Universität Hamburg
DER FORSCHUNG | DER LEHRE | DER BILDUNG

KlimaCampus

Energetically based mixing schemes

Carsten Eden, Dirk Olbers, Lars Czeschel, Antonija Rimac, Jin-Song v. Storch
Institut für Meereskunde, Universität Hamburg

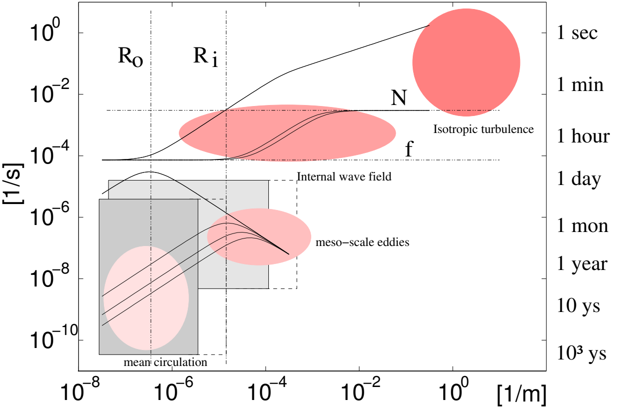
Universität Hamburg
DER FORSCHUNG | DER LEHRE | DER BILDUNG

KlimaCampus

Energetically based mixing schemes
or
how to construct a consistent ocean model

Carsten Eden, Dirk Olbers, Lars Czeschel, Antonija Rimac, Jin-Song v. Storch

Institut für Meereskunde, Universität Hamburg



- solid lines: dispersion relations of linear wave solutions
- red ellipses: dynamical regimes, grey boxes: ocean models
- R_o, R_i: Rossby radii of deformation,
  N buoyancy frequency, f Coriolis frequency

IDEMIX

- assume field of weakly interacting internal (WKB) waves

IDEMIX

- assume field of weakly interacting internal (WKB) waves
- split spectrum $\mathcal{E}$ into up/downward propagating part with

  $$\mathcal{E}^+ = \mathcal{E}(\mathbf{k}_h, m < 0) \quad , \quad \mathcal{E}^- = \mathcal{E}(\mathbf{k}_h, m > 0)$$

  where m is vertical wavenumber

IDEMIX

- assume field of weakly interacting internal (WKB) waves
- split spectrum $\mathcal{E}$ into up/downward propagating part with

  $$\mathcal{E}^+ = \mathcal{E}(\mathbf{k}_h, m < 0) \quad , \quad \mathcal{E}^- = \mathcal{E}(\mathbf{k}_h, m > 0)$$

  where m is vertical wavenumber
- note that waves with negative m move upward (with $\dot{z} = c$), those with positive m move downward (with $\dot{z} = -c$)

IDEMIX

- assume field of weakly interacting internal (WKB) waves
- split spectrum $\mathcal{E}$ into up/downward propagating part with

$$\mathcal{E}^+ = \mathcal{E}(\mathbf{k}_h, m < 0) \quad , \quad \mathcal{E}^- = \mathcal{E}(\mathbf{k}_h, m > 0)$$

  where m is vertical wavenumber
- note that waves with negative m move upward (with $\dot{z} = c$), those with positive m move downward (with $\dot{z} = -c$)
- integrate over all vertical wavenumbers $m > 0$ and $m < 0$

$$\begin{aligned} E^+ &= \int_{-\infty}^{0} dm \int d^2 k_h \mathcal{E}^+(\mathbf{k}_h, m, z, t) \\ E^- &= \int_{0}^{\infty} dm \int d^2 k_h \mathcal{E}^-(\mathbf{k}_h, m, z, t) \end{aligned}$$

IDEMIX

- integration of spectral energy $\mathcal{E}(\mathbf{k}, \mathbf{x}, t)$

$$\frac{\partial \mathcal{E}}{\partial t} + \nabla_h \cdot (\dot{\mathbf{x}}_h \mathcal{E}) + \frac{\partial}{\partial \mathbf{k}_h} \cdot (\dot{\mathbf{k}}_h \mathcal{E}) + \frac{\partial}{\partial z}(\dot{z}\mathcal{E}) + \frac{\partial}{\partial m}(\dot{m}\mathcal{E}) = \omega S$$

- integration of spectral energy $\mathcal{E}(\mathbf{k}, \mathbf{x}, t)$

$$\frac{\partial \mathcal{E}}{\partial t} + \nabla_h \cdot (\dot{\mathbf{x}}_h \, \mathcal{E}) + \frac{\partial}{\partial \mathbf{k}_h} \cdot (\dot{\mathbf{k}}_h \, \mathcal{E}) + \frac{\partial}{\partial z}(\dot{z}\mathcal{E}) + \frac{\partial}{\partial m}(\dot{m}\mathcal{E}) \quad = \quad \omega S$$

yields

$$\frac{\partial E^+}{\partial t} + \frac{\partial J^+}{\partial z} = \int_{-\infty}^{0} dm \int d^2 k_h \, \omega (S_{diss} + S_{ww})$$

with $J^+ = \int_{-\infty}^{0} dm \int d^2 k_h \, c\mathcal{E}^+$

IDEMIX

- integration of spectral energy $\mathcal{E}(\mathbf{k}, \mathbf{x}, t)$

$$\frac{\partial \mathcal{E}}{\partial t} + \nabla_h \cdot (\dot{\mathbf{x}}_h \mathcal{E}) + \frac{\partial}{\partial \mathbf{k}_h} \cdot (\dot{\mathbf{k}}_h \mathcal{E}) + \frac{\partial}{\partial z}(\dot{z}\mathcal{E}) + \frac{\partial}{\partial m}(\dot{m}\mathcal{E}) = \omega S$$

yields

$$\frac{\partial E^+}{\partial t} + \frac{\partial J^+}{\partial z} = \int_{-\infty}^{0} dm \int d^2 k_h \, \omega (S_{diss} + S_{ww})$$

with $J^+ = \int_{-\infty}^{0} dm \int d^2 k_h \, c\mathcal{E}^+$ and

$$\frac{\partial E^-}{\partial t} + \frac{\partial J^-}{\partial z} = \int_{0}^{\infty} dm \int d^2 k_h \, \omega (S_{diss} + S_{ww})$$

with $J^- = -\int_{0}^{\infty} dm \int d^2 k_h \, c\mathcal{E}^-$

IDEMIX

- integration of spectral energy $\mathcal{E}(\mathbf{k}, \mathbf{x}, t)$

$$\frac{\partial \mathcal{E}}{\partial t} + \nabla_h \cdot (\dot{\mathbf{x}}_h \mathcal{E}) + \frac{\partial}{\partial \mathbf{k}_h} \cdot (\dot{\mathbf{k}}_h \mathcal{E}) + \frac{\partial}{\partial z}(\dot{z}\mathcal{E}) + \frac{\partial}{\partial m}(\dot{m}\mathcal{E}) \quad = \quad \omega S$$

yields

$$\frac{\partial E^+}{\partial t} + \frac{\partial J^+}{\partial z} = \int_{-\infty}^{0} dm \int d^2 k_h \, \omega (S_{diss} + S_{ww})$$

with $J^+ = \int_{-\infty}^{0} dm \int d^2 k_h \, c\mathcal{E}^+$ and

$$\frac{\partial E^-}{\partial t} + \frac{\partial J^-}{\partial z} = \int_{0}^{\infty} dm \int d^2 k_h \, \omega (S_{diss} + S_{ww})$$

with $J^- = -\int_{0}^{\infty} dm \int d^2 k_h \, c\mathcal{E}^-$

- work with total energy $E = E^+ + E^-$ and $\Delta E = E^+ - E^-$

IDEMIX

- integration of spectral energy $\mathcal{E}(\mathbf{k}, \mathbf{x}, t)$

$$\frac{\partial \mathcal{E}}{\partial t} + \nabla_h \cdot (\dot{\mathbf{x}}_h \mathcal{E}) + \frac{\partial}{\partial \mathbf{k}_h} \cdot (\dot{\mathbf{k}}_h \mathcal{E}) + \frac{\partial}{\partial z}(\dot{z}\mathcal{E}) + \frac{\partial}{\partial m}(\dot{m}\mathcal{E}) = \omega S$$

yields

$$\frac{\partial E^+}{\partial t} + \frac{\partial J^+}{\partial z} = \int_{-\infty}^{0} dm \int d^2 k_h \, \omega (S_{diss} + S_{ww})$$

with $J^+ = \int_{-\infty}^{0} dm \int d^2 k_h \, c\mathcal{E}^+$ and

$$\frac{\partial E^-}{\partial t} + \frac{\partial J^-}{\partial z} = \int_{0}^{\infty} dm \int d^2 k_h \, \omega (S_{diss} + S_{ww})$$

with $J^- = -\int_{0}^{\infty} dm \int d^2 k_h \, c\mathcal{E}^-$

- work with total energy $E = E^+ + E^-$ and $\Delta E = E^+ - E^-$
- S_{ww} conserves total energy E

IDEMIX

- integration of spectral energy $\mathcal{E}(\mathbf{k}, \mathbf{x}, t)$

$$\frac{\partial \mathcal{E}}{\partial t} + \nabla_h \cdot (\dot{\mathbf{x}}_h \mathcal{E}) + \frac{\partial}{\partial \mathbf{k}_h} \cdot (\dot{\mathbf{k}}_h \mathcal{E}) + \frac{\partial}{\partial z}(\dot{z}\mathcal{E}) + \frac{\partial}{\partial m}(\dot{m}\mathcal{E}) = \omega S$$

yields

$$\frac{\partial E^+}{\partial t} + \frac{\partial J^+}{\partial z} = \int_{-\infty}^{0} dm \int d^2 k_h \, \omega (S_{diss} + S_{ww})$$

with $J^+ = \int_{-\infty}^{0} dm \int d^2 k_h \, c\mathcal{E}^+$ and

$$\frac{\partial E^-}{\partial t} + \frac{\partial J^-}{\partial z} = \int_{0}^{\infty} dm \int d^2 k_h \, \omega (S_{diss} + S_{ww})$$

with $J^- = -\int_{0}^{\infty} dm \int d^2 k_h \, c\mathcal{E}^-$

- work with total energy $E = E^+ + E^-$ and $\Delta E = E^+ - E^-$
- S_{ww} conserves total energy E
- assume that S_{ww} damps ΔE and that S_{diss} is symmetric in m

IDEMIX

- total energy E and of difference of up/downward waves ΔE

$$\frac{\partial E}{\partial t} + \frac{\partial}{\partial z} c_0 \Delta E = -\epsilon_{iw} \ , \ \ \frac{\partial \Delta E}{\partial t} + \frac{\partial}{\partial z} c_0 E = -\Delta E / \tau_v$$

IDEMIX

- total energy E and of difference of up/downward waves ΔE

$$\frac{\partial E}{\partial t} + \frac{\partial}{\partial z} c_0 \Delta E = -\epsilon_{iw} \ , \ \frac{\partial \Delta E}{\partial t} + \frac{\partial}{\partial z} c_0 E = -\Delta E / \tau_v$$

- c_0 is computed assuming a "GM spectrum" (Munk, 1981)

IDEMIX

- total energy E and of difference of up/downward waves ΔE

$$\frac{\partial E}{\partial t} + \frac{\partial}{\partial z} c_0 \Delta E = -\epsilon_{iw} \ , \ \frac{\partial \Delta E}{\partial t} + \frac{\partial}{\partial z} c_0 E = -\Delta E / \tau_v$$

- c_0 is computed assuming a "GM spectrum" (Munk, 1981)
- flux in wavenumber space by wave-wave interaction at high vertical wavenumber end of $\mathcal{E}$ is assumed to be dissipated

IDEMIX

- total energy E and of difference of up/downward waves ΔE

$$\frac{\partial E}{\partial t} + \frac{\partial}{\partial z} c_0 \Delta E = -\epsilon_{iw} \; , \; \frac{\partial \Delta E}{\partial t} + \frac{\partial}{\partial z} c_0 E = -\Delta E / \tau_v$$

- c_0 is computed assuming a "GM spectrum" (Munk, 1981)
- flux in wavenumber space by wave-wave interaction at high vertical wavenumber end of $\mathcal{E}$ is assumed to be dissipated
- Olbers (1976), McComas and Müller (1981) and Henyey et al (1986) found (for GM spectrum)

$$\epsilon_{iw} = \mu f \, E^2 / c_\star^2$$

with $c_\star = (j_\star \pi)^{-1} \int_{-h}^{0} N(z) dz$, vertical mode bandwidth $j_\star$, and a parameter $\mu = O(1)$

IDEMIX

- total energy E and of difference of up/downward waves ΔE
$$\frac{\partial E}{\partial t} + \frac{\partial}{\partial z} c_0 \Delta E = -\epsilon_{iw} \ , \ \frac{\partial \Delta E}{\partial t} + \frac{\partial}{\partial z} c_0 E = -\Delta E / \tau_v$$
- c_0 is computed assuming a "GM spectrum" (Munk, 1981)
- flux in wavenumber space by wave-wave interaction at high vertical wavenumber end of $\mathcal{E}$ is assumed to be dissipated
- Olbers (1976), McComas and Müller (1981) and Henyey et al (1986) found (for GM spectrum)
$$\epsilon_{iw} = \mu f \, E^2 / c_\star^2$$
with $c_\star = (j_\star \pi)^{-1} \int_{-h}^{0} N(z) dz$, vertical mode bandwidth $j_\star$, and a parameter $\mu = O(1)$
- similar form of ϵ_{iw} is used for analysis of fine-structure measurements (Gregg, 1989)

IDEMIX

- total energy E and of difference of up/downward waves ΔE

$$\frac{\partial E}{\partial t} + \frac{\partial}{\partial z} c_0 \Delta E = -\mu f \, E^2 / c_\star^2 \; , \; \frac{\partial \Delta E}{\partial t} + \frac{\partial}{\partial z} c_0 E = -\Delta E / \tau_v$$

IDEMIX

- total energy E and of difference of up/downward waves ΔE

$$\frac{\partial E}{\partial t} + \frac{\partial}{\partial z} c_0 \Delta E = -\mu f\, E^2/c_\star^2 \ , \ \ \frac{\partial \Delta E}{\partial t} + \frac{\partial}{\partial z} c_0 E = -\Delta E/\tau_v$$

- for $t \gg \tau_v$

$$\frac{\partial E}{\partial t} = \frac{\partial}{\partial z} c_0 \tau_v \frac{\partial}{\partial z} c_0 E + \boldsymbol{\nabla}_h \cdot v_0 \tau_h \boldsymbol{\nabla}_h v_0 E - \mu f\, E^2/c_\star^2$$

IDEMIX

- total energy E and of difference of up/downward waves ΔE

$$\frac{\partial E}{\partial t} + \frac{\partial}{\partial z} c_0 \Delta E = -\mu f \, E^2/c_\star^2 \; , \;\; \frac{\partial \Delta E}{\partial t} + \frac{\partial}{\partial z} c_0 E = -\Delta E/\tau_v$$

- for $t \gg \tau_v$

$$\frac{\partial E}{\partial t} = \frac{\partial}{\partial z} c_0 \tau_v \frac{\partial}{\partial z} c_0 E + \nabla_h \cdot v_0 \tau_h \nabla_h v_0 E - \mu f \, E^2/c_\star^2$$

- assumptions:
  - S_{diss} is nearly symmetric in m
  - effect of S_{ww} is to eliminate asymmetries, i.e. to damp ΔE
  - $\epsilon_{iw} = \mu f \, E^2/c_\star^2$ is flux at/to large m by wave-wave interaction
  - $c_0^{\pm}$ are identical and are calculated assuming a GM spectrum

IDEMIX

- total energy E and of difference of up/downward waves ΔE

$$\frac{\partial E}{\partial t} + \frac{\partial}{\partial z} c_0 \Delta E = -\mu f \, E^2 / c_\star^2 \; , \; \frac{\partial \Delta E}{\partial t} + \frac{\partial}{\partial z} c_0 E = -\Delta E / \tau_v$$

- for $t \gg \tau_v$

$$\frac{\partial E}{\partial t} = \frac{\partial}{\partial z} c_0 \tau_v \frac{\partial}{\partial z} c_0 E + \nabla_h \cdot v_0 \tau_h \nabla_h v_0 E - \mu f \, E^2 / c_\star^2$$

- assumptions:
  - S_{diss} is nearly symmetric in m
  - effect of S_{ww} is to eliminate asymmetries, i.e. to damp ΔE
  - $\epsilon_{iw} = \mu f \, E^2 / c_\star^2$ is flux at/to large m by wave-wave interaction
  - $c_0^\pm$ are identical and are calculated assuming a GM spectrum
- parameters are τ_v, $j_\star$ and μ and τ_h

IDEMIX

- total energy E and of difference of up/downward waves ΔE

$$\frac{\partial E}{\partial t} + \frac{\partial}{\partial z} c_0 \Delta E = -\mu f\, E^2/c_\star^2 \ , \ \frac{\partial \Delta E}{\partial t} + \frac{\partial}{\partial z} c_0 E = -\Delta E/\tau_v$$

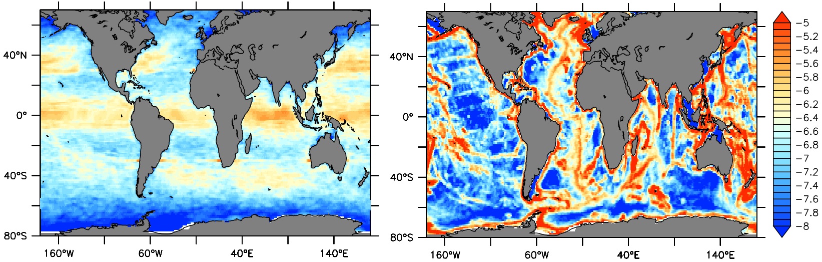
- for $t \gg \tau_v$

$$\frac{\partial E}{\partial t} = \frac{\partial}{\partial z} c_0 \tau_v \frac{\partial}{\partial z} c_0 E + \nabla_h \cdot v_0 \tau_h \nabla_h v_0 E - \mu f\, E^2/c_\star^2$$

- assumptions:
  - S_{diss} is nearly symmetric in m
  - effect of S_{ww} is to eliminate asymmetries, i.e. to damp ΔE
  - $\epsilon_{iw} = \mu f\, E^2/c_\star^2$ is flux at/to large m by wave-wave interaction
  - $c_0^\pm$ are identical and are calculated assuming a GM spectrum
- parameters are τ_v, $j_\star$ and μ and τ_h
- forcing by energy flux $c_0 \tau_v \partial(c_0 E)/\partial z$ at surface/bottom

Forcing IDEMIX

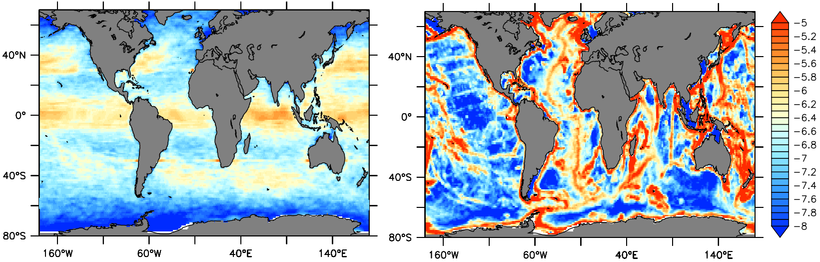
- external forcing functions in $log_{10}(F/m^3s^{-3})$



- wind-driven inertial oscillations in the surface mixed layer
  fraction of it (0.2) propagate into interior
  0.14 TW from Rimac et al (2012)

Forcing IDEMIX

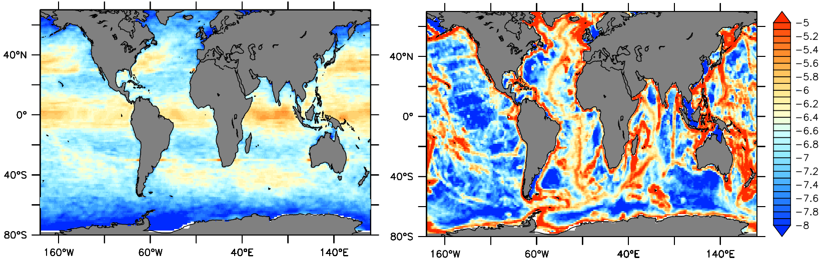
- external forcing functions in $log_{10}(F/m^3s^{-3})$



- wind-driven inertial oscillations in the surface mixed layer
  fraction of it (0.2) propagate into interior
  0.14 TW from Rimac et al (2012)
- tidal flow over topography
  1.7 TW from Jayne (2009)

Forcing IDEMIX

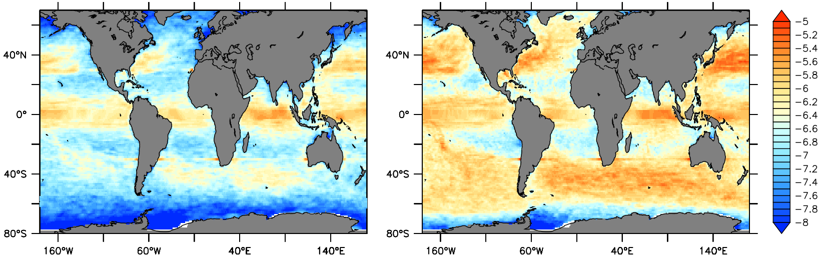
- external forcing functions in $log_{10}(F/m^3s^{-3})$



- wind-driven inertial oscillations in the surface mixed layer
  fraction of it (0.2) propagate into interior
  0.14 TW from Rimac et al (2012)
- tidal flow over topography
  1.7 TW from Jayne (2009)
- and other sources

Forcing IDEMIX

- external forcing functions in $log_{10}(F/m^3 s^{-3})$



0.14 TW from NCEP (6 hourly, 1.875^o)
but 0.38 TW from CVFS (1 hourly, 0.35^o) (Rimac et al, 2012)

TKE model

- small-scale turbulent kinetic energy (TKE) equation

$$\frac{D\bar{E}}{Dt} = -\frac{\partial}{\partial z}\,(\text{fluxes}) - \overline{\mathbf{u}'w'}\frac{\partial \bar{\mathbf{u}}}{\partial z} + \overline{w'b'} - \nu\overline{(\nabla \mathbf{u})^2}$$

  - with $\overline{E} = \overline{(u'^2 + v'^2 + w'^2)}/2$
  - vertical shear production $\overline{\mathbf{u}'w'}\partial\bar{\mathbf{u}}/\partial z$
  - exchange with potential energy $\overline{w'b'}$
  - dissipation, conversion to internal energy

TKE model

- small-scale turbulent kinetic energy (TKE) equation

$$\frac{D\bar{E}}{Dt} = -\frac{\partial}{\partial z}\,(\text{fluxes}) - \overline{\mathbf{u}'w'}\frac{\partial \bar{\mathbf{u}}}{\partial z} + \overline{w'b'} - \nu\overline{(\nabla\mathbf{u})^2}$$

  - with $\overline{E} = \overline{(u'^2 + v'^2 + w'^2)}/2$
  - vertical shear production $\overline{\mathbf{u}'w'}\partial\bar{\mathbf{u}}/\partial z$
  - exchange with potential energy $\overline{w'b'}$
  - dissipation, conversion to internal energy

- assume that

$$\overline{b'w'} = -c_b K\frac{\partial \bar{b}}{\partial z} \quad , \quad \overline{\mathbf{u}'w'} = -c_u K\frac{\partial \bar{\mathbf{u}}}{\partial z} \quad \text{and} \quad \nu\overline{(\nabla\mathbf{u})^2} \approx c_\epsilon \bar{E}^{3/2}L^{-1}$$

  with dimensionless parameters $c_u, c_b, c_\epsilon = O(1)$
  and mixing length L

TKE model

- small-scale turbulent kinetic energy (TKE) equation

$$\frac{D\bar{E}}{Dt} = -\frac{\partial}{\partial z}\,(\text{fluxes}) - \overline{\mathbf{u}'w'}\frac{\partial \bar{\mathbf{u}}}{\partial z} + \overline{w'b'} - \nu\overline{(\boldsymbol{\nabla}\mathbf{u})^2}$$

  - with $\overline{E} = \overline{(u'^2 + v'^2 + w'^2)}/2$
  - vertical shear production $\overline{\mathbf{u}'w'}\partial\bar{\mathbf{u}}/\partial z$
  - exchange with potential energy $\overline{w'b'}$
  - dissipation, conversion to internal energy

- assume that

$$\overline{b'w'} = -c_b K\frac{\partial \bar{b}}{\partial z} \quad , \quad \overline{\mathbf{u}'w'} = -c_u K\frac{\partial \bar{\mathbf{u}}}{\partial z} \quad \text{and} \quad \nu\overline{(\boldsymbol{\nabla}\mathbf{u})^2} \approx c_\epsilon \bar{E}^{3/2}L^{-1}$$

with dimensionless parameters $c_u, c_b, c_\epsilon = O(1)$
and mixing length L

$$\frac{D\bar{E}}{Dt} = \frac{\partial}{\partial z}c_E K\frac{\partial}{\partial z}\bar{E} + c_u K\left(\frac{\partial \bar{\mathbf{u}}}{\partial z}\right)^2 - c_b K N^2 - c_\epsilon \bar{E}^{3/2}L^{-1}$$

TKE model

- TKE based mixed layer closure model

$$\frac{D\bar{E}}{Dt} = -\frac{\partial}{\partial z}\left(\text{fluxes}\right) + c_u K \left(\frac{\partial \bar{\mathbf{u}}}{\partial z}\right)^2 - c_b K N^2 - c_\epsilon \bar{E}^{3/2} L^{-1}$$

TKE model

- TKE based mixed layer closure model

$$\frac{D\bar{E}}{Dt} = -\frac{\partial}{\partial z}(\text{fluxes}) + c_u K \left(\frac{\partial \bar{\mathbf{u}}}{\partial z}\right)^2 - c_b K N^2 - c_\epsilon \bar{E}^{3/2} L^{-1}$$

- close with $K = \bar{E}^{1/2} L$ and $L = \sqrt{2E/N}$ (Gaspar et al, 1990)

TKE model

- TKE based mixed layer closure model

$$\frac{D\bar{E}}{Dt} = -\frac{\partial}{\partial z}(\text{fluxes}) + c_u K \left(\frac{\partial \bar{\mathbf{u}}}{\partial z}\right)^2 - c_b K N^2 - c_\epsilon \bar{E}^{3/2} L^{-1}$$

- close with $K = \bar{E}^{1/2} L$ and $L = \sqrt{2E/N}$ (Gaspar et al, 1990)
- in the interior production by simulated $\partial \bar{\mathbf{u}}/\partial z$ vanishes: use minimal E threshold and it follows $K = \sqrt{2} E_{min}/N$

TKE model

- TKE based mixed layer closure model

$$\frac{D\bar{E}}{Dt} \quad = \quad -\frac{\partial}{\partial z}(\text{fluxes}) + c_u K \left(\frac{\partial \bar{\mathbf{u}}}{\partial z}\right)^2 - c_b K N^2 - c_\epsilon \bar{E}^{3/2} L^{-1}$$

- close with $K = \bar{E}^{1/2} L$ and $L = \sqrt{2E/N}$ (Gaspar et al, 1990)
- in the interior production by simulated $\partial \bar{\mathbf{u}}/\partial z$ vanishes: use minimal E threshold and it follows $K = \sqrt{2} E_{min}/N$
- but shear production is related to breaking internal waves $\rightarrow$ add ϵ_{iw} from IDEMIX and set $E_{min} = 0$

Global model

- global model, MITgcm, with $1^o \times 1^o$ horizontal resolution, 115 vertical layers and thus small implicit numerical diffusion

Global model

- global model, MITgcm, with $1^o \times 1^o$ horizontal resolution, 115 vertical layers and thus small implicit numerical diffusion
- forced by surface heat, freshwater fluxes and wind stress

Global model

- global model, MITgcm, with $1^o \times 1^o$ horizontal resolution, 115 vertical layers and thus small implicit numerical diffusion
- forced by surface heat, freshwater fluxes and wind stress

- internal wave closure (IDEMIX) forced by
  - inertial oscillations at the surface
  - tidal forcing at the bottom

$$\frac{DE_{iw}}{Dt} = \nabla \cdot (\text{fluxes}) - \epsilon_{iw}$$

Global model

- global model, MITgcm, with $1^o \times 1^o$ horizontal resolution, 115 vertical layers and thus small implicit numerical diffusion
- forced by surface heat, freshwater fluxes and wind stress

- internal wave closure (IDEMIX) forced by
  - inertial oscillations at the surface
  - tidal forcing at the bottom

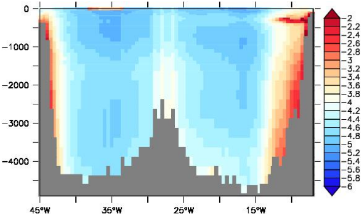
$$\frac{DE_{iw}}{Dt} = \nabla \cdot (\text{fluxes}) - \epsilon_{iw}$$

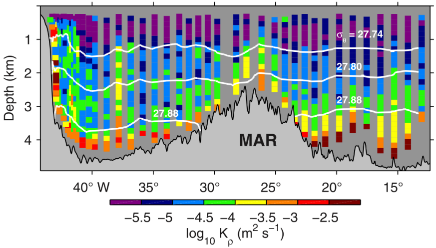
- small-scale turbulence closure with TKE equation forced by
  - direct wind input of TKE
  - internal wave breaking

$$\frac{DE_{tke}}{Dt} = \nabla \cdot (\text{fluxes}) + c_u K \left(\frac{\partial \bar{\mathbf{u}}}{\partial z}\right)^2 + \epsilon_{iw} - c_b K N^2 - \epsilon_{tke}$$

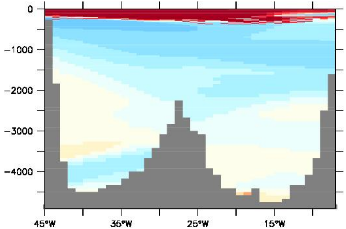
Performance of IDEMIX

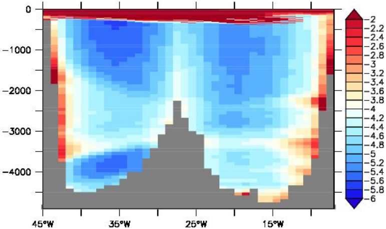
- $log_{10}(K/m^2 s^{-1})$ using N^2 from WOCE climatology at 48^oN





- K using TKE eq. (left) and TKE + IDEMIX (right) in model





More energy for mixing

- meso-scale eddy kinetic energy

$$\frac{D\bar{E}}{Dt} = -\boldsymbol{\nabla}\cdot(\text{fluxes}) + \bar{S} + \overline{b'w'} - \epsilon_{eke}$$

  - with meso-scale eddy kinetic energy (EKE) $\bar{E} = \overline{(u'^2 + v'^2)}/2$
  - and barotropic instability production $\bar{S} = -\overline{\mathbf{u}'\mathbf{u}'}\cdot\boldsymbol{\nabla}_h\bar{\mathbf{u}}$
  - and baroclinic instability production $\overline{b'w'}$
  - and dissipation of EKE ϵ_{eke}

More energy for mixing

- meso-scale eddy kinetic energy

$$\frac{D\bar{E}}{Dt} = -\boldsymbol{\nabla}\cdot(\text{fluxes}) + \bar{S} + \overline{b'w'} - \epsilon_{eke}$$

  - with meso-scale eddy kinetic energy (EKE) $\bar{E} = \overline{(u'^2 + v'^2)}/2$
  - and barotropic instability production $\bar{S} = -\overline{\mathbf{u}'\mathbf{u}'}\cdot\boldsymbol{\nabla}_h\bar{\mathbf{u}}$
  - and baroclinic instability production $\overline{b'w'}$
  - and dissipation of EKE ϵ_{eke}

- meso-scale eddy potential energy $\bar{P} = \overline{b'^2}/(2N^2)$

$$\frac{D\bar{P}}{Dt} = -\boldsymbol{\nabla}\cdot\text{flux} - \overline{\mathbf{u}'_h b'}\cdot\boldsymbol{\nabla}_h\bar{b}N^{-2} - \overline{b'w'}$$

More energy for mixing

- meso-scale eddy kinetic energy

$$\frac{D\bar{E}}{Dt} = -\boldsymbol{\nabla} \cdot (\text{fluxes}) + \bar{S} + \overline{b'w'} - \epsilon_{eke}$$

  - with meso-scale eddy kinetic energy (EKE) $\bar{E} = \overline{(u'^2 + v'^2)}/2$
  - and barotropic instability production $\bar{S} = -\overline{\mathbf{u}'\mathbf{u}'} \cdot \boldsymbol{\nabla}_h \bar{\mathbf{u}}$
  - and baroclinic instability production $\overline{b'w'}$
  - and dissipation of EKE ϵ_{eke}

- meso-scale eddy potential energy $\bar{P} = \overline{b'^2}/(2N^2)$

$$\frac{D\bar{P}}{Dt} = -\boldsymbol{\nabla} \cdot \text{flux} - \overline{\mathbf{u}'_h b'} \cdot \boldsymbol{\nabla}_h \bar{b} N^{-2} - \overline{b'w'}$$

- adding and assuming $\overline{\mathbf{u}'_h b'} = -K_{gm} \boldsymbol{\nabla}_h \bar{b}$

$$\frac{D}{Dt}\left(\bar{E} + \bar{P}\right) = -\boldsymbol{\nabla} \cdot (\text{fluxes}) + \bar{S} + K_{gm}(\boldsymbol{\nabla}_h \bar{b})^2 N^{-2} - \epsilon_{eke}$$

More energy for mixing

- meso-scale eddy kinetic energy

$$\frac{D\bar{E}}{Dt} = -\boldsymbol{\nabla}\cdot(\text{fluxes}) + \bar{S} + \overline{b'w'} - \epsilon_{eke}$$

  - with meso-scale eddy kinetic energy (EKE) $\bar{E} = \overline{(u'^2 + v'^2)}/2$
  - and barotropic instability production $\bar{S} = -\overline{\mathbf{u}'\mathbf{u}'}\cdot\boldsymbol{\nabla}_h\bar{\mathbf{u}}$
  - and baroclinic instability production $\overline{b'w'}$
  - and dissipation of EKE ϵ_{eke}
- meso-scale eddy potential energy $\bar{P} = \overline{b'^2}/(2N^2)$

$$\frac{D\bar{P}}{Dt} = -\boldsymbol{\nabla}\cdot\text{flux} - \overline{\mathbf{u}_h' b'}\cdot\boldsymbol{\nabla}_h\bar{b}N^{-2} - \overline{b'w'}$$

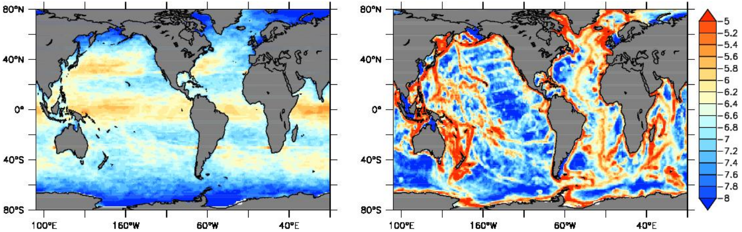
- adding and assuming $\overline{\mathbf{u}_h' b'} = -K_{gm}\boldsymbol{\nabla}_h\bar{b}$

$$\frac{D}{Dt}\left(\bar{E} + \bar{P}\right) = -\boldsymbol{\nabla}\cdot(\text{fluxes}) + \bar{S} + K_{gm}(\boldsymbol{\nabla}_h\bar{b})^2 N^{-2} - \epsilon_{eke}$$

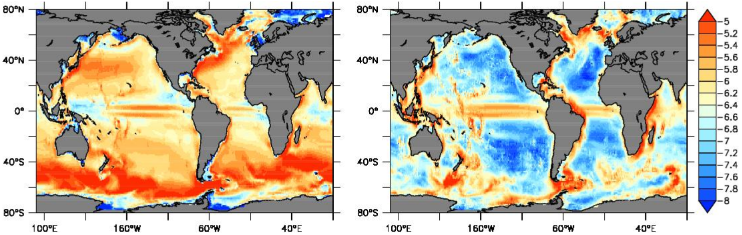
- close with $K_{gm} = c_g\sqrt{\bar{E}}L$, and
  $\epsilon_{eke} = c_\epsilon \bar{E}^{3/2}/L$ as in (Eden + Greatbatch, 2008)

consistent ocean model

- external forcing functions in $log_{10}(F/m^3s^{-3})$



0.14 (0.38) TW wind forcing, 1.7 TW tidal forcing



0.67 TW of $K_{gm}(\nabla_h \bar{b})^2 N^{-2}$ and 0.49 TW of $\bar{S}$

IDEMIX forced with eddies

- internal wave closure (IDEMIX)

$$\frac{DE_{iw}}{Dt} = \nabla \cdot (\text{fluxes}) + \epsilon_{eke} - \epsilon_{iw}$$

- meso-scale eddy closure

$$\frac{DE_{eke}}{Dt} = \nabla \cdot (\text{fluxes}) + \bar{S} + K_{gm}(\nabla_h \bar{b})^2 N^{-2} - \epsilon_{eke}$$

IDEMIX forced with eddies

- internal wave closure (IDEMIX)

$$\frac{DE_{iw}}{Dt} = \nabla \cdot (\text{fluxes}) + \epsilon_{eke} - \epsilon_{iw}$$

- meso-scale eddy closure

$$\frac{DE_{eke}}{Dt} = \nabla \cdot (\text{fluxes}) + \bar{S} + K_{gm}(\nabla_h \bar{b})^2 N^{-2} - \epsilon_{eke}$$

- assume local dissipation (interior loss of balance?)

IDEMIX forced with eddies

- internal wave closure (IDEMIX)

$$\frac{DE_{iw}}{Dt} = \nabla \cdot (\text{fluxes}) + \epsilon_{eke} - \epsilon_{iw}$$

- meso-scale eddy closure

$$\frac{DE_{eke}}{Dt} = \nabla \cdot (\text{fluxes}) + \bar{S} + K_{gm}(\nabla_h \bar{b})^2 N^{-2} - \epsilon_{eke}$$

- assume local dissipation (interior loss of balance?)
- or assume $\int_{-h}^{0} \epsilon_{eke} dz$ is injected at bottom only (generation of lee waves by meso-scale eddies?)

IDEMIX forced with eddies

- internal wave closure (IDEMIX)

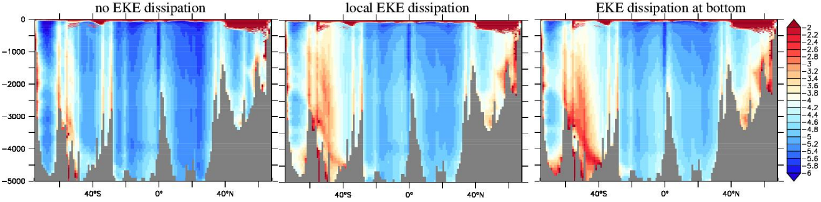
$$\frac{DE_{iw}}{Dt} = \nabla \cdot (\text{fluxes}) + \epsilon_{eke} - \epsilon_{iw}$$

- meso-scale eddy closure

$$\frac{DE_{eke}}{Dt} = \nabla \cdot (\text{fluxes}) + \bar{S} + K_{gm}(\nabla_h \bar{b})^2 N^{-2} - \epsilon_{eke}$$

- assume local dissipation (interior loss of balance?)
- or assume $\int_{-h}^{0} \epsilon_{eke} dz$ is injected at bottom only (generation of lee waves by meso-scale eddies?)

$log_{10}(K/m^2 s^{-1})$ at 30^oW



Summary

- construct a consistent ocean model
  without spurious energy sources and sinks

$$\frac{DE_m}{Dt} = \nabla \cdot (\text{fluxes}) + \bar{b}\bar{w} - \bar{S} - c_u K \left(\frac{\partial \bar{\mathbf{u}}}{\partial z}\right)^2$$

$$\frac{DP_m}{Dt} = \nabla \cdot (\text{fluxes}) - \bar{b}\bar{w} - K_{gm}(\nabla_h \bar{b})^2 N^{-2} + c_b K N^2$$

$$\frac{DE_{eke}}{Dt} = \nabla \cdot (\text{fluxes}) + \bar{S} + K_{gm}(\nabla_h \bar{b})^2 N^{-2} - \epsilon_{eke}$$

$$\frac{DE_{iw}}{Dt} = \nabla \cdot (\text{fluxes}) + \epsilon_{eke} - \epsilon_{iw}$$

$$\frac{DE_{tke}}{Dt} = \nabla \cdot (\text{fluxes}) + c_u K \left(\frac{\partial \bar{\mathbf{u}}}{\partial z}\right)^2 + \epsilon_{iw} - c_b K N^2 - \epsilon_{tke}$$

Summary

- construct a consistent ocean model without spurious energy sources and sinks
- IDEMIX connects different forcing functions, small-scale, and meso-scale turbulence closures

$$
\begin{aligned}
\frac{DE_m}{Dt} &= \nabla \cdot (\text{fluxes}) + \bar{b}\bar{w} - \bar{S} - c_u K \left(\frac{\partial \bar{\mathbf{u}}}{\partial z}\right)^2 \\
\frac{DP_m}{Dt} &= \nabla \cdot (\text{fluxes}) - \bar{b}\bar{w} - K_{gm} (\nabla_h \bar{b})^2 N^{-2} + c_b K N^2 \\
\frac{DE_{eke}}{Dt} &= \nabla \cdot (\text{fluxes}) + \bar{S} + K_{gm} (\nabla_h \bar{b})^2 N^{-2} - \epsilon_{eke} \\
\frac{DE_{iw}}{Dt} &= \nabla \cdot (\text{fluxes}) + \epsilon_{eke} - \epsilon_{iw} \\
\frac{DE_{tke}}{Dt} &= \nabla \cdot (\text{fluxes}) + c_u K \left(\frac{\partial \bar{\mathbf{u}}}{\partial z}\right)^2 + \epsilon_{iw} - c_b K N^2 - \epsilon_{tke}
\end{aligned}
$$

Summary

- construct a consistent ocean model without spurious energy sources and sinks
- IDEMIX connects different forcing functions, small-scale, and meso-scale turbulence closures
- unknown dissipation of meso-scale eddy energy

$$
\begin{aligned}
\frac{DE_m}{Dt} &= \nabla \cdot (\text{fluxes}) + \bar{b}\bar{w} - \bar{S} - c_u K \left(\frac{\partial \bar{\mathbf{u}}}{\partial z}\right)^2 \\
\frac{DP_m}{Dt} &= \nabla \cdot (\text{fluxes}) - \bar{b}\bar{w} - K_{gm}(\nabla_h \bar{b})^2 N^{-2} + c_b K N^2 \\
\frac{DE_{eke}}{Dt} &= \nabla \cdot (\text{fluxes}) + \bar{S} + K_{gm}(\nabla_h \bar{b})^2 N^{-2} - \epsilon_{eke} \\
\frac{DE_{iw}}{Dt} &= \nabla \cdot (\text{fluxes}) + \epsilon_{eke} - \epsilon_{iw} \\
\frac{DE_{tke}}{Dt} &= \nabla \cdot (\text{fluxes}) + c_u K \left(\frac{\partial \bar{\mathbf{u}}}{\partial z}\right)^2 + \epsilon_{iw} - c_b K N^2 - \epsilon_{tke}
\end{aligned}
$$