INSTITUTE FOR GEOPHYSICS
JACKSON SCHOOL OF GEOSCIENCES

THE UNIVERSITY OF TEXAS AT AUSTIN

A metric of CAM performance that includes field dependences

Charles Jackson

(The University of Texas at Austin)

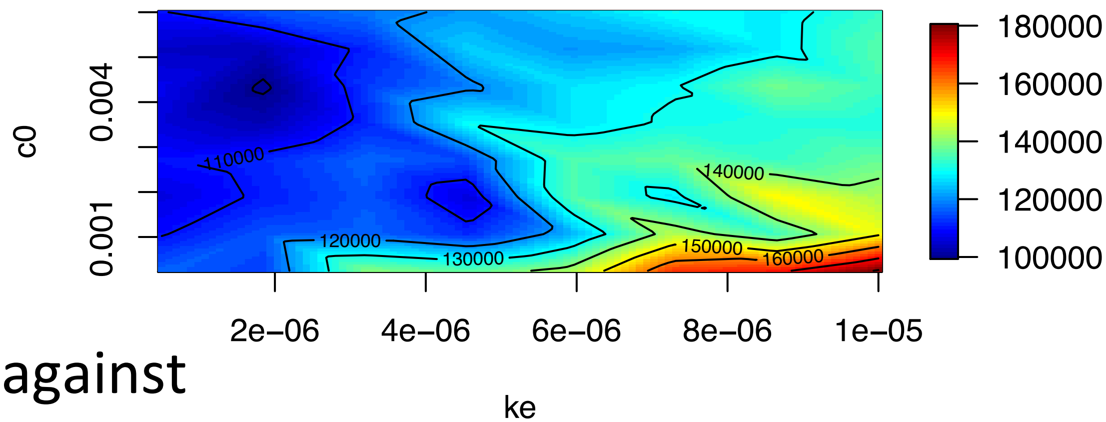
Alvaro Nosedal Sanchez and Gabriel Huerta

(The University of New Mexico)

Gabriel Huerta Alvaro Nosedal-Sanchez
Dept. of Mathematics and Statistics, University of New Mexico

Total "Cost"

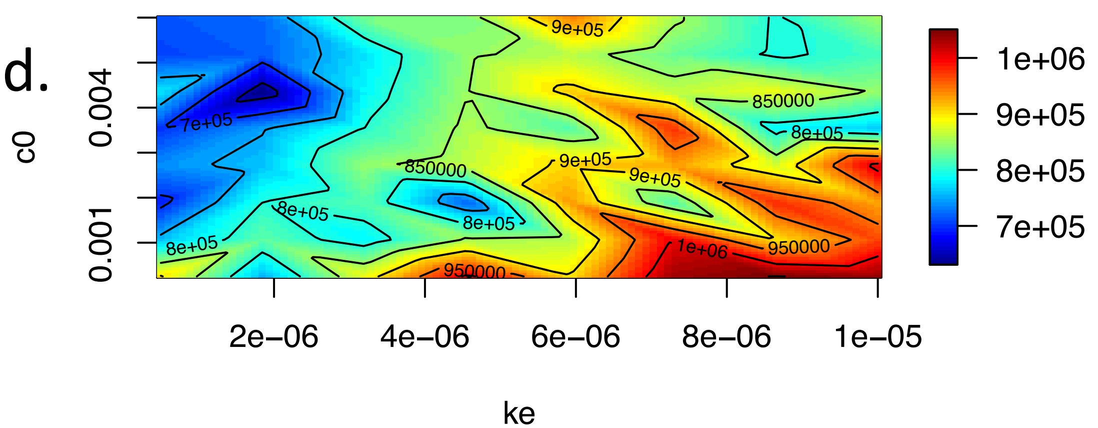
Intuitive Approach



Metric of CAM compared against observational data for JJA 30S to 30N as a function two model parameters.

64 experiments on 8x8 grid.

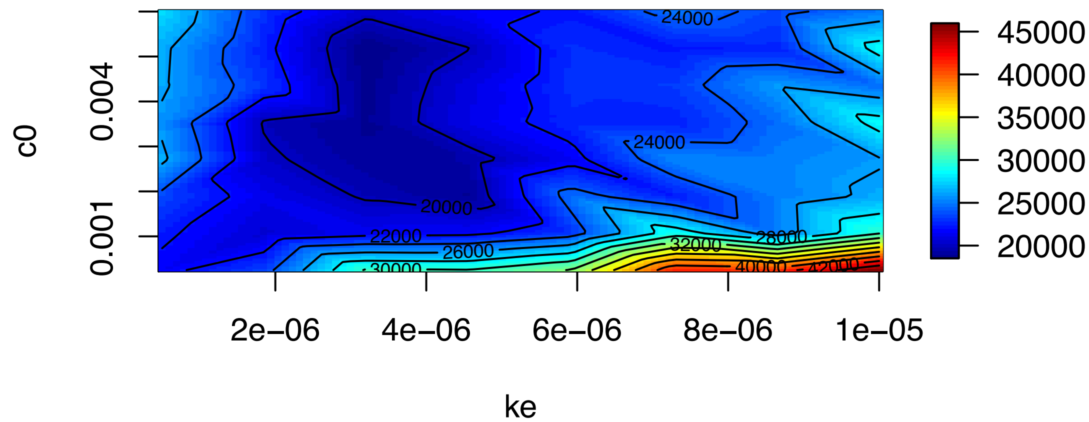
Uses Gaussian Markov Random Fields



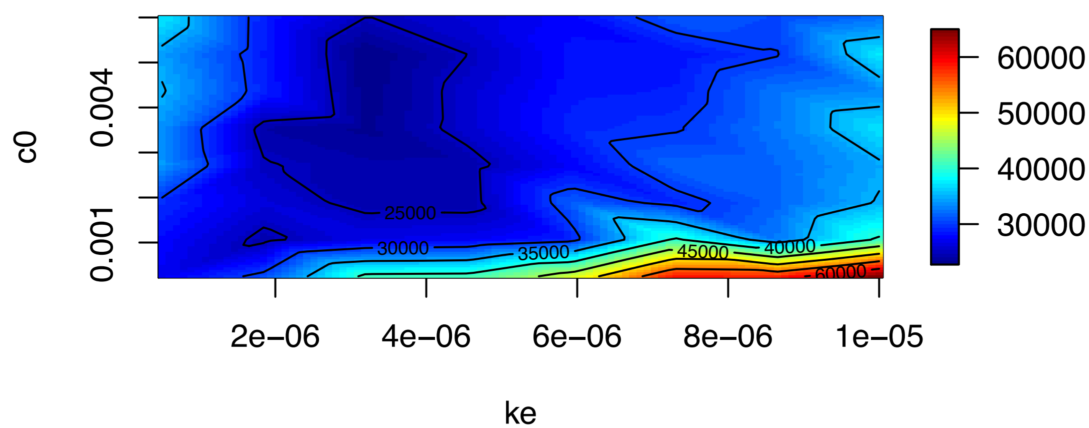
Significance of changes in cost is much larger in "intuitive approach", perhaps because it ignores spatial dependencies but not field dependencies.

Precipitation Cost

Intuitive Approach

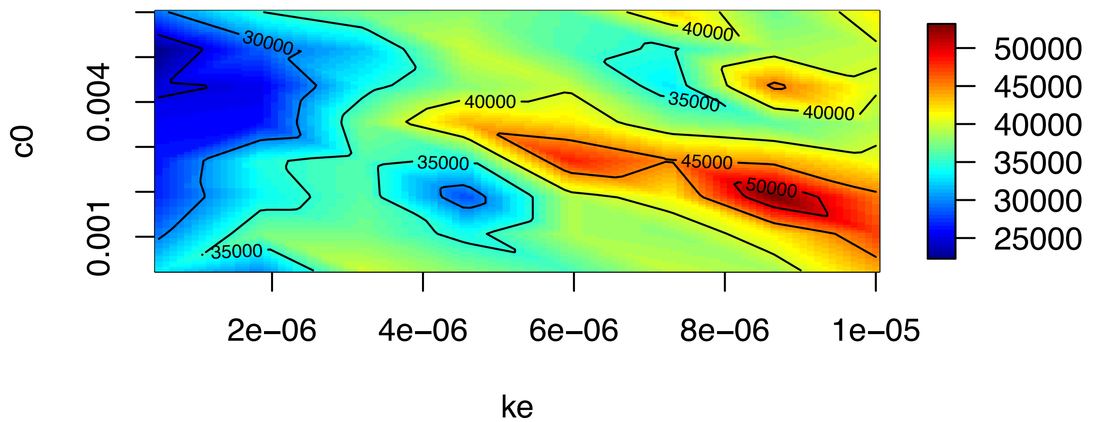


Uses Gaussian Markov Random Fields

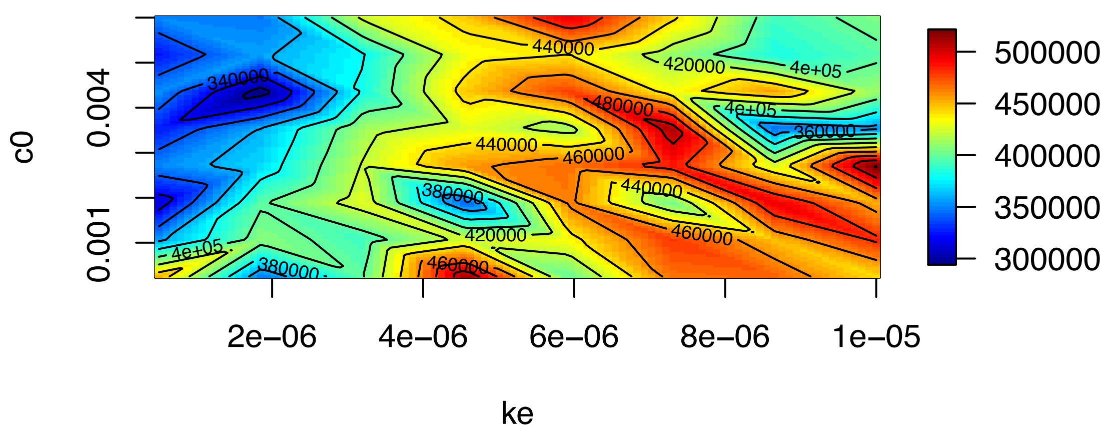


Sea Level Pressure Cost

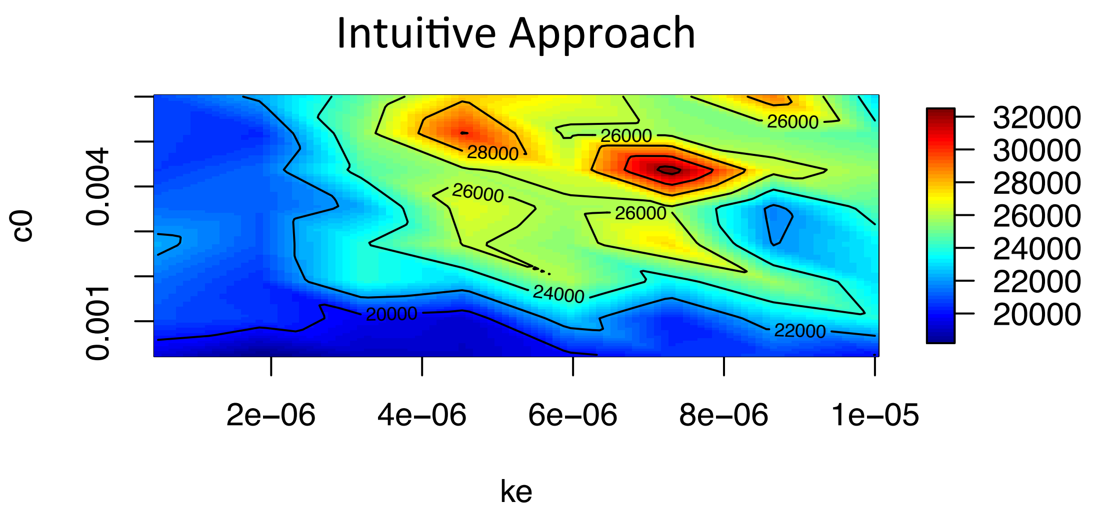
Intuitive Approach

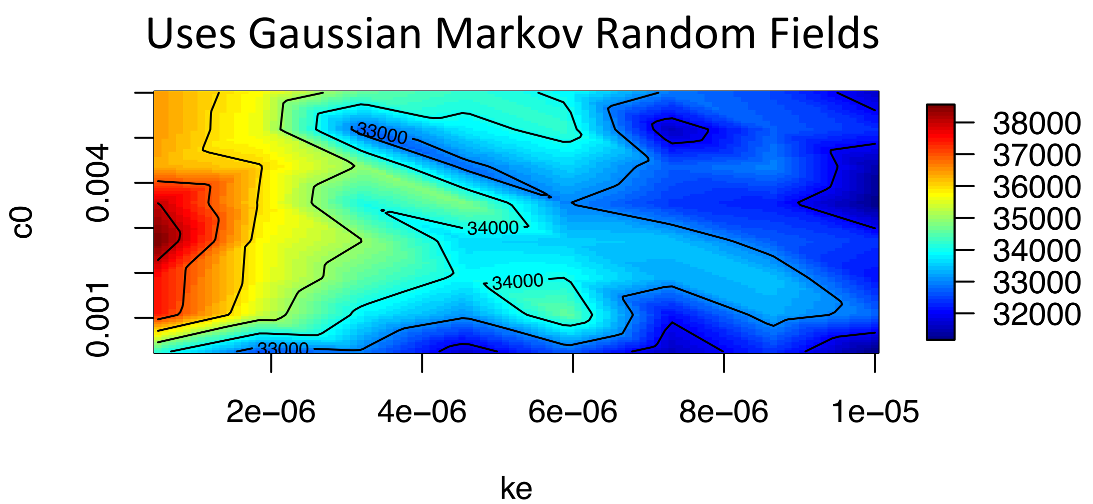


Uses Gaussian Markov Random Fields



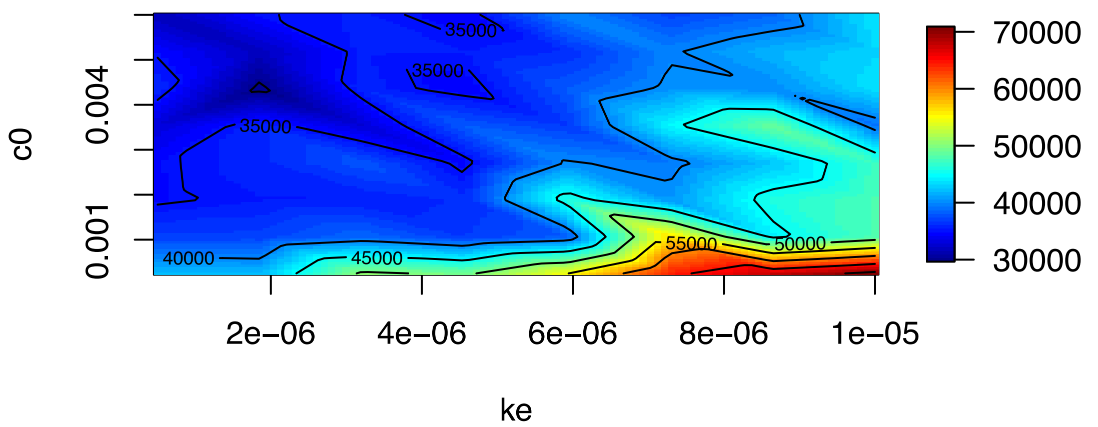
2 m air temperature Cost



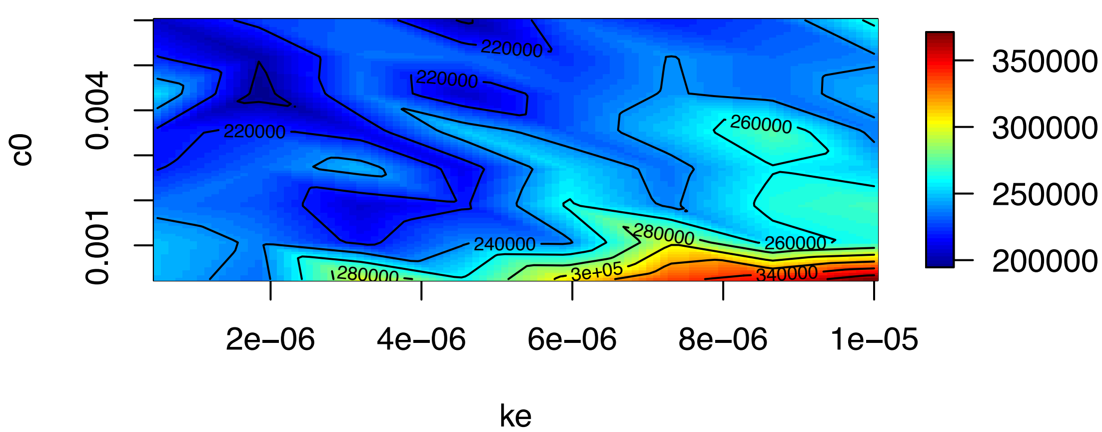


U winds at 300 mb Cost

Intuitive Approach



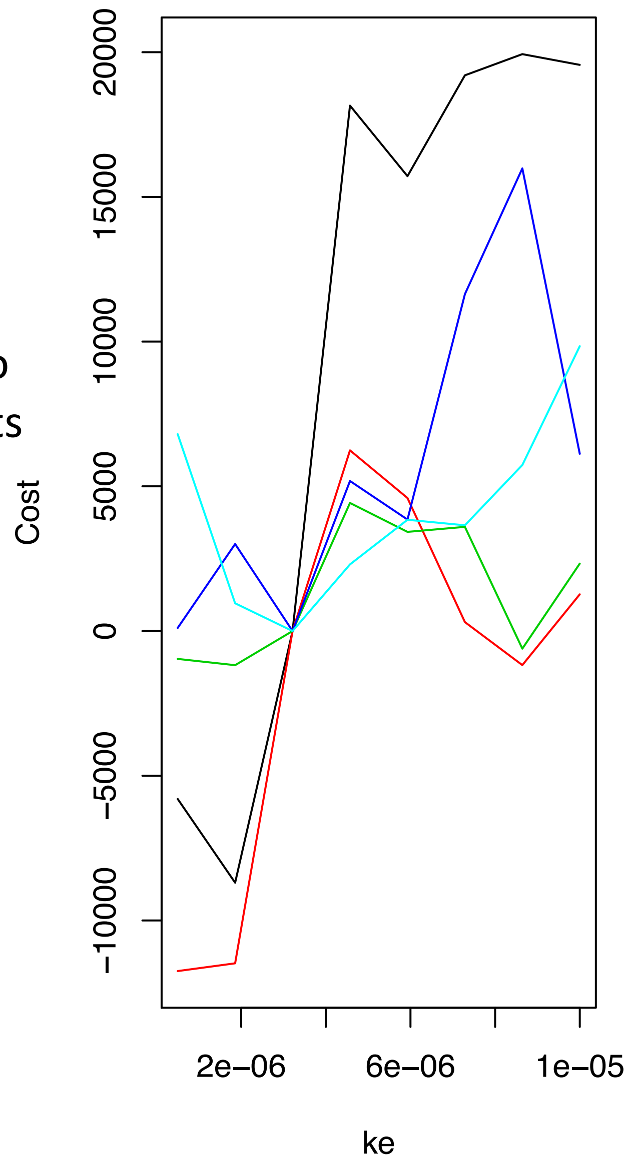
Uses Gaussian Markov Random Fields



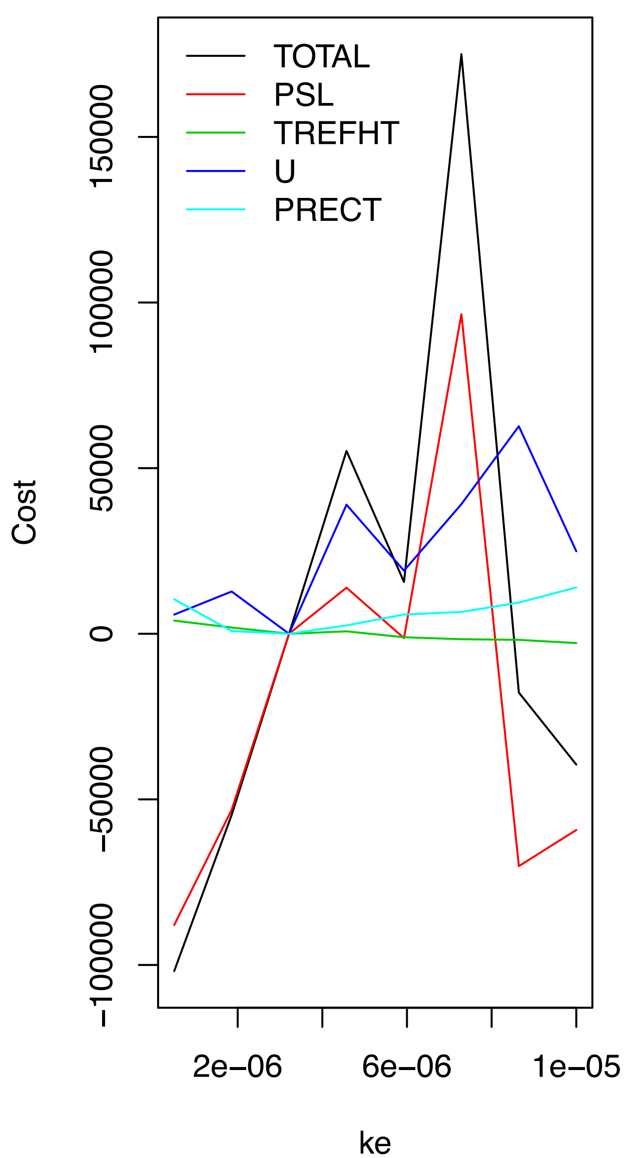
"Slice" anomalies of total Cost and its components

Field dependencies are incorporated into how component costs are combined.

Intuitive Approach



Uses Gaussian Markov Random Fields



$\hat{x}$ is a model estimate of observations x

with representation error ε_x

$$\hat{x} = x + \varepsilon_x$$

$$\bar{x} = \left\langle x + \varepsilon_x \right\rangle$$

$$\sigma_x^2 = \mathrm{var}\left(\hat{x}\right)$$

$$pdf\left(\hat{x}\right) = \frac{1}{\sigma_x \sqrt{2\pi}} \exp\left(-\frac{\left(\hat{x} - \bar{x}\right)^2}{2\sigma_x^2} \right)$$

The joint probability of two correlated quantities is given by: $\mathrm{prob}(\hat{x} \text{ and } \hat{y}) = \mathrm{prob}(\hat{x}) \cdot \mathrm{prob}(\hat{y} \mid \hat{x})$

$$pdf(\hat{x},\hat{y}) = \frac{1}{\sigma_x \sigma_y 2\pi(1-\rho^2)} \exp\left(-\frac{1}{2(1-\rho^2)}\left[\frac{(\hat{x}-\bar{x})^2}{\sigma_x^2} + \frac{(\hat{y}-\bar{y})^2}{\sigma_y^2} - 2\rho\frac{(\hat{x}-\bar{x})(\hat{y}-\bar{y})}{\sigma_x \sigma_y}\right]\right)$$

$$pdf(\mathbf{X}) = \frac{1}{2\pi|\mathbf{C}|^{\frac{1}{2}}} \exp\left(-\frac{1}{2}(\mathbf{X}-\bar{\mathbf{X}})^T \mathbf{C}^{-1}(\mathbf{X}-\bar{\mathbf{X}})\right)$$

Matrix notation

$$\mathbf{X} = \begin{bmatrix} \hat{x} \\ \hat{y} \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} \sigma_x^2 & \rho\sigma_x\sigma_y \\ \rho\sigma_x\sigma_y & \sigma_y^2 \end{bmatrix} = \begin{bmatrix} \mathbf{eof}_1 & \mathbf{eof}_2 \end{bmatrix} \cdot \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$

$$\text{cost} = D^T \mathbf{C}^{-1} D$$

Correct, but issues exist for inverting covariance **C**.

$$\text{cost} = D^T S^{-1} \otimes \left(\alpha I + (1-\alpha) Q\right) D$$

$$D_i = \left(Model_i - \overline{Obs_i}\right)$$

$$S_{ij} = \frac{1}{Nseg - 1} \sum_{k=1}^{Nseg} \left\{O_i^T\right\}_k \left(\alpha I + (1-\alpha) Q\right) \left\{O_j\right\}_k$$

$$O_i = \left(Obs_i - \overline{Obs_i}\right)$$

$$i = \text{field "a"}$$

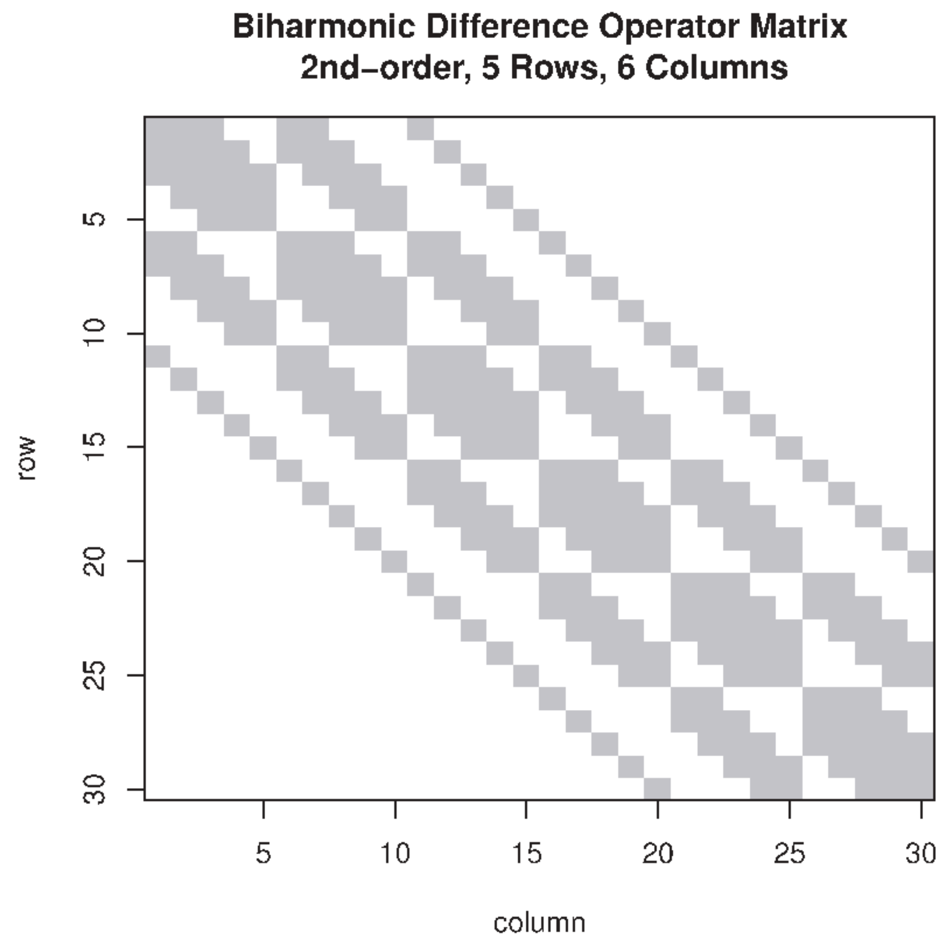
Based on CAR theory Besag (1974)
And Brook (1964)

$$j = \text{field "b"}$$

$$k = 2 \text{ year segment from 30 years of observations}$$

$$\text{cost} = D^T S^{-1} \otimes \left(\alpha I + (1-\alpha) Q\right) D$$

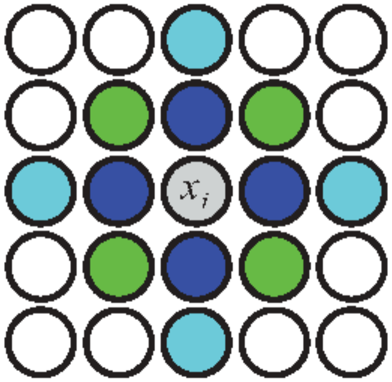
$S_{M\times M}$: observational covariance $\quad \mathbf{Q}_{N\times N} =$



M fields

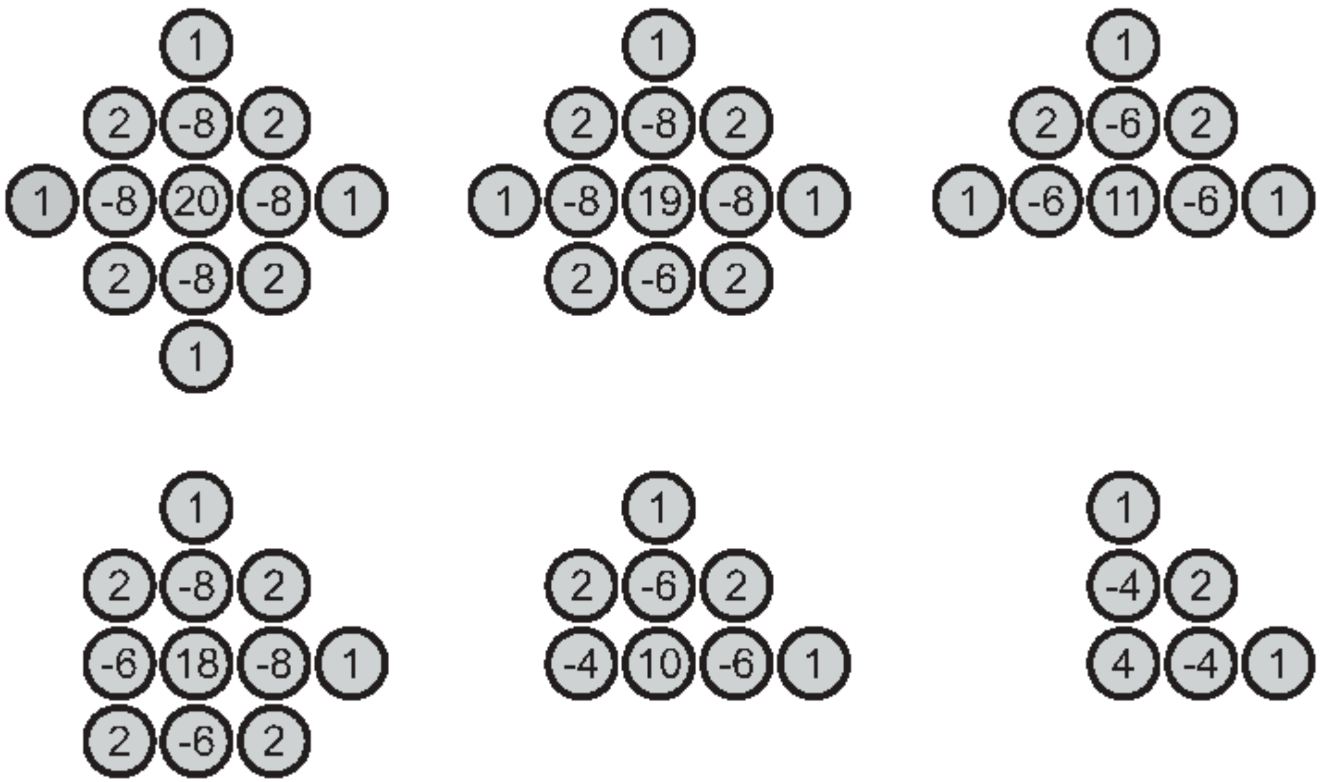
N grid points

Gaussian Markov Random Field data transform



$$E\left(x_i | x_{-i}\right) = \frac{1}{20}\left(8 \bullet - 2 \bullet + 1 \bullet\right)$$

$$Prec\left(x_i | x_{-i}\right) = 20\text{k}$$



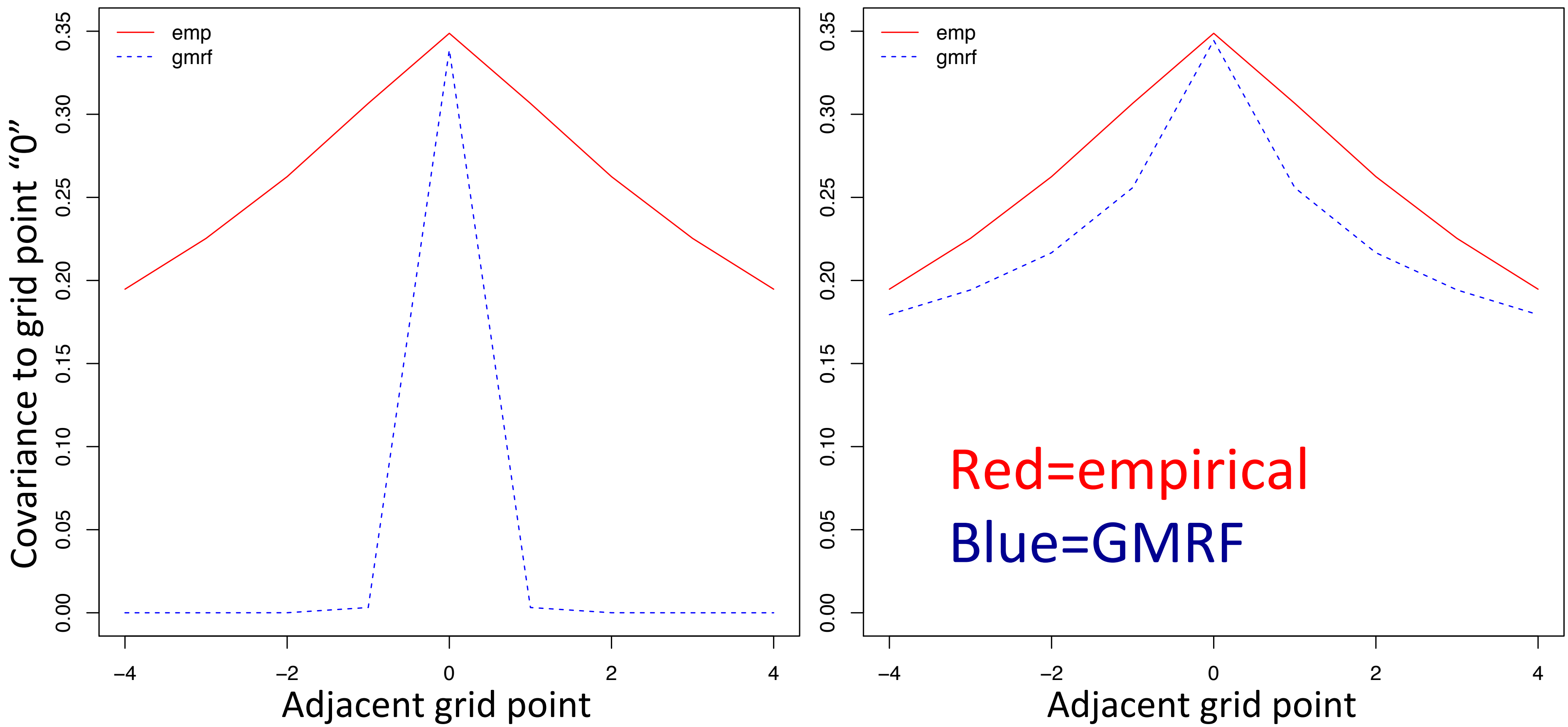
Spatial covariances represented in 2m air temperature

$$\alpha = 1$$

Intuitive Approach

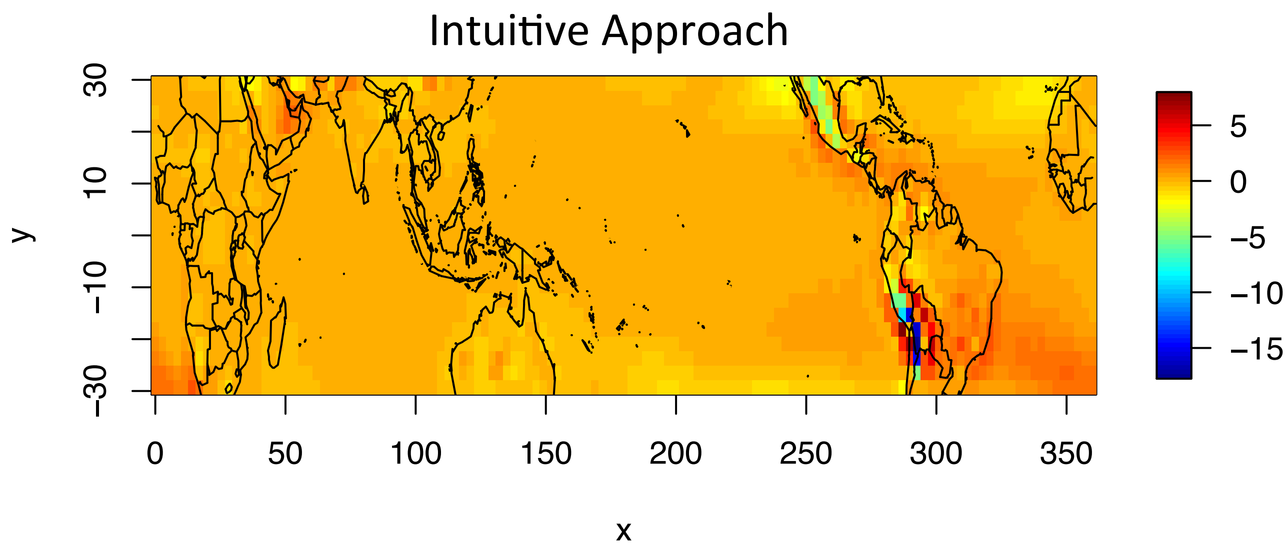
$$\alpha = 0.003$$

Uses Gaussian Markov Random Fields



Interpretation of anomalies in temperature component Cost

GMRF transform does well at representing structures. Large scale differences are not as significant.



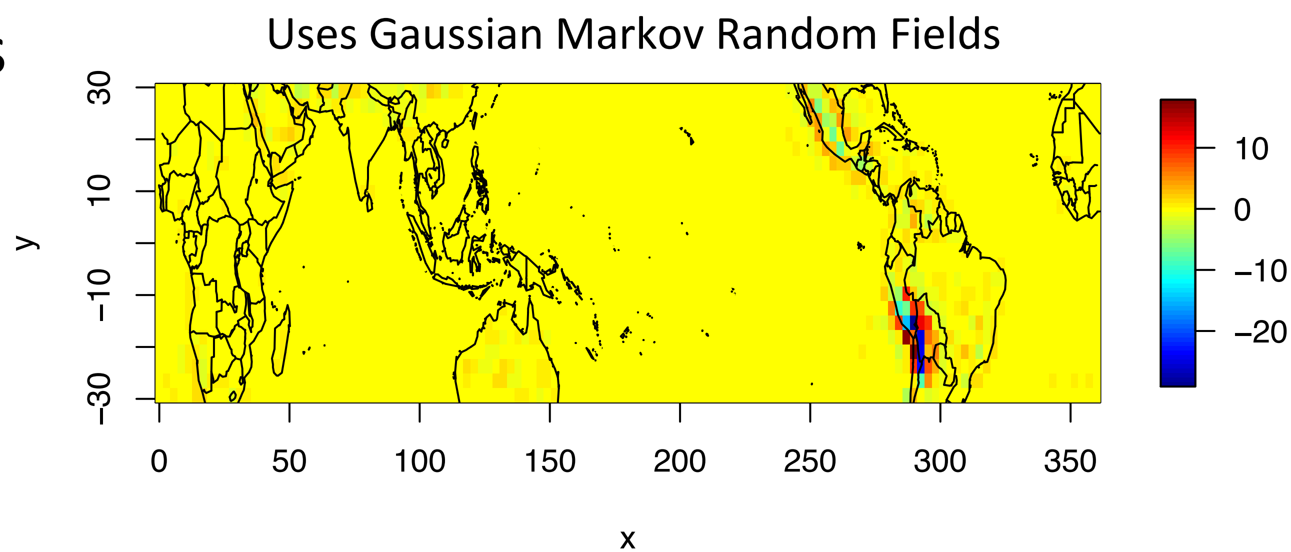
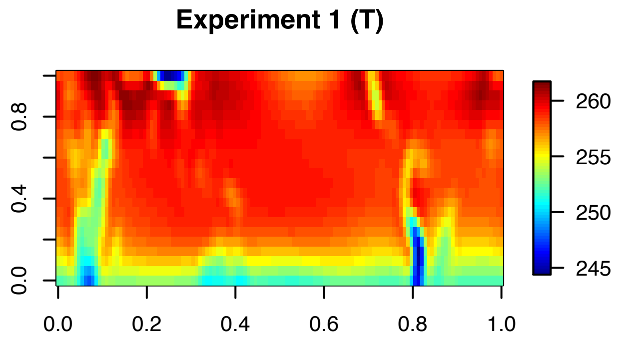
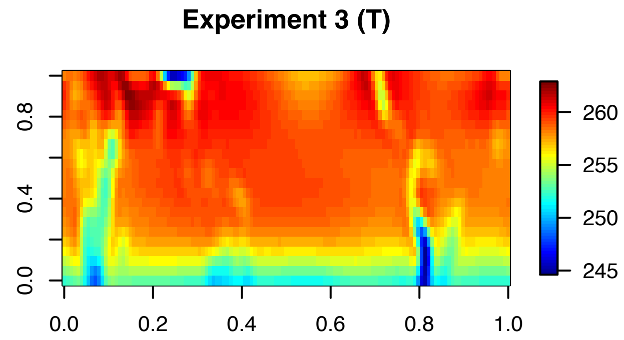
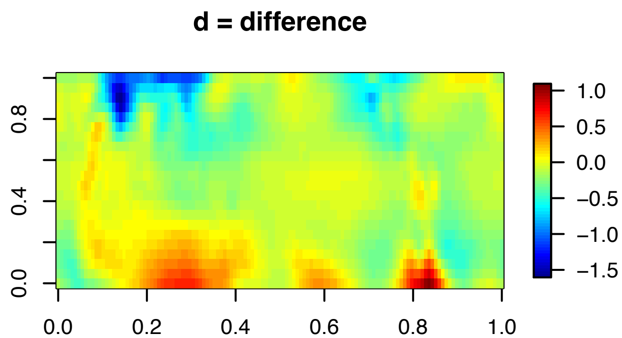
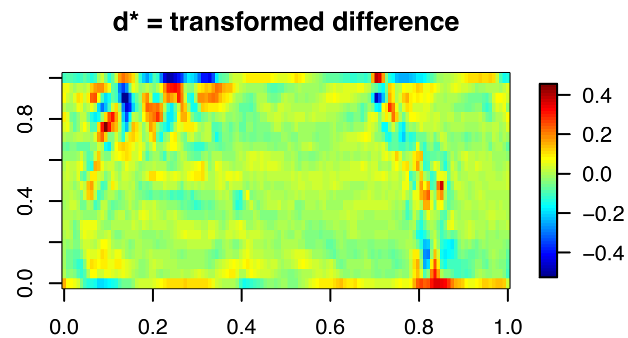


Illustration of effect of GMRF transform on Temperature

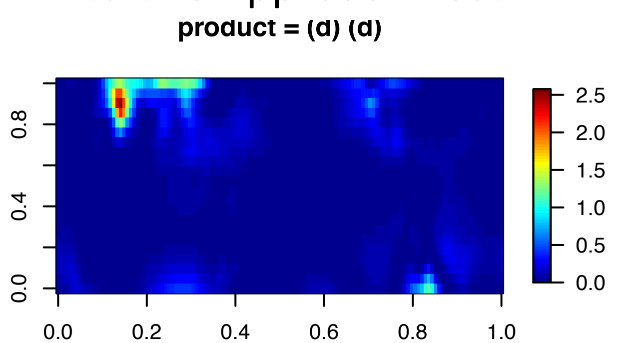








Intuitive Approach Cost



Gaussian Markov Random Field Cost

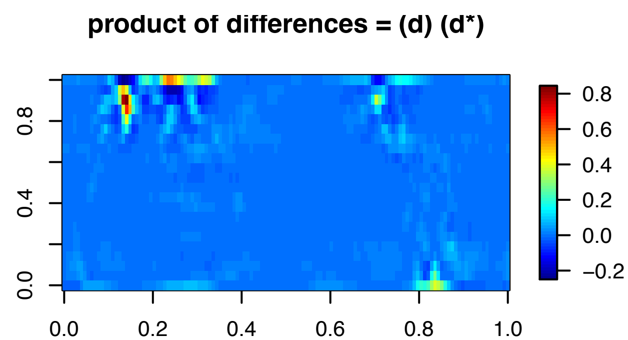
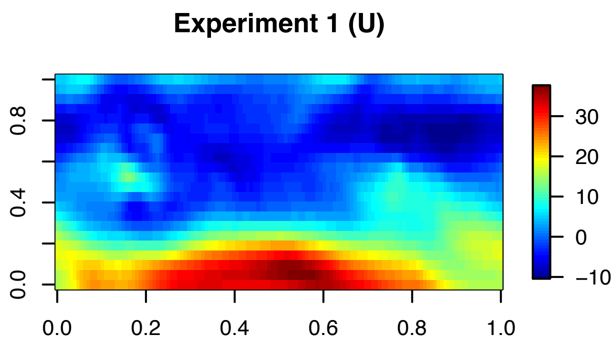
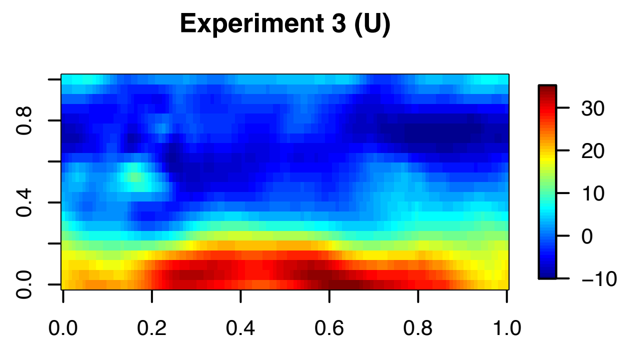
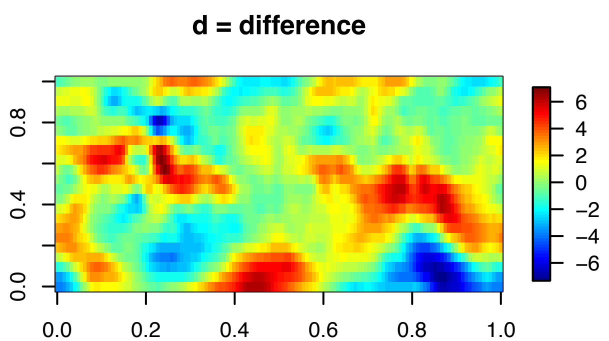
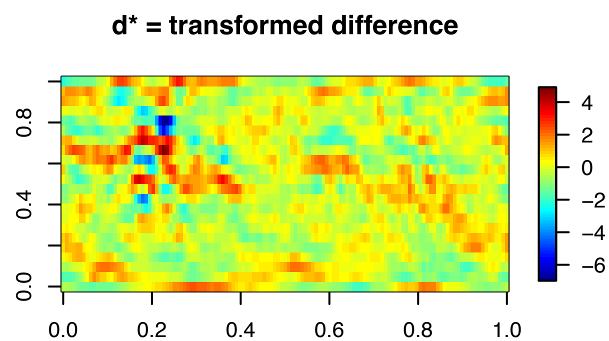


Illustration of effect of GMRF transform on winds

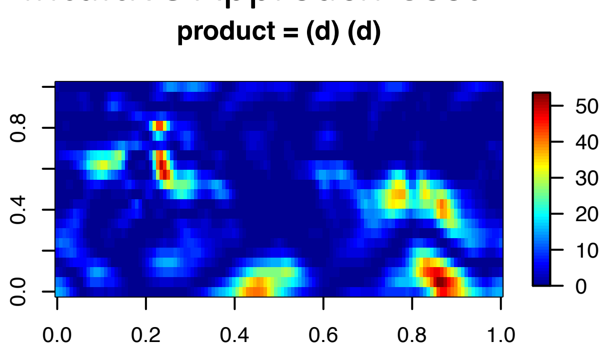




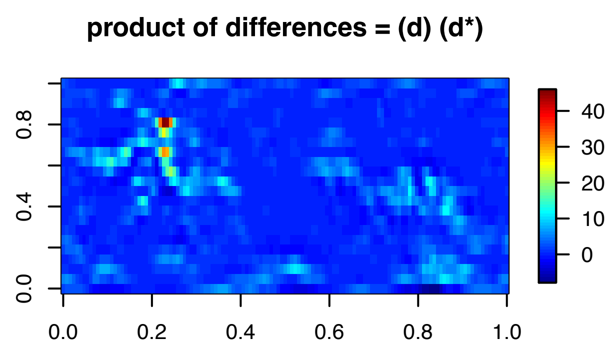




Intuitive Approach Cost



Gaussian Markov Random Field Cost



Imposing observed covariances in "S"

$$\text{cost} = D^T S^{-1} \otimes \left(\alpha I + (1-\alpha) Q\right) D$$

$$\eta = \begin{matrix} \phi_1 & \phi_2 \\ \phi_3 & \phi_4 \end{matrix} \text{ and } \begin{matrix} \psi_1 & \psi_2 \\ \psi_3 & \psi_4 \end{matrix}$$

$$D = \left(\eta_1, \eta_{2,}, \eta_3, \eta_{4,}, \eta_5, \eta_{6,}, \eta_7, \eta_8\right)$$

$$S^{-1} \otimes \left(\alpha I + (1-\alpha) Q\right) =$$

$$\begin{bmatrix} \dfrac{1}{\sigma_1^2} & \dfrac{-1}{\sigma_1 \sigma_2} \\ \dfrac{-1}{\sigma_1 \sigma_2} & \dfrac{1}{\sigma_2^2} \end{bmatrix} \otimes \left(\alpha I + (1-\alpha) Q\right)_{4\times 4} =$$

$$S^{-1} \otimes \left(\alpha I + (1-\alpha) Q \right) =$$

$$\begin{bmatrix} \frac{1}{\sigma_1^2} & \frac{-1}{\sigma_1 \sigma_2} \\ \frac{-1}{\sigma_1 \sigma_2} & \frac{1}{\sigma_2^2} \end{bmatrix} \otimes \left(\alpha I + (1-\alpha) Q \right)_{4\times 4} =$$

$$\begin{bmatrix} \frac{1}{\sigma_1^2} \left(\alpha I + (1-\alpha) Q \right)_{4\times 4} & \frac{-1}{\sigma_1 \sigma_2} \left(\alpha I + (1-\alpha) Q \right)_{4\times 4} \\ \frac{-1}{\sigma_1 \sigma_2} \left(\alpha I + (1-\alpha) Q \right)_{4\times 4} & \frac{1}{\sigma_2^2} \left(\alpha I + (1-\alpha) Q \right)_{4\times 4} \end{bmatrix}$$

$$S_{ij} = \frac{1}{Nseg-1}\sum_{k=1}^{Nseg}\left\{\mathbf{O}_i^T\right\}_k\left(\alpha I + (1-\alpha)Q\right)\left\{\mathbf{O}_j\right\}_k$$

$$\mathbf{O}_i = \left(Obs_i - \overline{Obs_i}\right)$$

$k = 2$ year segment from 30 years of observations

Correlation Matrix

Observed JJA 30S to 30N

Intuitive Approach

$$\alpha = 1$$

	precip	psl	trefht	u
precip	1.00	-0.22	-0.05	0.01
psl	-0.22	1.00	-0.31	-0.11
trefht	-0.05	-0.31	1.00	-0.15
u	0.01	-0.11	-0.15	1.00

Uses Gaussian Markov Random Fields

$$\alpha = 0.003$$

	precip	psl	trefht	u
precip	1.00	-0.01	-0.09	0.01
psl	-0.01	1.00	-0.52	-0.05
trefht	-0.09	-0.52	1.00	-0.02
u	0.01	-0.05	-0.02	1.00

Conclusions

- Presented a flexible structure for combining fields into a single metric.
- Gaussian Markov Random Fields can better represent spatial dependencies within a single field.
  - Reduces significance of large scale differences.
  - Hammers field dependencies (relative to intuitive approach) … correct? desirable?