A finite element solver for ice-sheet dynamics to be integrated with MPAS

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February 16, CESM LIWG Meeting, Boulder (CO), 2012

Outline

- Introduction of finite element ice-sheets component
- Integration of finite element ice-sheet component with MPAS
- Results on ISMIP-HOM and Greenland/Antarctica geometries
- Non linear solvers: features and issues

Main components of an ice model:

Ice flow equations (momentum and mass balance)

$$-\nabla \cdot \sigma = \rho \mathbf{g} \qquad \text{and} \qquad \nabla \cdot \mathbf{u} = 0,$$

with $\sigma = \tau - pI = 2\mu(\dot{\varepsilon})\,\dot{\varepsilon} - pI$,
where μ viscosity, $\dot{\varepsilon}$ shear rate

Model for the evolution of the boundaries (thickness evolution equation)

$$\frac{\partial H}{\partial t} = H_{flux} - \nabla \cdot \int_z \mathbf{u}\, dz$$

Temperature equation

$$\rho c \frac{\partial T}{\partial t} = \frac{\partial}{\partial z}\left(k \frac{\partial T}{\partial z}\right) - \rho c \mathbf{u} \cdot \nabla T + 2\dot{\varepsilon}\sigma$$

Ice Sheet Modeling

"Reference" model: FULL ***STOKES***[1]

Approximations based on their accuracy with respect to the ice-sheet aspect ratio δ

$O(\delta^2)$ ***FO,*** Blatter-Pattyn first order model[2] (3D PDE, in horizontal velocities)

$O(\delta)$ Zeroth order, depth integrated models:
SIA, Shallow Ice Approximation (slow sliding regimes) ,
SSA Shallow Shelf Approximation (2D PDE) (fast sliding regimes)

$\simeq O(\delta^2)$ High order, depth integrated (2D) models: ***L1L2***[3], (L1L1)...

[1]Gagliardini and Zwinger, 2008. The Cryosphere.
[2]Dukowicz, Price and Lipscomb, 2010. J. Glaciol.
[3]Schoof and Hindmarsh, 2010. Q. J. Mech. Appl. Math.

Implementation Overview

ice-sheets FE dynamics component

LifeV: Parallel, object oriented, C++ Finite Element Library:

- linear and quadratic finite elements
- assembling of finite element matrices
- handling of boundary conditions

Implementation Overview

Trilinos:

- Parallel Data Structures (EPETRA)
- Parallel Linear Solvers (GMRES, CG...)
- Preconditioners (Multilevel, Multigrid, Incomplete LU)
- Nonlinear Solvers (**NOX** package: Newton, JFNK methods)

⇧

ice-sheets FE dynamics component

LifeV: Parallel, object oriented, C++ Finite Element Library:

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⇧ **Work in Progress**

MPAS (land ice component):

- Voronoi unstructured grids
- Evolution equation solvers (temperature and thickness equation)
- ...

Interface MPAS-LIFEV

MPAS

LIFEV

Land ice component

2D CVT mesh
(Stereographic projection)

thickness/elevation/layers

temperature/ice flow factor

bedrock sliding coefficient

Solver options:
model (FO, L1L2, SSA, SIA)
nonlinear solver (Newton, Picard, JFNK)
Boundary condition (free-slip, no-slip, robin, coulomb)

→

←

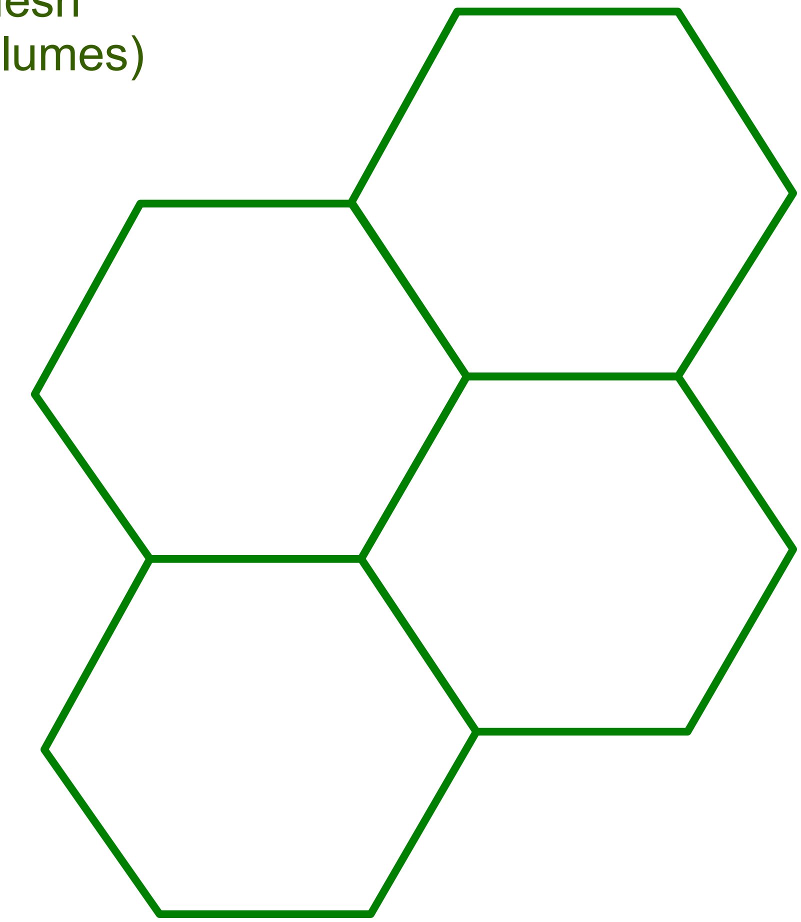
velocity

heat dissipation

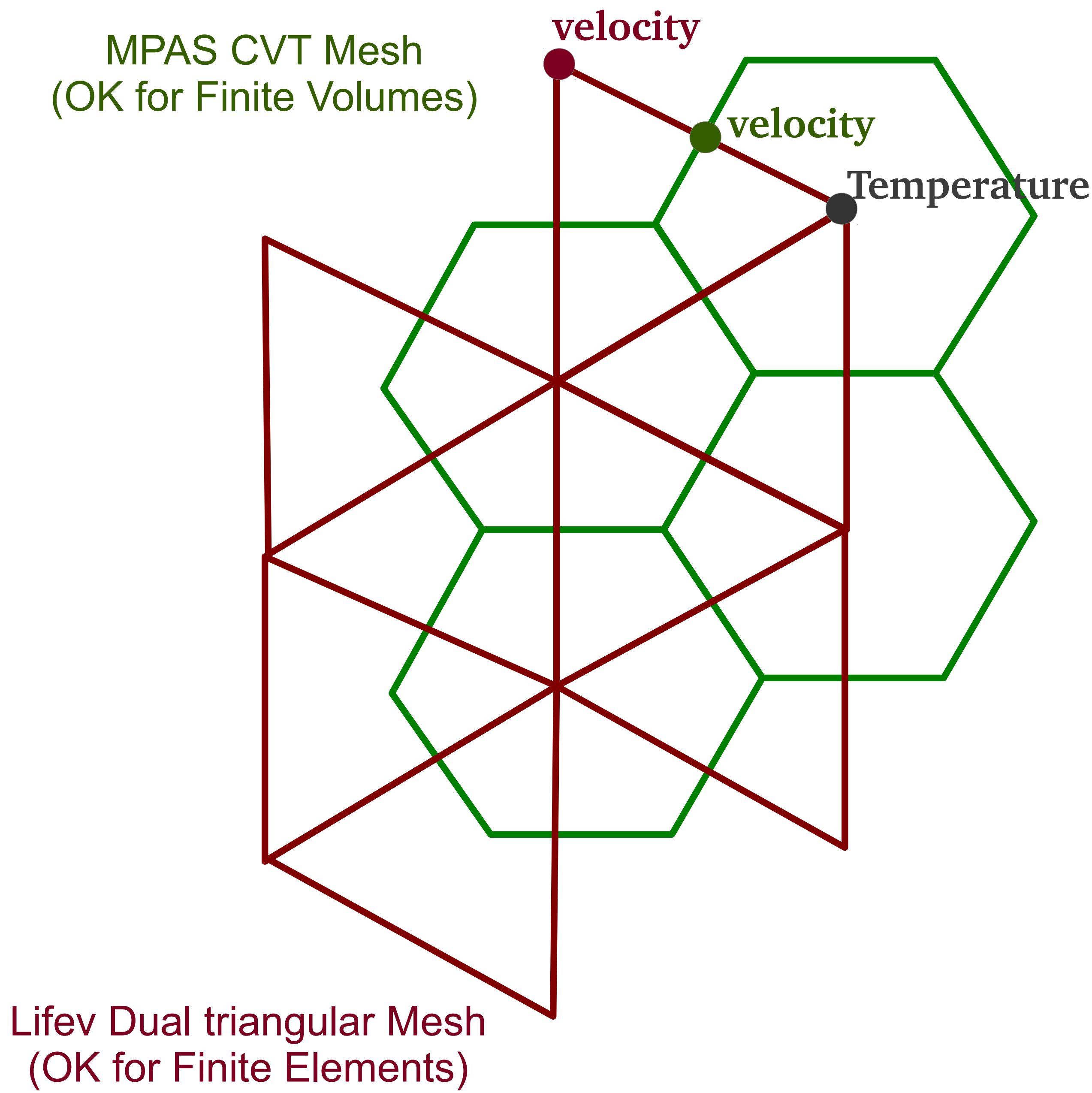
viscosity

ice-sheets component

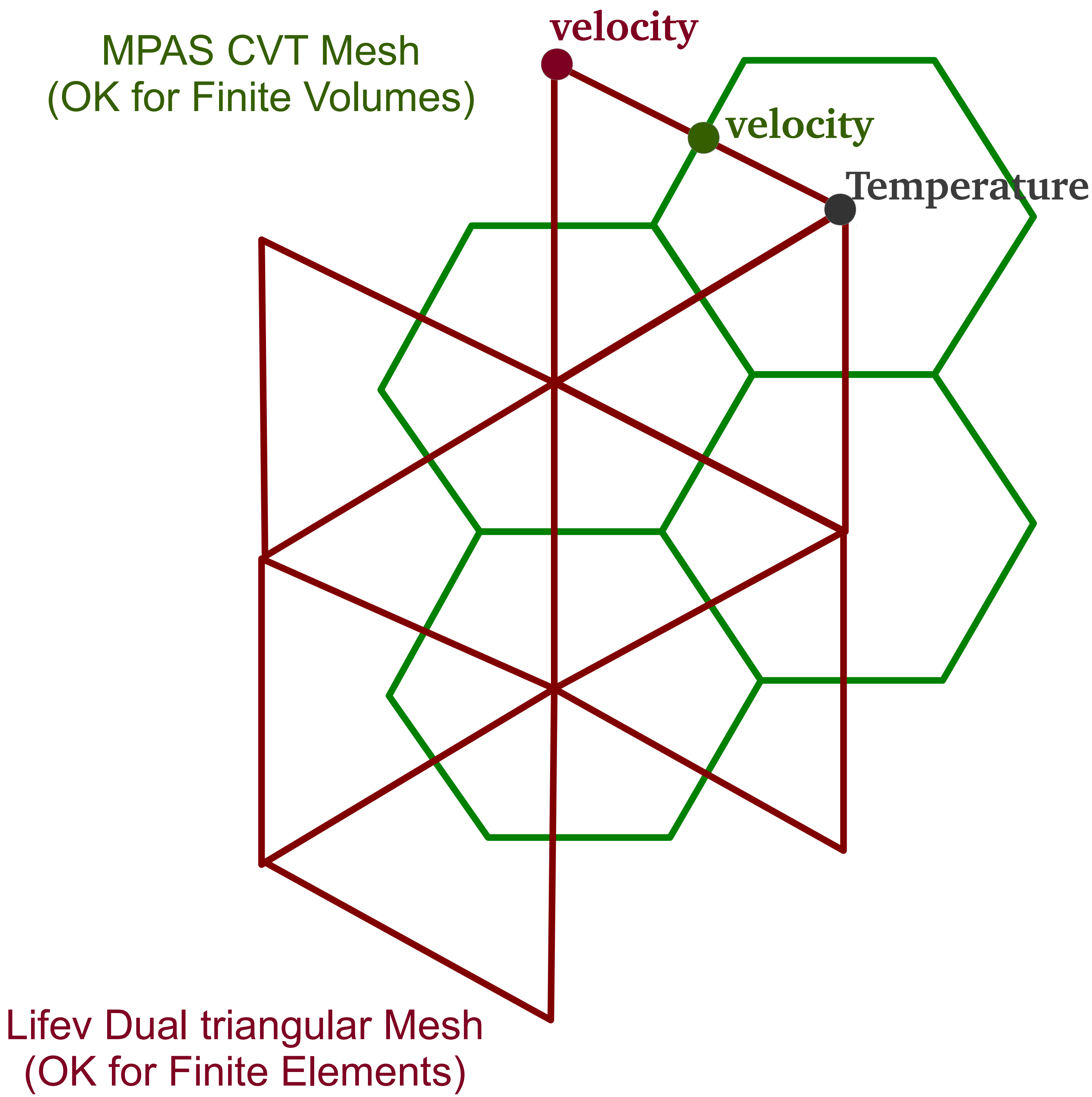
MPAS CVT Mesh
(OK for Finite Volumes)



Interface - Grids

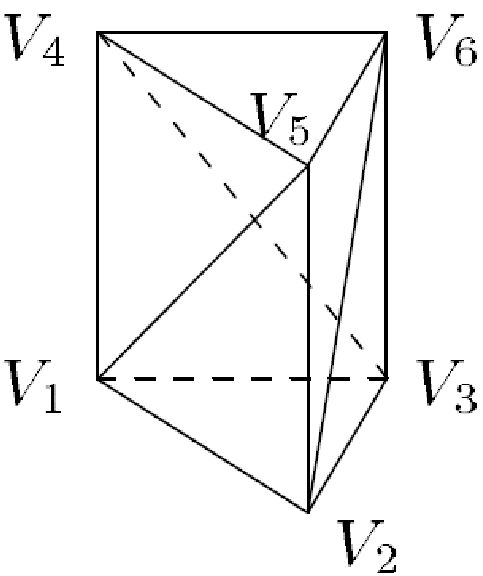


Interface - Grids



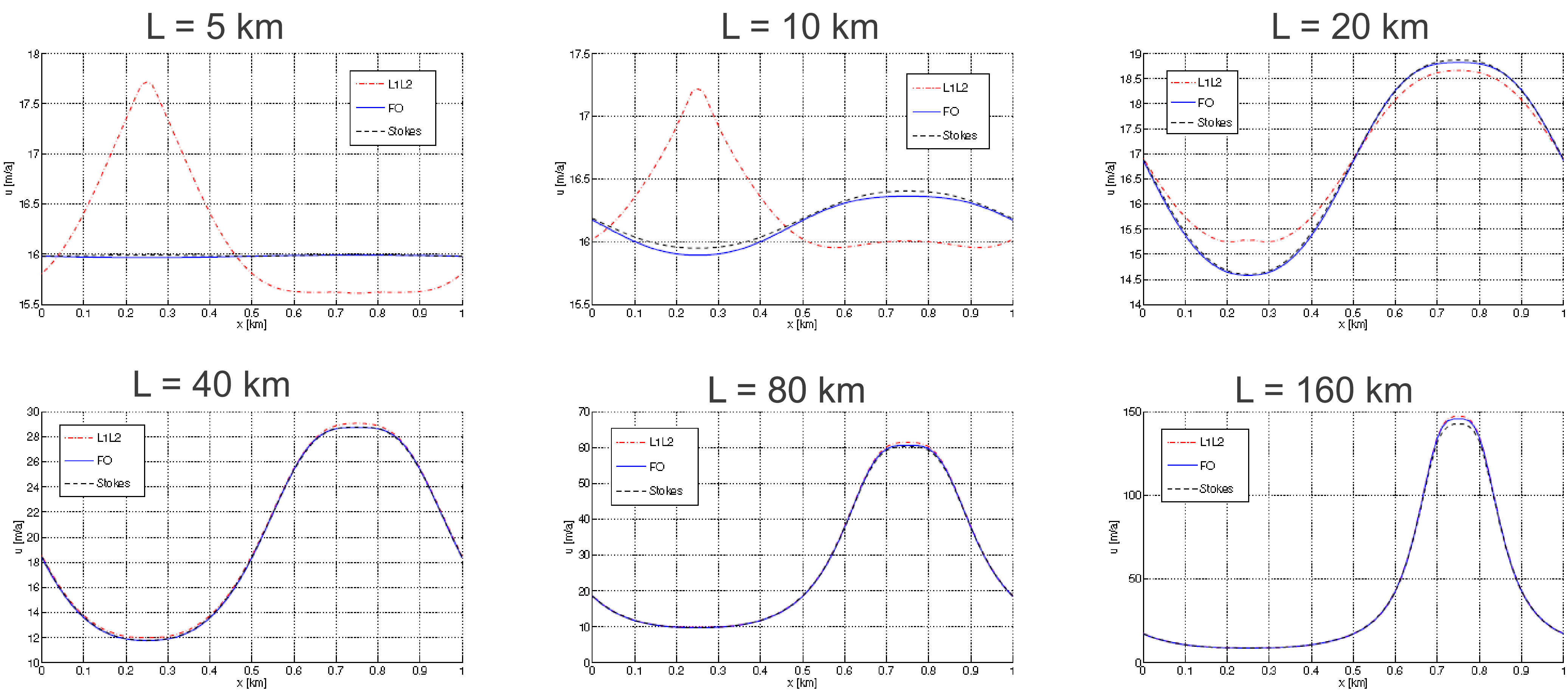
Based on 2D grid and thickness and layers build vertically structured **3D grid.**

Build prisms with triangular base and split them in tetrahedra.



ISMIP-HOM: Test C

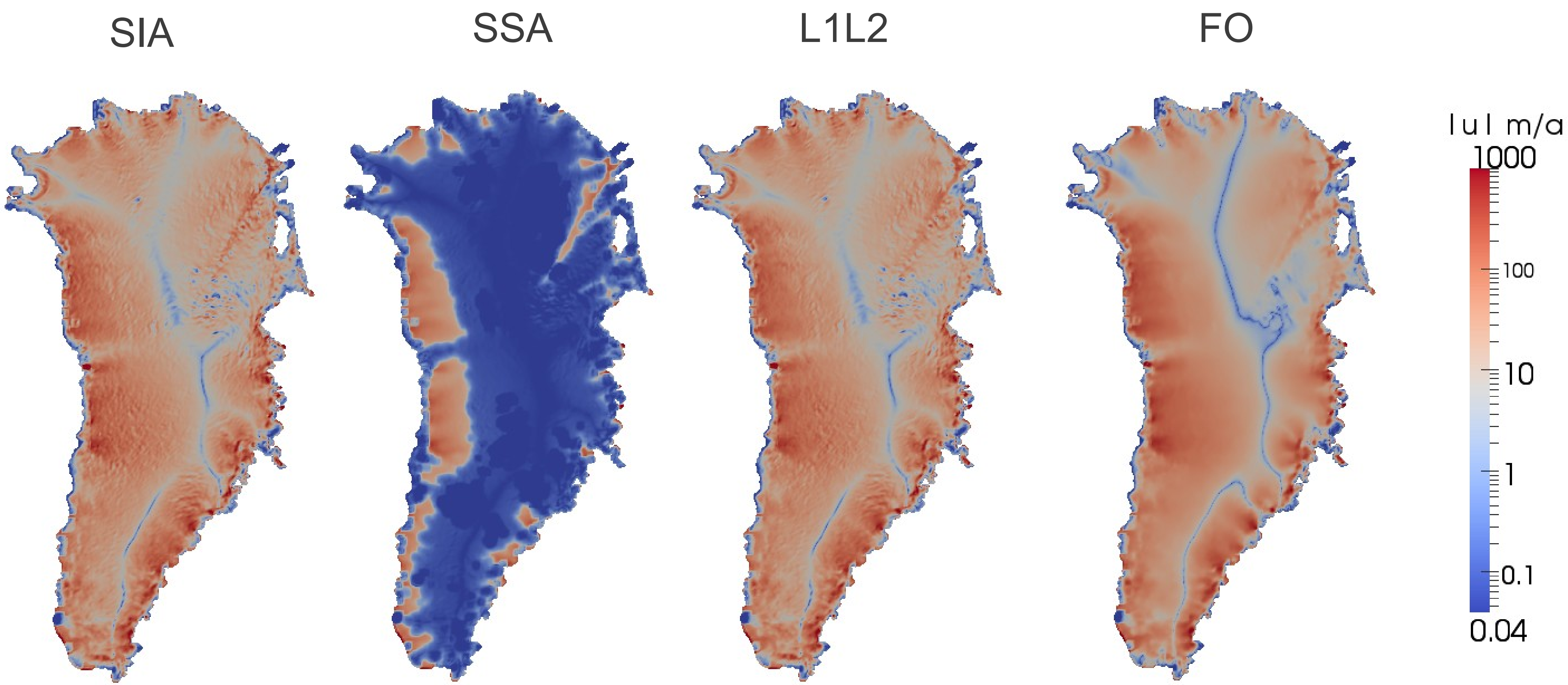
Surface velocity



Perego, Gunzburger, Burkardt, Journal of Glaciology, 2012.

Towards realistic simulations:
Greenland, 5km resolution, sliding case

Comparisons between different models



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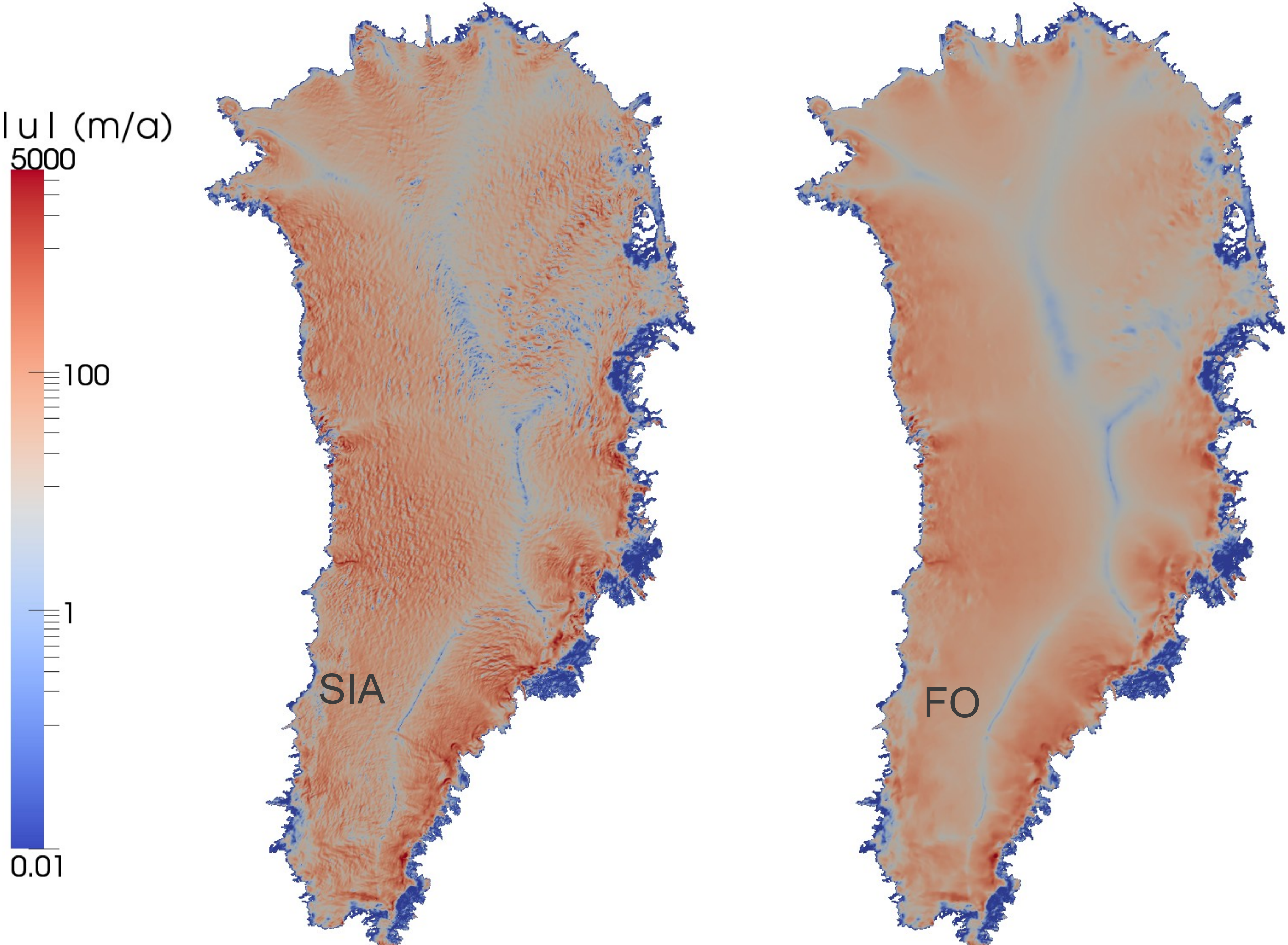
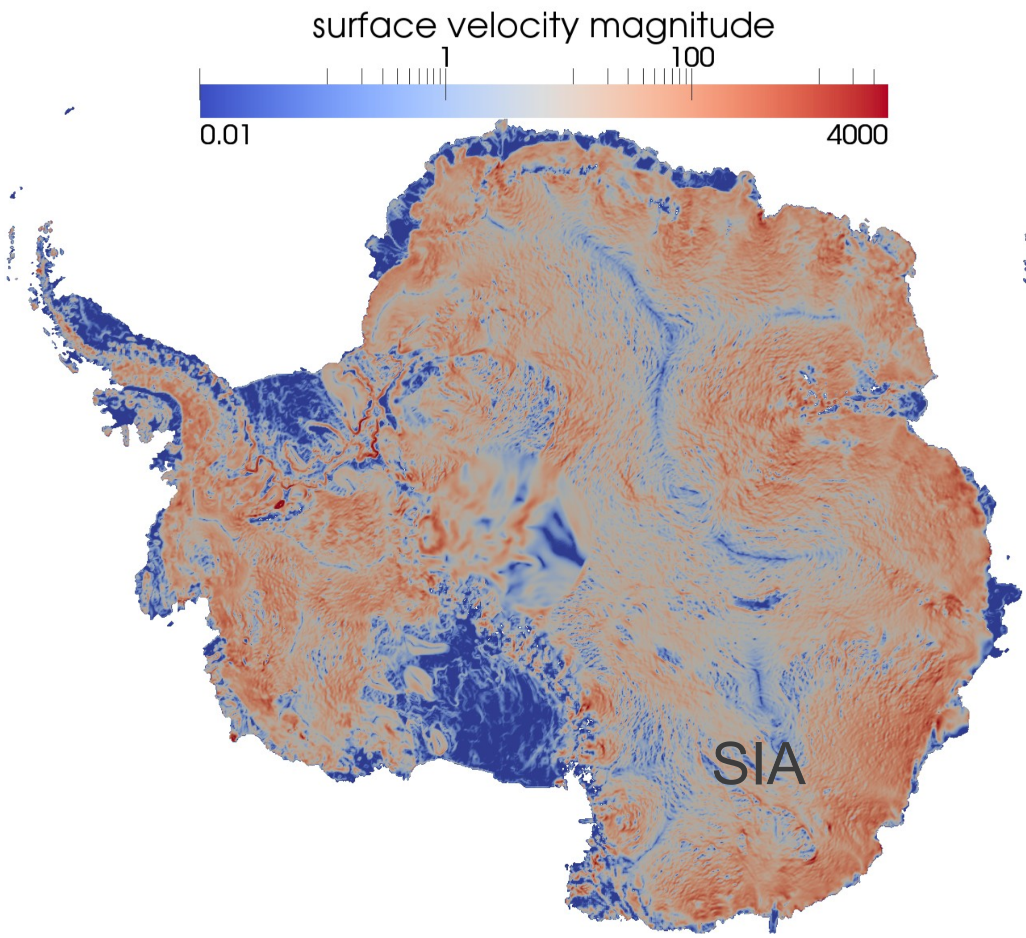
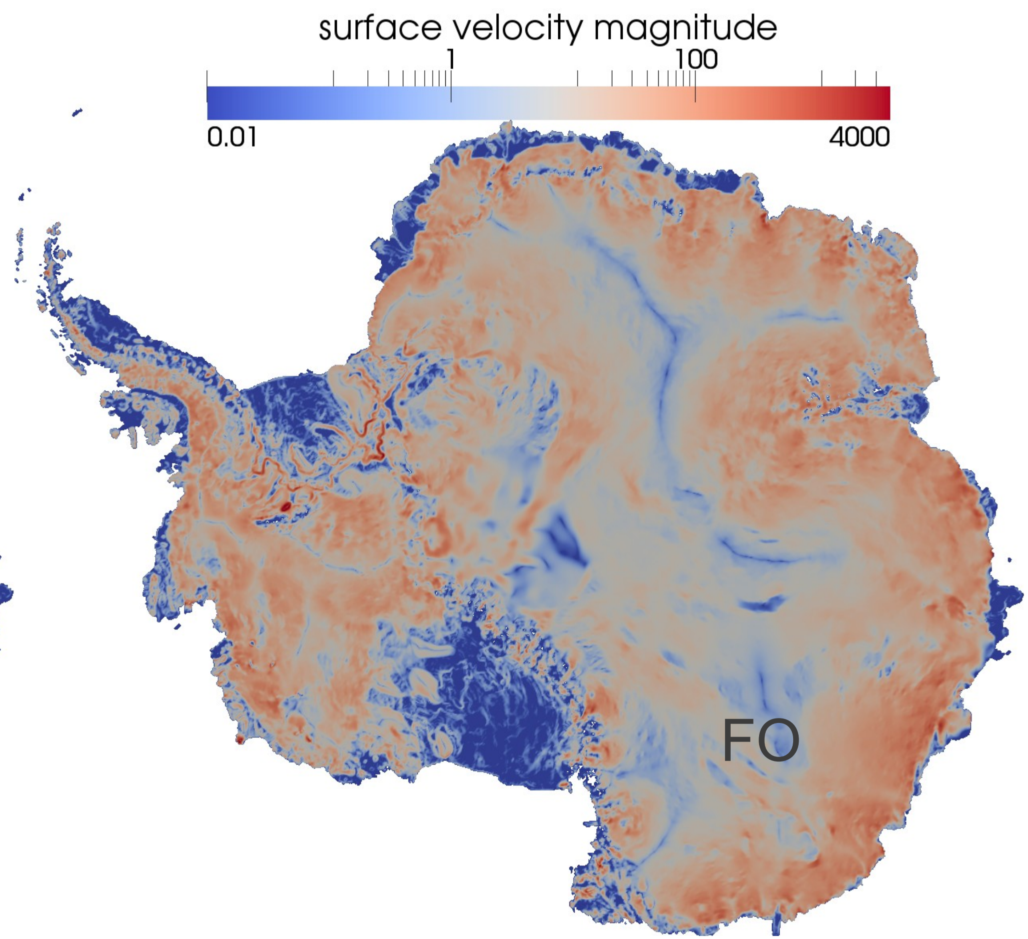
Towards realistic simulations:
Greenland, 2km resolution, no-slip



M.Perego, mperego@fsu.edu

Towards realistic simulations:
Antarctica, 5km resolution, no-slip





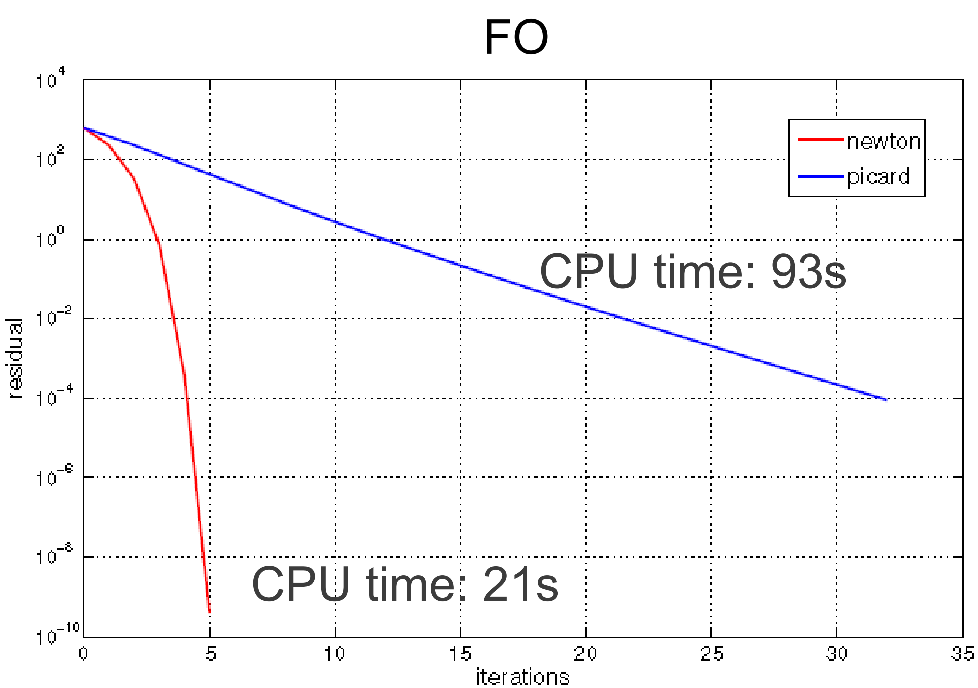
Convergence of the nonlinear method (test C ISMIP-HOM)

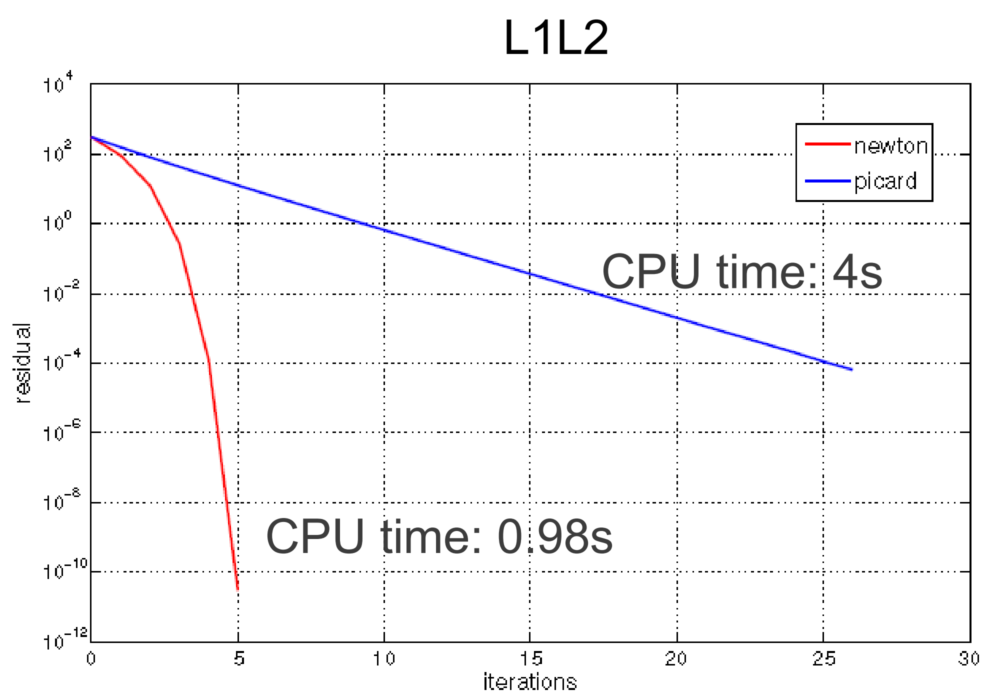
Solve $F(u) = 0.$

Newton method: $J^k\,(\delta u)^{k+1} = -F(u^k).$ (NOX library)

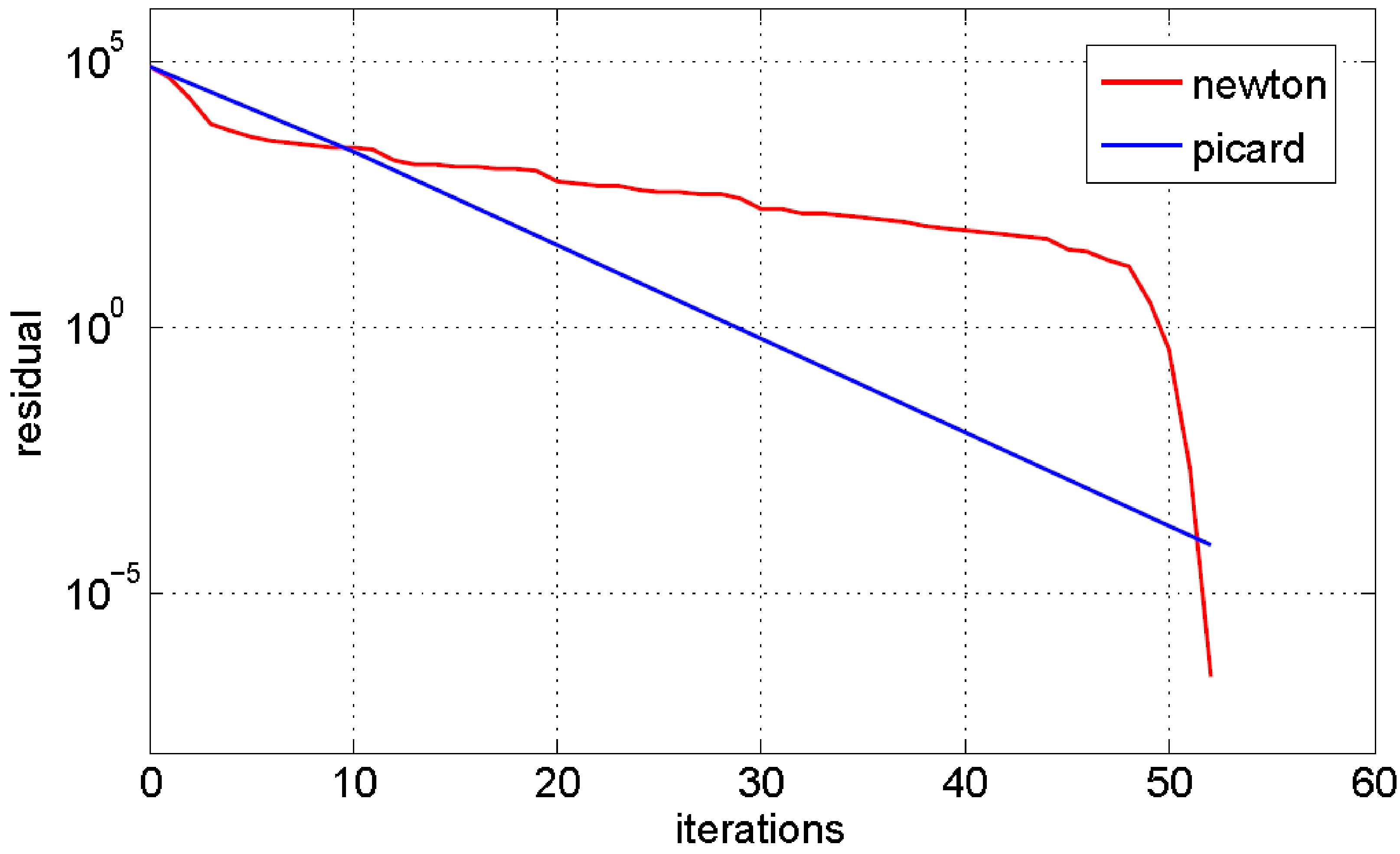
The exact Jacobian matrix must be assembled at each nonlinear iteration.

In order to increase robustness of Newton method, the Newton step is halved to achieve monotonic convergence.





Greenland, FO: convergence of the nonlinear method



First Order Equations

FO is a nonlinear system of elliptic equations in the horizontal velocities:

$$\begin{cases} -\nabla \cdot (2\mu\, \dot{\boldsymbol{\varepsilon}}_1) = -\rho g \dfrac{\partial s}{\partial x} \\ -\nabla \cdot (2\mu\, \dot{\boldsymbol{\varepsilon}}_2) = -\rho g \dfrac{\partial s}{\partial y}, \end{cases}$$

$$\mu = \frac{1}{2} A^{-\frac{1}{n}} \dot{\varepsilon}_e^{\left(\frac{1}{n}-1\right)}$$

$$\dot{\varepsilon}_e = \sqrt{\dot{\varepsilon}_{xx}^2 + \dot{\varepsilon}_{yy}^2 + \dot{\varepsilon}_{xx}\dot{\varepsilon}_{yy} + \dot{\varepsilon}_{xy}^2 + \dot{\varepsilon}_{xz}^2 + \dot{\varepsilon}_{yz}^2}$$

where s is the ice surface and,

$$\dot{\varepsilon}_{i,j} = \frac{1}{2}\left(\partial_j u_i + \partial_i u_j\right), \quad i,j \in \{x,y,z\}, \quad \dot{\boldsymbol{\varepsilon}}_1 = \begin{bmatrix} 2\dot{\varepsilon}_{xx} + \dot{\varepsilon}_{yy} \\ \dot{\varepsilon}_{xy} \\ \dot{\varepsilon}_{xz} \end{bmatrix}, \quad \dot{\boldsymbol{\varepsilon}}_2 = \begin{bmatrix} \dot{\varepsilon}_{yx} \\ \dot{\varepsilon}_{xx} + 2\dot{\varepsilon}_{yy} \\ \dot{\varepsilon}_{yz} \end{bmatrix}$$

Remark The nonlinear viscosity μ is singular when $\dot{\varepsilon}_e = 0$, however, $\mu\, \dot{\boldsymbol{\varepsilon}}_1$ is not singular and the PDE is well defined.

Viscosity[1] regularization: $$\dot{\varepsilon}_e^{-\left(1-\frac{1}{n}\right)} \approx \left(\sqrt{\dot{\varepsilon}_e^2 + \delta^2}\right)^{-\left(1-\frac{1}{n}\right)}$$

[1] *Goldsby D. L. Kohlsted, JGR, 2001.*

Greenland, FO: convergence of the nonlinear method

Simplified Problem (similar kind of nonlinearity):

$$x\,|x|^{-\left(1-\frac{1}{n}\right)} = C. \qquad \text{solution: } \alpha = C|C|^{n-1}$$

Picard method: $x^{k+1} = C|x^k|^{\left(1-\frac{1}{n}\right)}$.

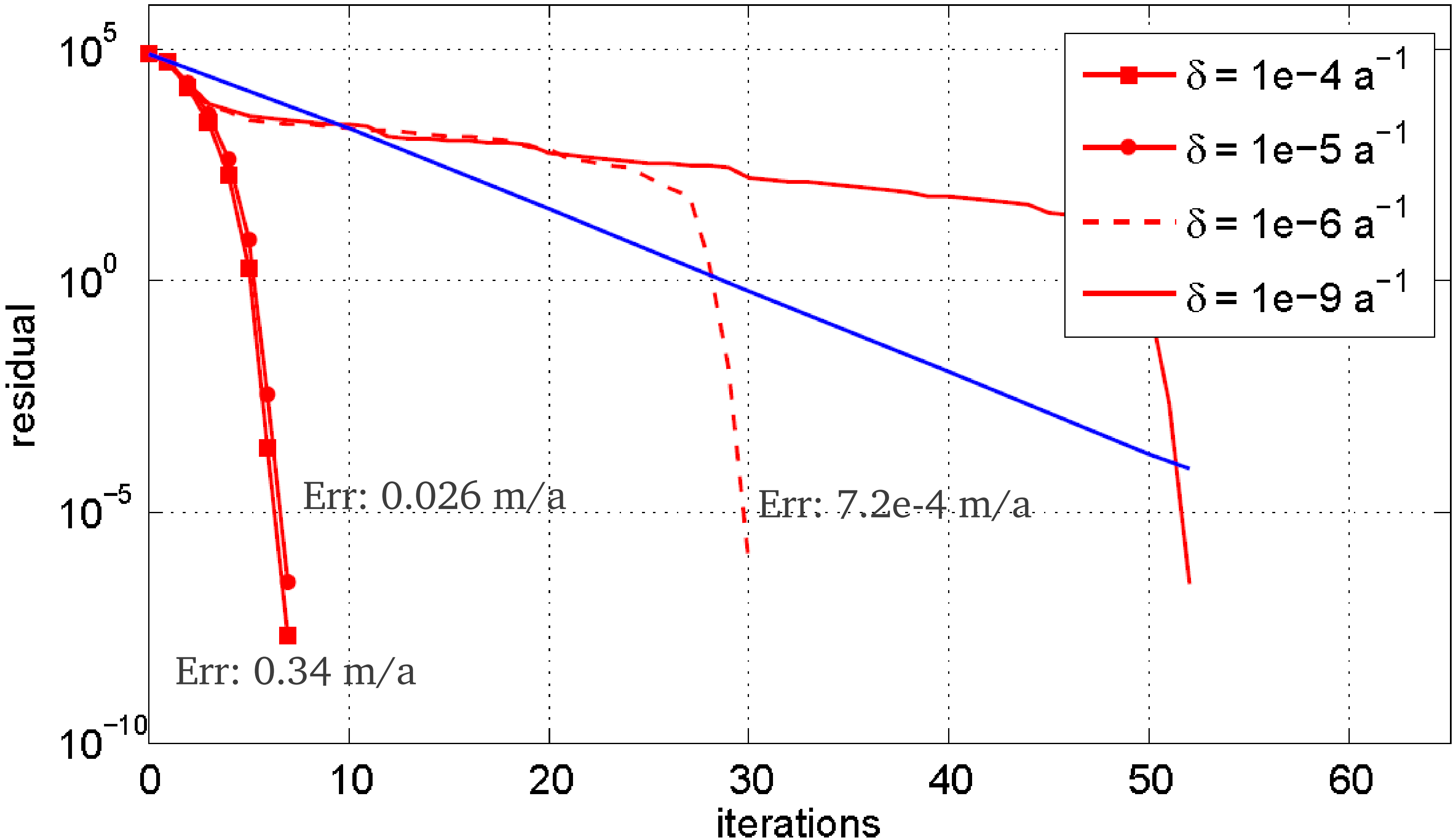
Newton method: $x^{k+1} = (1-n)x^k + nC|x^k|^{\left(1-\frac{1}{n}\right)}$.

Picard convergence: $\displaystyle\lim_{k\to\infty} \frac{x^{k+1}-\alpha}{x^k-\alpha} = 1 - \frac{1}{n} \quad \left(= \frac{2}{3} \text{ when } n = 3\right)$

Newton convergence: $\displaystyle\lim_{k\to\infty} \frac{x^{k+1}-\alpha}{(x^k-\alpha)^2} = \frac{1}{2\alpha}\left(\frac{1}{n}-1\right)$

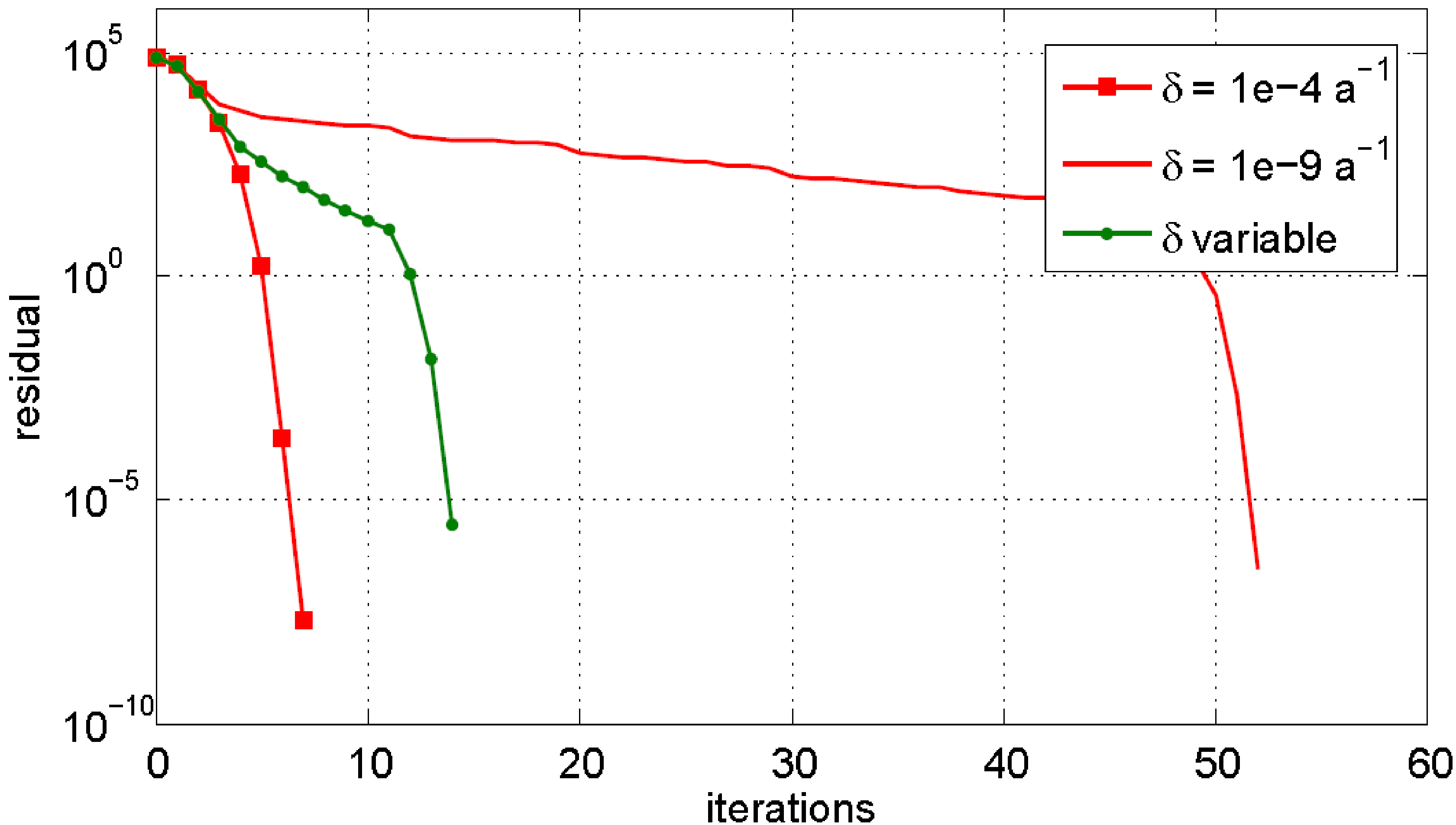
regularization: $\displaystyle x\,|x|^{-\left(1-\frac{1}{n}\right)} \approx x\,\left(\sqrt{x^2+\delta^2}\right)^{-\left(1-\frac{1}{n}\right)}$

Greenland, FO: convergence of the nonlinear method (comparison using different regularizations)



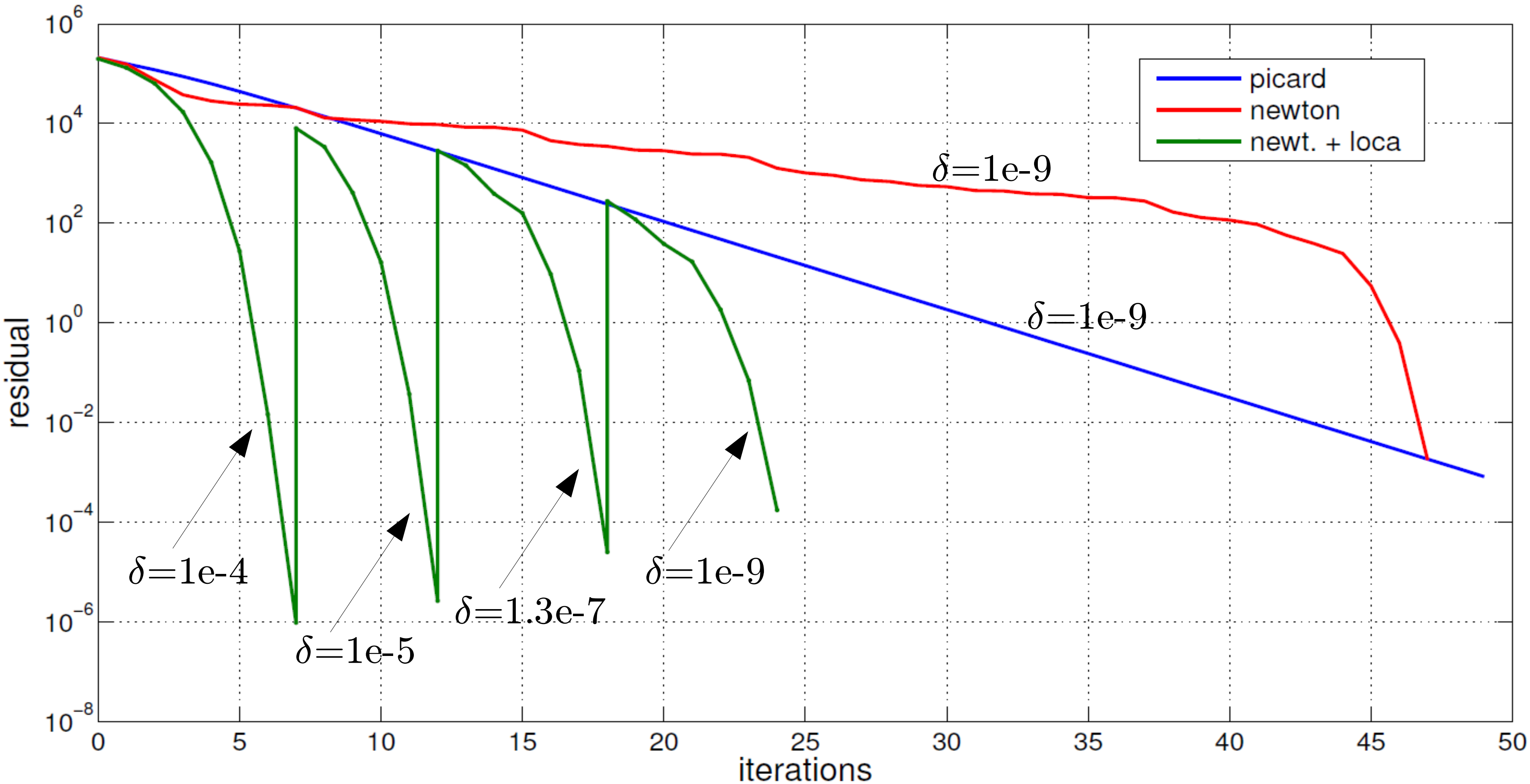
Viscosity regularization: $\dot{\varepsilon}_e^{-\left(1-\frac{1}{n}\right)} \approx \left(\sqrt{\dot{\varepsilon}_e^2 + \delta^2}\right)^{-\left(1-\frac{1}{n}\right)}$

Greenland, FO: convergence of the nonlinear method (comparison using different regularizations)



δ variable : At each newton iteration δ is decreased from 1e-4 to 1e-9

Greenland, FO: convergence of the nonlinear method (LOCA continuation method)



The parameter δ is decreased by LOCA from 1e-4 to 1e-9

Acknowledgment

J. Burkardt, M. Gunzburger (FSU)

B. Lipscomb, S. Price, X. Asay-Davis, T. Ringler (LANL)

K. Evans, J. Nichols (ORNL)

A. Salinger (Sandia)