A Topographic Internal Lee Wave Drag Parameterization: Evaluation of an Eddying Global Ocean Model and Its Energy Budget

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## Outline



## Introduction

- Motivation and what wave drag is
- The model and observations for comparison

## 2 Energy budget wave drag estimates: offline and inline

- Mechanical energy budget from the continuity and momentum equations
- Offline estimates of wave drag
- Inline estimates of wave drag and other energy budget terms
- What happens to the velocity and stratification fields?

## **3** Model evaluation

- Comparison with satellite altimetry measurements
- Comparison with current meter measurements
- Taylor (2001) diagrams of all five diagnostics

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A truncated history of topographic wave drag studies

#### **Previous studies**

• Atmospheric general circulation models improved with wave drag (e.g., Palmer et al., 1986)

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- Atmospheric general circulation models improved with wave drag (e.g., Palmer et al., 1986)
- ∃ ample observational evidence that vertical diffusivity is enhanced in regions with rough topography (e.g., Polzin et al., 1997; ...; St. Laurent et al., 2012)

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- ∃ ample observational evidence that vertical diffusivity is enhanced in regions with rough topography (e.g., Polzin et al., 1997; ...; St. Laurent et al., 2012)
- Wave drag boosts vertical diffusivity (e.g., St. Laurent et al., 2002) and improves all considered tidal constituent amplitudes (e.g., Jayne and St. Laurent, 2001) in barotropic tidal models

Motivation and what wave drag is

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- Wave drag boosts vertical diffusivity (e.g., St. Laurent et al., 2002) and improves all considered tidal constituent amplitudes (e.g., Jayne and St. Laurent, 2001) in barotropic tidal models
- Offline estimates suggest wave drag dissipates energy at 0.2 – 0.49 TW in abyssal hill regions (e.g., Nikurashin and Ferrari, 2011; Scott et al., 2011)

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A history of topographic wave drag improving models (contd...)

### Our goals

• How much wave drag energy dissipation is there in non-abyssal hill regions?

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Motivation and what wave drag is

## A history of topographic wave drag improving models (contd...)

## Our goals

- How much wave drag energy dissipation is there in non-abyssal hill regions?
- How does wave drag impact the abyssal currents, stratification, and in turn the energy dissipation rates?

Motivation and what wave drag is

## A history of topographic wave drag improving models (contd...)

## Our goals

- How much wave drag energy dissipation is there in non-abyssal hill regions?
- How does wave drag impact the abyssal currents, stratification, and in turn the energy dissipation rates?
- Are general circulation ocean models forced only by winds and air-sea fluxes improved when wave drag is included?

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Motivation and what wave drag is

What is topographic wave drag and how do parameterizations work? Bell (1975) versus Garner (2005) [Froude number= U/NH]



- Garner (2005) allows for topographic blocking, but does not depend on Coriolis
- Bell (1975) does not allow for topographic blocking, but does depend on Coriolis
  - Both schemes depend on stratification, velocity, and underlying topographic features

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#### Our model: HYbrid Coordinate Ocean Model (HYCOM)

## Resolution

- 32 hybrid layers
- 1/12°, 1/25° resolutions

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## Inputs

 Air-sea fluxes - monthly mean ECMWF Re-Analysis (ERA-40; Kallberg et al., 2004)

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## Inputs

- Air-sea fluxes monthly mean ECMWF Re-Analysis (ERA-40; Kallberg et al., 2004)
- Winds monthly mean ERA-40 supplemented with 6-hourly 2003 fields of the Navy Operational Global Atmospheric Prediction System (NOGAPS; Rosmond et al., 2002)

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### Our model: HYbrid Coordinate Ocean Model (HYCOM) (contd...)

## **Dissipators**

• Horizontal viscosity - ( $\sim 10^2 - 10^3 \text{ m}^2 \text{ s}^{-1}$ ) includes the maximum of a Laplacian and a Smagorinsky (1993) parameterization with an additional biharmonic term

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- Vertical viscosity (~ 10<sup>-4</sup> 10<sup>-3</sup> m<sup>2</sup> s<sup>-1</sup>) multiply the vertical diffusivities from KPP (Large et al., 1994) by a Prandtl number (ten)

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- **Bottom drag** quadratic in the momentum equations with coefficient,  $C_d = 0.0025$  (Taylor, 1919; ...; Arbic et al., 2009)

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- Wave drag Garner (2005) scheme is used with parameters from Goff and Arbic (2010) and Goff (2010) where there are abyssal hills; a Generalized Additive Model (Wood, 2006) is used to predict the parameters elsewhere

Energy budget wave drag estimates: offline and inline

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The model and observations for comparison

# Diagnostics informed by observations and compared with model output

Current meters (Global Multi-Archive Current Meter Database; http://stockage.univ-brest.fr/scott/GMACMD/updates.html)

Mean vertical structure of kinetic energy

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Diagnostics informed by observations and compared with model output

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Mean vertical structure of kinetic energy

Satellite altimetry (Archiving, Validation and Interpretation of Satellite Oceanographic; http://www.aviso.oceanobs.com/es/data/index.html)

- Surface kinetic energy
- Eddy length scales (inverse first centroid of kinetic energy power spectrum)
- Sea surface height variance
- Intensified jet positions (via Kelly et al., 2007)

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Continuit	y and momentum equations		
Mechanical energy	budget from the continuity and momentum equations		
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$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{u}) = 0 \tag{1}$$

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \vec{\nabla})\vec{u} + \frac{1}{\rho}\vec{\nabla}\rho + f\hat{k} \times \vec{u} + g\hat{k} =$$
(2)

$$\frac{\delta_{s}}{\rho} \frac{\vec{\tau}_{wind}}{H_{s}} - \delta_{b,H_{BD}} \frac{C_{d}}{H_{BD}} |\vec{u}|\vec{u} - \delta_{b,H_{WD}} \frac{|r_{drag}|}{H_{WD}} \vec{u} \\ - \frac{\partial}{\partial z} (\nu_{z} \frac{\partial}{\partial z} \vec{u}) - \vec{\nabla} \cdot (\nu_{h,2} \vec{\nabla} \vec{u} + \nu_{h,4} \vec{\nabla} \nabla^{2} \vec{u})$$

Continuit	y and momentum equations		
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wind + buoyancy = (3)

bottom drag + wave drag +

vertical viscosity + horizontal viscosity.

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#### Offline estimates of wave drag

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 Mechanical energy budget from the continuity and momentum equations

## • Offline estimates of wave drag

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Offline estimates of wave drag

## Bell (1975) versus Garner (2005)

Using the same bottom velocity fields from a 1/12<sup>o</sup> HYCOM simulation without wave drag and two different stratification fields: one from the World Ocean Atlas (WOA) and one from the same HYCOM simulation (HYCOM)...



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Offline estimates of wave drag

**Global Integrals of Offline Wave Drag Estimates in TW**= 10<sup>12</sup>W

wave drag scheme	WOA	HYCOM
Garner (2005)	0.45	0.57
Bell (1975)	0.47	0.52

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Bottom drag and wave drag				
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Averaging the energy dissipation rate per unit area every two minute baroclinic time step in a  $1/12^{\circ}$  simulation with wave drag...



Inline estimates of wave drag and other energy budget terms

#### Can we just substitute wave drag with a boost in bottom drag?

See Waterman et al. (in press, JPO) for observational evidence that wave drag is mostly a non-local dissipative process, while bottom drag (see, e.g., Sen et al., 2008) is a local dissipative process; also...



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Inline estimates of wave drag and other energy budget terms

#### Vertical viscosity and horizontal viscosity



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#### Wind power inputs and thermodynamic work



Energy budget wave drag estimates: offline and inline

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Inline estimates of wave drag and other energy budget terms

## Global Integrals of Energy Budget Terms in TW = 10<sup>12</sup>W

Res.	WD?	Wind	Buoy	BD	WD	VV	HV
1/12 <sup>0</sup>	no	0.87	0.066	0.31	N/A	0.29	0.29
1/12 <sup>0</sup>	yes	0.87	0.066	0.14	0.40	0.28	0.26

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What happens to the velocity and stratification fields?

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What happens to the velocity and stratification fields?

# Bottom kinetic energy and stratification differences (with and without wave drag)



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What happens to the velocity and stratification fields?

# Zonally averaged kinetic energy and stratification differences (with and without wave drag)



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## Vertical and horizontal viscosity differences (with and without wave drag)



a) Depth-integrated vertical viscosity difference (without-with wave drag) b) Zonally averaged vertical viscosity difference (without-with wave drag)

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#### Surface kinetic energy



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#### **Eddy length scales**



Energy budget wave drag estimates: offline and inline

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Comparison with satellite altimetry measurements

#### Sea surface height variance



Energy budget wave drag estimates: offline and inline

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Comparison with satellite altimetry measurements

#### Intensified jet positions

Observations 1/12° HYCOM without wave drag 1/12° HYCOM with wave drag

1/25° HYCOM without wave drag



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Average vertical kinetic energy

Observations 1/12° HYCOM without wave drag 1/12° HYCOM with wave drag 1/25° HYCOM without wave drag



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#### Taylor (2001) diagrams of all five diagnostics

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Taylor (2001) diagrams of all five diagnostics

# Does wave drag ever make the model simulations in worse agreement with diagnostics informed by observations?

#### Observations

- 1/12° HYCOM without wave drag
- 1/12° HYCOM with wave drag
- 1/25° HYCOM without wave drag



#### Summary

- Dissipation in non-abyssal hill regions≈dissipation in abyssal hill regions
- Dissipation inline≈ <sup>1</sup>/<sub>2</sub>dissipation offline (due to active feedback on velocities and stratification)
- These roughly cancel so that what Scott et al. (2011) found  $\approx$  our inline estimates
- Wave drag dissipates energy at a larger rate than any other dissipative term
- Wave drag cannot be substituted for by boosting bottom drag

#### Summary

- Simulation without wave drag: dissipation≈ 95% inputs (likely due to non-conservation of a tracer quantity [Griffies et al., 2000; Leclair and Madec, 2009] and the ocean is not in steady-state)
- Simulation with wave drag: dissipation ≈ 115% inputs (the model ocean is not in steady-state)

#### Summary

- Simulation without wave drag: dissipation≈ 95% inputs (likely due to non-conservation of a tracer quantity [Griffies et al., 2000; Leclair and Madec, 2009] and the ocean is not in steady-state)
- Simulation with wave drag: dissipation ≈ 115% inputs (the model ocean is not in steady-state)
- Both the addition of a wave drag parameterization and going to a higher resolution (1/12° to 1/25° HYCOM) improves the model and never makes the model worse

## Why I'm here

- Let's do the same with CESM's ocean component (POP)!
- There are several minor details with putting this into a model like: 1) the range of relevant wavenumbers for the internal waves to not be evanescent, 2) estimating the parameters in non-abyssal hill regions an alternative way, 3) relaxing the assumption of small off-diagonal components to the topographic information tensor, and 4) using a depth-dependent momentum deposition procedure that Garner (2005) used (rather than using the bottom 500 meters)

## How $N^2$ in $1/10^o$ POP compares with Argo data



$$\mathbf{T}(x,y) = \begin{bmatrix} \frac{\rho N}{(2\pi)^2} \int dk dl |P(k,l)| \frac{k^2}{|\vec{k}|} & \frac{\rho N}{(2\pi)^2} \int dk dl |P(k,l)| \frac{kl}{|\vec{k}|} \\ \frac{\rho N}{(2\pi)^2} \int dk dl |P(k,l)| \frac{kl}{|\vec{k}|} & \frac{\rho N}{(2\pi)^2} \int dk dl |P(k,l)| \frac{l^2}{|\vec{k}|} \end{bmatrix} = (4)$$
$$\begin{bmatrix} T_{1,1} & T_{1,2} \\ T_{2,1} & T_{2,2} \end{bmatrix},$$

$$\chi_0(x,y) = -\frac{\rho N}{2\pi} \int d\vec{x}' \frac{h(\vec{x}')}{|\vec{x} - \vec{x}'|} = -\rho N \int d\vec{k}' \frac{\hat{h}(\vec{k}')}{|\vec{k}'|} \exp[i\vec{k}' \cdot \vec{x}]$$
(5)

 $\beta$  is set to be proportional to the magnitudes of the spatial gradients in the bathymetry

## Spatially varying input parameter plots for the Garner (2005) scheme



#### Implementation of wave drag in the momentum equations

$$r_{drag} = \frac{\vec{\tau} \cdot \vec{u}_d}{\rho |\vec{u}_d|^2} \tag{6}$$

where  $\vec{u}_d$  is the velocity field averaged over the bottom 500 meters,

$$\vec{\tau} = (\tau_x, \tau_y) = \left(\frac{D_p}{D^*} + \frac{D_{np}}{D^*}\right) (\mathbf{T}\vec{u}_d),\tag{7}$$

$$D^{*} = a_{0} \frac{\rho V_{\tau}^{3}}{NL_{r}} h_{r}^{\gamma} \left[ \frac{(2\gamma - \epsilon)(H_{max}^{2+\gamma - \epsilon} - H_{min}^{2+\gamma - \epsilon})}{NL_{r}} \right]$$
(8)  
$$D_{p} = a_{0} h_{r}^{\gamma} \frac{\rho V_{\tau}^{3}}{NL_{r}} \frac{2\gamma - \epsilon}{H_{max}^{2-\gamma - \epsilon}} - H_{min}^{2-\gamma - \epsilon} \left( \frac{H_{clip}^{2+\gamma - \epsilon} - H_{min}^{2+\gamma - \epsilon}}{2+\gamma - \epsilon} + H_{crit}^{2+\beta} \frac{H_{max}^{2-\epsilon - \beta} - H_{clip}^{2-\epsilon - \beta}}{\gamma - \epsilon - \beta} \right),$$
$$D_{np} = a_{1} h_{r}^{\gamma} \frac{\rho V_{\tau}^{3}}{NL_{r}(1+\beta)} \frac{2\gamma - \epsilon}{H_{max}^{2-\gamma - \epsilon}} \left( \frac{H_{max}^{1+\gamma - \epsilon} - H_{clip}^{1+\gamma + \epsilon}}{1+\gamma - \epsilon} - H_{clip}^{1+\gamma + \epsilon}} - H_{crit}^{1+\beta} \frac{H_{max}^{\gamma - \epsilon - \beta} - H_{clip}^{2-\epsilon - \beta}}{\gamma - \epsilon - \beta} \right),$$

and

$$V_{\tau} = -\frac{\bar{u}_{d} \cdot (\mathbf{T}\bar{u}_{d})}{\sqrt{(u_{d}T_{1,1} + v_{d}T_{2,1})^{2} + (u_{d}T_{1,2} + v_{d}T_{2,2})^{2}}}.$$
(9)



#### Curls of drags and wind stress



b) Wave drag curl  $[\log_{10}(s^{-2})]$ 





#### Relationships between curls of drags and wind stress



## Is the model spun-up?



## Is the model spun-up (cont...)?



a) Center of gravity without wave drag [fraction of water column]



## Is the model spun-up (cont...)?

