A 2-D subglacial drainage system model combining channels with sheet flow and water storage

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About water flow

Now some theory...

About water flow

Water flows down the hydraulic potential

$$\underbrace{\phi}_{\text{hydraulic potential}} = \underbrace{p_w}_{\text{pressure potential}} + \underbrace{\rho_w g H}_{\text{elevation potential}}$$

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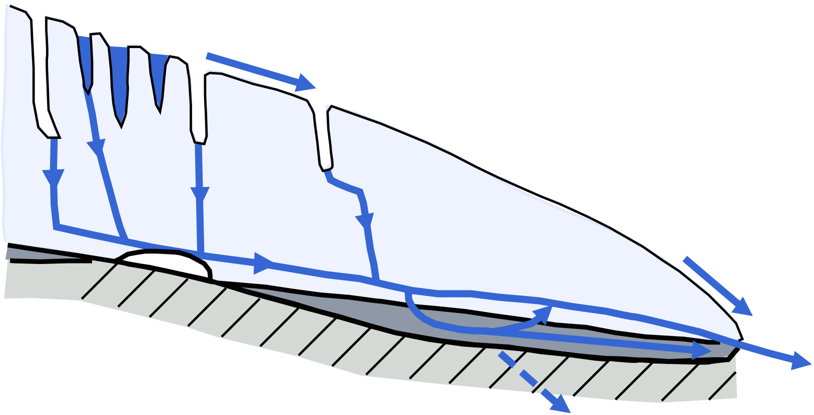
$$\underbrace{\phi}_{\text{hydraulic potential}} = \underbrace{p_w}_{\text{pressure potential}} + \underbrace{\rho_w g H}_{\text{elevation potential}}$$

For turbulent flow discharge is

$$q \propto -\sqrt{\nabla \phi}$$

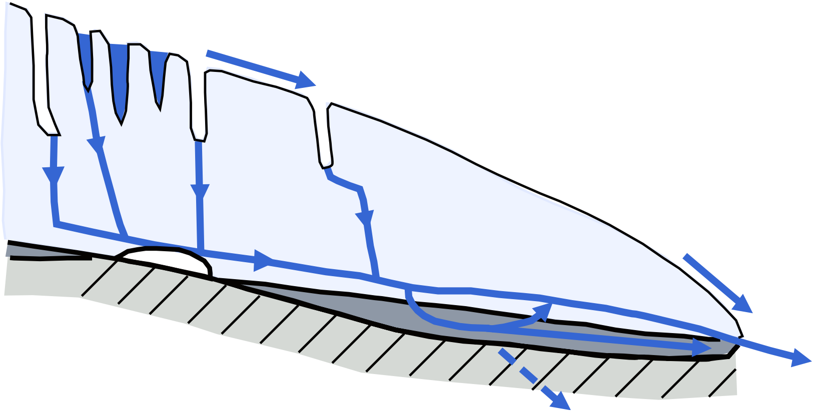
Water flow through a glacier

Cross-section of ablation area of glacier/ice sheet:



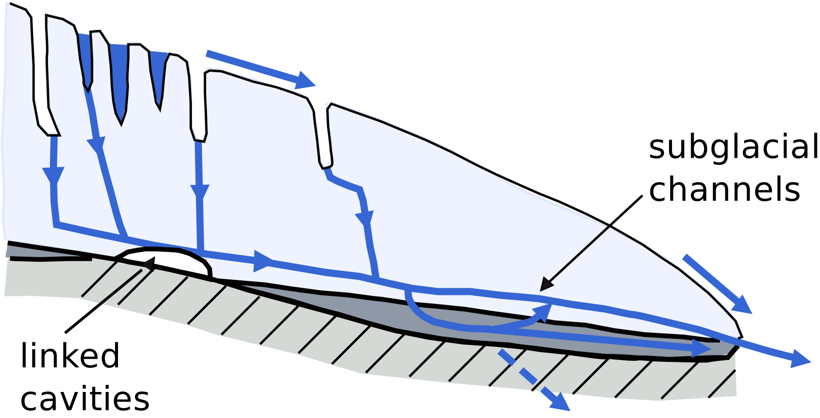
Water flow through a glacier

I'll focus on **a few components** of the drainage system,



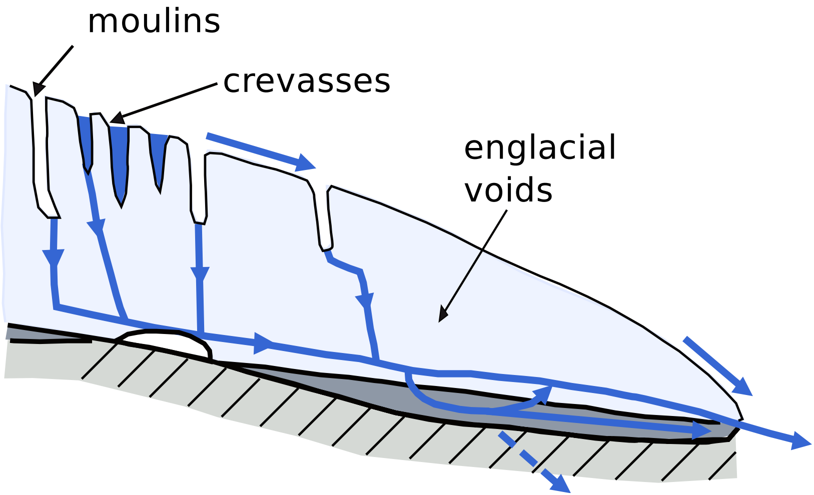
Water flow through a glacier

mainly on the **subglacial drainage** system



Water flow through a glacier

and a little on the **englacial drainage** system.



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This model

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$$\text{This model} = \underbrace{\text{sheet flow}}_{\text{linked cavities}} + \underbrace{\text{channel flow}}_{\text{R-channels}}$$

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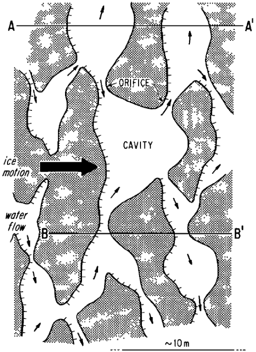
Two subglacial systems:
distributed: sheet flow
localized: R-channels

Sheet model

Porous sheet consisting of linked cavities:



Kamb 1987

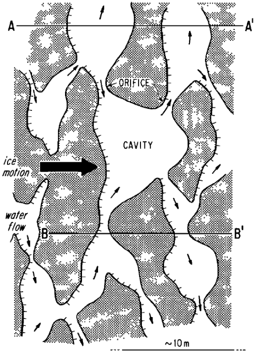
Sheet model

Porous sheet consisting of linked cavities:

mass conservation $$\frac{\partial h}{\partial t} + \nabla.\mathbf{q} = m$$

turbulent flow $$\mathbf{q} = -k_s h^\alpha \left|\nabla\phi\right|^{\beta-2} \nabla\phi$$

opening and closure $$\frac{\partial h}{\partial t} = v_{os}(u_b, h) - v_{cs}(\phi, h)$$



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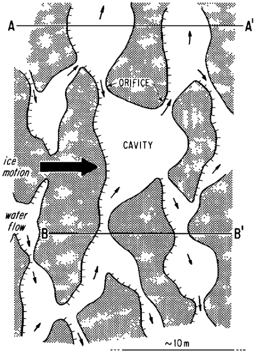

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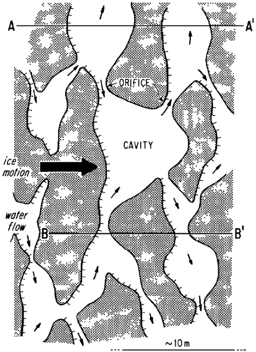

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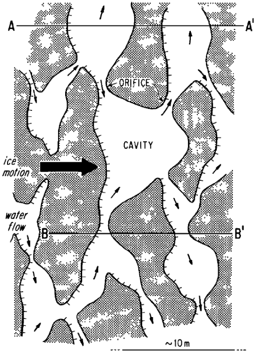

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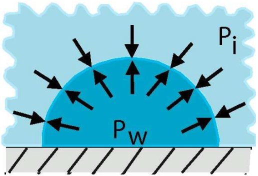
→ elliptic equation for ϕ, ODE for h

Aside: u_b is the basal sliding velocity; with which two-way coupling to ice flow could be done (together with ϕ).

Channel model

Channels are modelled as R-channels:



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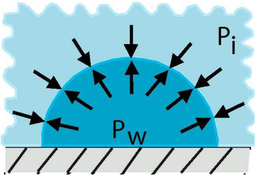

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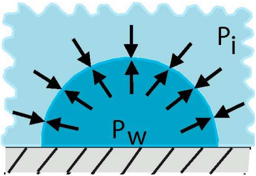

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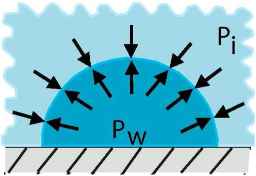

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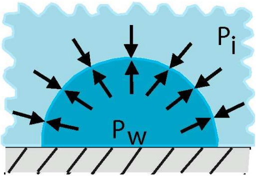

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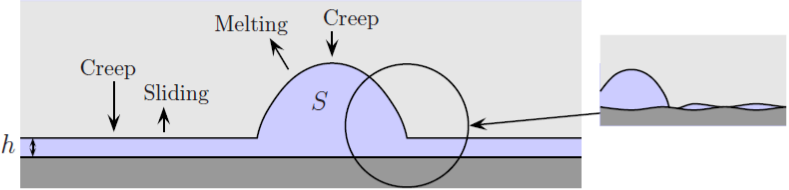
Note that channels are 1-D creatures!

Channels vs sheet



Sheet opens by sliding

Channel opens by melt

Both close by creep

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A network of *potential* R-channels is put on top of the sheet:

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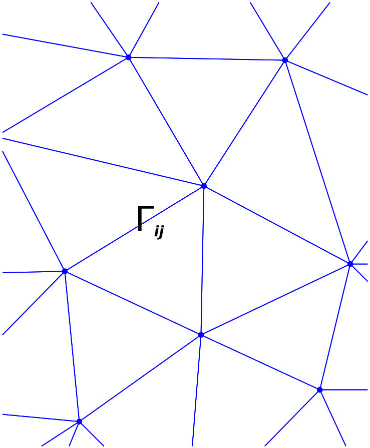

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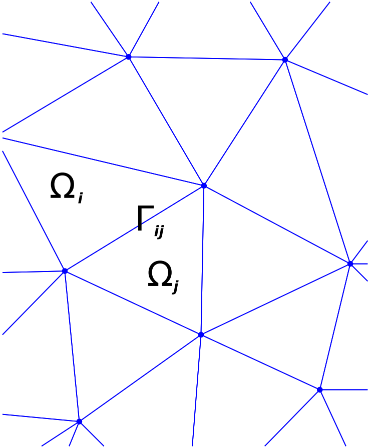

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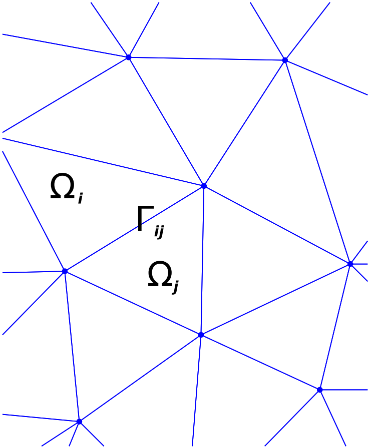

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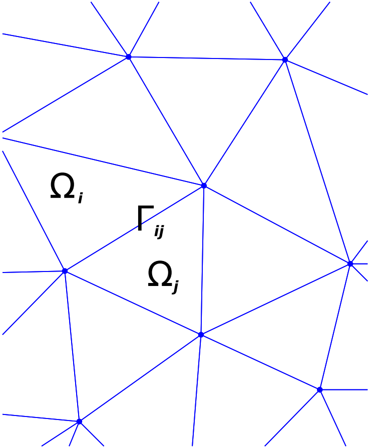

Englacial storage

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$\rightarrow$ now ϕ equation parabolic

And the last bit: moulins

Surface meltwater often collects in streams and enters the glacier through moulins. Presumably each **moulin connects to a subglacial channel** and also has some associated (storage) volume.

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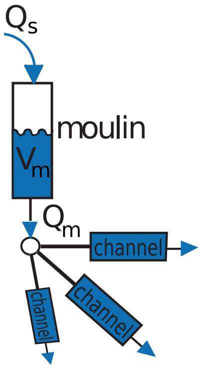
Q_s surface input

V_m volume of water stored in moulin

Q_m discharge into channels



This concludes the description of the physical model

Numerics

Parabolic hydraulic potential (ϕ) equation solved:

- with finite elements
- on unstructured triangular mesh
  (mesh = network of potential channels)
- backward Euler time step using hybrid Picard-Newton method

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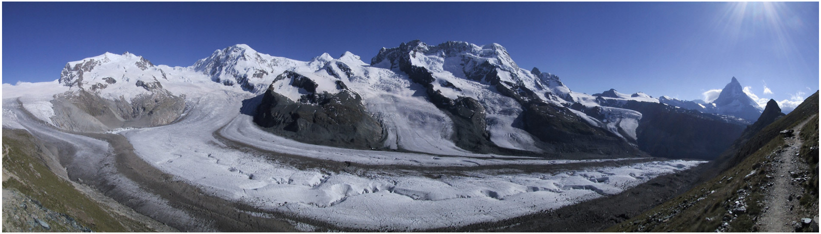
- up to 20 000 total DOF, 8 000 ϕ DOF
- model runs $\sim$10 min on laptop for 4000 DOF

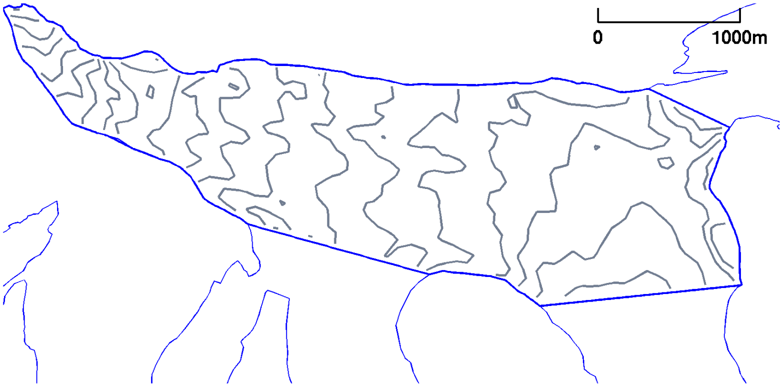
Real geometry: Gornergletscher

Model application to Gornergletscher, Switzerland



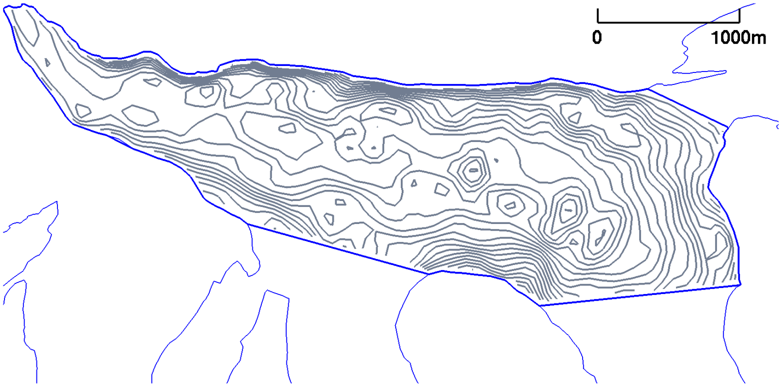
5 km by 1.5 km trunk
up to 400 m deep ice

Gornergletscher: Surface



Model domain with
surface elevation contours (interval 20m)

Gornergletscher: Bed



Bed elevation (contour interval 20m)

Run to steady state

Initial conditions

- Uniform sheet thickness = 5cm
- Channel diameters = 0m

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- sheet: uniform 0.3m/day ($\approx$ 20m^3/s total)
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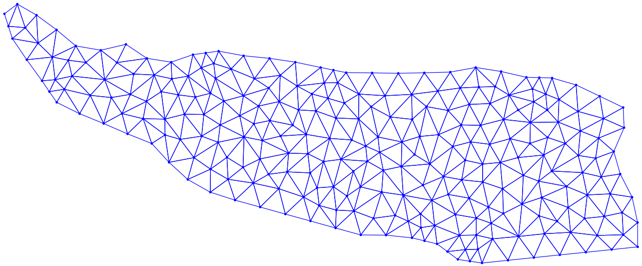
No storage

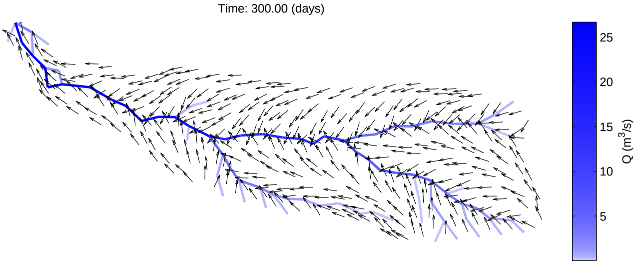
Example coarse mesh

Example (coarse) mesh with ∼1000 DOF produced with Triangle



Example output

We'll be looking at animations of this form



Blue edges:

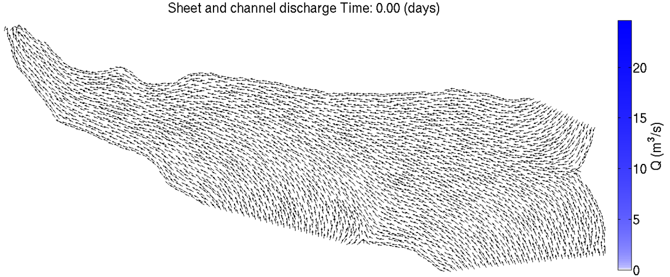
channel discharge

Arrows:

flow direction in sheet

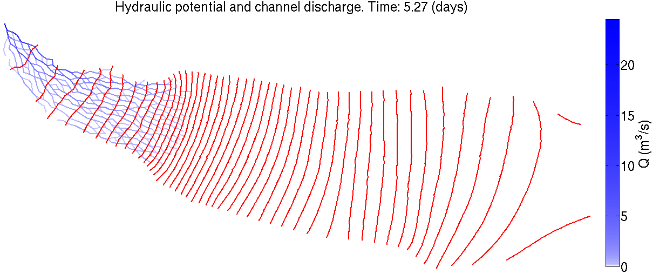
Run to steady state

Channel and sheet discharge



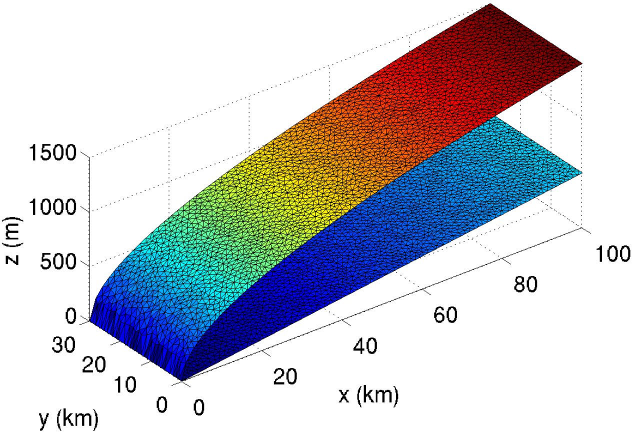
Run to steady state

Channel discharge and contours of ϕ



Model run on a square ice sheet catchment

Model application to 100x30km square ice sheet margin.



Topography

Model run on a square glacier

IC

Uniform sheet thickness = 5cm

Channel diameters = 0m

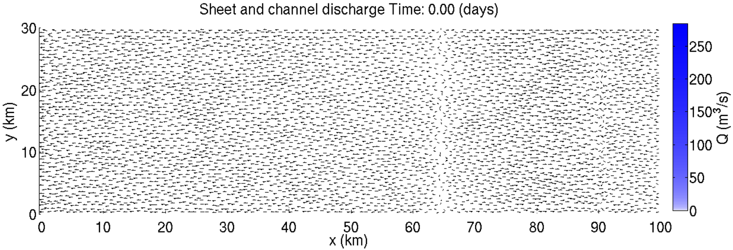
Sources

Sheet: input with lapse rate (0.14-0.0m/day ≈ 1400m^3/s total)

Channels: no input

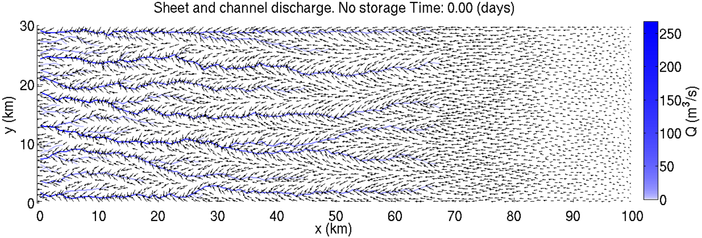
No storage

Water flow


Sheet and channel discharge Time: 0.00 (days)
y (km)
x (km)
0
10
20
30
0
10
20
30
40
50
60
70
80
90
100
Q (m^3/s)
0
50
100
150
200
250

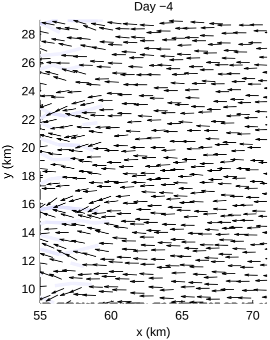
Switching on a moulin

Here a moulin is switched on ($50\,\mathrm{m}^3/\mathrm{s}$) draining into a steady state drainage system.



Switching on a moulin

Zoomed plots around moulin (slightly different run):



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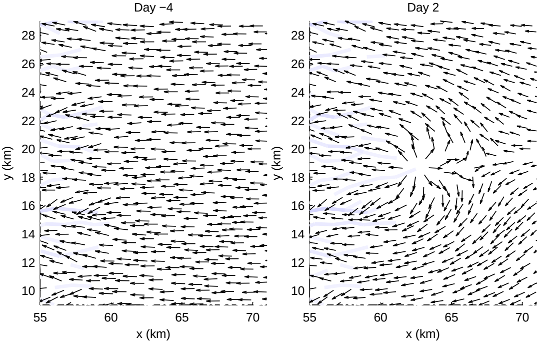

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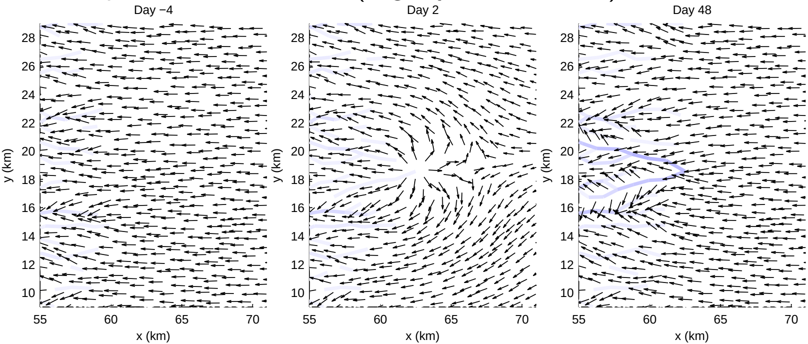
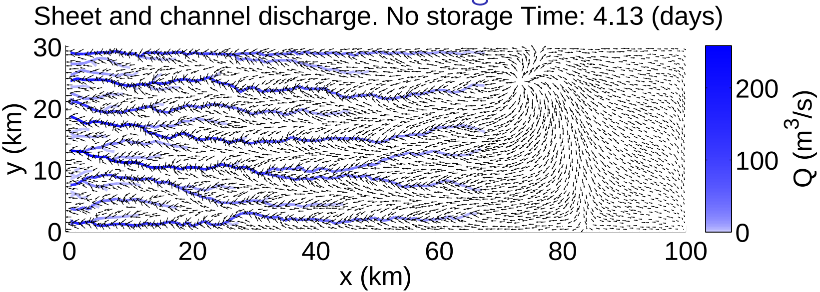


Storage

The "spooky action at a distance" when switching on the moulin is a reason to have storage.

Storage: moulin switch



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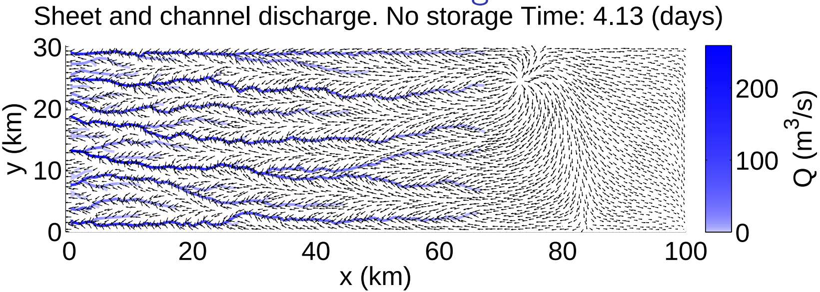

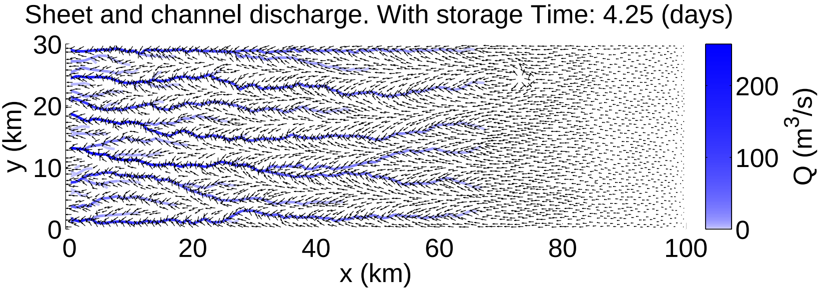

Storage: diurnal meltwater forcing

Observations show a **lag** between peak surface **melt** and peak proglacial **discharge**.

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The model can reproduce this lag between melt and proglacial discharge **with the storage term**

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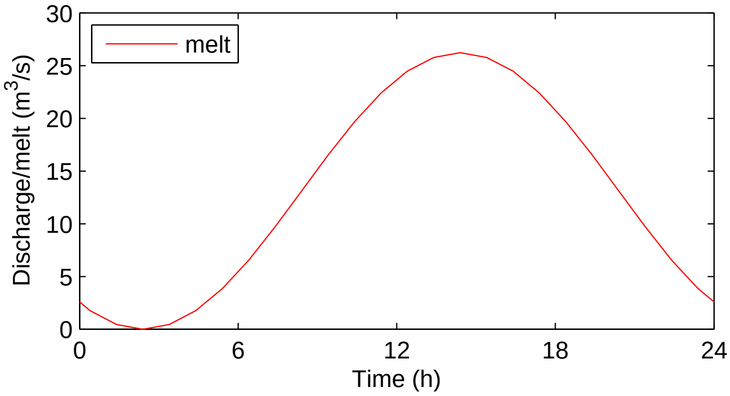

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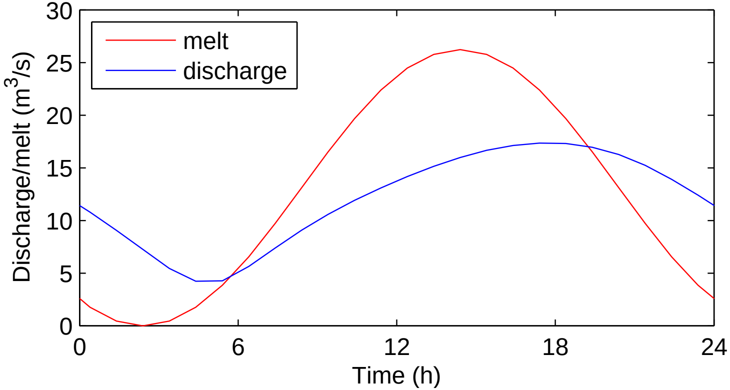
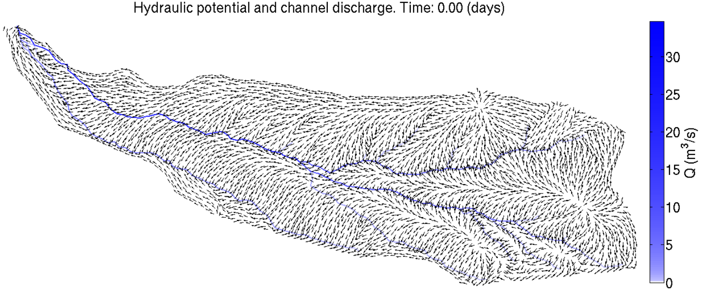


Storage: diurnal meltwater forcing

Animation of what the hydraulic potential does:



Conclusions

Model

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- **2D** subglacial drainage system model

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- formation of arborescent channel network

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Results

- formation of arborescent channel network
- works on real topographies
- point source input: ok
- diurnal meltwater forcings: ok

Outlook

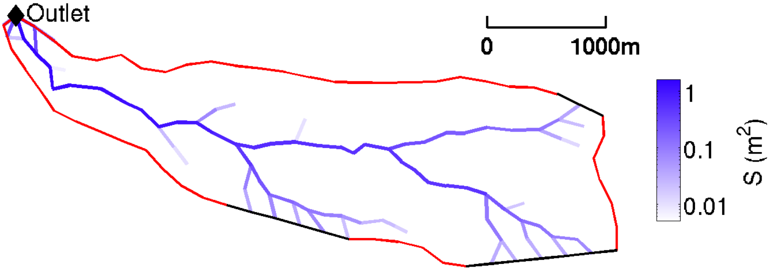
- Test it thoroughly: convergence, statistics of channels
- Explore parameters
- Add external model components: melt model, supraglacial water routing, etc.
- Rewrite of model in faster language
- Inclusion of subglacial lakes
- Model validation with Gornergletscher observations
- Application to ice sheet catchments
- Coupling to ice flow model: CISM

Does the model converge

Question:

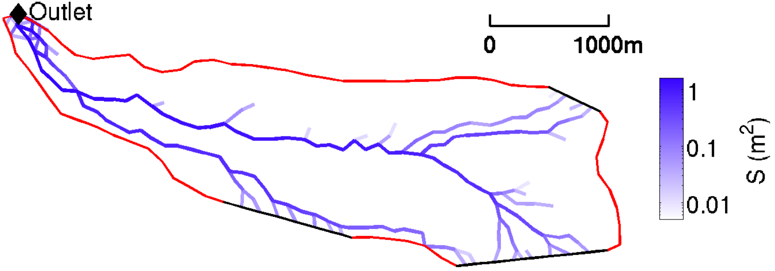
Does the predicted location of channels converge when the mesh is refined?
(Other types of convergence of course also interesting.)

Steady state channels



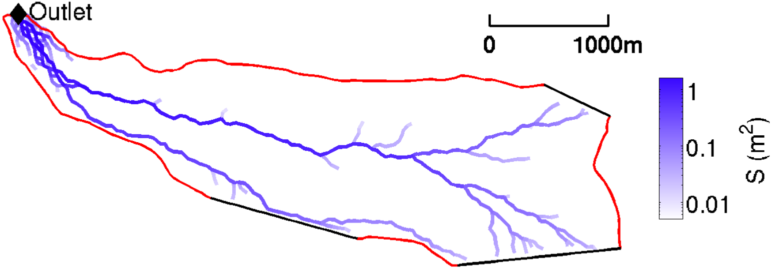
Mesh: 444 elements, 700 edges

Steady state channels



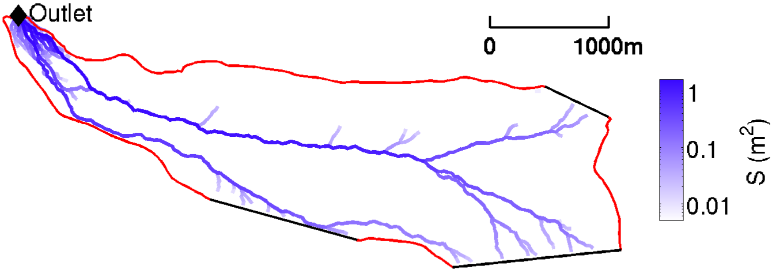
Mesh: 935 elements, 1458 edges

Steady state channels



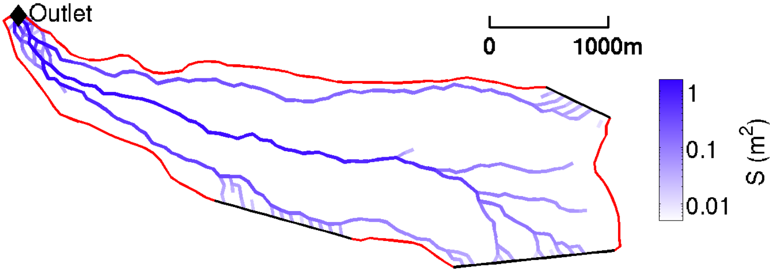
Mesh: 4502 elements, 6878 edges

Steady state channels



Mesh: 8933 elements, 13574 edges

Steady state channels



Mesh: 2203 elements, 3382 edges   Northern channel different!

Summary of equations

Standard equations describing linked cavity sheet and R-channels:

	Sheet	**R-channels**
Mass conserv.	$\frac{\partial h}{\partial t} + \nabla.\mathbf{q} = m$	$\frac{\partial S}{\partial t} + \frac{\partial Q}{\partial s} = \frac{\Xi}{\rho_w L} + m_C$
Turbulent flow	$\mathbf{q} = -k_s h^\alpha \left\vert\nabla\phi\right\vert^{\beta-2} \nabla\phi$	$Q = -k_C S^\alpha \left\vert\frac{\partial \phi}{\partial s}\right\vert^{\beta-2} \frac{\partial \phi}{\partial s}$
Time evolution	$\frac{\partial h}{\partial t} = v_{os} - v_{cs}$	$\frac{\partial S}{\partial t} = \frac{\Xi}{\rho_i L} - v_{cC}$
Opening	$v_{os}(u_b, h) \propto u_b(h_r - h)$	$\Xi(\nabla\phi, S) = \left\vert Q \frac{\partial \phi}{\partial s}\right\vert + \vert l_r \mathbf{q}.\nabla\phi\vert$
Closure	$v_{cs}(N, h) \propto h\vert N\vert^{n-1} N$	$v_{cC}(N, S) \propto S\vert N\vert^{n-1} N$

Steady state for different meshes

Does the model reach a comparable steady state for different meshes?

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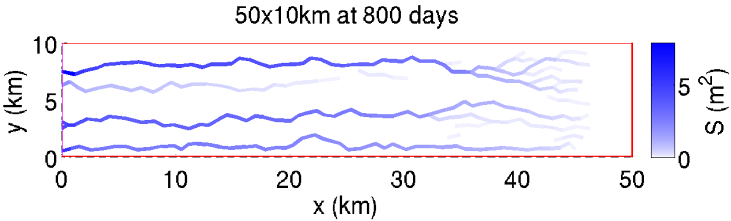
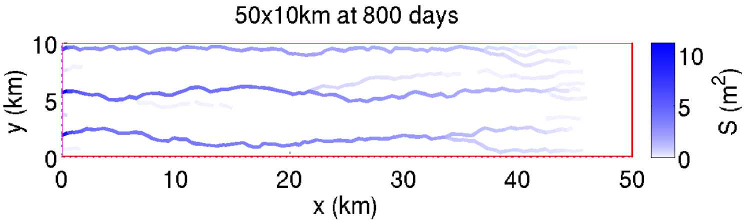
In particular for the channels?

Steady state for different meshes



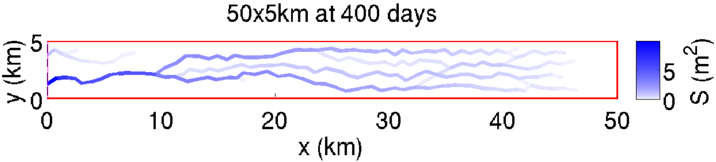
Mesh: 1269 elements, 1969 edges

Steady state for different meshes



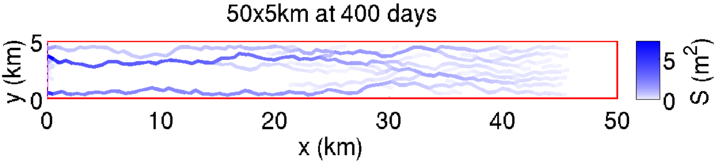
Mesh: 3189 elements, 4889 edges

Steady state for different meshes



Mesh: 635 elements, 1014 edges

Steady state for different meshes



Mesh: 1639 elements, 2557 edges

Steady state for different meshes

Looks ok: all have $\sim$1 channel per 3km width.