Data assimilation and prognostic whole ice sheet modelling with the variationally derived, higher order, open source, and fully parallel ice sheet model VarGlaS

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Advertisement.

Capabilities.

- Stokes' and first-order.
- Enthalpy balance.
- Advanced time stepping.
- Data assimilation.

Features.

- Parallel.
- Built on open-source FEM platform FEniCS.
- Mesh refinement.

Better living through the calculus of variations.

$\dot{V} = -\rho \mathbf{g} \cdot \mathbf{u}$ Potential energy of ice.

$\dot{D} = \frac{2n}{n+1}\eta(\dot{\epsilon}^2)\dot{\epsilon}^2$ Conversion to heat through internal friction.

$\dot{F} = \beta^2 h^r \mathbf{u} \cdot \mathbf{u}$ Conversion to heat through basal friction.

$\mathbf{Ic} = P(\nabla \cdot \mathbf{u})$ Incompressability.

$\mathbf{Ip} = P(\mathbf{u} \cdot \mathbf{n})$ Impenetrability.

The Stokes' problem

$$\min \int_\Omega \dot{V} + \dot{D} + \mathbf{Ic} \, \mathrm{d}\Omega + \int_{\Gamma_B} \dot{F} + \mathbf{Ip} \, \mathrm{d}\Gamma \tag{1}$$

The FO problem

$$\min \int_\Omega \dot{V}_1 + \dot{D}_1 + \not{\mathbf{Ic}} \, \mathrm{d}\Omega + \int_{\Gamma_B} \dot{F}_1 + \not{\mathbf{Ip}} \, \mathrm{d}\Gamma \tag{2}$$

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Newton's method

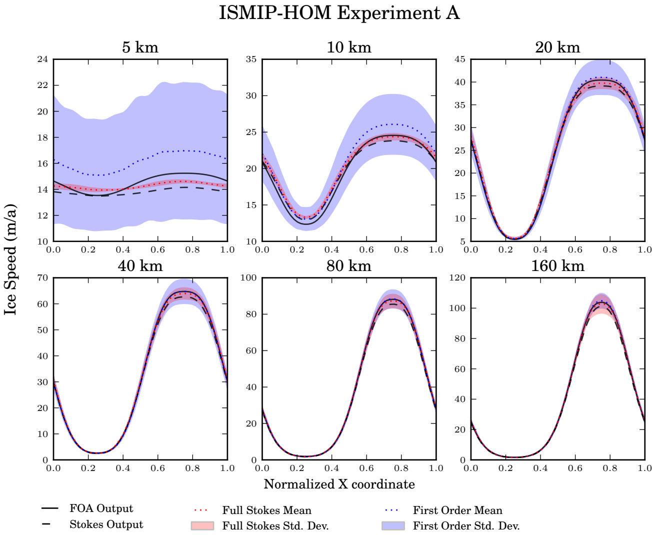
$$\delta\mathcal{A}(\mathbf{u}) = 0 \tag{3}$$

$$\delta^2\mathcal{A}\Delta\mathbf{u} = -\delta\mathcal{A} \tag{4}$$

solvers and efficiency.

- Currently using gmres and hypre-euclid (ILU) for Stokes'.
- Currently using gmres and hypre-amg (AMG) for FO.
- Ongoing research into what makes an efficient solver here.

ISMIP-HOM A



ISMIP-HOM Experiment A
5 km
10 km
20 km
40 km
80 km
160 km
Ice Speed (m/a)
Normalized X coordinate
FOA Output
Stokes Output
Full Stokes Mean
Full Stokes Std. Dev.
First Order Mean
First Order Std. Dev.

Enthalpy versus temperature.

$$\rho(\partial_t + \mathbf{u} \cdot \nabla)H = \rho\nabla \cdot \kappa(H)\nabla H + Q \tag{5}$$

$$\kappa(H) = \begin{cases} \frac{k}{\rho C_p} & \text{if cold} \\ \frac{\nu}{\rho} & \text{if temperate,} \end{cases} \tag{6}$$

- Easy to implement.
- Yields water content.
- Avoids contact problem.
- Self consistent boundary conditions.

$$\rho\kappa(H)\nabla H \cdot n = q_{geo} + \dot{F} - \frac{\rho m_b}{L} \tag{7}$$

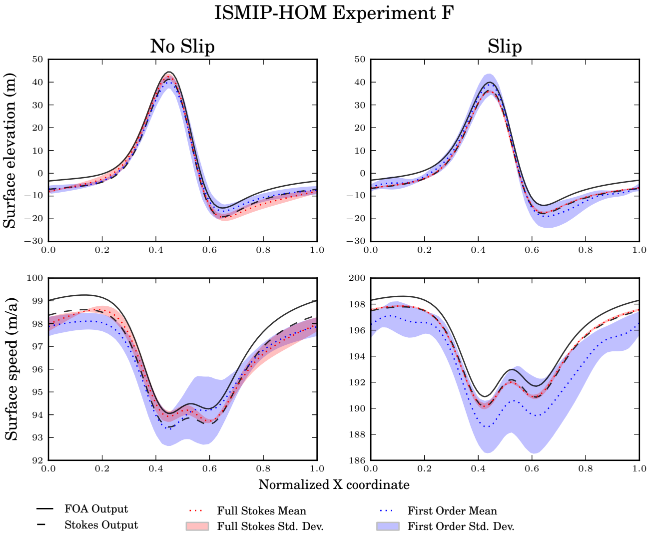
Dynamic boundaries

$$\frac{\partial S}{\partial t} = \dot{a} + \mathbf{u} \cdot \mathbf{n} \tag{8}$$

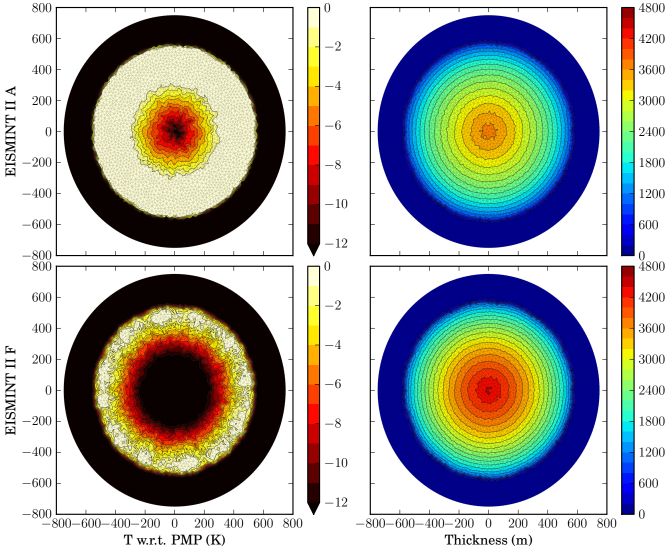
Implementation.

- Total Variation Diminishing Runge-Kutta (TVD-RK).
- Shock capturing.
- Streamline upwind Petrov-Galerkin (SUPG).

ISMIP-HOM F


ISMIP-HOM Experiment F
No Slip
Slip
Surface elevation (m)
Surface speed (m/a)
Normalized X coordinate
FOA Output
Stokes Output
Full Stokes Mean
Full Stokes Std. Dev.
First Order Mean
First Order Std. Dev.

EISMINT-II


EISMINT II A
EISMINT II F
T w.r.t. PMP (K)
Thickness (m)
800
600
400
200
0
−200
−400
−600
−800
0
−2
−4
−6
−8
−10
−12
4800
4200
3600
3000
2400
1800
1200
600
0

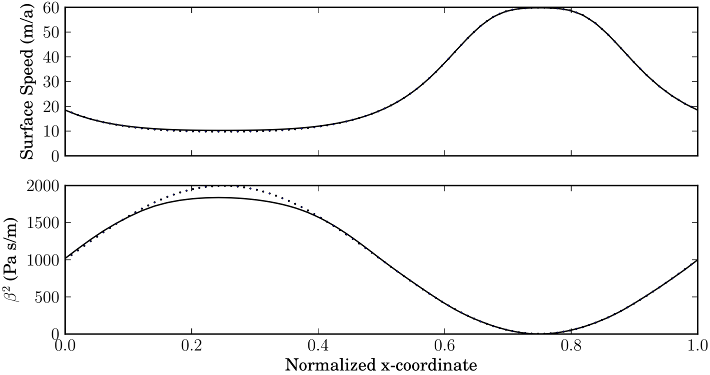
Data assimilation.

- u Model variable.
- d Data.
- p Unknown parameter.
- $\mathcal{I}$ Objective (Cost) functional.
- $\delta\mathcal{A}$ Forward model.
- λ Lagrange multiplier (Adjoint variable).

The problem

$$\min \mathcal{F} = \min \int_\Omega \mathcal{I}(u, d, p)\, \mathrm{d}\Omega + \int_\Omega \lambda \delta\mathcal{A}\, \mathrm{d}\Omega$$

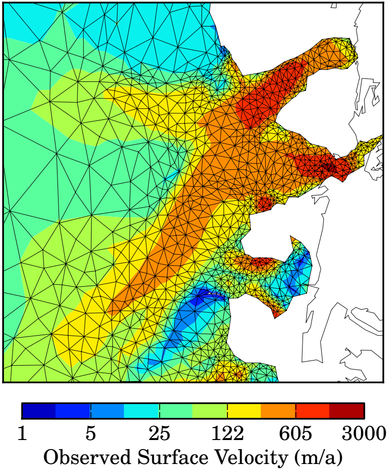
Inverting ISMIP-HOM C.



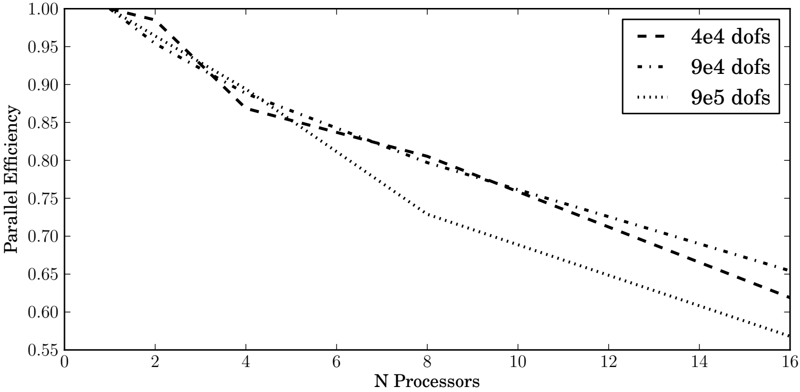
Mesh refinement.

$$\mathbf{e}(c) \propto \max_{\mathrm{i} \in E} \mathbf{x}_i^T \mathbf{M} \mathbf{x}_i \tag{9}$$

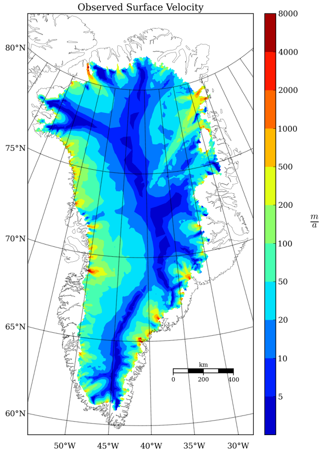
$$\mathbf{M} = \mathbf{V}^T |\mathbf{\Lambda}| \mathbf{V}. \tag{10}$$



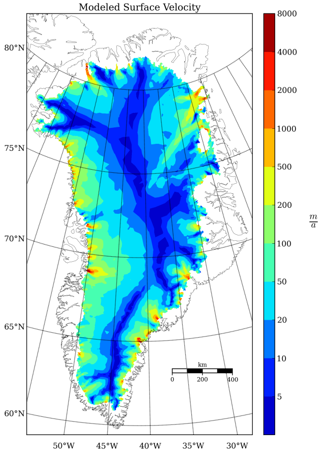
Parallelism.



Inverting *Joughin 2007-2008 two year average velocity*



Observed velocity 2007-2008



Modeled velocity

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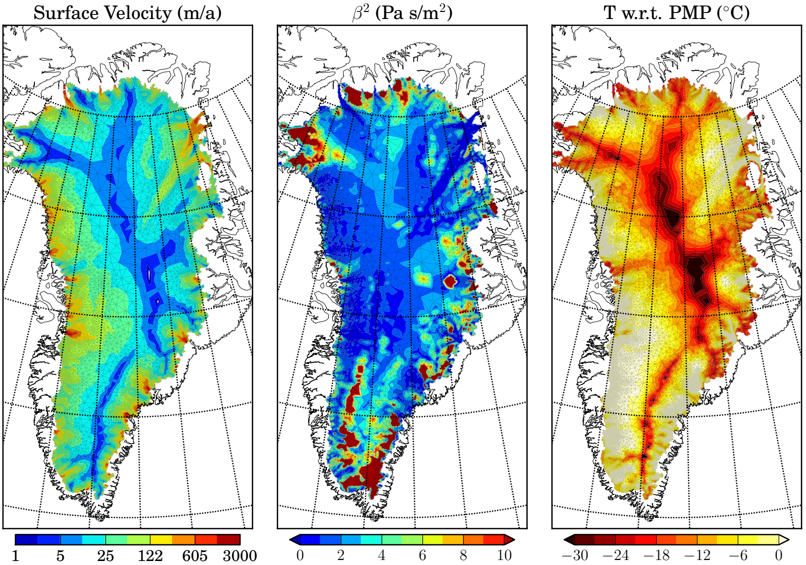
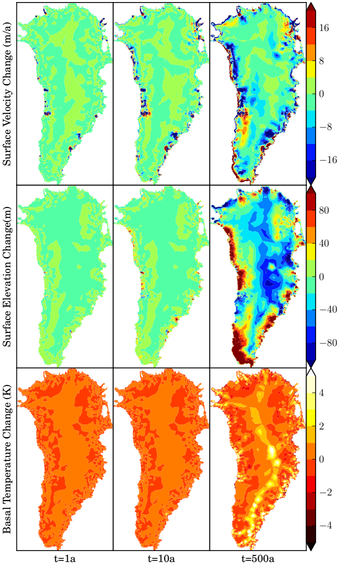
Greenland steady state, fixed geometry.

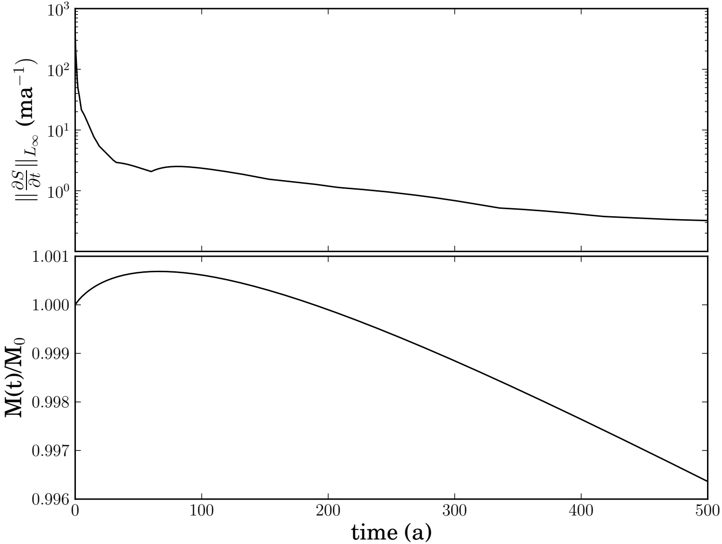


500 years with constant climate.



The University of Montana

Mass through time.



Next steps.

- Marine margin treatment (The equality constrained variational principle become *inequality* constrained).
- Adaptive mesh refinement.
- Mass continuity initialization.

How to try your own copy.

- Install FEniCS 1.1.0 stable
- bzr branch lp:up-feism
- Comment on The Cryosphere Discuss.