



*A Simple Inverse Method for Deducing the Large-Scale Distribution of Basal Sliding Coefficients beneath the Antarctic Ice Sheet*

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A satellite view of Earth's surface, showing a mix of blue oceans and brownish-green landmasses. A white rounded rectangle with a red border is overlaid on the image, containing a list of five items.

## *Outline*

1. Motivation, previous work
2. Simple inverse method # 1
3. Simple inverse method # 2 (with basal temperature)
4. Comparison with previous work
5. Summary

# Motivation

- Ice sheet geometry is sensitive to basal boundary conditions, mainly deformable sediment ( $C=10^{-5}$ ) vs. hard bedrock ( $C=10^{-10}$ )

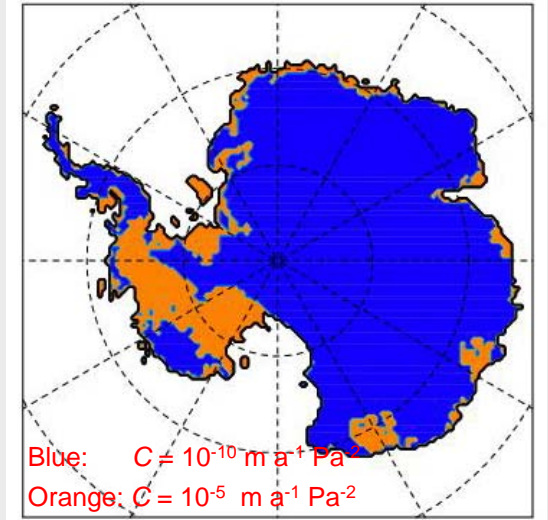
$$u_b = C(x,y) f(T_b) \tau_b^2$$

- Primary cause of O(500 m) elevation errors in Antarctic continental paleo ice-sheet models?

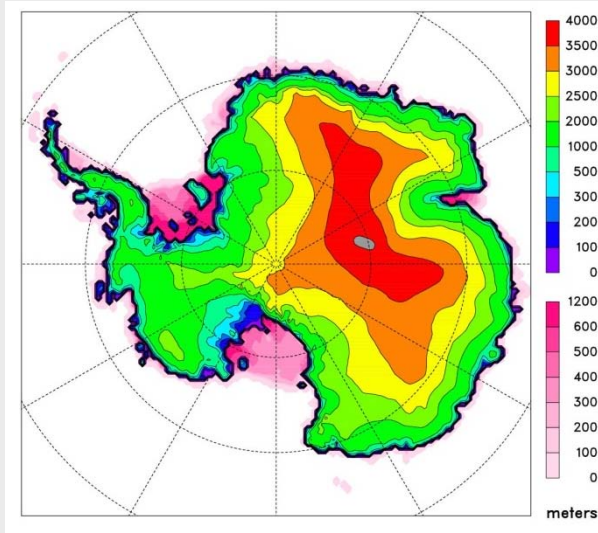
where

$u_b$  = basal ice velocity,  
 $\tau_b$  = basal shear stress ,  
 $T_b$  = basal temperature,  
 $f(T_b) = 0$  if bed is frozen,  
 1 if bed is at melt point

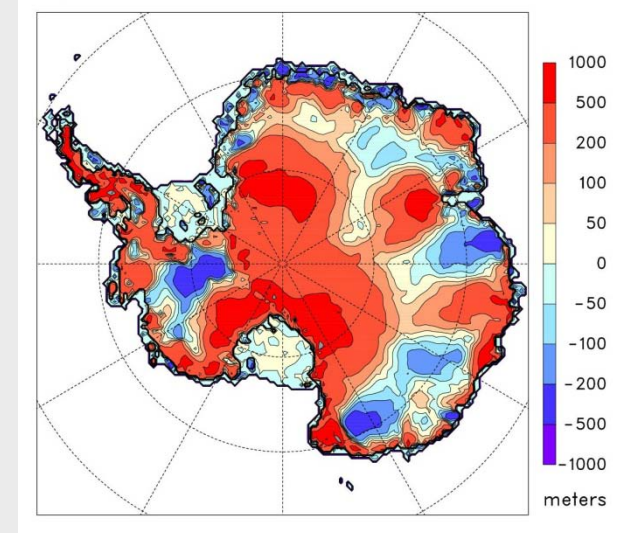
crude  $C(x,y)$  map:  
 sediment if rebounded bed is below sea level, hard bedrock if above



Observed surface elevation



Model minus observed surface elevation



# Typical surface elevation or thickness errors In continental (paleo) Antarctic models

Ritz et al.,  
JGR, 2001

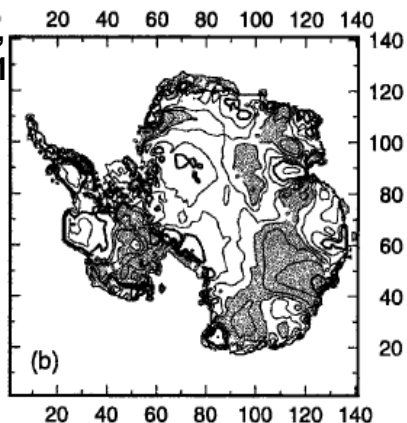
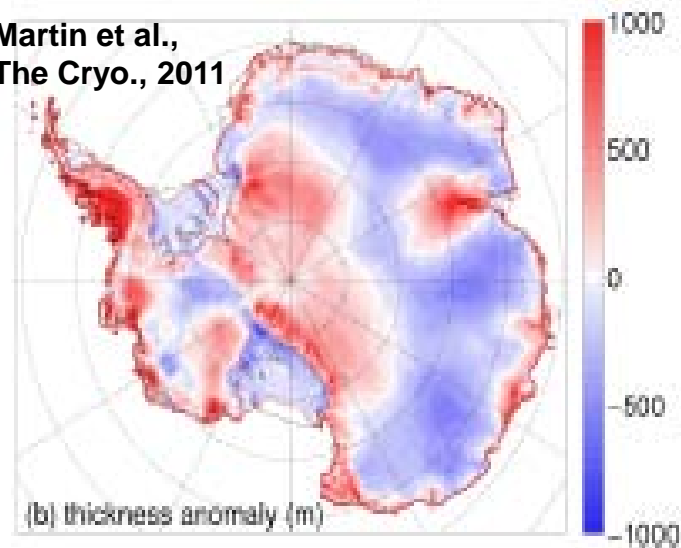


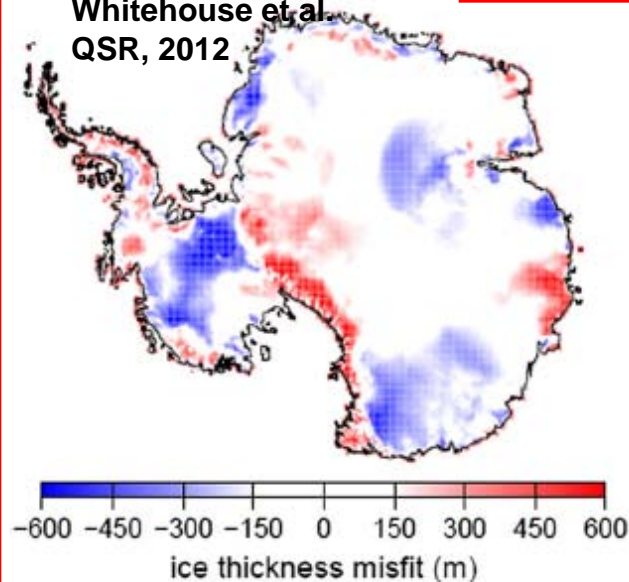
Figure 4. Comparison between observed and modeled surface elevation for the present-day geometry of the grounded ice. (a) Elevation contours: Continuous lines represent observed configuration and dotted lines represent modeled configuration. Thick lines represent grounding lines, thin lines represent isolines for every 1000 m intervals. (b) Altitude difference map: in grey the modeled surface is below the observed one (negative isolines). Isolines are for -1000, -500 (thick line), -250, -100, 0, 100, 250, 500 (thick line) and 1000 m. Horizontal scales are as in Figure 3.

Modern  
surface  
elevations,  
model minus  
observed

Martin et al.,  
The Cryo., 2011



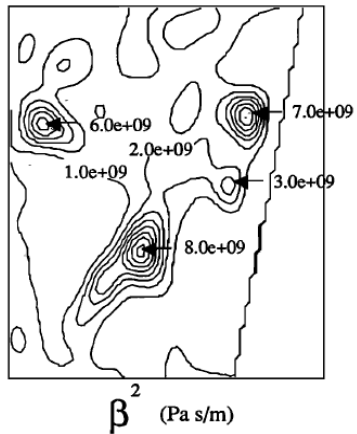
Whitehouse et al.  
QSR, 2012



# Previous basal inversions for Antarctica

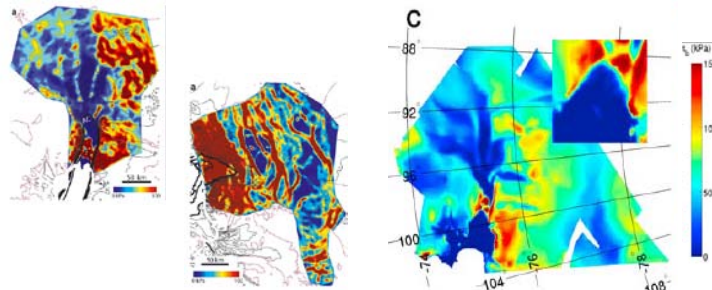
- Previous work has deduced basal-stress or sliding-coefficient maps using control theory (Lagrangian multiplier/adjoint) methods, fitting modeled vs. observed velocities, with ice geometry (thickness, elevation) fixed from observations.
- **Regional:** MacAyeal, 1992; Vieli and Payne, 2003; Joughin et al. 2009; Morlighem et al., 2010.  
**Continental:** ISSM, Larour et al., ISSM, [issm.jpl.nasa.gov](http://issm.jpl.nasa.gov); Bueler et al., PISM, [www.pism-docs.org](http://www.pism-docs.org).  
Also Price et al. (PNAS, 2011), Greenland, local method.

Ice Stream E  
(MacAyeal, 1992):



Basal drag coefficient, Ice Stream E. Macayéal JGR, 1992.

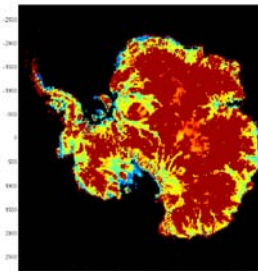
Pine Island and Thwaites Glaciers  
(Joughin et al., 2009; Morlighem et al., 2010):



Basal stress, Pine Isl;and and Thwaites Glaciers. Joughin et al., J. Glac., 2009

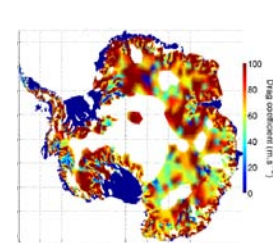
Basal stress, Pine Island GI: Morlighem et al., GRL, 2010

PISM (U. Alaska):



PISM basal drag coefficient ( $\text{Pa s m}^{-1}$ ). Lingle et al., JPL PARCA meeting, 2007

ISSM (JPL):



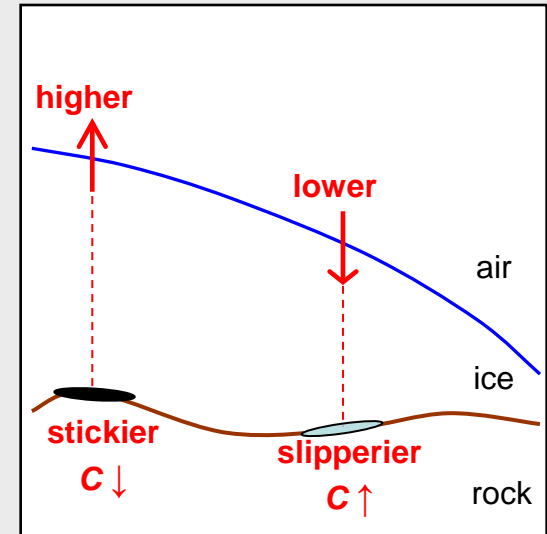
ISSM basal drag coefficient ( $\text{ms}^{-1/2}$ ). Larour et al., JPL PARCA meeting, 2009

# The Inversion Method

- Very simple procedure to deduce basal sliding coefficients  $C(x,y)$ , fitting to observed ice geometry (surface elevation).
- Run model forward, and every 5 kyrs adjust  $C$  locally depending on current ice surface elevation mismatch with observed

## Details:

- Every 5000 years, decrease (stiffen)  $C(x,y)$  if the *local* ice surface is too low, or increase (soften)  $C(x,y)$  if *local* surface is too high:
  - $C_{new} = C 10^{\Delta z / 500}$   
where  $\Delta z = \text{model} - \text{observed surface elevation (m)}$
  - Constrain  $C$  to remain in range  $10^{-20}$  to  $10^{-5} \text{ m a}^{-1} \text{ Pa}^{-2}$
- Run model forward for ~200,000 years until convergence

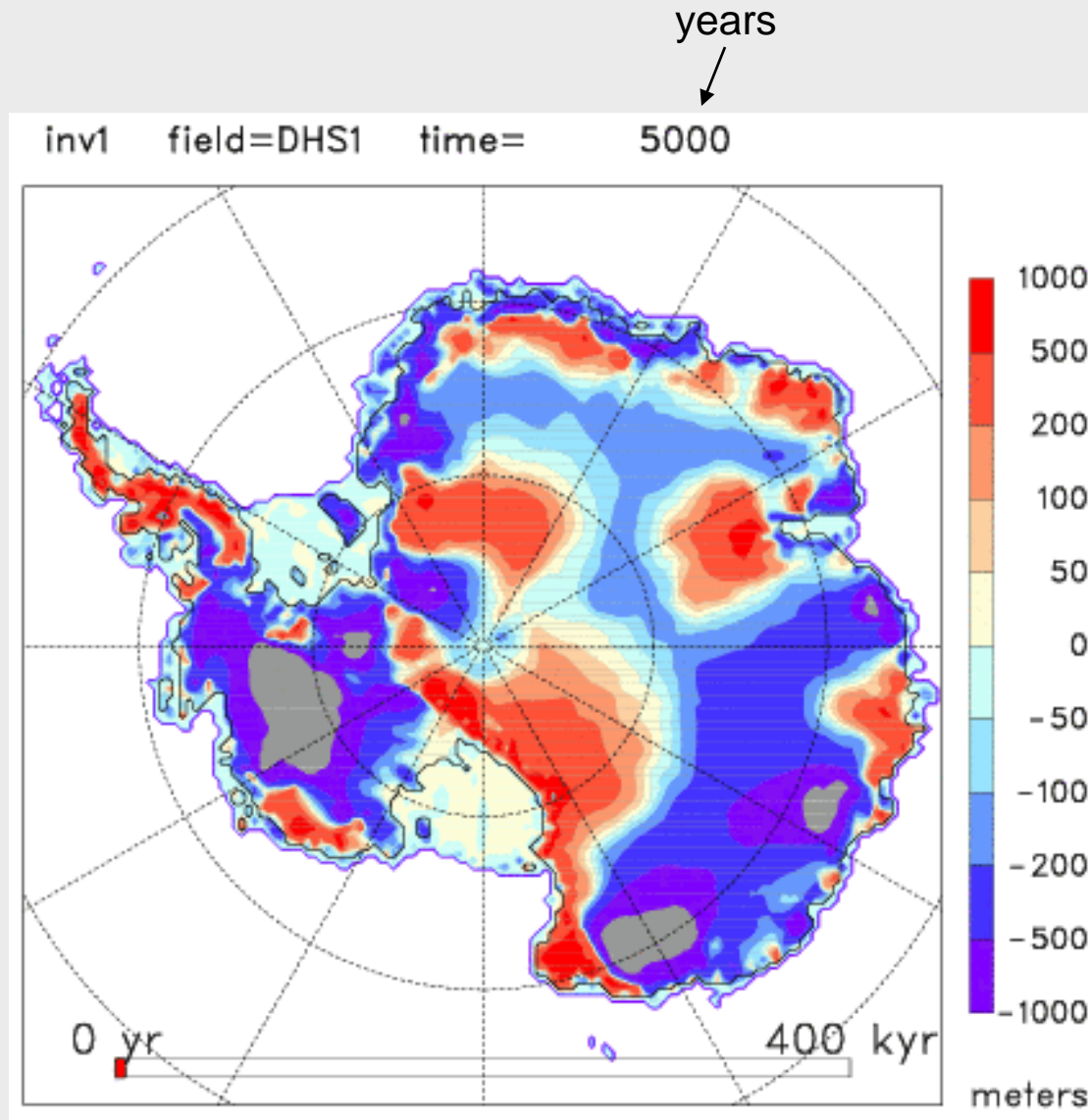


Ignore  $\partial/\partial x$ ,  $\partial/\partial y$ 's....as if effects are local

Ignore all other potentially canceling model errors ! (e.g. internal deformation  $\partial u/\partial z$ )

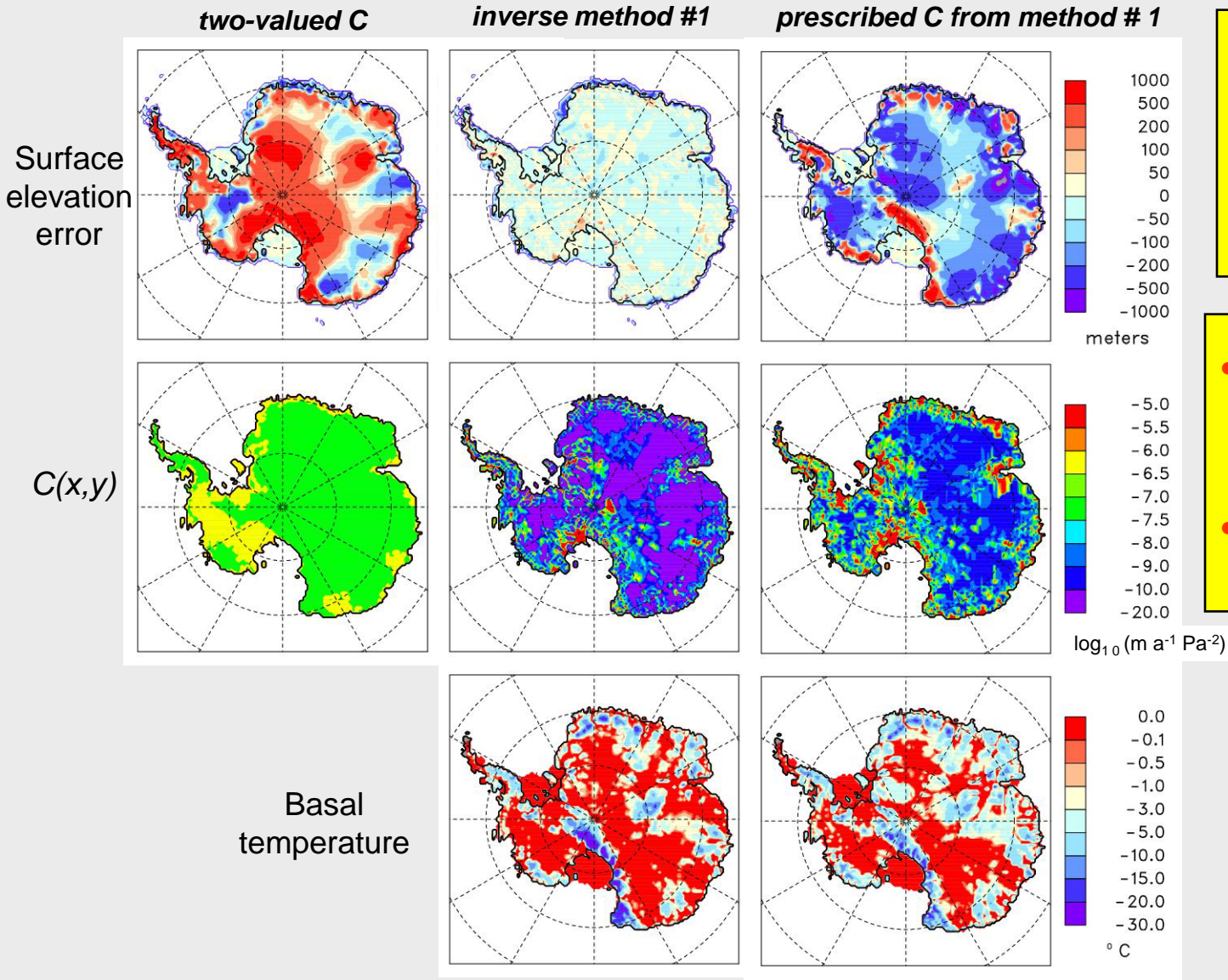
Ignore GIA - assume modern ice sheet is in equilibrium

# Spinup in a 400,000 year run. The method converges!



Model surface  
elevations minus  
observed

# Results of Method # 1 (no basal temperature effect)



- Turn off effect of basal temperature on sliding
- Allow minimum  $C = 10^{-20}$ , so inverse procedure can find "frozen" (stuck) areas

- But when run full model with  $C(x,y)$  prescribed, frozen areas differ from inverse-deduced stuck areas.
- Large surface elevation errors re-occur.



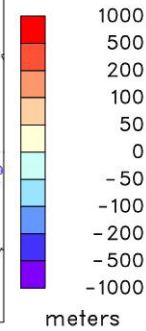
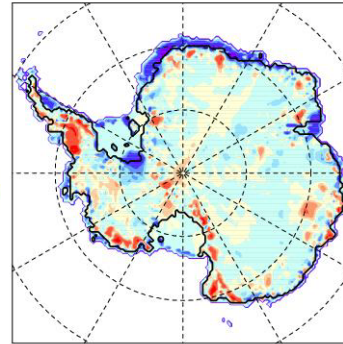
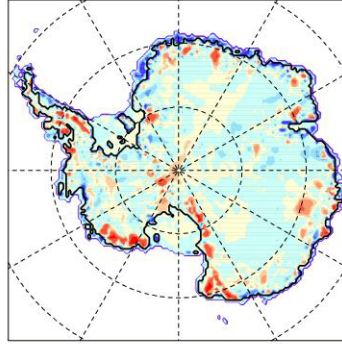
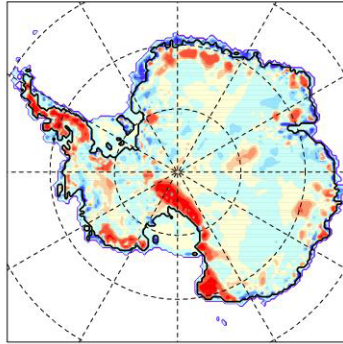
# Results of Method # 2 (with basal temperature effect)

inverse method # 2

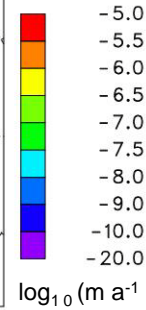
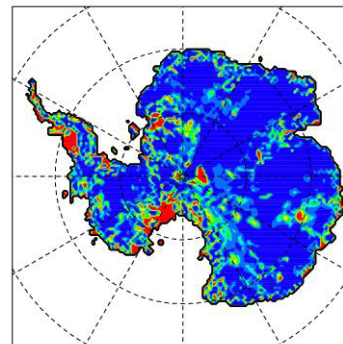
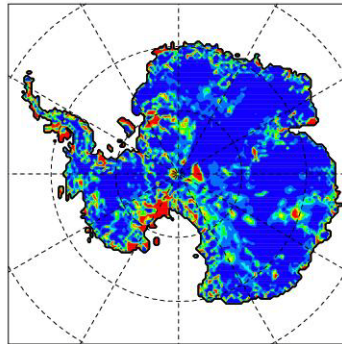
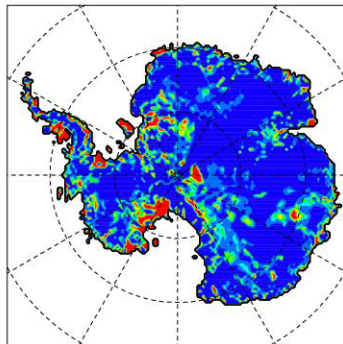
2<sup>nd</sup> method + s.a.

full model, prescribed C (2<sup>nd</sup> + s.a.)

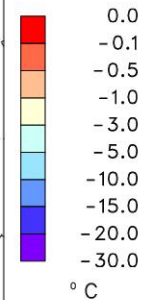
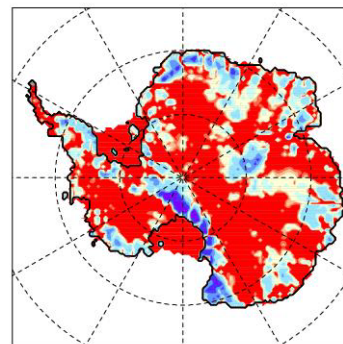
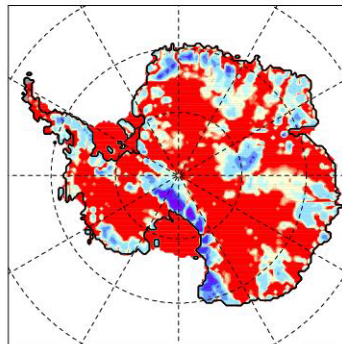
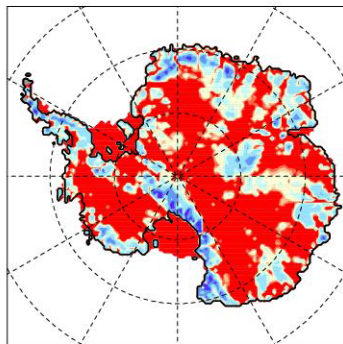
$\Delta h_s$



$C(x,y)$



$T_b$

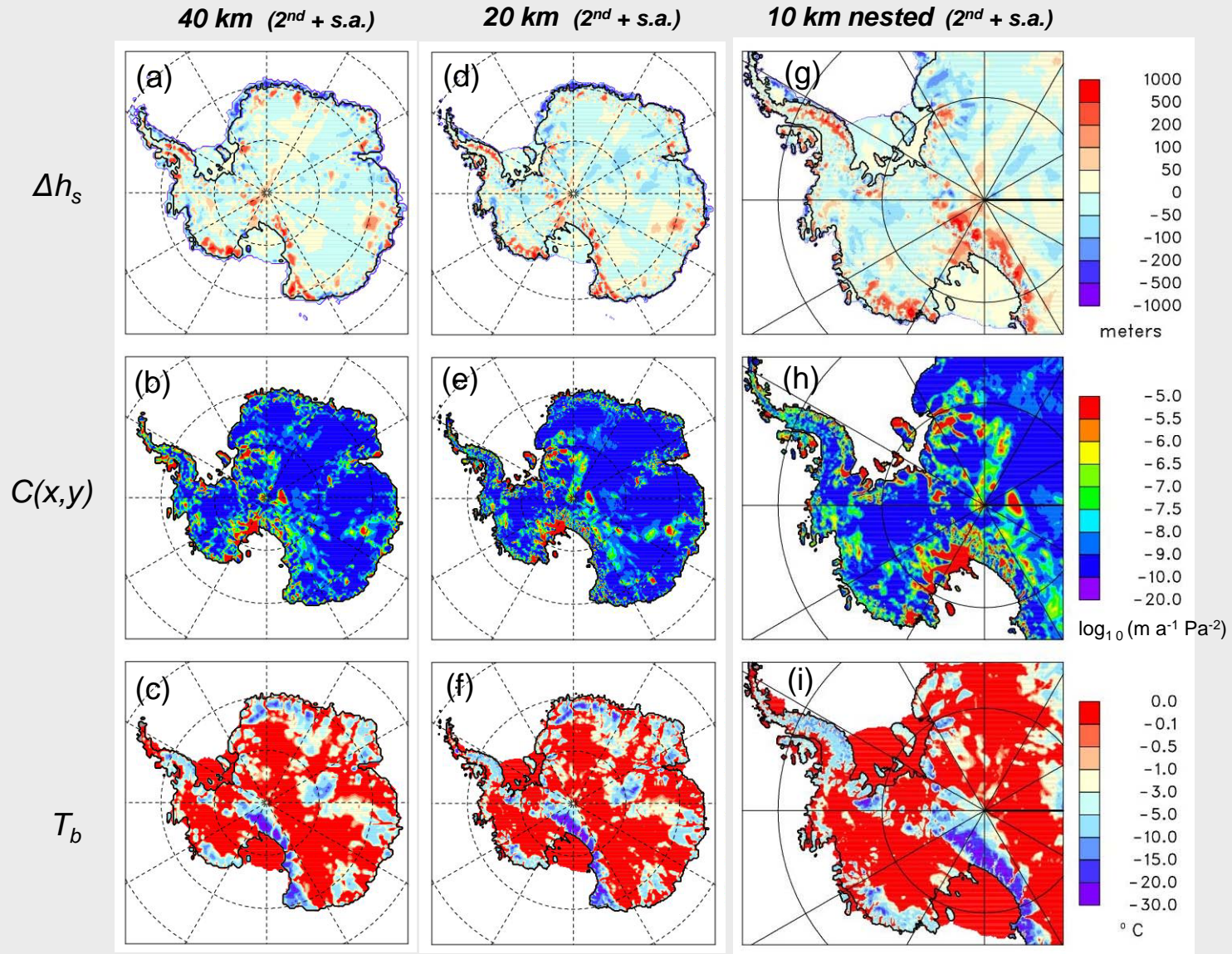


- $T_b$  affects sliding during inversion procedure
- Minimum  $C = 10^{-10}$  (hard bedrock)
- Guarantees same results when full model is run with resulting  $C(x,y)$  prescribed

- Remaining errors over mountain ranges
- Reduce further by modifying  $f(T_b)$  in sliding law, using sub-grid slope amplitude  $s.a.$

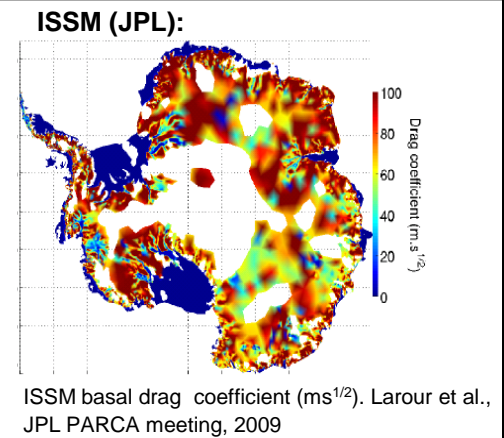
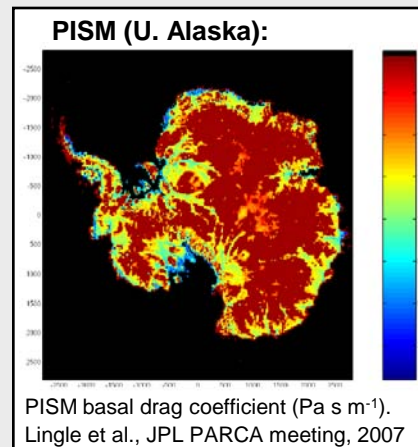
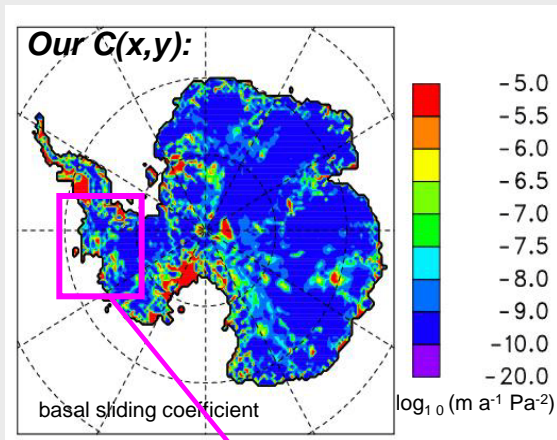
$$u_b = C(x,y) f(T_b, s.a.) \tau_b^2$$

# Different grid resolutions: results are ~unchanged

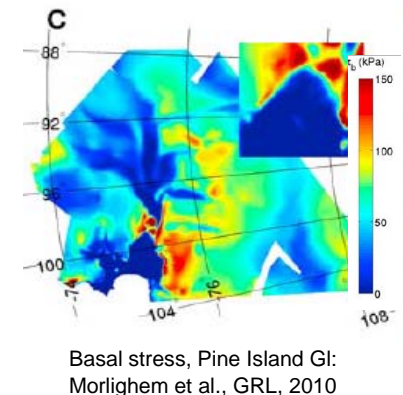
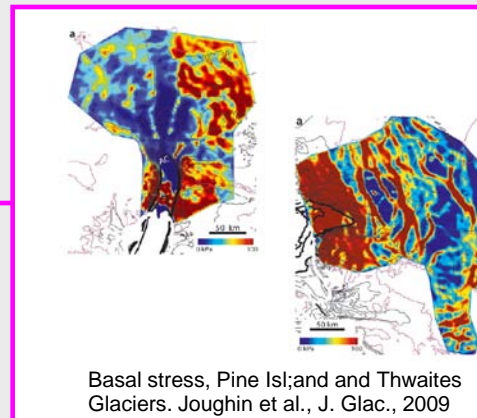
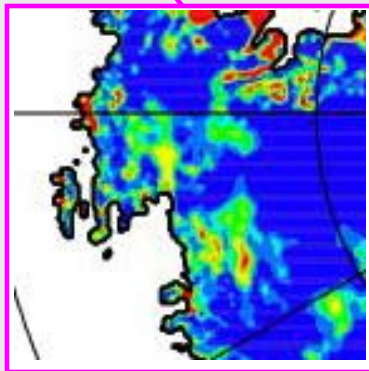


# Comparison with previous basal inversions

- Previous studies have fitted model to observed velocities, with ice geometry fixed from observations.
- The method here fits model to observed ice geometry, with no constraints on velocities.
- In principle, this should yield ~same results for  $C(x,y)$ , due to unique relationship between surface mass balance, ice thickness and balance velocity.



Pine Island and  
Thwaites Glaciers:



# Summary

- Simple inverse method “works”:
  - (a) converges, (b) reduces surface elevation errors, (c) deduces reasonable  $C(x,y)$  patterns.
- Independent of ice model. Just needs:
  - (a) run for ~200,000 years, (b) bedrock parameter(s) that make  $u_b$  increase or decrease.
- **BUT** some of the deduced  $C(x,y)$  *must* be due to other model errors, not real bed conditions.  
Lesser of two evils: **cancelling errors vs. O(500m) biases in surface elevation**
- Next steps:
  - Combine with large-ensemble techniques? (Stone et al., The Cryo. 2010; Tarasov et al., EPSL, 2011)
  - Apply to last deglaciation (Briggs et al., ISAES abs., 2011.; Whitehouse et al., QSR, 2012)