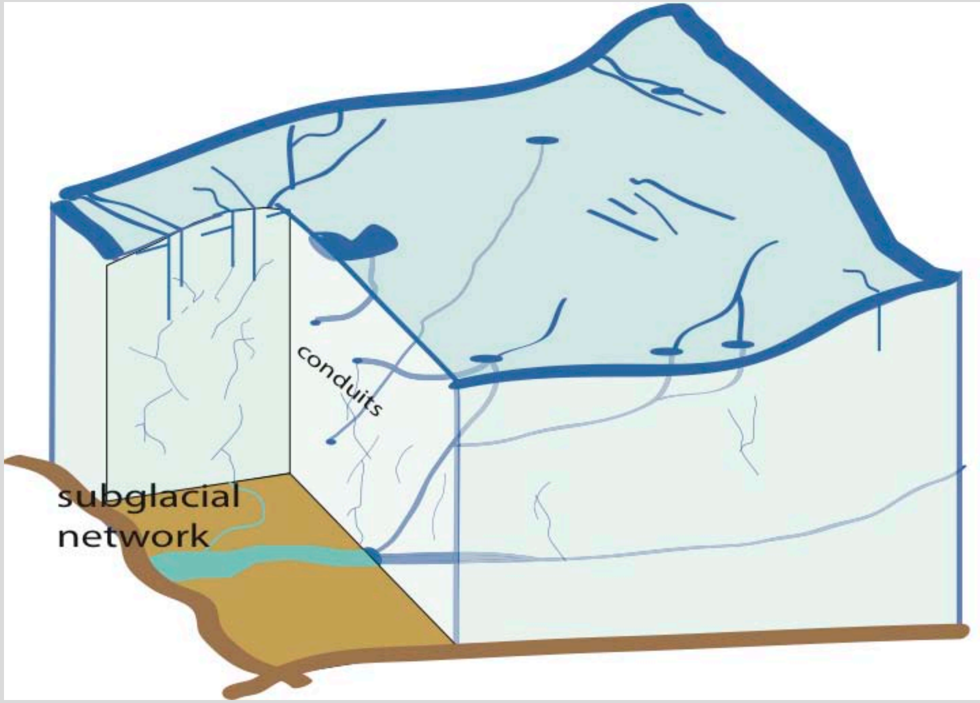
Enthalpy-Based Models for Ice Sheets and Improving Understanding of Cryo-Hydrologic Warming

Brian Macpherson

Harihar Rajaram



Motivation

- Rising ELA causes new areas of melt/crevassing (West Greenland)
- Englacial water rapid means of causing warmth in ice sheet
- Enthlapy methods provide efficient and precise way of dealing with phase change in thermodynamic models (Aschwanden et. al 2012)

Goals

- Compare temperature and enthalpy-based thermodynamic formulations for use in CISM
- Explore fundamental physics of englacial heat transfer (cryo-hydrologic warming) to improve parameterization

Temperature Based Approach

Advection-Conduction: $$\frac{\partial\theta}{\partial t}+u\frac{\partial\theta}{\partial x}+w\frac{\partial\theta}{dz}-\frac{\partial}{\partial z}\left(\mathrm{K}_{ice}\frac{\partial\theta}{\partial z}\right)=\frac{Q}{\rho C}$$

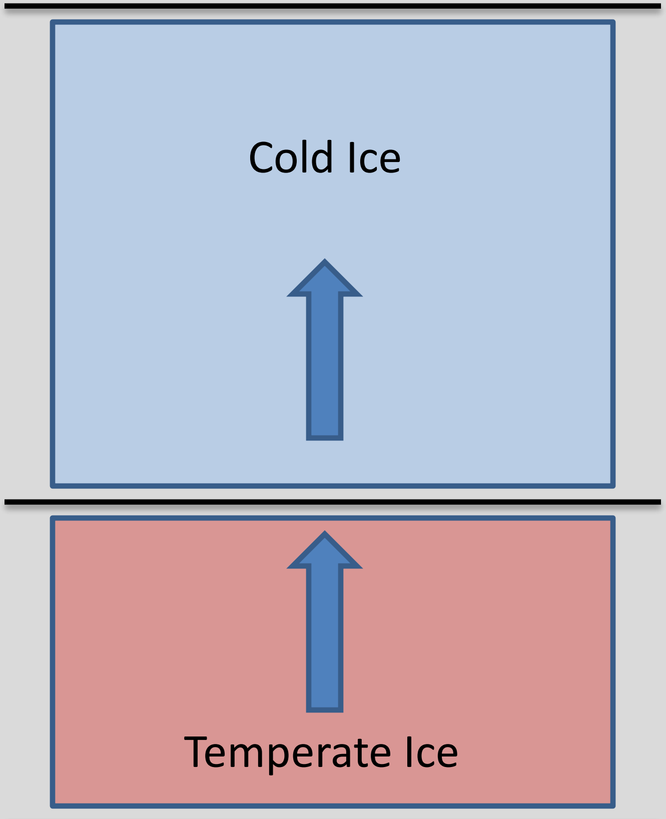
K=diffusivity
Q=strain heat
C=heat capacity

- straightforward for cold ice
- requires CTS tracking in polythermal conditions

Heat flux balance:

$$-\kappa\frac{\partial\theta}{\partial z}\bigg|_{+}=-\kappa\frac{\partial\theta}{\partial z}\bigg|_{-}+\rho_w\phi_w wL$$

κ=conductivity
L=latent heat of fusion
ϕ_w=water content


Cold Ice

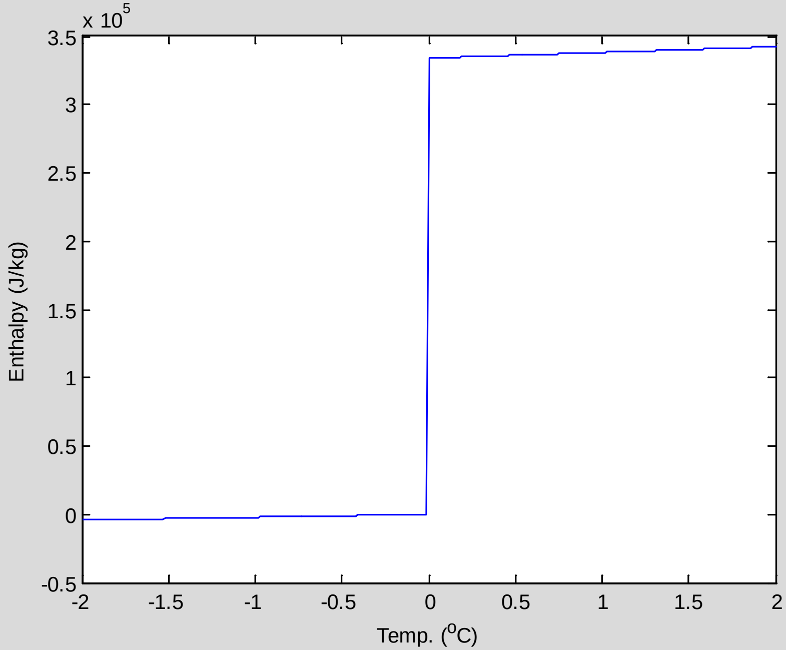
CTS Interface

Temperate Ice

Apparent Heat Capacity Method

$$\underbrace{\frac{\partial\left(\overline{\rho H}\right)}{\partial\theta}}_{\rho C}\frac{\partial\theta}{\partial t}+w\cdot\underbrace{\frac{\partial\overline{\rho H}}{\partial\theta}}_{\rho C}\frac{\partial\theta}{\partial z}-\frac{\partial}{\partial z}\left(\kappa_{ice}\frac{\partial\theta}{\partial z}\right)=Q-u(z)\rho C\alpha\lambda$$

H=specific enthalpy
κ=conductivity
C=heat capacity

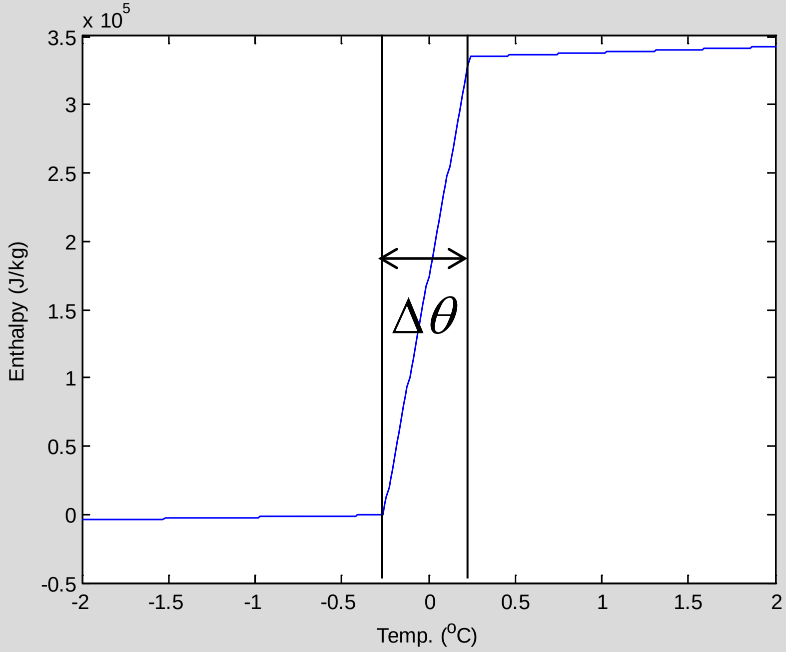


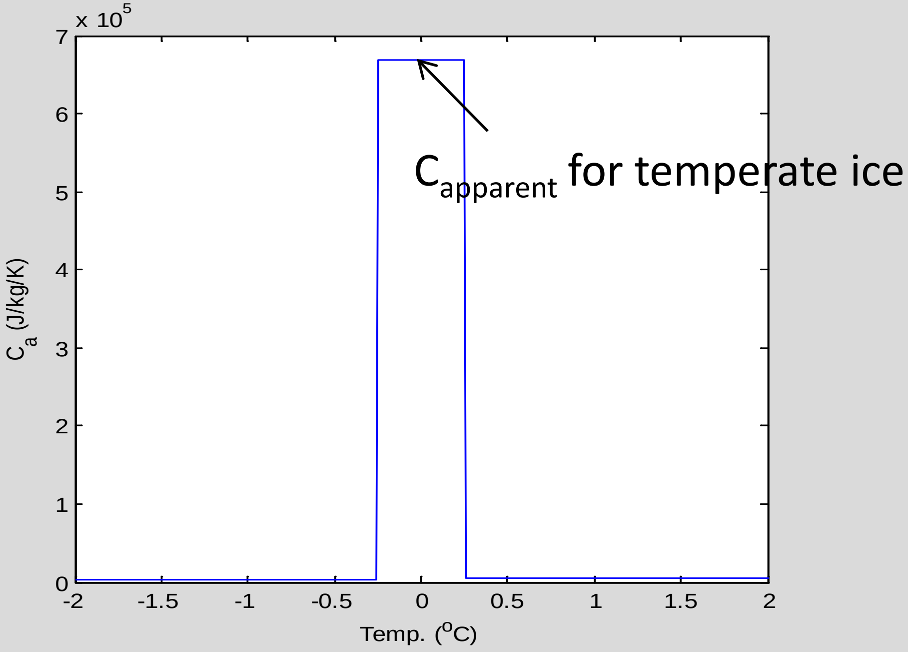
Apparent Heat Capacity Method

$$\underbrace{\frac{\partial\left(\overline{\rho H}\right)}{\partial\theta}}_{\rho C_{apparent}}\frac{\partial\theta}{\partial t}+w\cdot\underbrace{\frac{\partial\,\overline{\rho H}}{\partial\theta}}_{\rho C_{apparent}}\frac{\partial\theta}{\partial z}-\frac{\partial}{\partial z}\left(\kappa_{ice}\frac{\partial\theta}{\partial z}\right)=Q-u(z)\rho C\alpha\lambda$$

H=specific enthalpy
κ=conductivity
C=heat capacity

- works well for low ϕ_w
- high ϕ_w causes artificial sensible heat flux





Enthalpy Gradient Method

$$\overline{\rho H} = (1-\phi_w)\rho_{ice} C_{ice}\theta + \phi_w \rho_w (C_{ice}\theta_{PMP} + L)$$

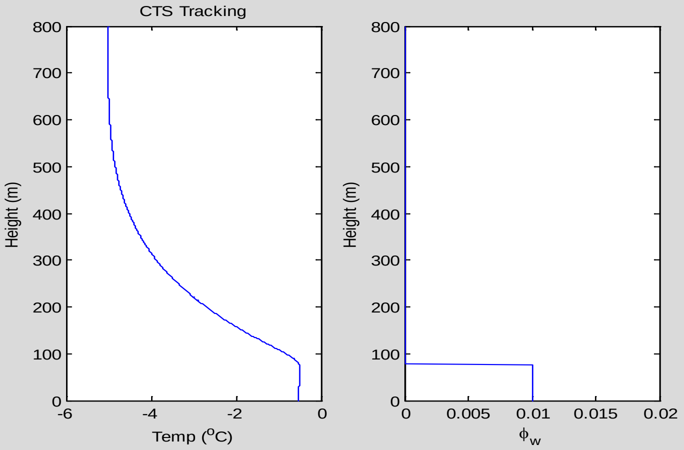
$$\frac{\partial \overline{\rho H}}{\partial t} + u\frac{\partial \overline{\rho H}}{\partial x} + w\frac{\partial \overline{\rho H}}{dz} - \frac{\partial}{\partial z}(J_{conduction}) = Q$$

H=specific enthalpy
ϕ_w=water content
C=heat capacity
L=latent heat of fusion

$$J_{conduction} = \begin{cases} \overbrace{\mathrm{K}_i \frac{\partial \overline{\rho H}}{\partial z}}^{\text{cold sensible heat flux}} , & \overline{\rho H} \le \overline{\rho H}_{PMP} \\ \underbrace{\kappa_i \frac{\partial \theta_{PMP}}{\partial z}}_{\text{PMP sensible heat flux}} + \underbrace{\mathrm{K}_0 \frac{\partial \overline{\rho H}}{\partial z}}_{\text{“Darcy-like” diffusion of liquid water}} , & \overline{\rho H} > \overline{\rho H}_{PMP} \end{cases}$$

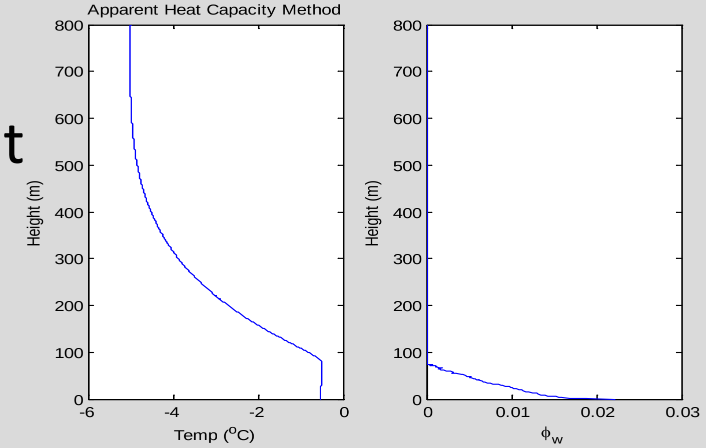
- For small ϕ_w, K_o small positive diffusivity
- For large ϕ_w, K_o chosen to match analytical solutions for crevasse refreezing in semi-infinite domains

Temp. Approach



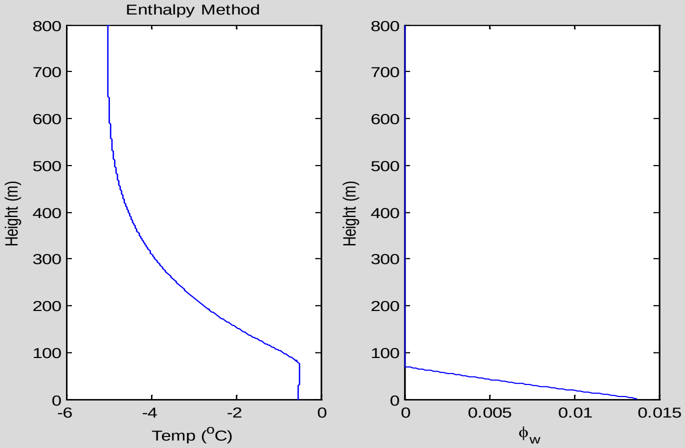
- CTS found through iteration
- ϕ_w=0.01 in temperate ice, assumed value

App. Heat Capacity



- Similar polythermal temperature profile
- Estimate of ϕ_w by assuming water content linear across $\Delta\theta$

Enthalpy Gradient

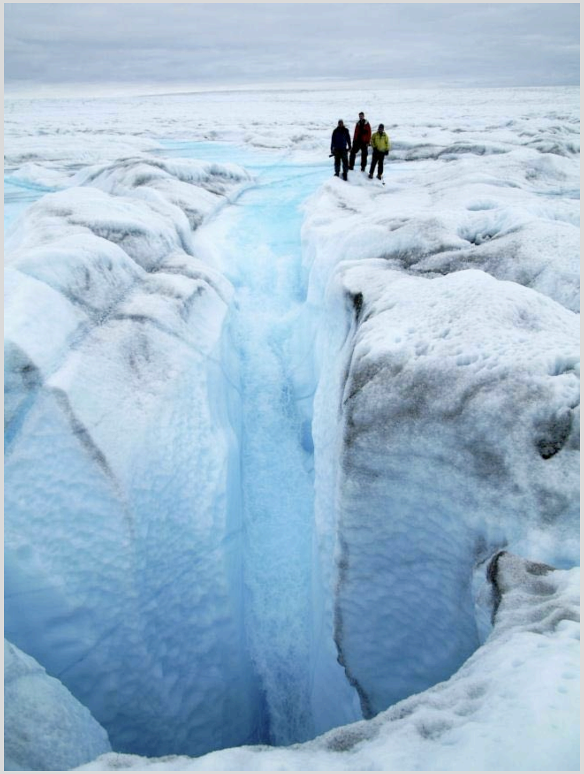


- Exact solution for θ and ϕ_w since both a function of enthalpy

Cryo-Hydrologic Warming

Standing water in crevasses

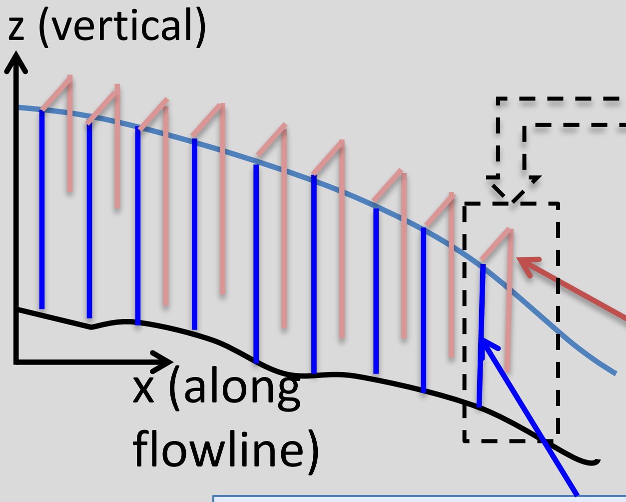
Moulin

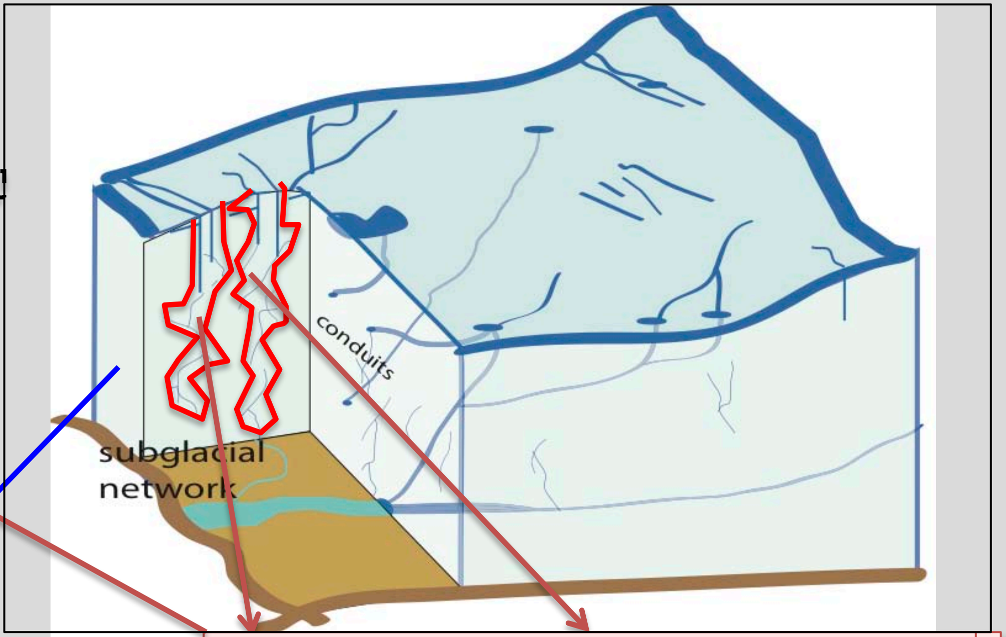


Cryo-Hydrologic Warming (CHW):

- Warming generated by meltwater refreezing and/or persistence within the englacial cryo-hydrologic system (**CHS**)….substantially faster than conduction of warmth from surface
- Potentially rapid thermo-mechanical feedback mechanism

DUAL-CONTINUUM MODEL





“Background” Ice

$$\rho_{ice}C_{ice}\frac{\partial\theta_{ice}}{\partial t}+\rho_{ice}C_{ice}u\frac{\partial\theta_{ice}}{\partial x}+\rho_{ice}C_{ice}w\frac{\partial\theta_{ice}}{\partial z}$$

Horiz. advection Vert. advection

$$-k_{ice}\frac{\partial^2\theta_{ice}}{\partial z^2}=Q+\frac{k_{ice}}{R^2}\left(\theta_{CHS}-\theta_{ice}\right)$$

Vert. conduction Strain heating CHW (warming)

Cryo-Hydrologic System (CHS)

$$\theta_{CHS}=\theta_{PMP}\quad\text{(melt season)}$$

enthalpy change (winter refreezing)

$$\frac{\partial\left(\overline{\rho H}\right)}{\partial t}-\frac{\partial}{\partial z}\left(k_{CHS}\frac{\partial\theta_{CHS}}{\partial z}\right)=$$

Vert. conduction

$$-\frac{k_{ice}}{R^2}\left(\theta_{CHS}-\theta_{ice}\right)$$

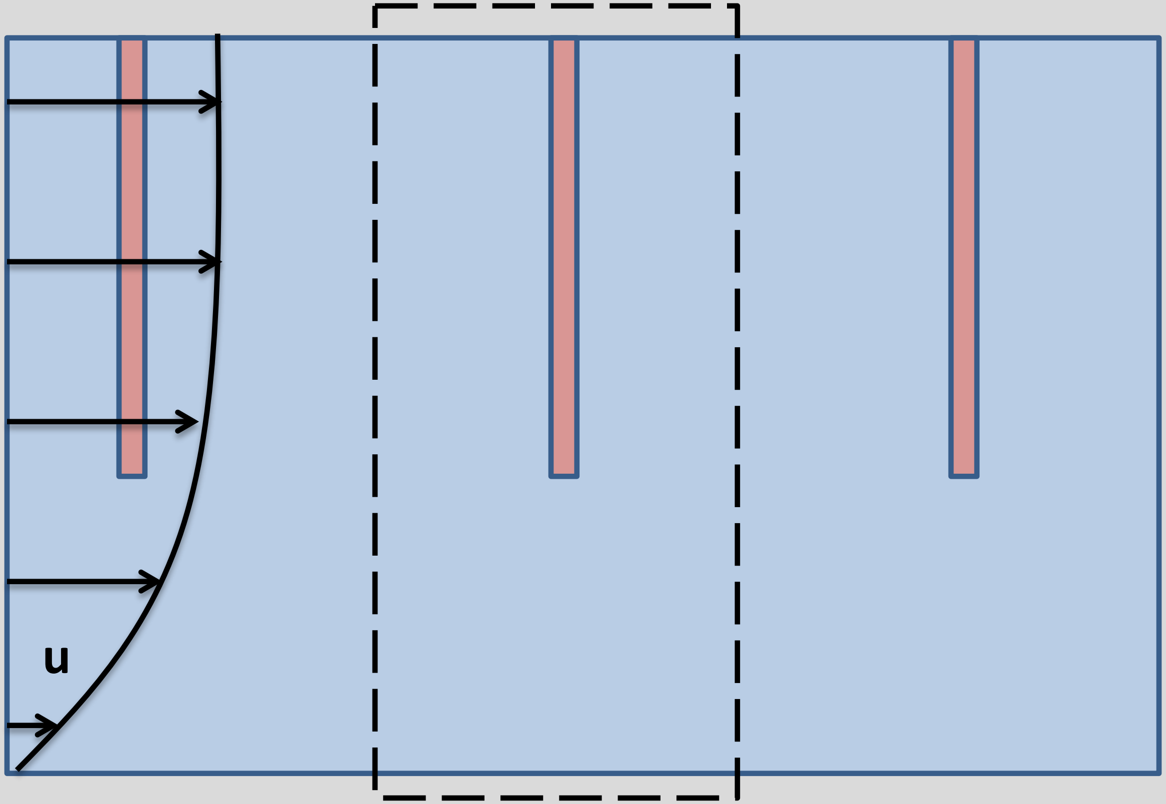
- CHW (cooling, heat loss)

Conventional thermodynamic model augmented with CHW

R – average half-spacing between elements of CHS (varies with x and z, and possibly time)

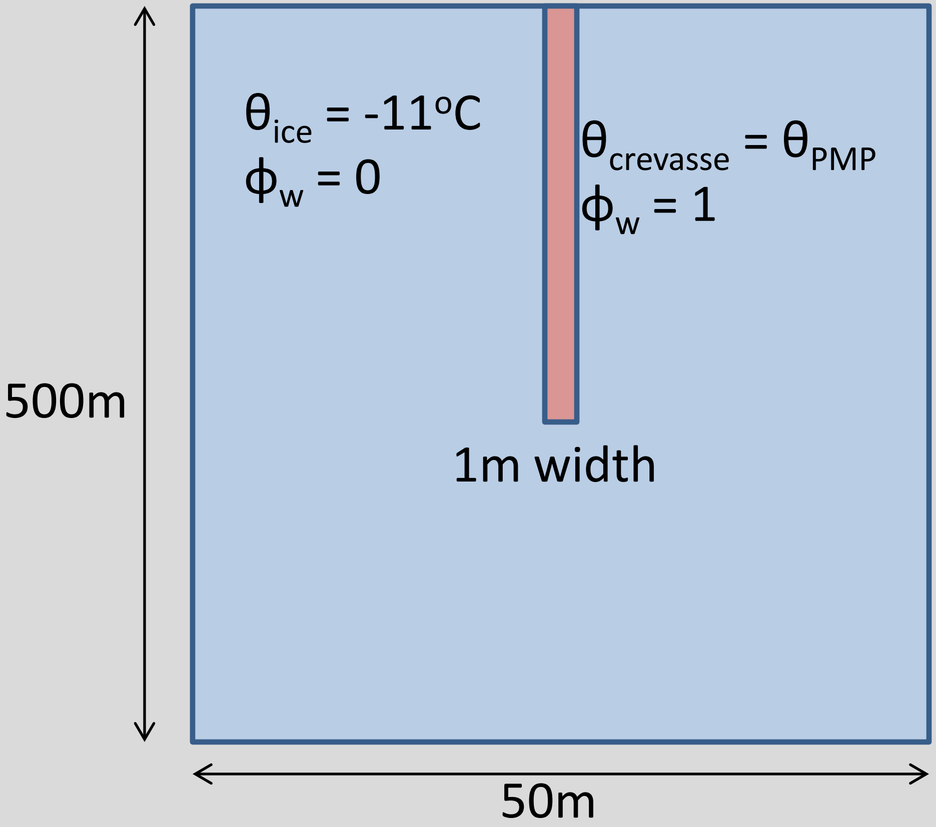
High-Resolution CHS Modeling

$$\frac{\partial\left(\overline{\rho H}\right)}{\partial t}+u(\zeta)\frac{\partial\left(\overline{\rho H}\right)}{\partial x}+w'(\zeta)\frac{\partial\left(\overline{\rho H}\right)}{\partial \zeta}-\frac{\partial}{\partial x}\left(\begin{matrix} \mathrm{K}_i\dfrac{\partial\overline{\rho H}}{\partial x} \\ \kappa_i\dfrac{\partial\theta_{PMP}}{\partial x}+\mathrm{K}_0\dfrac{\partial\overline{\rho H}}{\partial x}\end{matrix}\right)-\frac{1}{H^2}\frac{\partial}{\partial\zeta}\left(\begin{matrix} \mathrm{K}_i\dfrac{\partial\overline{\rho H}}{\partial \zeta} \\ \kappa_i\dfrac{\partial\theta_{PMP}}{\partial \zeta}+\mathrm{K}_0\dfrac{\partial\overline{\rho H}}{\partial \zeta}\end{matrix}\right)=Q(\zeta)$$



- Eulerian reference frame
- Specified velocity field, IC temp., englacial water body geometry and spacing
- One-time filling of englacial water bodies

CHS – Conduction Case



θ_{ice} = -11°C
ϕ_w = 0

$\theta_{crevasse} = \theta_{PMP}$
ϕ_w = 1

500m

1m width

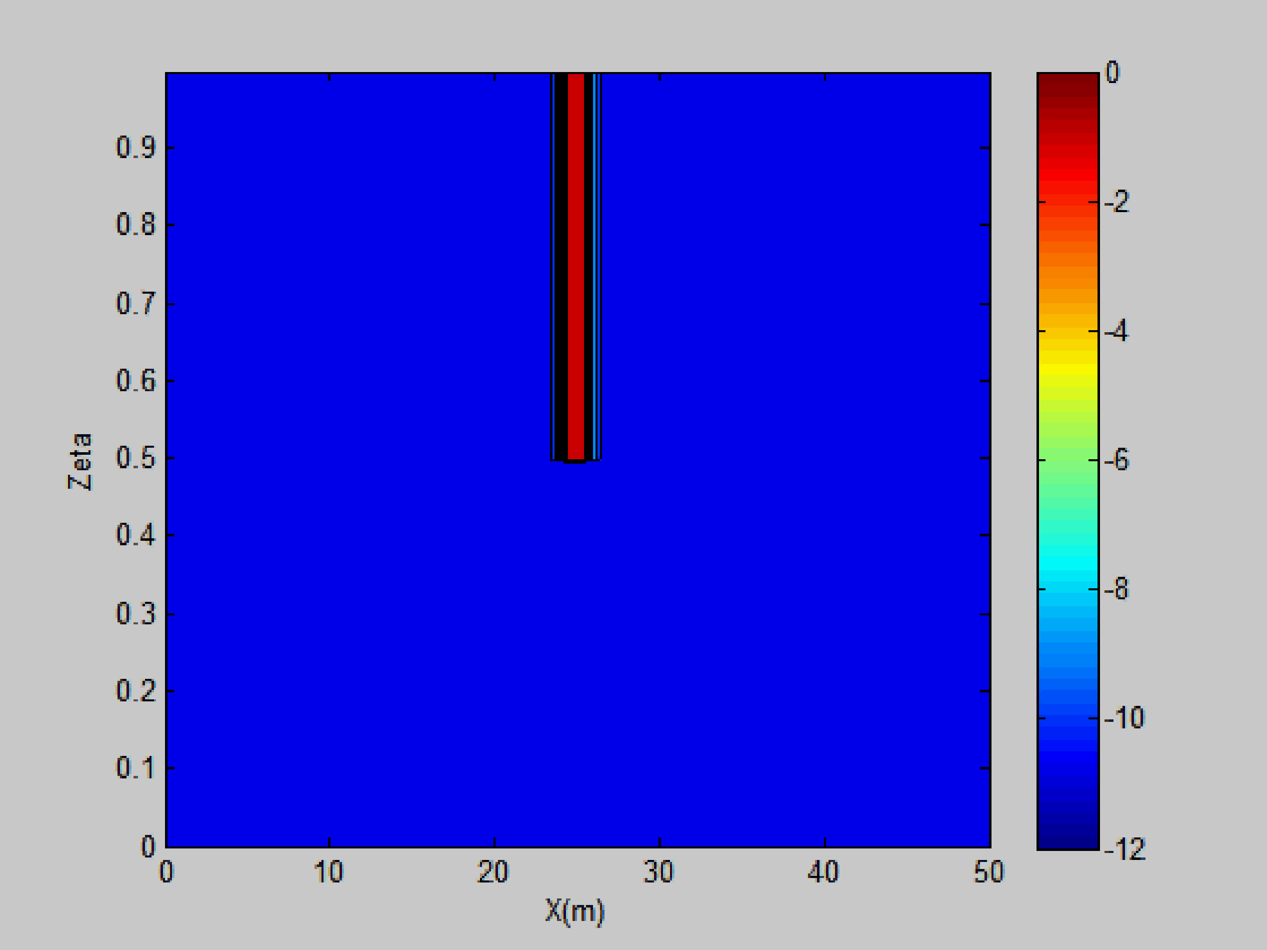
50m

- Comparable with Stefan Problem analytical solutions
- Rate of thermal response affected by englacial water body features (width, depth, spacing) and background ice temperature

Ultimate warming after full refreezing:

$$\theta_{final} = \frac{w_{crevasse}\left(\overline{\rho H}\right)_{crevasse} + \left(R_{crevasse} - w_{crevasse}\right)\left(\overline{\rho H}\right)_{ice}}{\rho_i C_i R_{crevasse}} = -7.24^{o}\,C$$

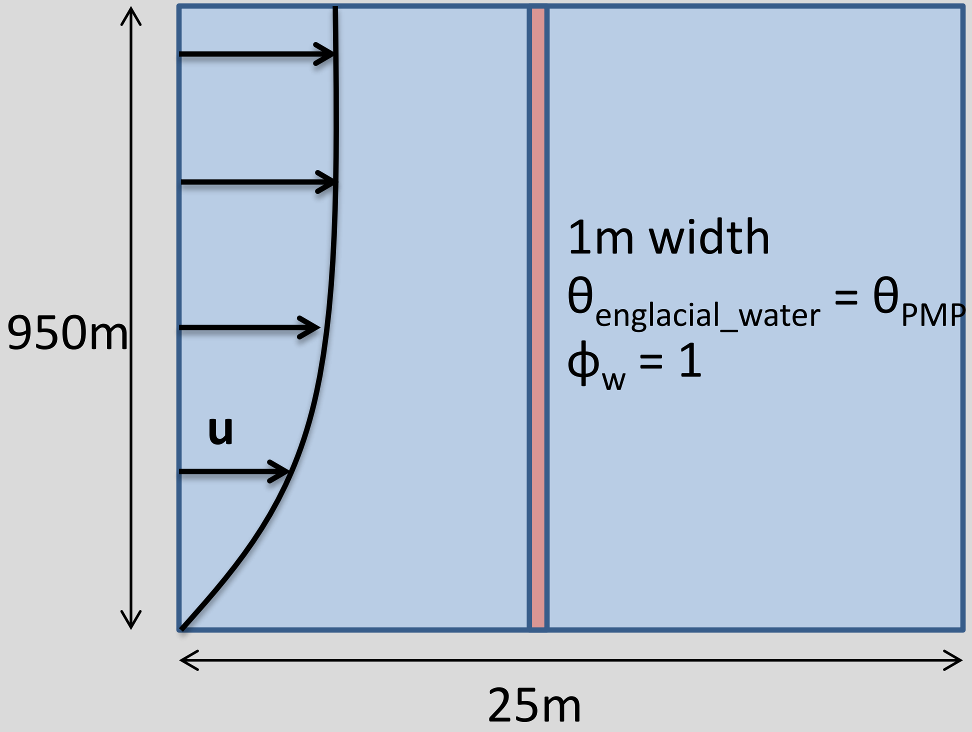
CHS – Conduction Case



t=5 years

CHS in TD5 Test Case

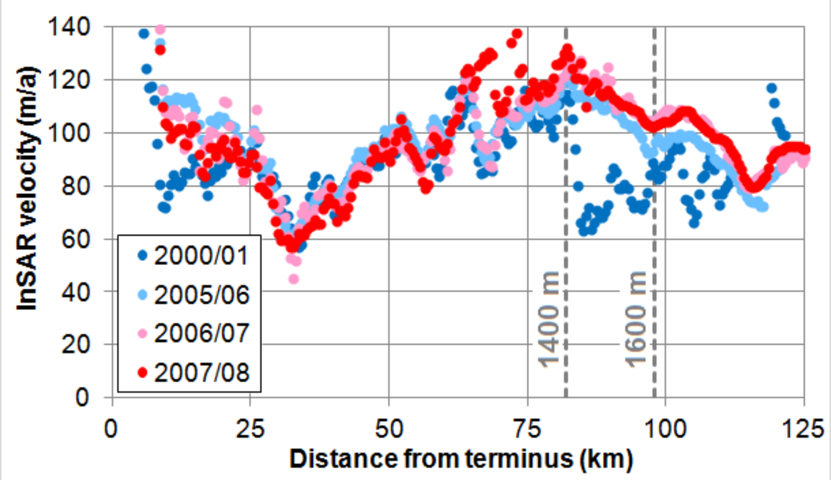
- Sermeq Avannarleq – upward ascent of ELA
- Accumulation to ablation zone transition (new melt input)
- InSAR data shows velocity acceleration

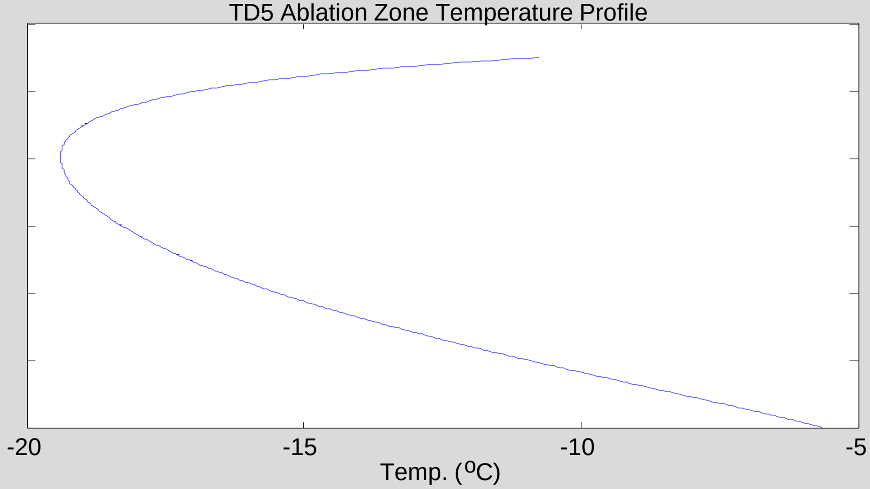


1m width

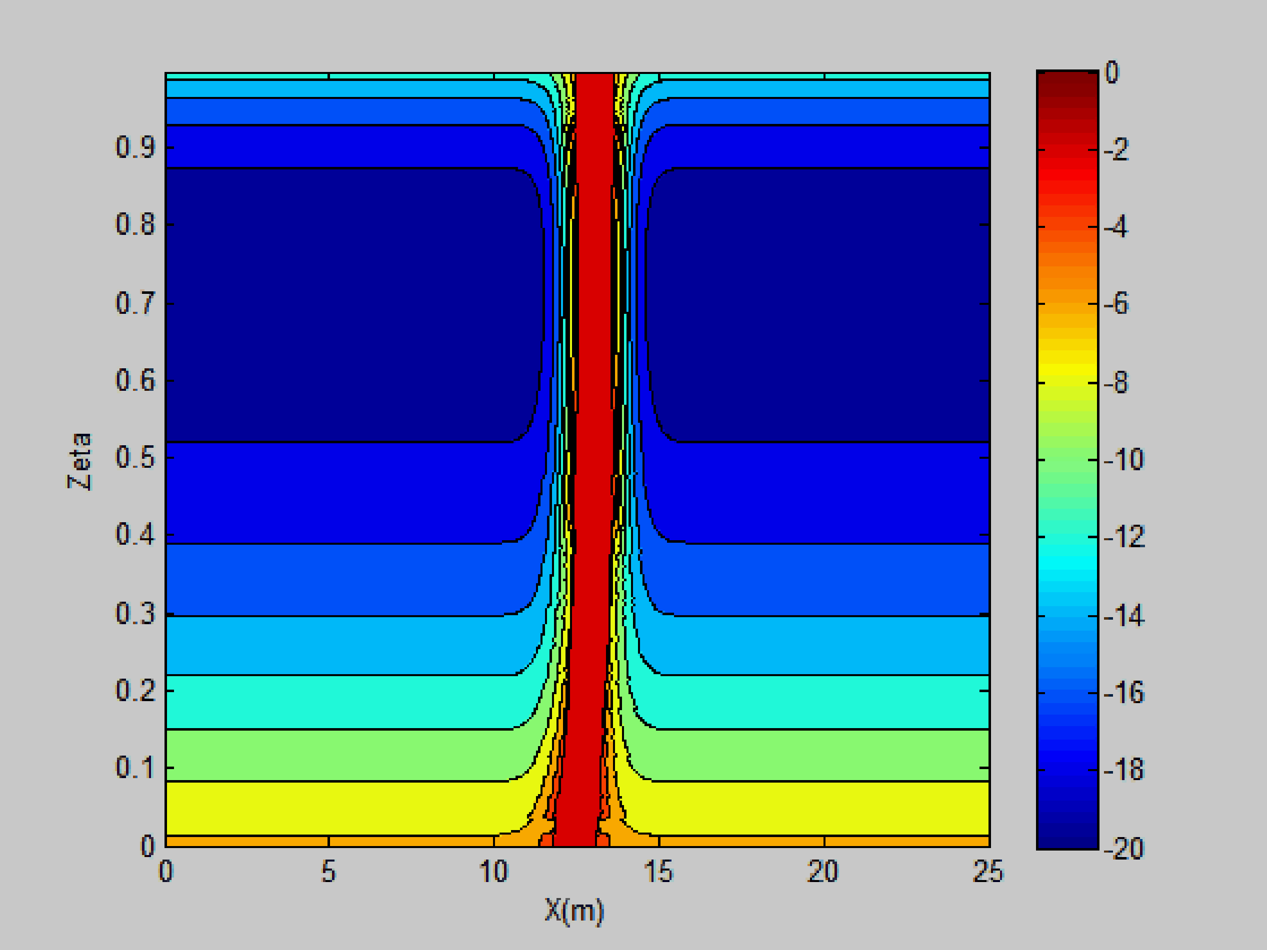
$\theta_{englacial\_water} = \theta_{PMP}$

$\phi_w = 1$



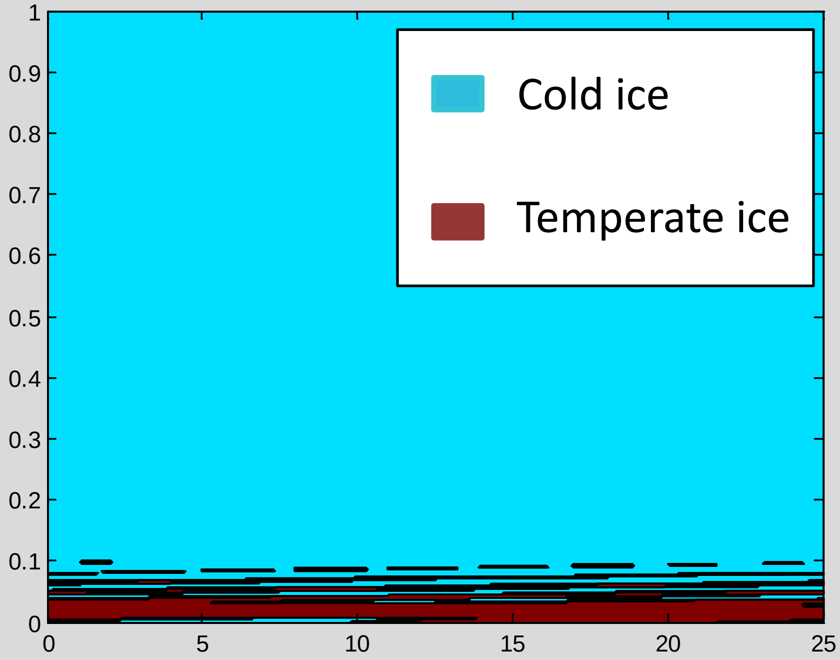


CHS in TD5 Test Case



t=4 years

TD5 Post-CHW



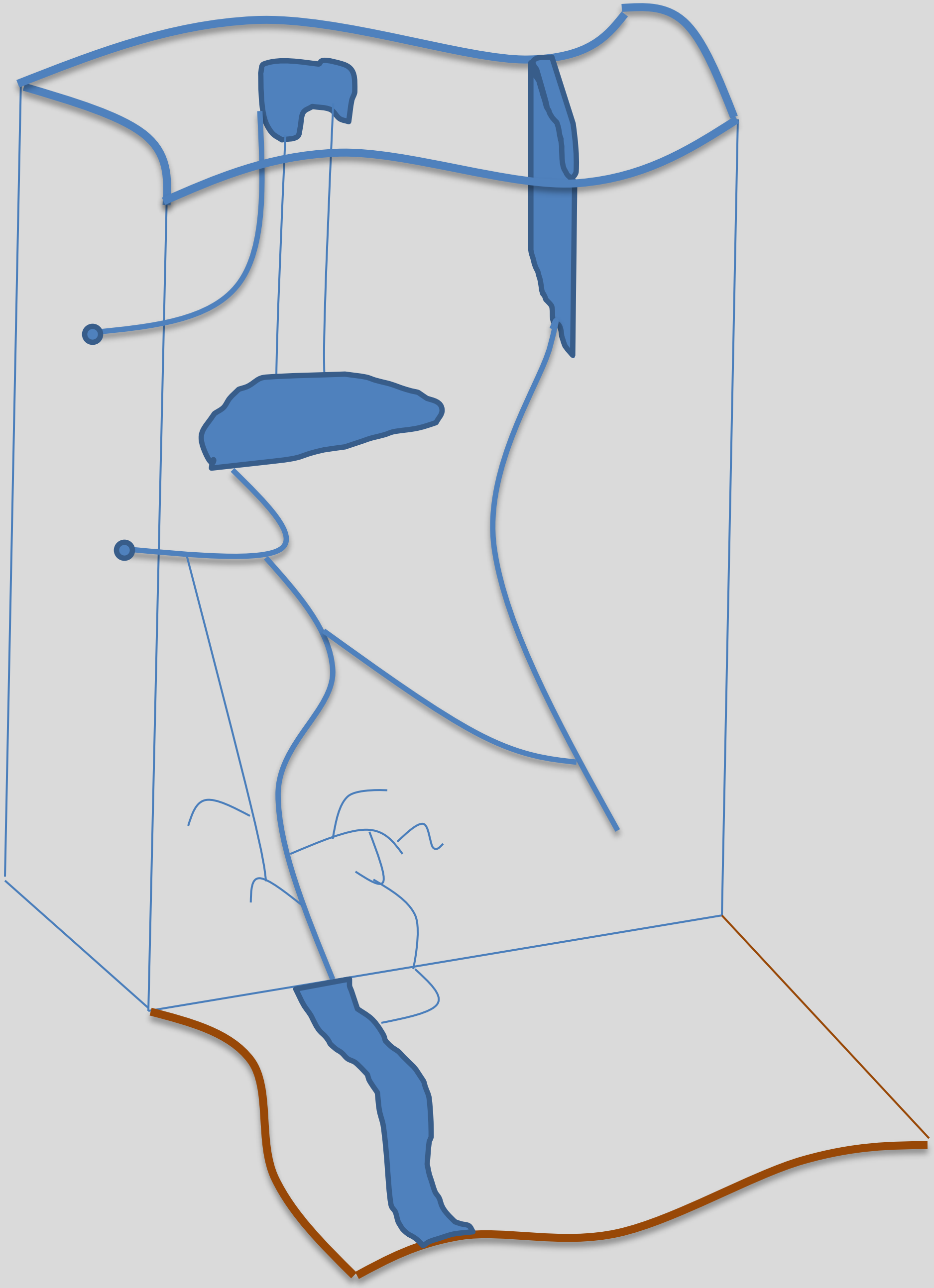
- Illustrative example of CHW causing transition to temperate ice
- Competing mechanisms affect time-scale of thermal response
  - englacial water body width
  - englacial water body spacing
  - stretching in high shear regions
  - background ice temperatures
  - strain heating

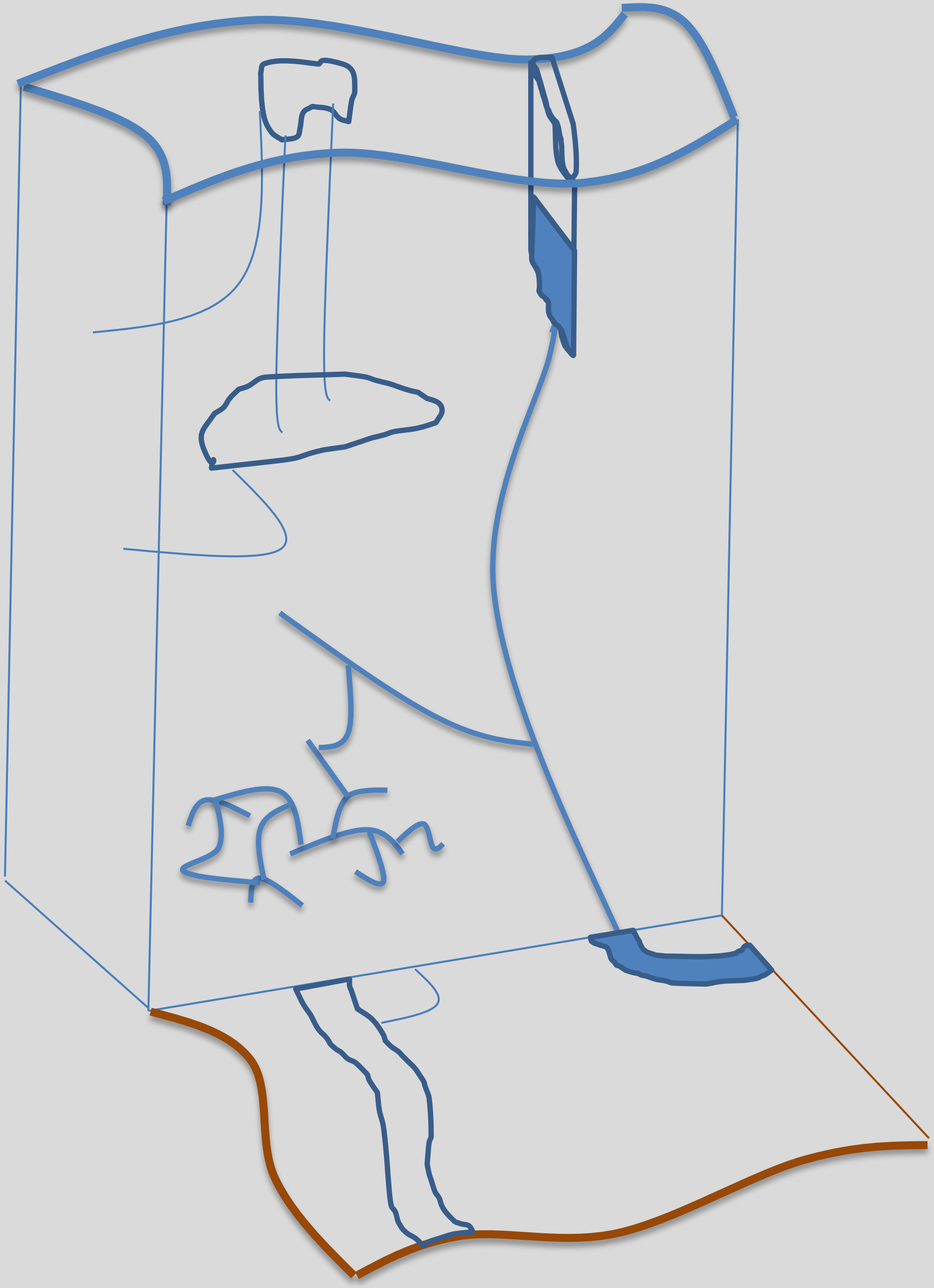
CHS Modeling Conclusions

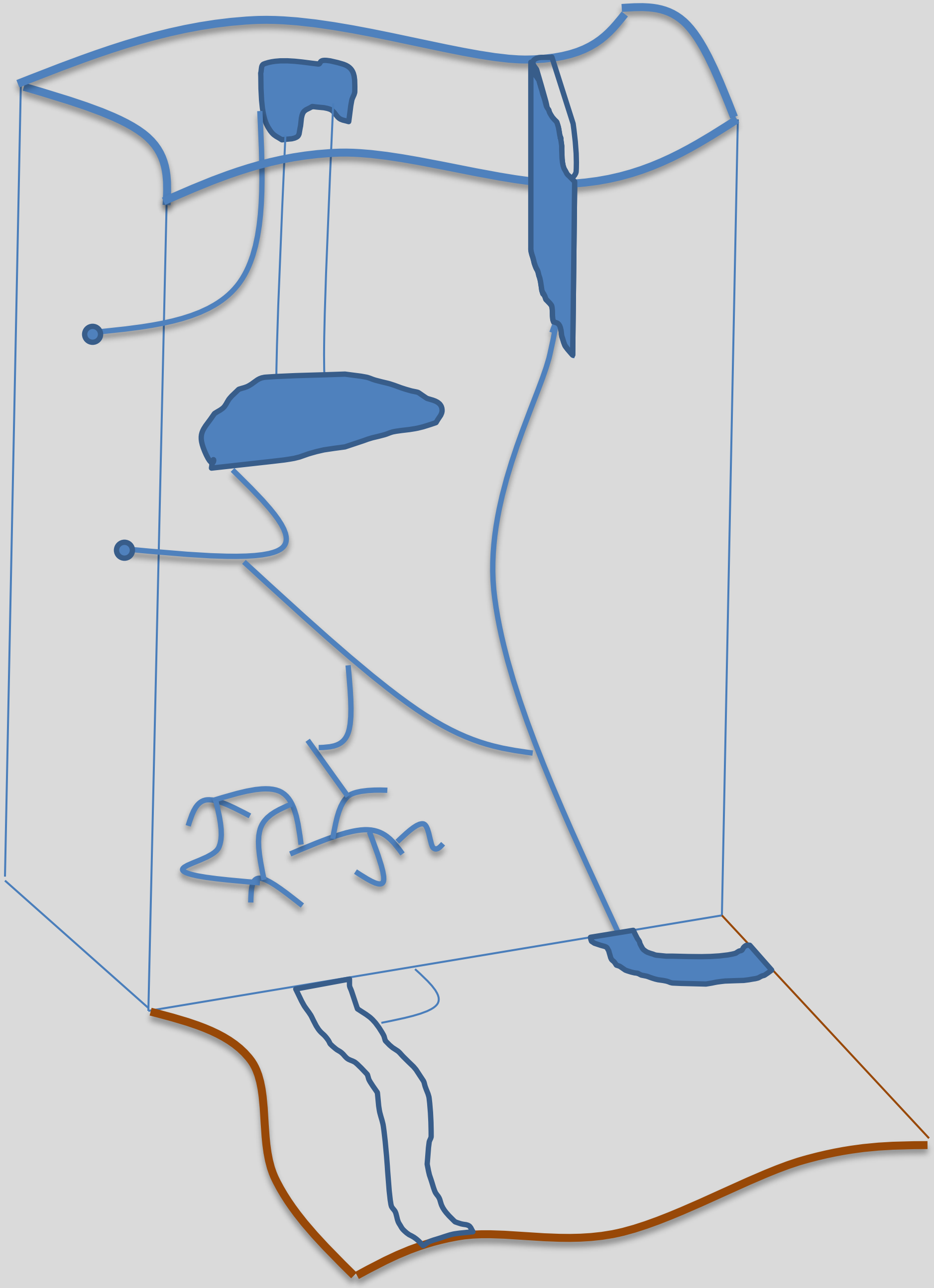
- Enthalpy gradient method can be applied to 2D model involving phase change
- Englacial water can cause significant warmth with time-scale dependent on factors such as:
  - width
  - spacing
  - background ice temperature
  - stretching in high shear flow

Future Work

- Improve understanding of CHW time-scale and competing thermal phenomenon
- Use understanding of small-scale physics to improve CHW parameterization
- Implement enthalpy-based thermodynamic formulation and CHW parameterization in CISM
- Explore other englacial water body geometries and dynamic process of draining, refilling, and fracturing

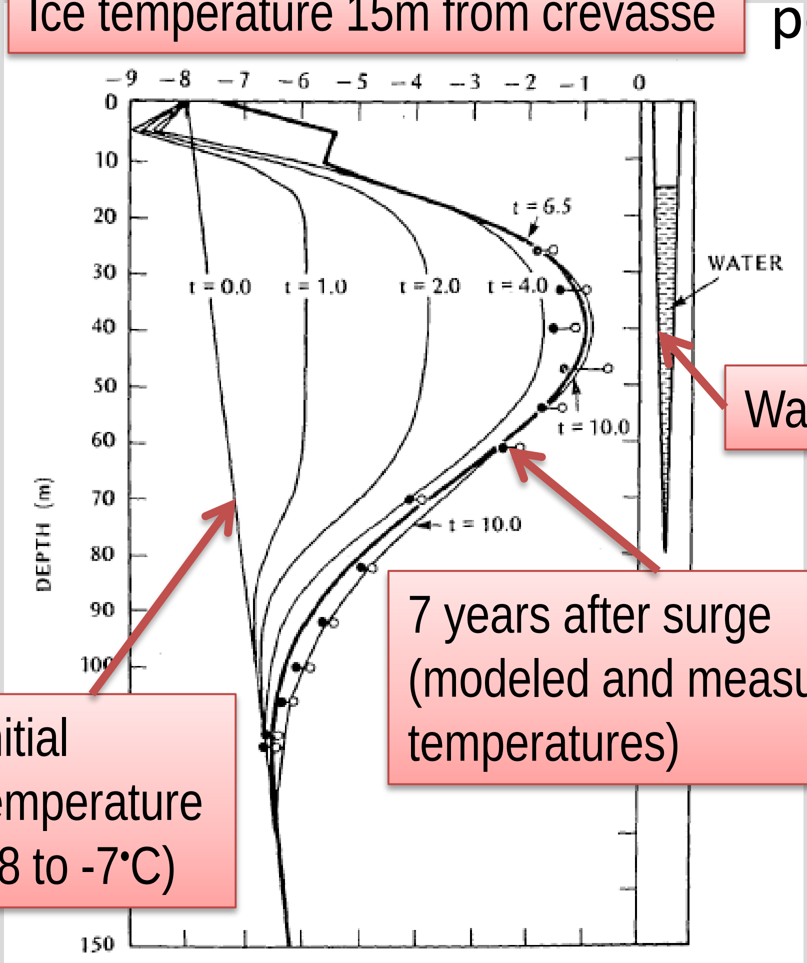






- Direct evidence for CHW: Jarvis and Clarke [1974], Steele Glacier - crevasses formed during surge, filled with water
- Model calculations suggested persistence of liquid water for 20-30 yrs

Ice temperature 15m from crevasse

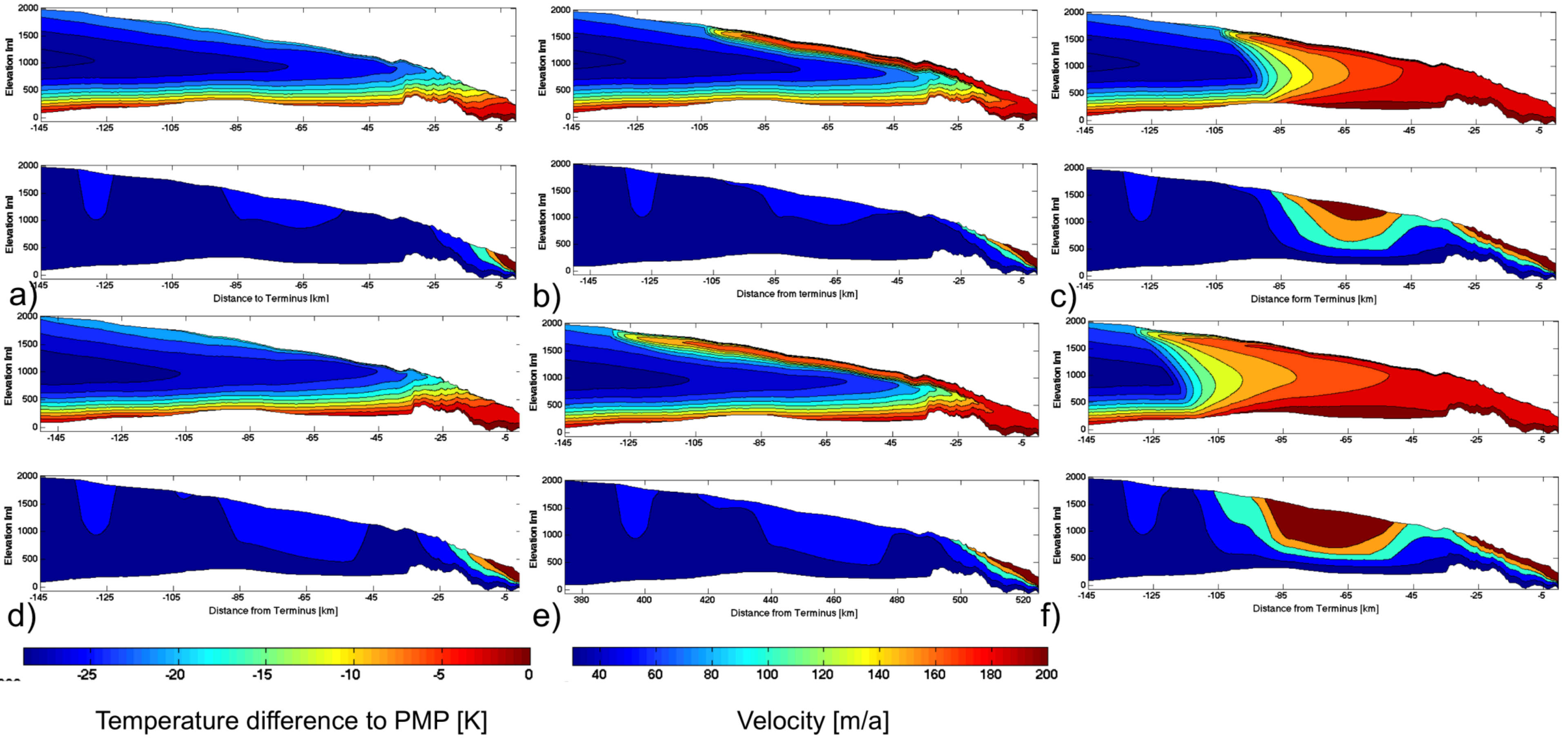


Water filled crevasse

7 years after surge (modeled and measured temperatures)

Initial temperature (-8 to -7•C)

On Greenland, annual filling and drainage (substantial water input) multi-year persistence of liquid water (Catania and Neumann, 2010)


a)
b)
c)
d)
e)
f)
Elevation [m]
2000
1500
1000
500
0
-145
-125
-105
-85
-65
-45
-25
-5
380
400
420
440
460
480
500
520
Distance to Terminus [km]
Distance from terminus [km]
Distance form Terminus [km]
Distance from Terminus [km]
-25
-20
-15
-10
-5
0
40
60
80
100
120
140
160
180
200
Temperature difference to PMP [K]
Velocity [m/a]