Parameter estimation for grounding-line transition

Gunter Leguy, Xylar Asay-Davis, William Lipscomb

Los Alamos National Laboratory

February 15th, 2013

Introduction

We are exploring how basal physics influences resolution in a 1-D vertically integrated flowline model.

- Several models of the form $\tau_b = \beta^2 u$ have been investigated:
  - Schoof (2007): β^2 = - $C|u|^{\frac{1}{n}-1}$
  - Pattyn (2006): β^2 = - $\exp[\beta_0(x_g - x)]$

Goals:

- Present a new parametrization of the basal shear stress (in the case of a Marine ice sheet):
  - Physically motivated (ocean connection)
  - Transitions smoothly between finite basal friction in the ice sheet and zero basal friction in the ice shelf
- Hope for $\approx$ 1km resolution near the grounding line.

Los Alamos NATIONAL LABORATORY EST.1943

Model equations

Valid in both, the ice sheet and the ice shelf:

$$\begin{aligned}
\text{conservation of mass}: &\quad H_t + (uH)_x = a, \\
\text{stress} - \text{balance equation}: &\quad \tau_l + \tau_b + \tau_l = 0.
\end{aligned}$$

$$\begin{aligned}
\tau_l &= \left[2\bar{A}^{-\frac{1}{n}} H |u_x|^{\frac{1}{n}-1} |u_x| \right]_x, \\
\tau_d &= -\rho_i g H s_x, \\
\tau_b &= \text{basal shear stress}.
\end{aligned}$$

Assumption and Boundary conditions

Symmetric ice sheet at the ice divide

$$\left.\begin{aligned} u &= 0 \\ s_x = (H-b)_x &= 0 \end{aligned}\right\} \text{ at } x = 0,$$

At the grounding line, we have the flotation condition and the balance between τ_l and τ_d.

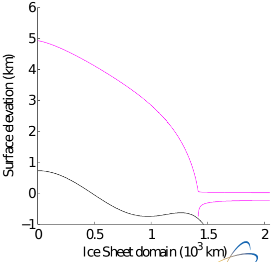
$$\left.\begin{aligned} H = & \quad \frac{\rho_w}{\rho_i} b, \\ 2\bar{A}^{-\frac{1}{n}} |u_x|^{\frac{1}{n}-1} u_x = & \quad \frac{1}{2}\rho_i g \left(1 - \frac{\rho_i}{\rho_w}\right) H \end{aligned}\right\} \text{ at } x = x_g.$$

Basal stress

We adopt the formulation from Schoof (2005):

$$\tau_b = -\gamma N \left(\frac{u}{u + \frac{\lambda_{\max}}{m_{\max}} A N^n} \right)^{\frac{1}{n}} .$$

- λ_{max} = wavelength of bedrock bumps
- m_{max} = maximum bed obstacle slope
- Effective pressure: $N \equiv p_i - p_w$
    - $p_i \equiv \rho_i g H$
    - p_w?



Los Alamos NATIONAL LABORATORY EST. 1943

Basal stress (continued)

N(p) $= \eta(p; H, b)\rho_i$ g H, η non-dimensional function

- $\eta \in [0, 1]$
- $p \in [0, 1]$

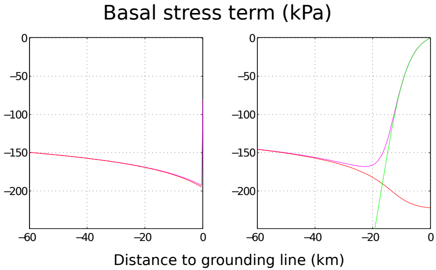
We want the following limit for η:

- When $p = 0$, $\eta = 1$: no water-pressure support.
- When $p = 1$, $\eta = 0$: subglacial water has a free connection to the ocean, Paterson(2010)
- At the grounding line, $\eta = 0$ (effective pressure goes to zero to allow continuity).

One expression with the right limit: $N(p) = \rho_i g H \left(1 - \frac{H_f}{H}\right)^p$, where $H_f = max(0, \frac{\rho_w}{\rho_i} b)$

Basal stress (continued)

- Right limits:
  - if $u \ll \kappa N(p)^n$ and $\tau_b \approx -\frac{\gamma}{\kappa^{1/n}}|u|^{\frac{1}{n}-1}u$ (frozen ice at the bed)
  - if $u \gg \kappa N(p)^n$ and $\tau_b \approx -CN(p)\frac{u}{|u|}$. (hydrological connection)
- Physically motivated
- Transitions smoothly from non-zero basal stress in the ice sheet to zero basal sliding in the ice shelf



a) p = 0 **b)** p=1

Numerics

Moving-grid:

- Numerically more accurate
- GL must lie on a grid point
- Could be used as a benchmark solution for fixed-grid model
- Harder to implement in 3-D models

Fixed-grid

- Suitable for 3-D model
- Constant resolution
- Numerically less accurate
- GL usually falls between two grid points leading to interpolation error

Numerics (continued)

How to compare our results for all p-values?

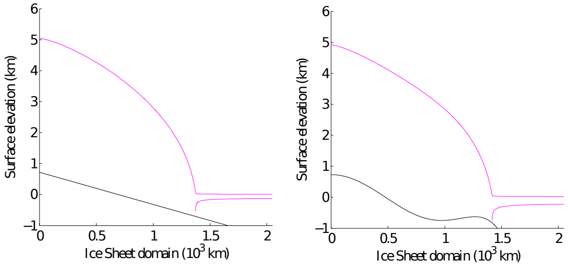
- Qualitative
  - Schoof (2007) boundary layer solution (in the case of frozen bed type basal sliding)
- Quantitative
  - Schoof (2007) boundary layer solution (good approximation for $p=0$)
  - Chebyshev polynomial numerical scheme
  - Moving grid with high enough resolution used as a benchmark for fixed-grid model

Los Alamos NATIONAL LABORATORY EST.1943

Numerics (end)

Experimental set-up:

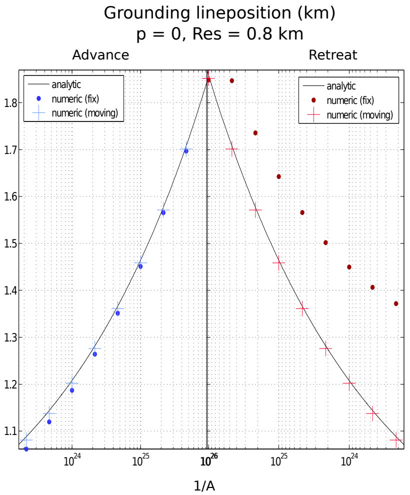
- Moving grid resolution benchmark: 156m $\leq$ Res $\leq$ 316 m.
- Fixed-grid resolution comparison: 1.6 km and 0.8 km.
- We will show results for 3 values of p.
- Constant accumulation rate: $a = 0.3$ m/yr.
- MISMIP experiment: neutral equilibrium experiments.
- Two different bed topographies (as in Schoof (2007a)).



Los Alamos NATIONAL LABORATORY EST. 1943

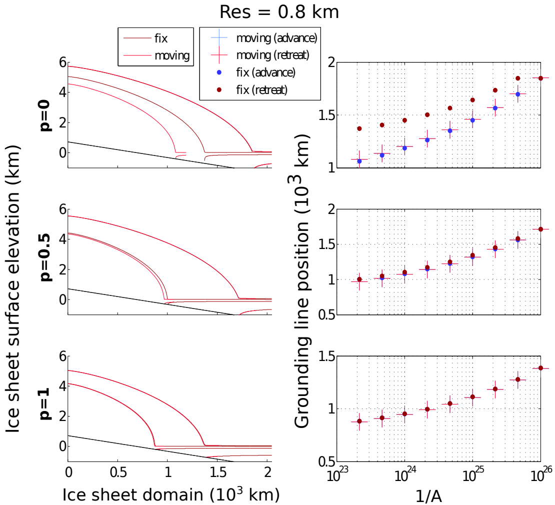
Results linear bed

The fixed-grid solution is inaccurate in capturing the retreat.



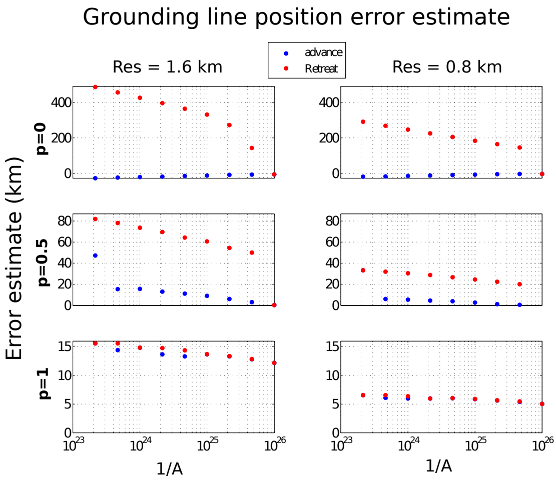
Results linear bed (continued)

MISMIP-type experiments 1 & 2



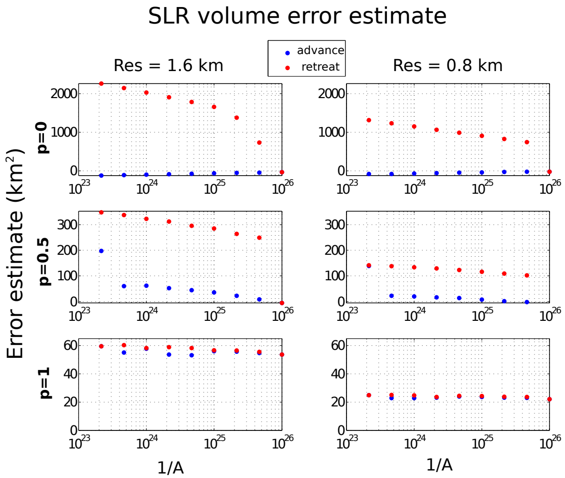
Results linear bed (continued)

MISMIP-type experiments 1 & 2 (note the different scales for the different p-values)



Results linear bed (end)

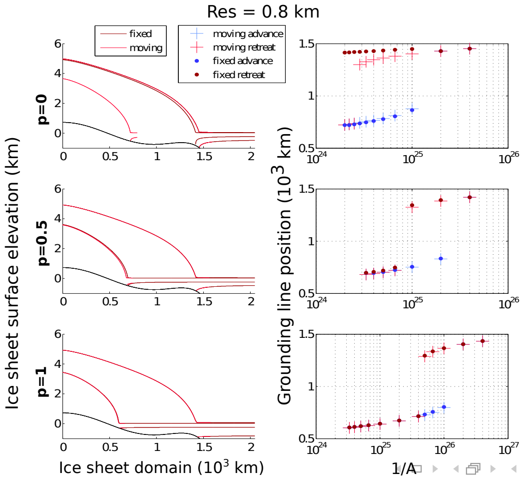
MISMIP-type experiments 1 & 2 (note the different scales for the different p-values)



Element of comparison: 2850 km^3 $\approx$ 7.2 mm.

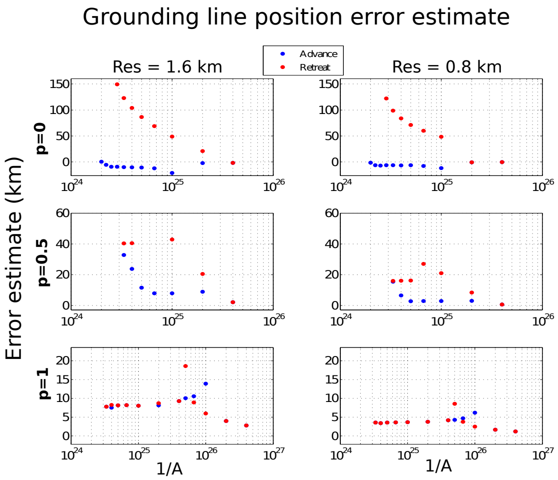
Results poly bed

MISMIP-type experiment 3



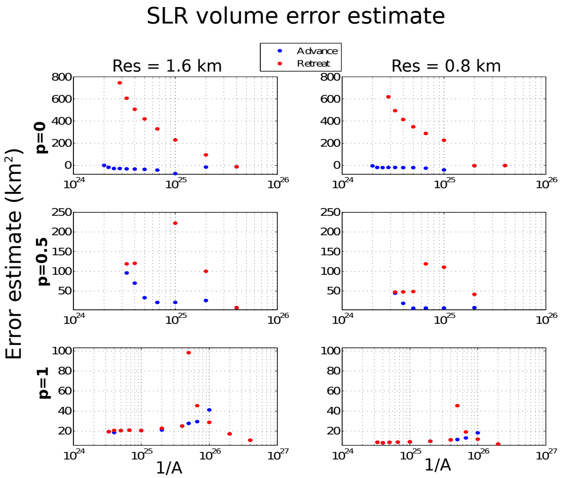
Results poly bed (continued)

MISMIP-type experiment 3 (note the different scales for the different p-values)



Results poly bed (continued)

MISMIP-type experiment 3 (note the different scales for the different p-values)



conclusion

- For low p-values the need of high resolution in the vicinity of the GL remains.
- Error estimate decreases substantially as the value of p increases and therefore the required model resolution decreases as p increases.
- p does not play any role in the bulk of the ice sheet. It impacts a relatively small distance (no more than 25 km from the grounding line in our experiments) which is enough to impact the solution.
- With p $\approx$ or $=1$, a resolution of $\approx$ 1km is sufficient to capture the solution.

Paper in preparation for the cryosphere

Future work

- 3-D implementation
- Add the missing physics (lateral drag, buttressing)
- Data comparison for admissible values of p (any value besides 0 and 1 valid?)


© Original Artist
Reproduction rights obtainable from
www.CartoonStock.com
search ID: raun68
"THE WINTERS ARE NOTHING LIKE THE OLD DAYS!"