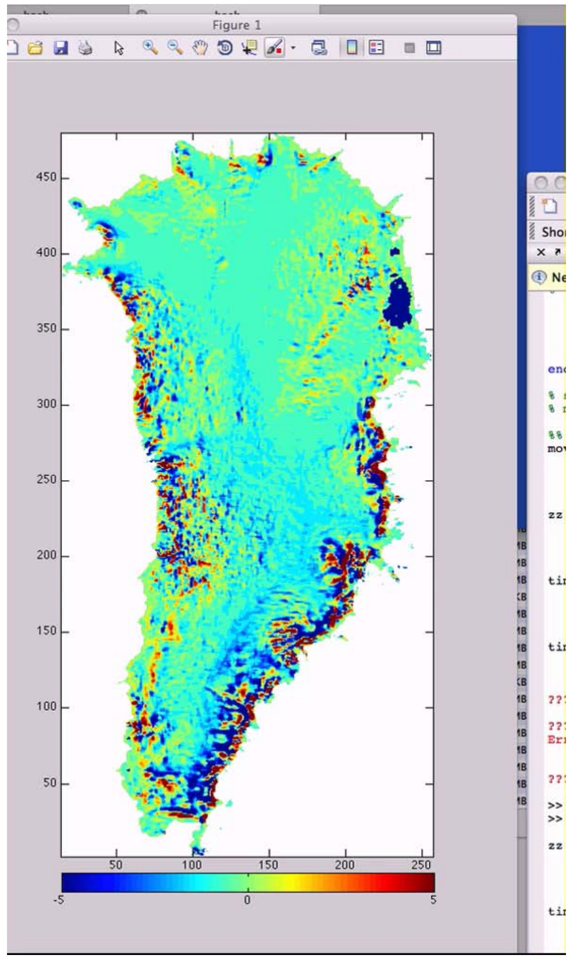
Conservation of mass

$$\nabla \cdot H\,\overline{\boldsymbol{v}} = \dot{M}_s - \dot{M}_b - \frac{\partial H}{\partial t}$$

- H is the glacier thickness (m)
- $\overline{\boldsymbol{v}}$ is the glacier depth-average velocity (m/yr)
- $\dot{M}_s$ is the surface accumulation rate (m/yr ice equivalent)
- $\dot{M}_b$ is the basal melting rate (m/yr ice equivalent, positive when melting)

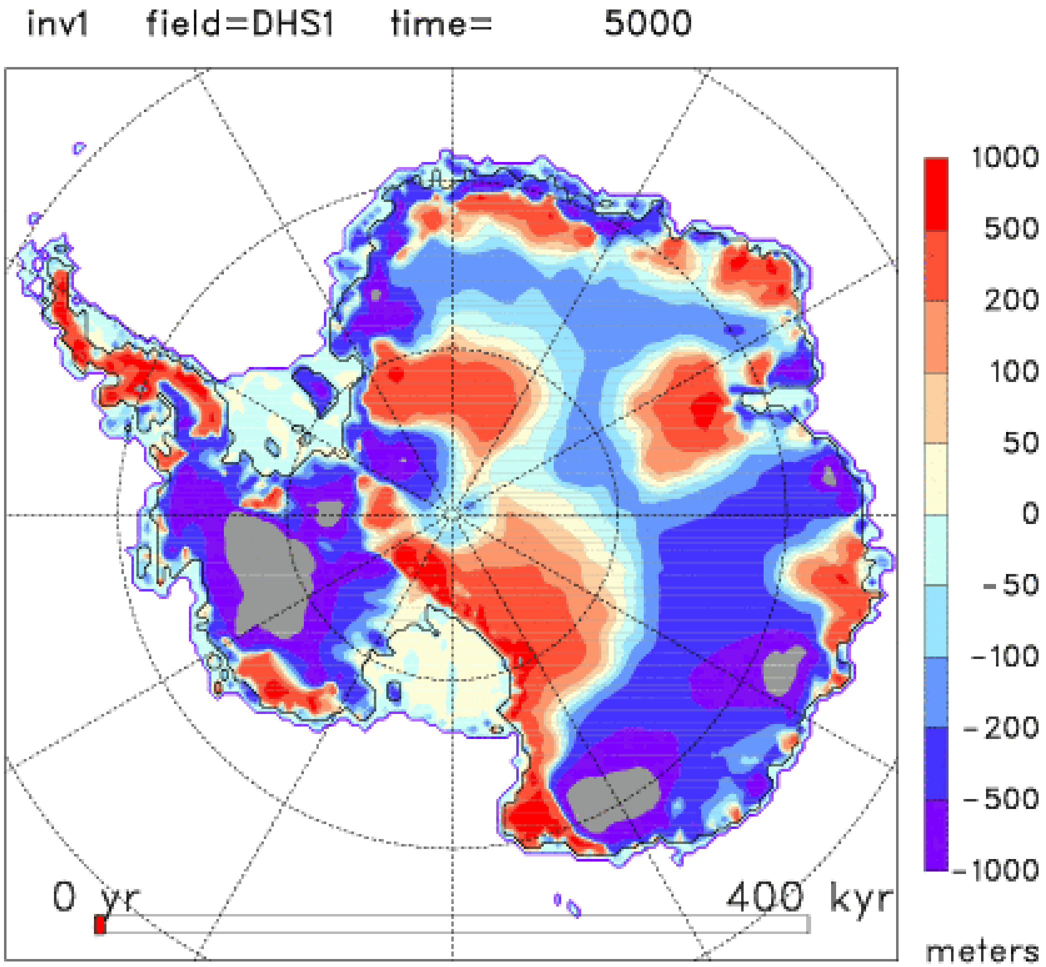


Vertical Velocity



wat, huminguins?

Incorrect Surface Elevations



PDE-constrained optimization

Minimize:

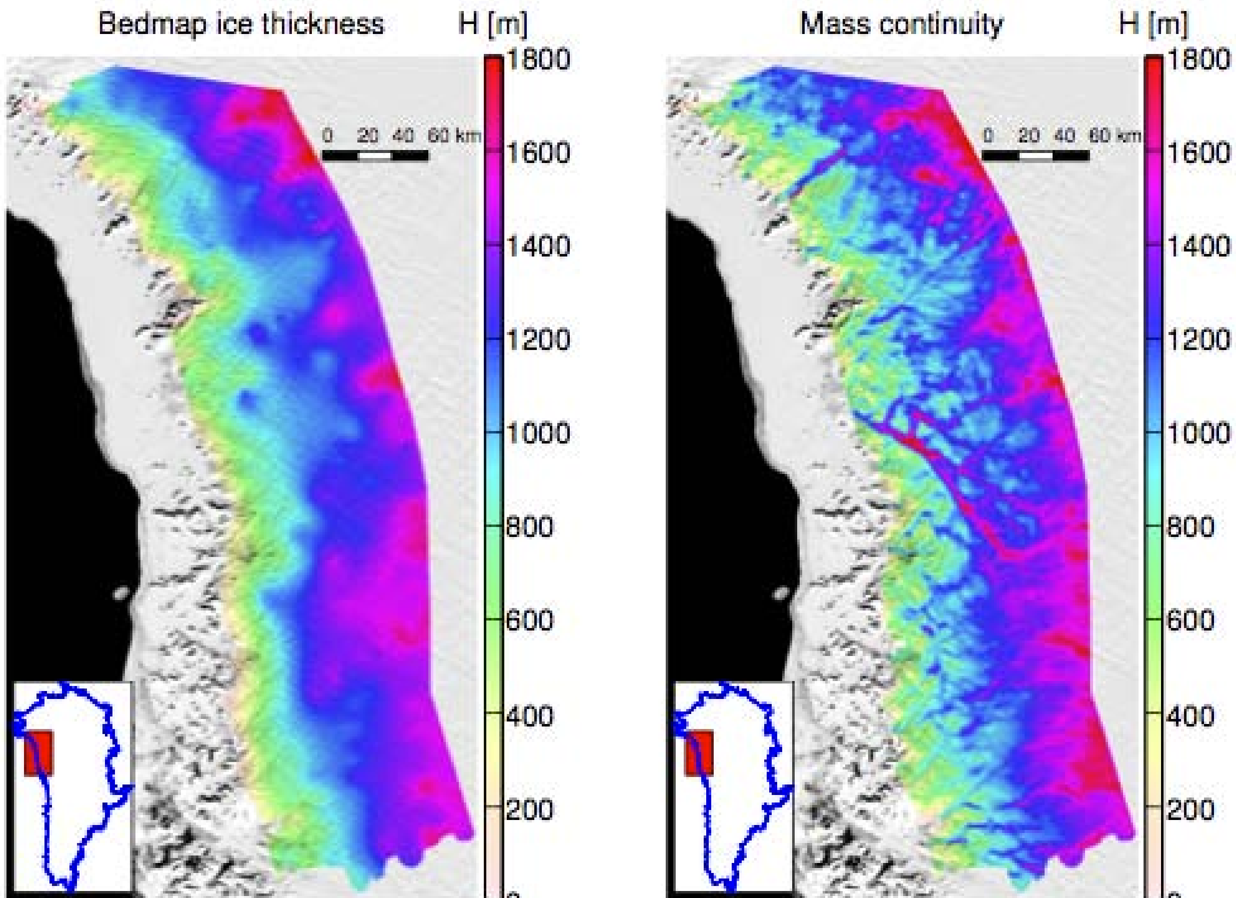
$$\mathcal{J}(\overline{\boldsymbol{v}}, \dot{a}) = \int_{\text{Tracks}} \frac{1}{2} (H - H_{obs})^2 \, dl$$

With the constraint:

$$\begin{cases} \nabla \cdot (H\overline{\boldsymbol{v}}) = \dot{a} & \text{in } \Omega \\ H = H_{obs} & \text{on } \Gamma_- \end{cases}$$

Controls:

- $\overline{\boldsymbol{v}} \in [0.95\,(\boldsymbol{v}_{obs} - 50) \quad \boldsymbol{v}_{obs} + 50]$ m/yr
- $\dot{a} = \dot{a}_{obs} \pm 1$ m/yr



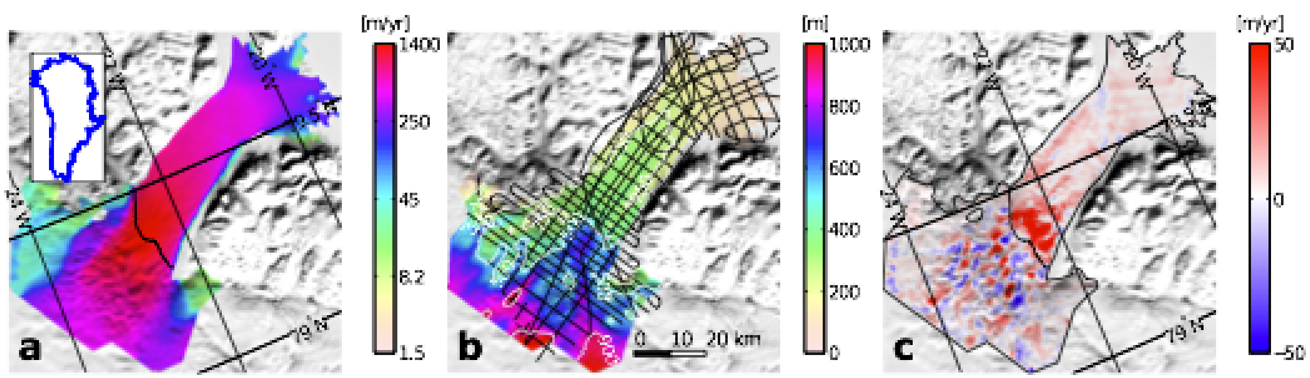
Not wat, but awesome!

Transport

- Ice Sheet Model evolution:

$$\frac{\partial H}{\partial t} = -\nabla \cdot H\,\overline{\boldsymbol{v}} + \dot{M}_s - \dot{M}_b$$

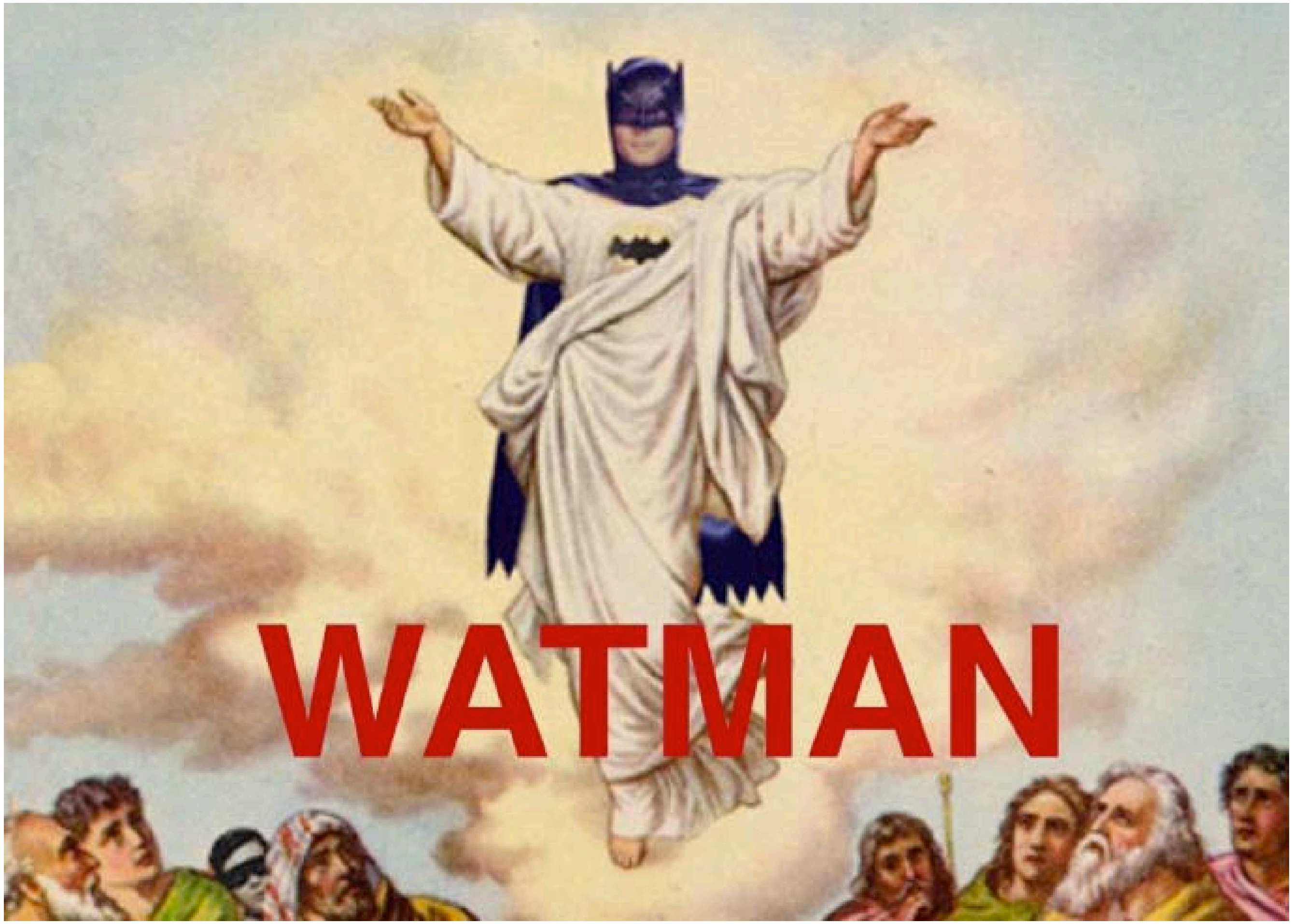
  - $\nabla \cdot H\,\overline{\boldsymbol{v}}$ is the ice flux divergence
  - $\dot{M}_s$ is the surface mass balance
  - $\dot{M}_b$ is the basal melting rate



(a) Ice velocity (b) ice thickness (c) ice flux divergence

Partition, ala PISM

$$\frac{\partial H}{\partial t} = \nabla \cdot \left(\tilde{D} \nabla h \right) - \tilde{\mathbf{v}} \cdot \nabla H - (\nabla \cdot \tilde{\mathbf{v}}) H + (M - S).$$


WATMAN