Relating inverse-derived basal sliding coefficients beneath ice sheets to other large-scale variables

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Outline

1. Deduce basal sliding coefficients *C(x,y)* by simple model inversion

   - (like last year)

2. Don't impose any constraints due to basal temperature or hydrology

   - (unlike last year)

3. Then compare *C(x,y)* patterns with basal temperature, melt, topography

   - new parameterization for *C(x,y)*?

4. Fails…*Why*?

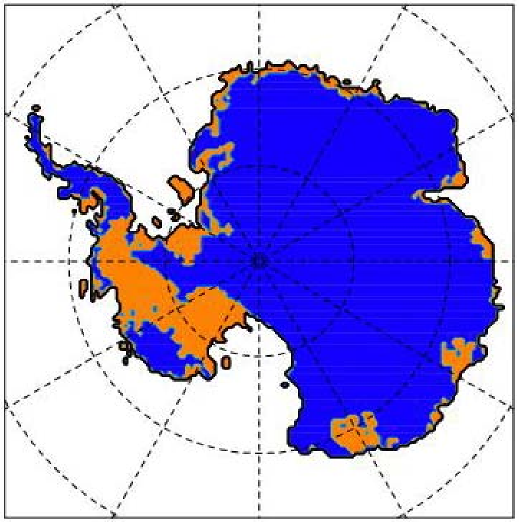
Common basal sliding laws in Antarctic-wide models

- Sliding velocity depends on basal shear stress, intrinsic bed conditions, and basal hydrology or temperature:

$u_b = C(x,y)\, N^{-q} \tau_b^n$ or $u_b = C(x,y)\, f(T_b)\, \tau_b^n$

where
u_b = basal ice velocity,
τ_b = basal shear stress ,
N = effective pressure,
T_b = basal temperature,
$f(T_b) = 0$ if bed is frozen, 1 if bed is at melt point

Crude $C(x,y)$ map:
sediment if rebounded bed is below sea level, hard bedrock if above



Blue: $C = 10^{-10}$ m a^{-1} Pa^{-2}
Orange: $C = 10^{-5}$ m a^{-1} Pa^{-2}

Typical surface elevation (or thickness) errors

model minus observed:

Ritz et al., JGR, 2001

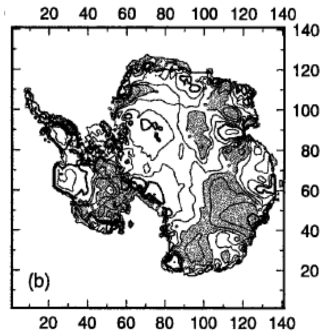
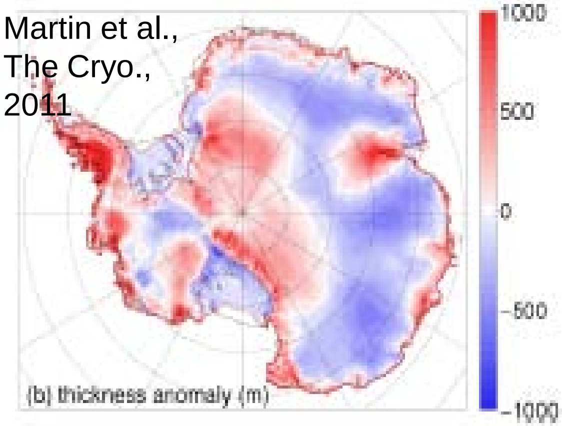
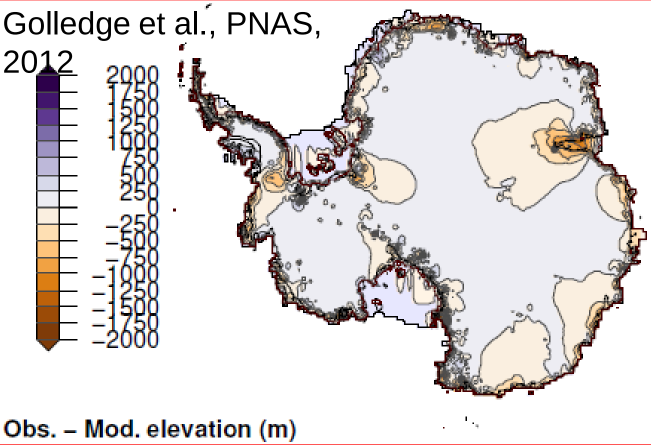


Figure 4. Comparison between observed and modeled surface elevation for the present-day geometry of the grounded ice. (a) Elevation contours: Continuous lines represent observed configuration and dotted lines represent modeled configuration. Thick lines represent grounding lines, thin lines represent isolines for every 1000 m intervals. (b) Altitude difference map: in grey the modeled surface is below the observed one (negative isolines). Isolines are for -1000,-500 (thick line),-250,-100,0,100,250,500 (thick line) and 1000 m. Horizontal scales are as in Figure 3.

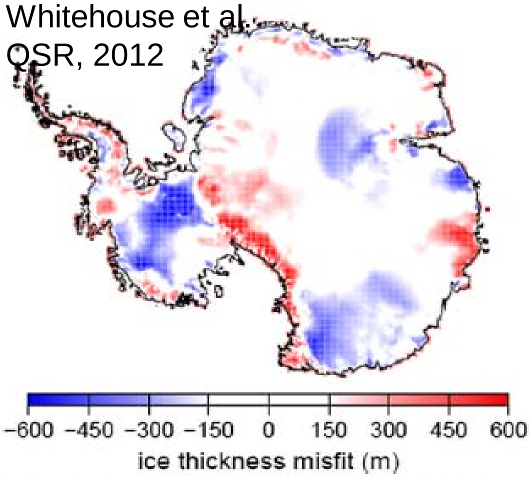
Martin et al., The Cryo., 2011



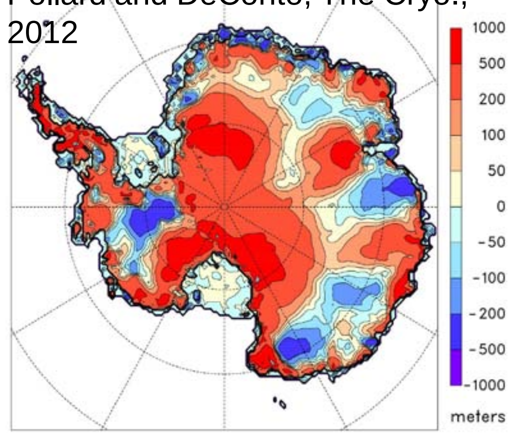
Golledge et al., PNAS, 2012



Whitehouse et al., QSR, 2012



Pollard and DeConto, The Cryo., 2012



- *Axiom of talk:*

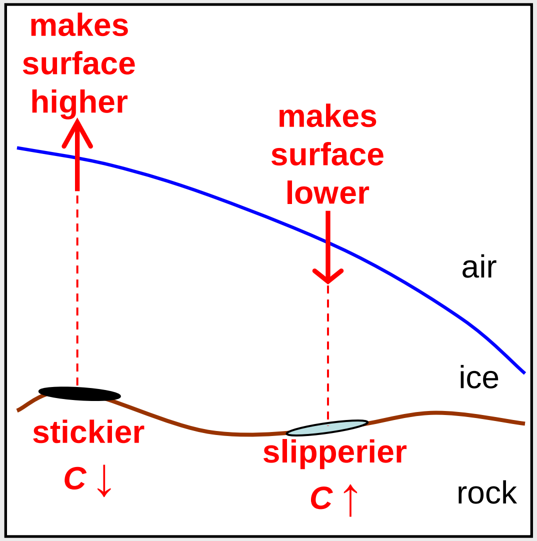
  C(x,y) is primary cause of *O*(500 m) elevation errors in Antarctic continental paleo ice-sheet models

$$u_b = C(x,y)\, f(T_b)\ \tau_b^{\,n}$$

Simple Inversion Method

Very simple procedure to deduce basal sliding coefficients *C(x,y)*, fitting to observed ice surface elevations

- Run model forward
- Every 2000 years, decrease (stiffen) *C(x,y)* if the *local* ice surface is too low, or increase (soften) *C(x,y)* if *local* surface is too high:
  - $C_{new} = C\ 10^{\Delta z/2000}$
    where Δz = model – observed surface elevation (m)
  - Constrain *C* to remain in range 10^{-15} to 10^{-4} m a^{-1} Pa^{-2}
- Run model for ~100,000 years until convergence



$u_b = C(x,y)\ f(T_b,\ldots)\ \tau_b^n$

Ignores ∂/∂x, ∂/∂y's….as if effects are local !

Ignores all other potentially canceling model errors !

2 strategies in using the inverse method

We can write the sliding law either as

$$u_b = C(x,y)\ f\,(T_b\,,\ hydrol.,\ topog.,\ \ldots)\ \tau_b$$

Imagine that we know *f(…),* and apply it during the inversion procedure, to deduce *C(x,y)* representing intrinsic bedrock properties.

(last year's talk, and The Cryo, 2012)

or as

$$u_b = C'(x,y)\ \tau_b$$

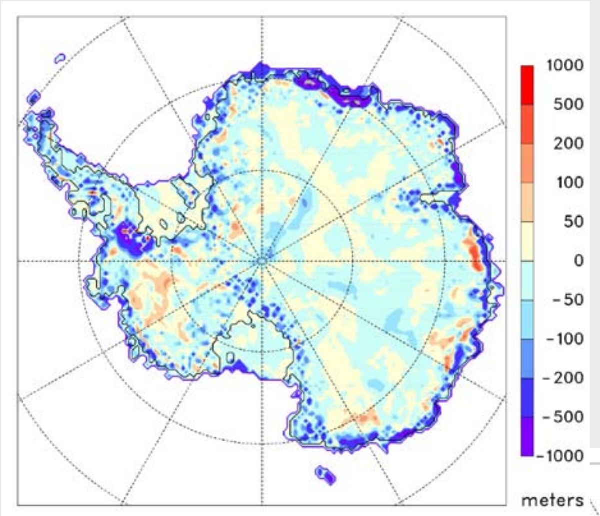
Don't apply *f(…)* during inversion. Invert for $C'(x,y)$.
Then try to find a function *f* so that $C' = C.f$,
i.e., $f\,(\ldots) \approx 0$ in regions with $C' \approx 0$, and $f = 1$ outside

this talk

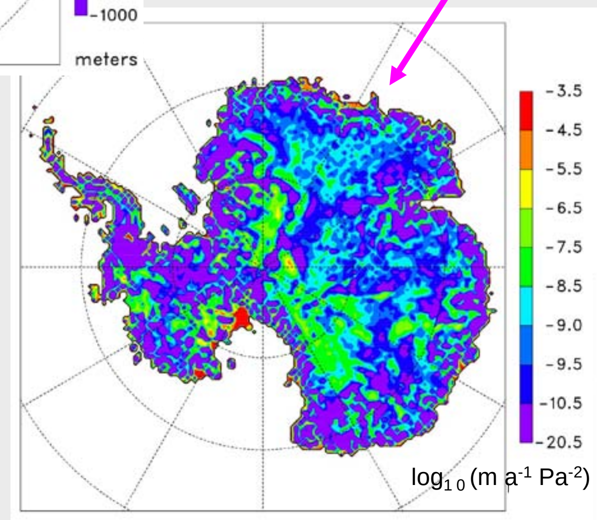
Results of inverse method, *no* basal temperature constraint

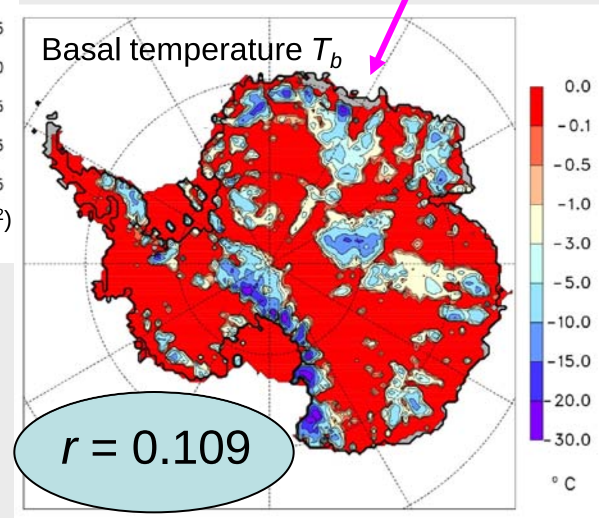
- Purple regions are where sliding ≈ 0
- Ideally, they correspond to frozen beds, or no basal water supply
- But they don't correspond to $T_b < 0$
- Can we find a function $f(T_b, \text{topog.}, \text{melt})$ that does?

Final elevation error Δh_s



Deduced sliding coefficients C'

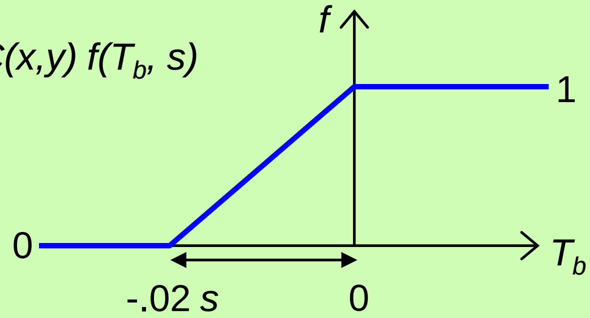




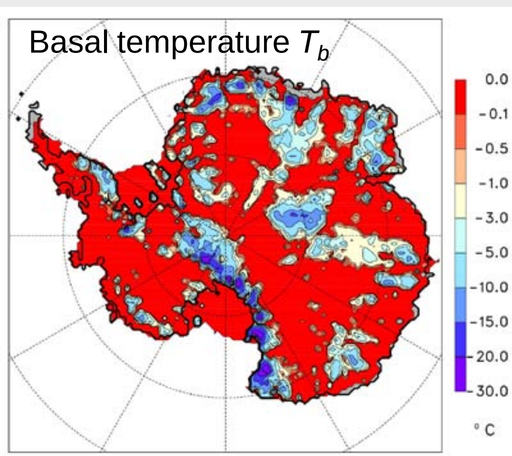
$r = 0.109$

Attempt at *f(…)* using basal temperature and sub-grid bed roughness

$C' = C(x,y)\, f(T_b, s)$



"Sub-grid valley bottoms may still be unfrozen even if $T_b < 0$"



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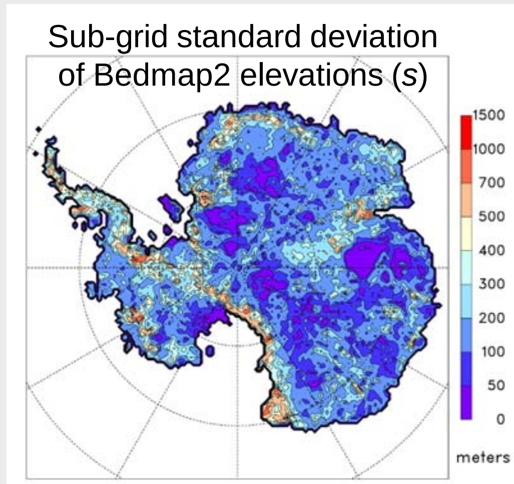

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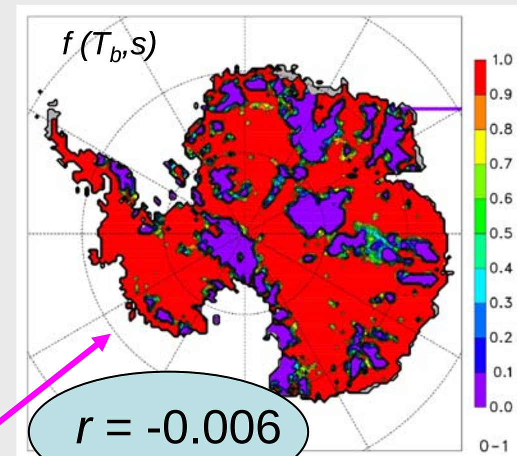
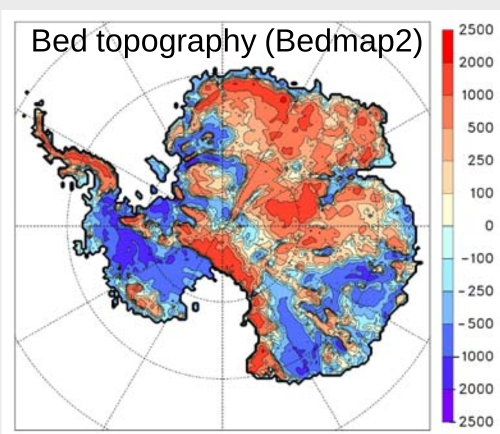
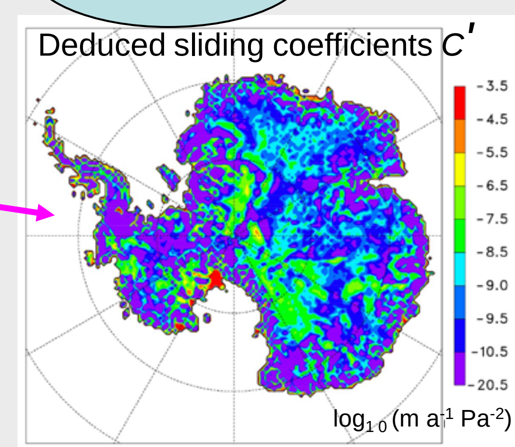


r = -0.006

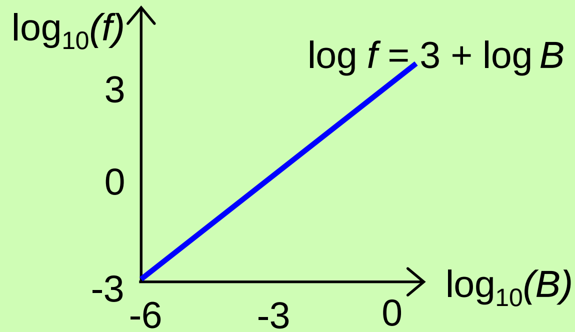


- But resulting "$f(T_b,s) \approx 0$" pattern does not resemble purple regions $C' \approx 0$
- Main problem is that T_b and *s* both resemble large-scale bed topography



Attempt at *f(…)* using basal liquid supply (m/yr)

$C' = C(x,y)\, f(B)$

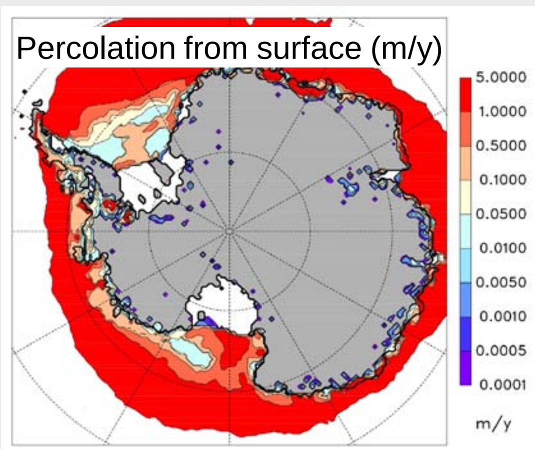


B (m/y) = basal liquid supply due to:

- melt (GHF+friction+conduction)

  plus

- percolation from surface



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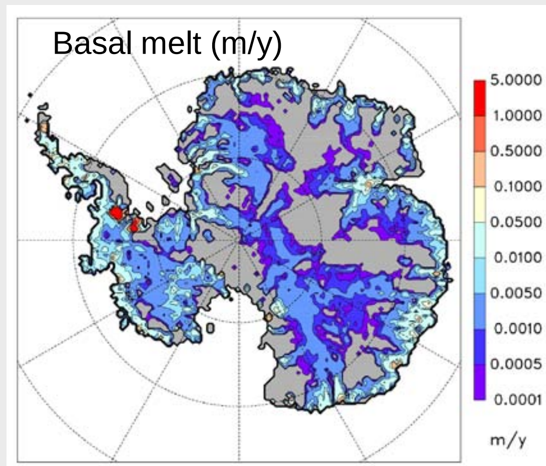

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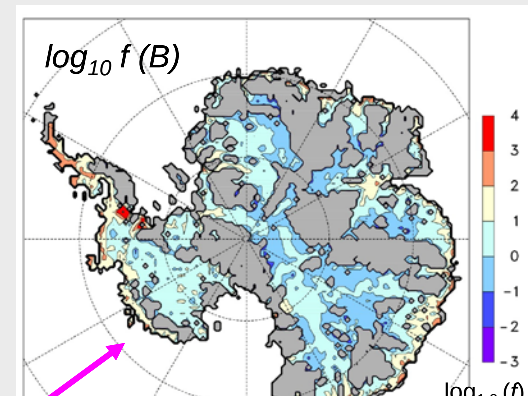
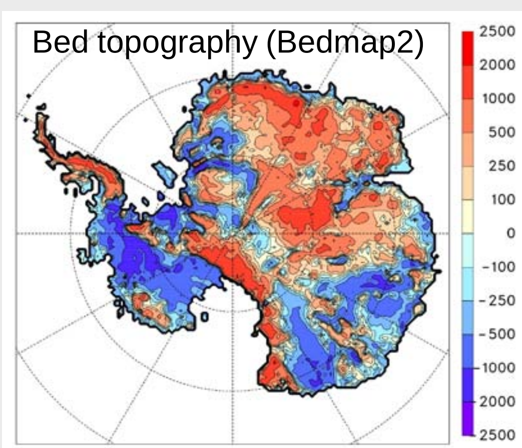
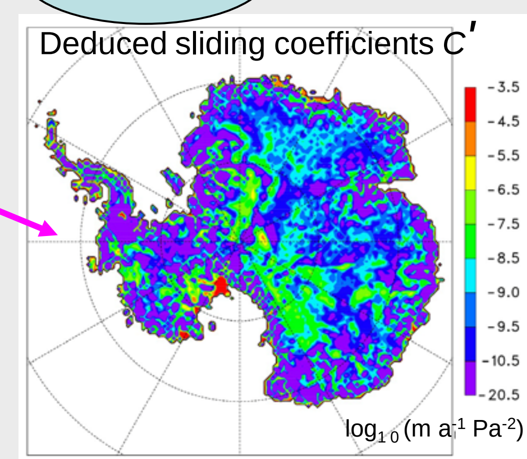


$r = 0.018$



- Again, resulting "*f(B)* ≈ 0" pattern does not resemble purple regions $C' \approx 0$
- Again, main problem is that *B* resembles large-scale bed topography



Why have these attempts at *f(…)* failed?

1) Incorrect internal deformation (mostly SIA) -
   incorrect enhancement factor *E ?*

2) Incorrect longitudinal stress dynamics (hybrid model)

3) Basal hydrologic flow system (re-arranges *B*)

4) Geothermal Heat Flux distribution

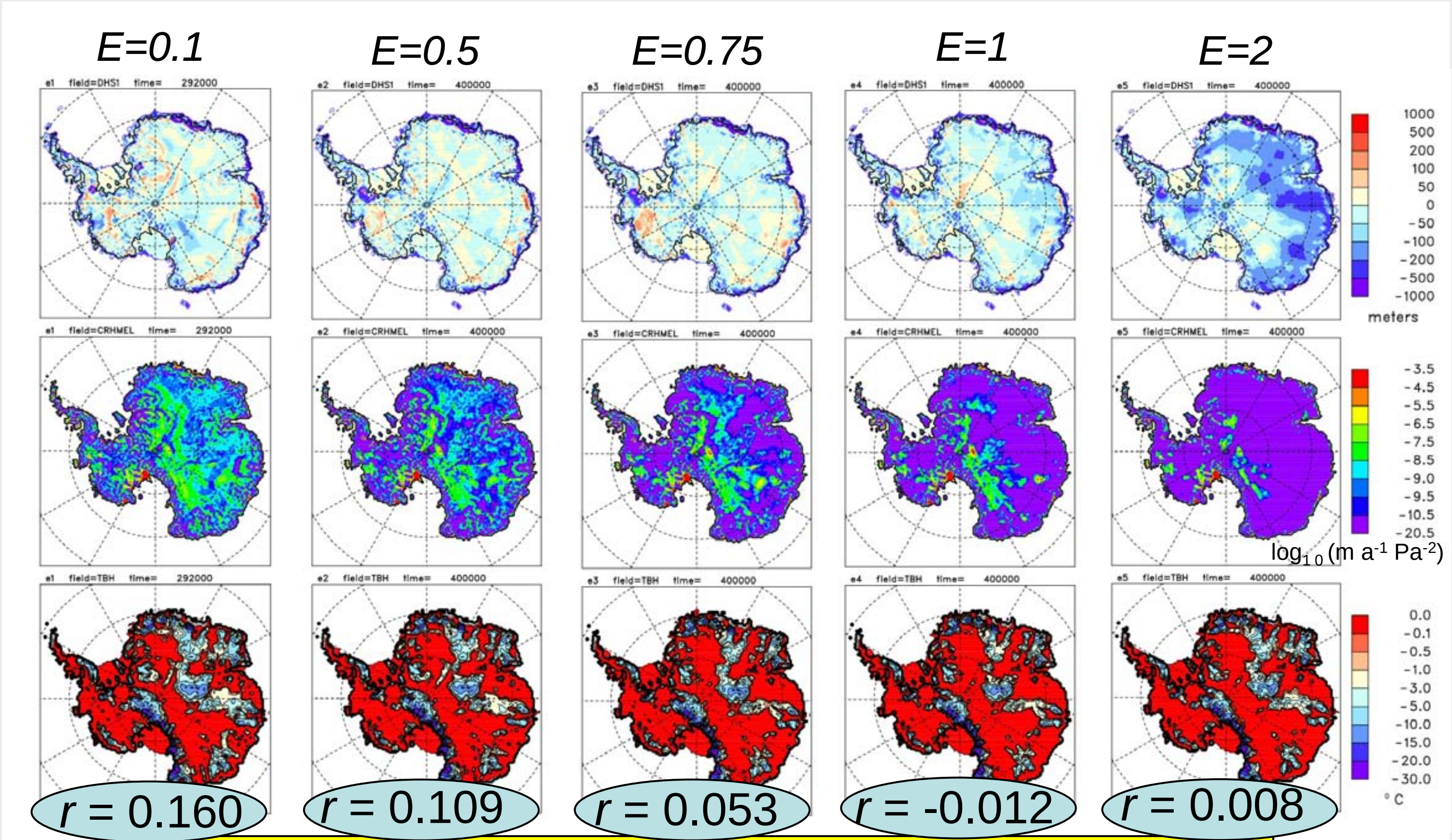
Why # 1: Results of inverse method, different enhancement factors *E*

Inverse with no basal temperature constraint on sliding

Final elevation error Δh_s

Deduced sliding coefficients C'

Basal temperature T_b

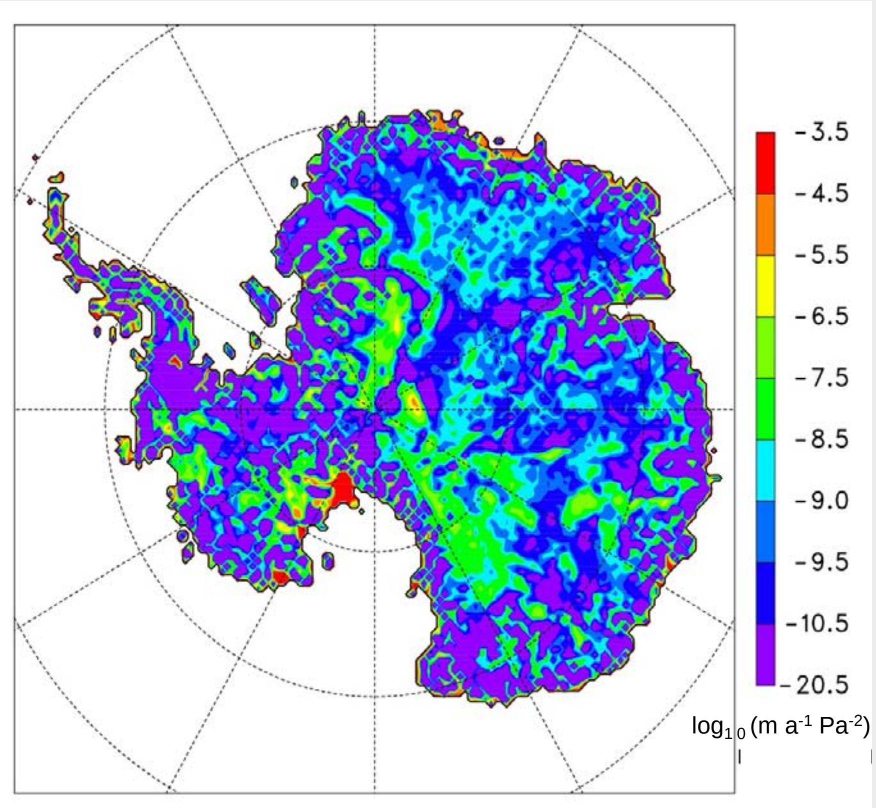


- Increasing *E* requires less sliding, more areas with $C' \approx 0$ (around EAIS flanks)
- None of the purple $C' \approx 0$ patterns resemble frozen-bed patterns $T_b<0$
- Fabric? …*E(x,y,z)*? …*Anisotropic models?*

  e.g., Wang and Warner, Ann. Glac, 1999; Seddick et al., TC 2011

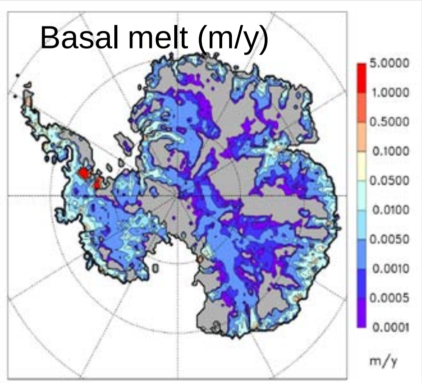
Why # 2: Incorrect longitudinal-stress dynamics (hybrid model)

Deduced sliding coefficients C'

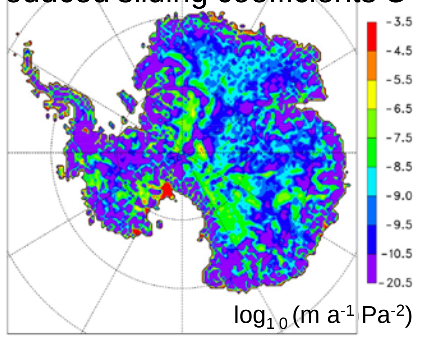


- Many areas with $C' \approx 0$ (purple) are close to ice sheet margins
- Could be compensating for dynamical errors in hybrid model – too much internal shear flow near margins ?
- Test with Full Stokes models

Why # 3: Basal hydrologic flow system (re-arranges *B*)



Deduced sliding coefficients C'



- Current model lacks basal hydrology
- Basal flow could transport water supply *B*, and produce patterns like purple C' > 0 regions (?)

A subglacial water-flow model for West Antarctica

A.M. Le Brocq, A.J. Payne, M.J. Siegert, R.B. Alley

J. Glaciol., 2009

$$\frac{\partial d}{\partial t} = M - \nabla . \overline{u}_w d,$$

$$\overline{u}_w = \frac{d^2}{12\mu} \nabla \Phi,$$

$$u_h = B\tau,$$

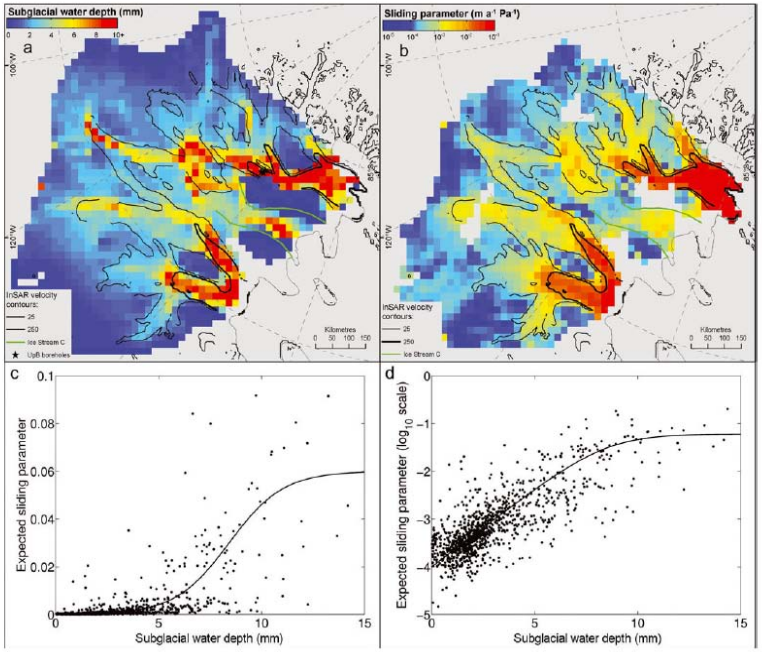
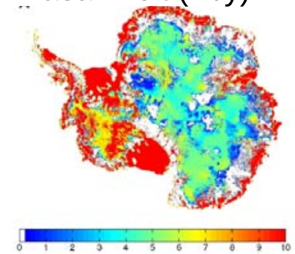


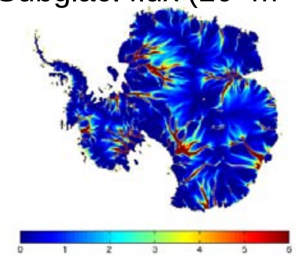
Fig. 4. Relationship between expected basal sliding parameter and subglacial water depth: (a) subglacial water depth; (b) expected basal sliding parameter; (c, d) expected basal sliding parameter plotted against subglacial water depth, all cells where d<15mm.

Pattyn, EPSL, 2010

Basal melt (m/y)



Subglac. flux (10^3 m^2 y^{-1})



Why # 4: Geothermal Heat Flux distribution

Geothermal heat flux (mW m^{-2}):

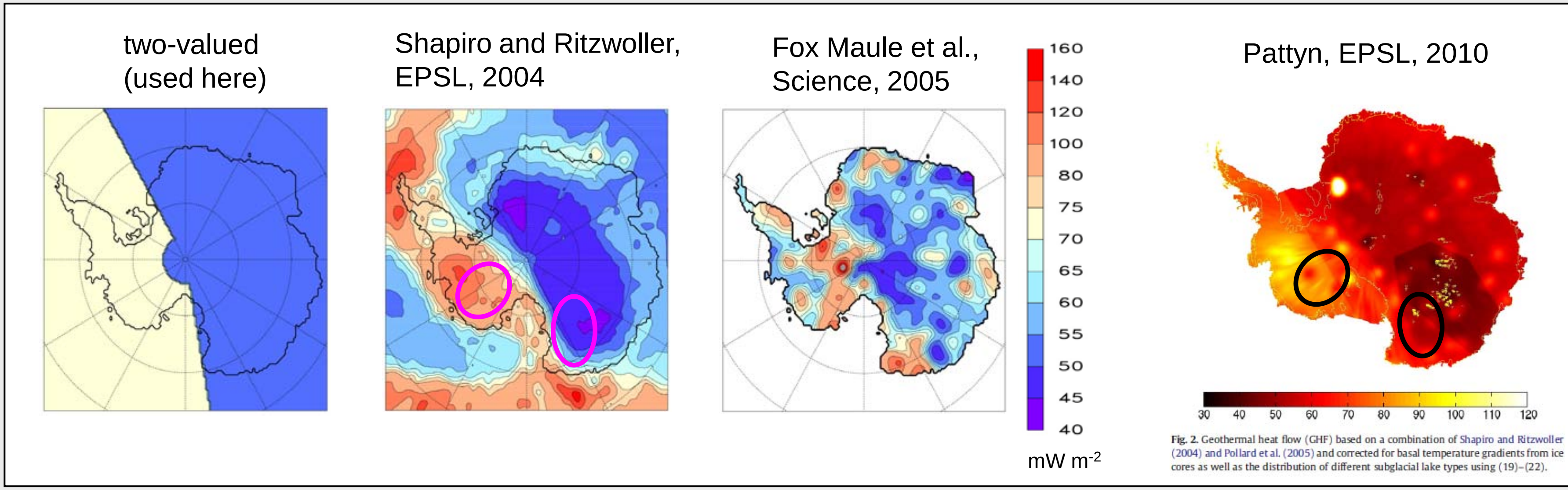
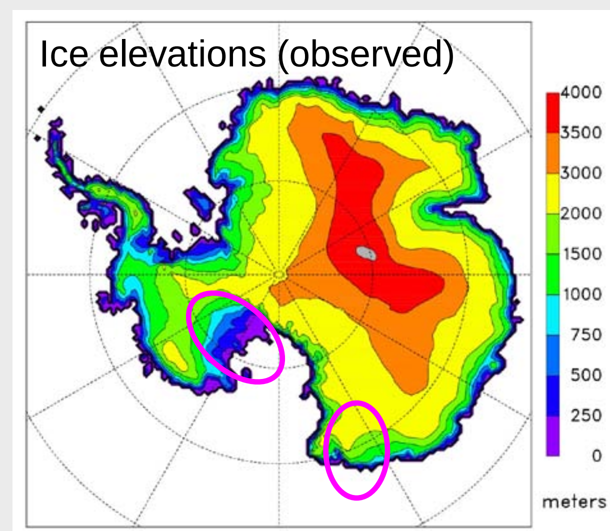


Fig. 2. Geothermal heat flow (GHF) based on a combination of Shapiro and Ritzwoller (2004) and Pollard et al. (2005) and corrected for basal temperature gradients from ice cores as well as the distribution of different subglacial lake types using (19)–(22).

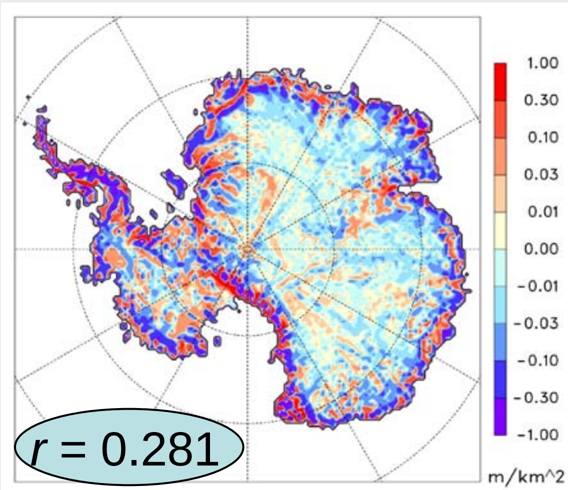
- Perhaps real GHF distribution has more structure, influencing basal melt
- Nb: Modern Siple coast is streaming, Wilkes basin outlet is not – due to high GHF and volcanism upstream of Siple? *

* Behrendt, GPC 2004; Blankenship et al., ARS 2001; Parizek et al., GRL 2002



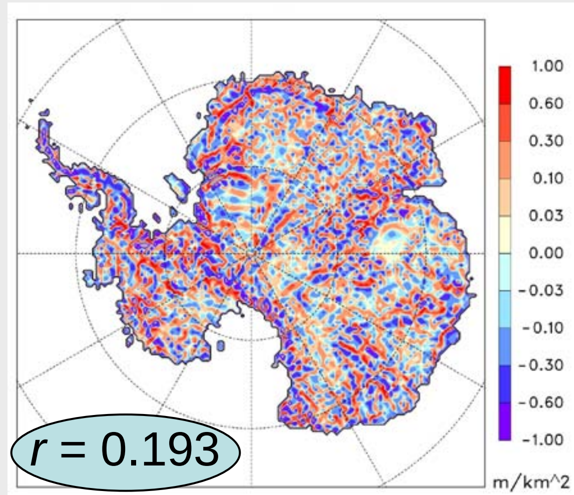
But…regardless of basal physics…the only input to the model with fine structure are Bedmap2 elevation maps

Surface: $\nabla^2 h_s = \partial^2 h_s/\partial x^2 + \partial^2 h_s/\partial y^2$

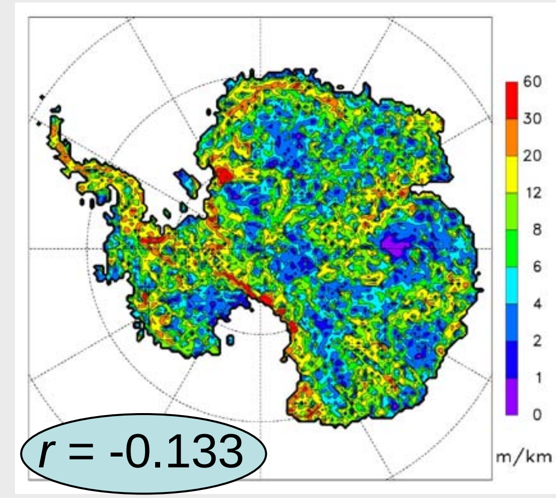


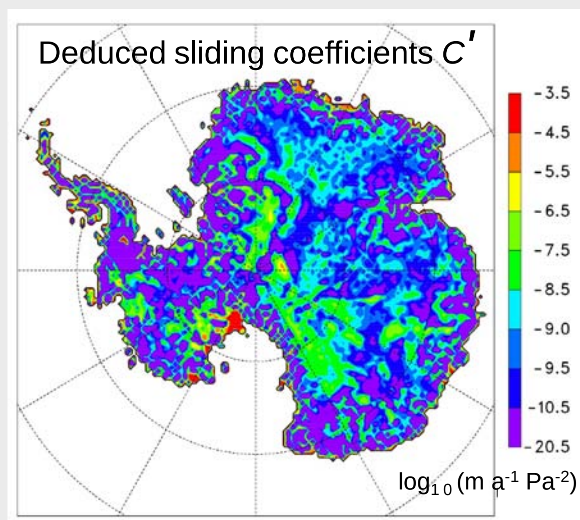
cf. Plan curvature (Le Brocq et al., GRL 2008)

Bed: $\nabla^2 h_b = \partial^2 h_b/\partial x^2 + \partial^2 h_b/\partial y^2$



Bed |slope|: $\sqrt{[\,(\partial h_b/\partial x)^2 + (\partial h_b/\partial y)^2\,]}$





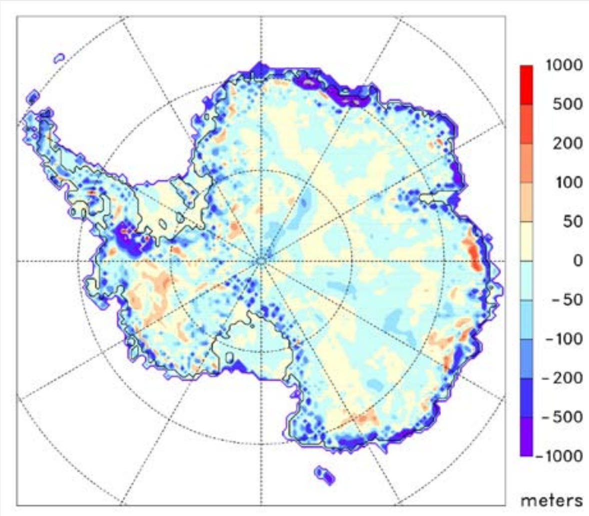
- Still no clear connection with $C' \approx 0$ (purple) patterns
- So where do the $C' \approx 0$ patterns come from in the model **?**
- Do they indicate any real physical process **?**

End

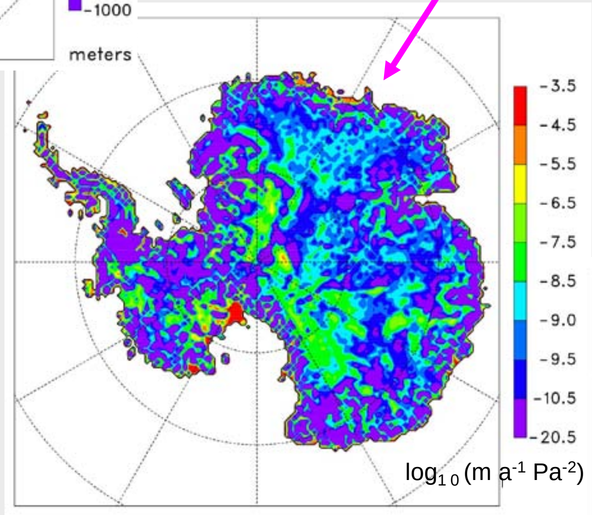
Results of inverse method, *no* basal temperature constraint

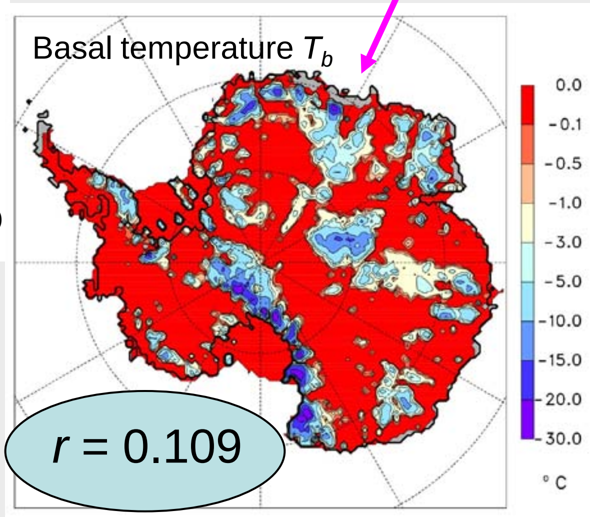
- Purple regions are where sliding ≈ 0
- Ideally, they correspond to frozen beds, or no basal water supply
- But they don't correspond to $T_b < 0$
- Can we find a function *f(T_b, topog., melt)* that does?

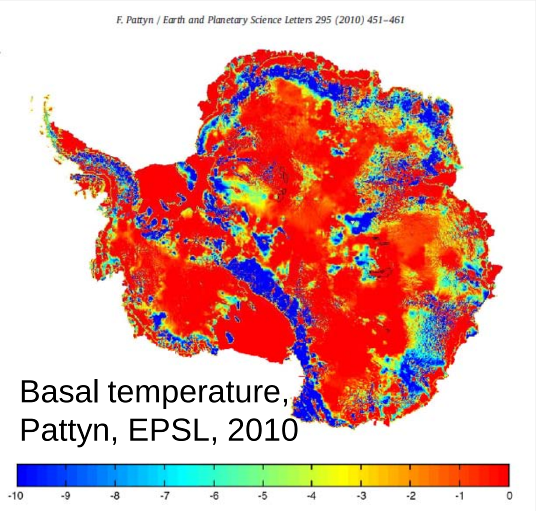
Final elevation error Δh_s



Deduced sliding coefficients C'



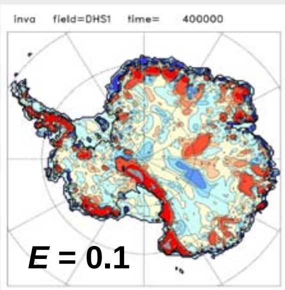


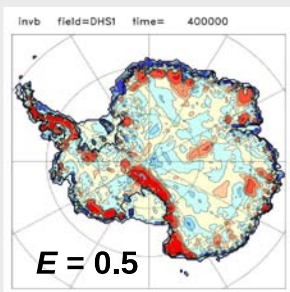


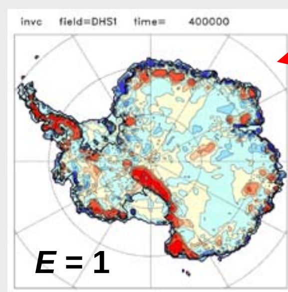
Basal temperature, Pattyn, EPSL, 2010

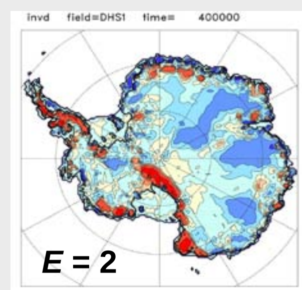
Constraining the internal-flow enhancement factor *E*

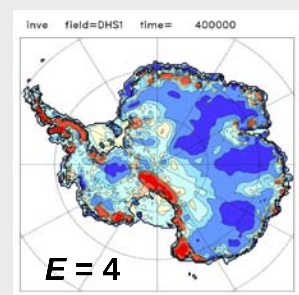
Model minus observed surface elevations:

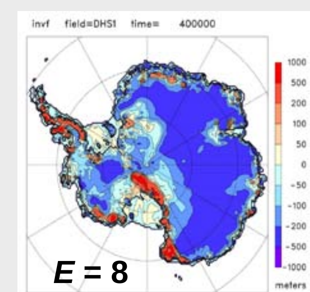












increasing *E*

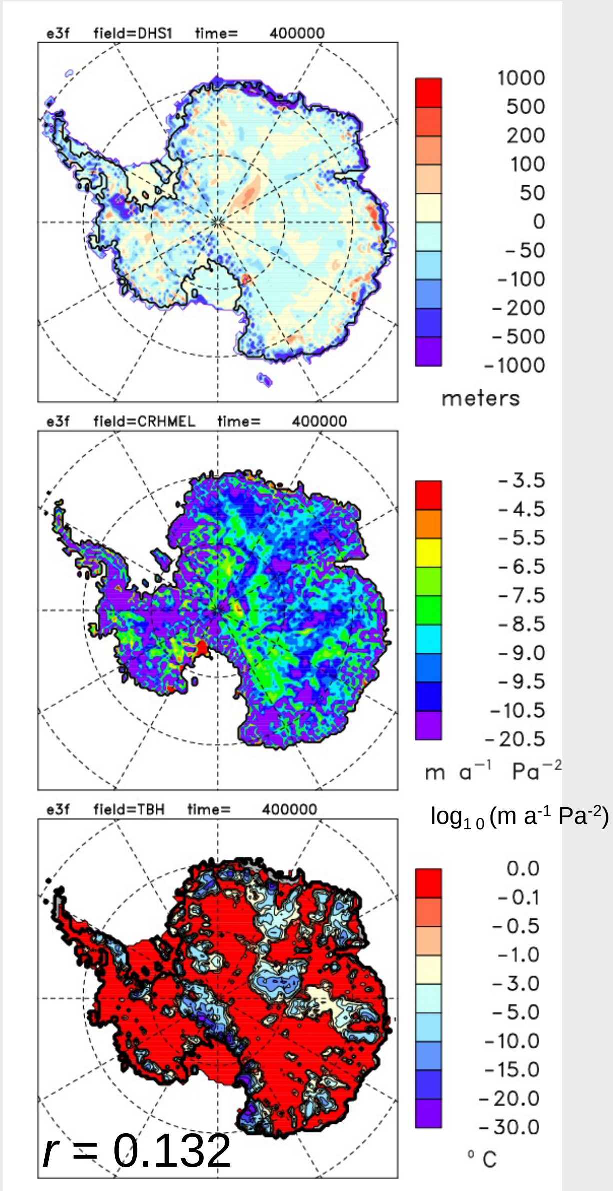
- If *E* is too small, nearly all motion has to be basal sliding ⇒ positive surface errors where base is frozen
- If *E* is too large, too much internal flow. If it exceeds the balance velocity, *C* inversion can't help ⇒ large ubiquitous negative surface errors
- Best results for *E* ≈ 1

Results of inverse method, no T_b effect, *E=0.75 x f(distance to dome)*

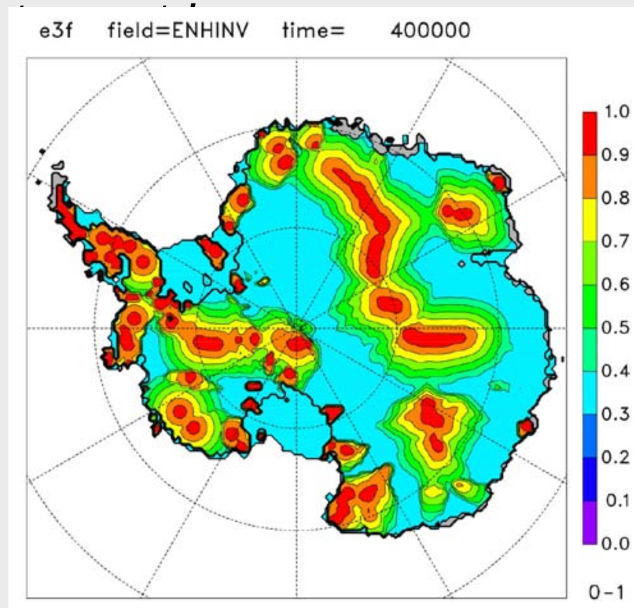
Δh_s

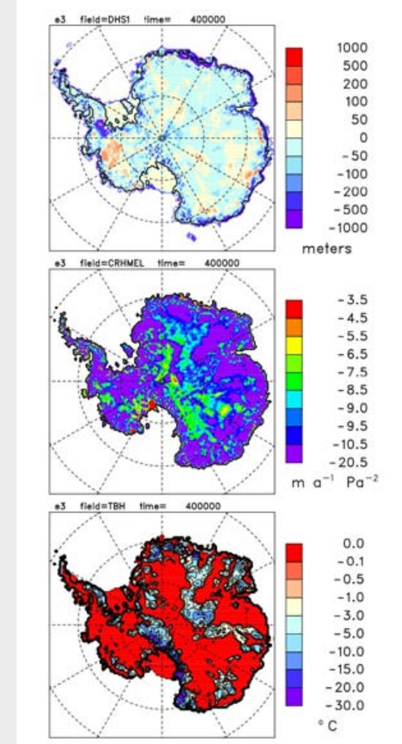
$C(x,y)$

T_b



Inverse with no basal T effect, E=0.75 x factor 1 to 0.3 depending on distance





Fabric, anisotropy, variable enhancement coefficients:

General or review: Alley et al., 1988, Nature. Gagliardini et al., 2009, Low Temp. Sci.

E = f (τ_{xz}, τ_{zz}): Wang and Warner, 1999, Ann. Glac.

Ren et al., 2011, JGR.

E = f(z): Mangeney and Califano, 1998, JGR

Graversen et al., 2011, Clim. Dyn.

Anisotropic models: Gillet-Chaulet et al., 2005, J. Glac. (GOLF law)

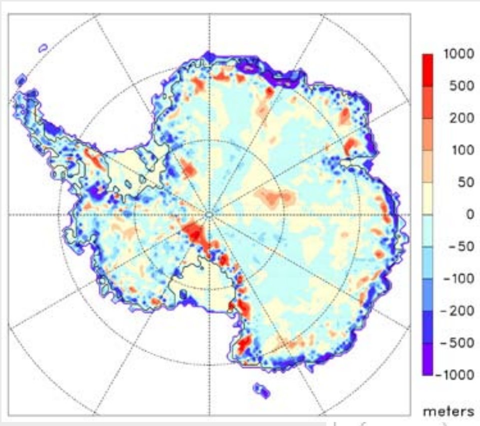
Ma et al., 2010, J. Glac. → *E (sheet vs. shelf).*

Seddick et al., 2011, The Cryo (CAFFE model)

Results of inverse method, *with* basal temperature constraint

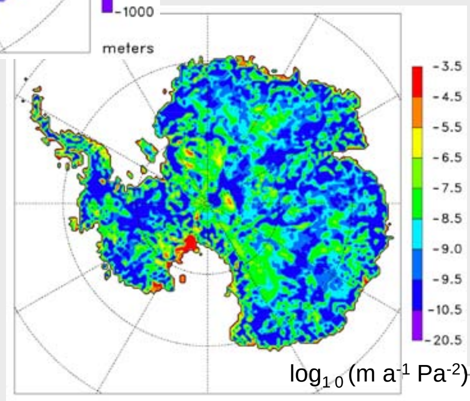
$u_b = C(x,y)\ f(T_b.\ s)\ \tau_b^{\ n}$ where $f(T_b)$ = 0 for frozen bed, ramps to 1 for bed at melt point, and width of ramp increases with sub-grid bed roughness *s*

Final elevation error Δh_s

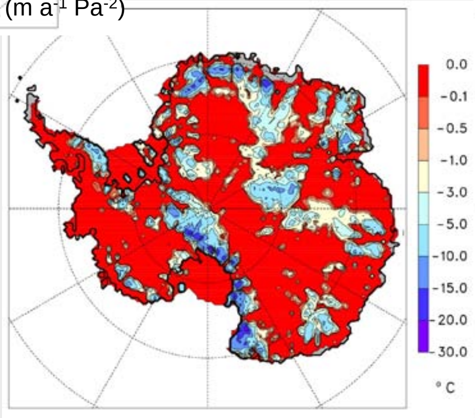


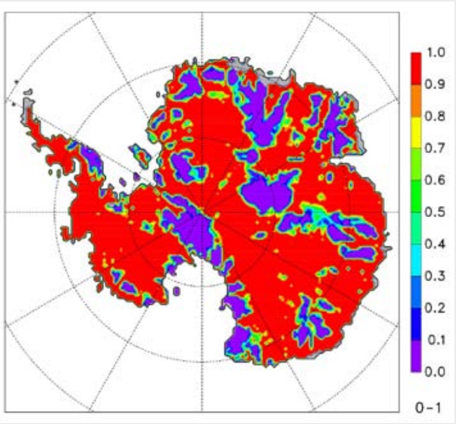
- Δh_s over mountain ranges is improved by dependence on *s*
- But not completely – h_s still too high over mountains

Deduced sliding coefficients *C(x,y)*



Basal temperature T_b

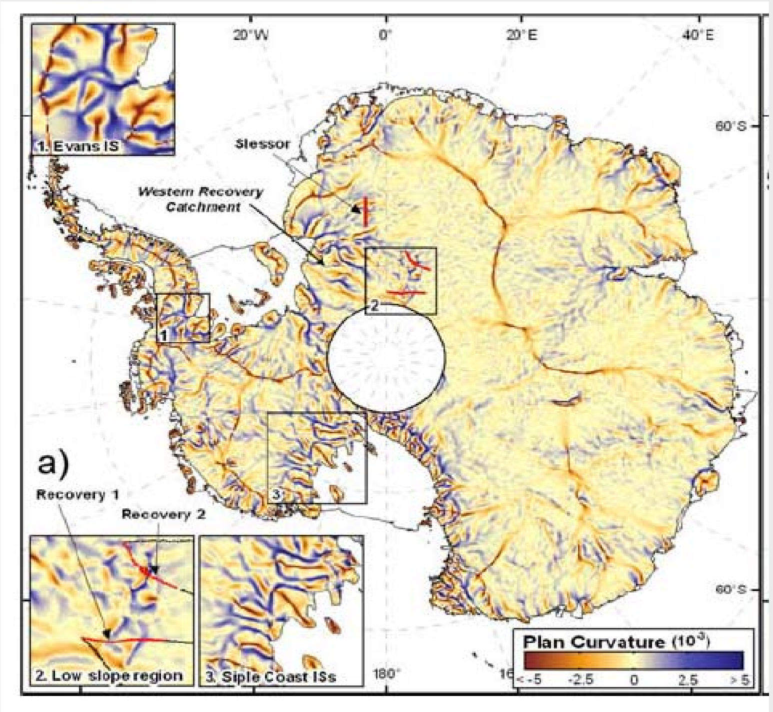




Basal fraction unfrozen (0 to 1)

Plan curvature

Le Brocq et al., GRL, 2008



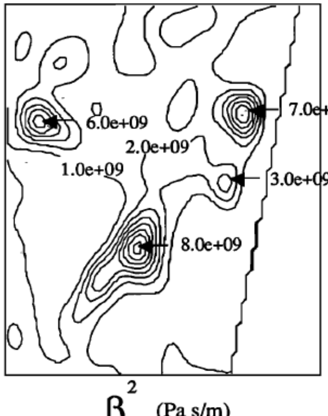
Previous basal inversions for Antarctica

- Previous work has deduced basal-stress or sliding-coefficient maps using control theory (Lagrangian multiplier/adjoint) methods, fitting modeled vs. observed velocities, with ice geometry (thickness, elevation) fixed from observations.
- **Regional:** MacAyeal,1992; Vieli and Payne, 2003; Joughin et al. 2009; Morlighem et al., 2010.

  Continental: ISSM, Larour et al., ISSM, issm.jpl.nasa.gov; Bueler et al., PISM, www.pism-docs.org.

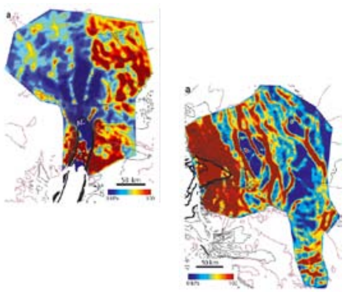
  Also Price et al. (PNAS, 2011), Greenland, local method.

Ice Stream E
(MacAyeal, 1992):

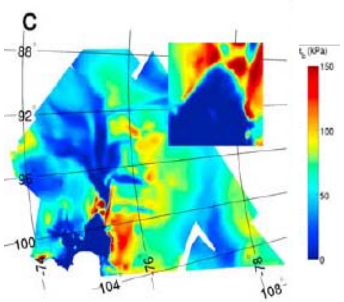


Basal drag coefficient, Ice Stream E. Macayeal JGR, 1992.

Pine Island and Thwaites Glaciers
(Joughin et al., 2009; Morlinghem et al., 2010):

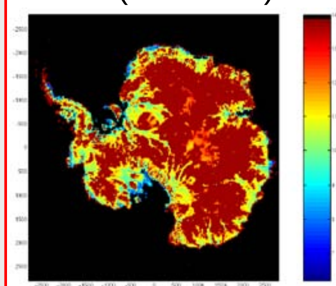


Basal stress, Pine Isl;and and Thwaites Glaciers. Joughin et al., J. Glac., 2009



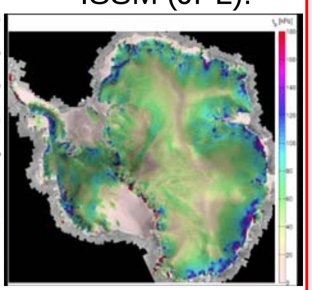
Basal stress, Pine Island Gl: Morlighem et al., GRL, 2010

PISM (U. Alaska):



PISM basal drag coefficient (Pa s m^{-1}). Lingle et al., JPL PARCA meeting, 2007

ISSM (JPL):



ISSM basal stress (Morlighem, pers. comm., 2012)

Relating inverse-derived basal sliding coefficients beneath ice sheets to other large-scale variables

David Pollard
Pennsylvania State University

Robert DeConto
University of Massachusetts

Land Ice Working Group/CESM meeting
NCAR, February 14-15 2013

Summary

- Simple inverse method "works":

  (a) converges, (b) reduces surface elevation errors, (c) deduces reasonable *C(x,y)* patterns.

- Independent of ice model. Just needs:

  (a) run for ~200,000 years, (b) bedrock parameter(s) that make u_b increase or decrease.

- ***BUT*** some of the deduced *C(x,y) must* be due to other model errors, not real bed conditions.

  Lesser of two evils: cancelling errors vs. O(500m) biases in surface elevation

- Next steps:

  - Combine with large-ensemble techniques? (Stone et al., The Cryo. 2010; Tarasov et al., EPSL, 2011)

  - Apply to last deglaciation (Briggs et al., ISAES abs., 2011.; Whitehouse et al., QSR, 2012)