Application of physics-based interpolation to cryospheric data

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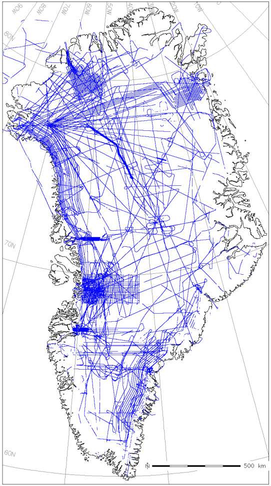
30 January, 2014
Winter LIWG Meeting, Boulder, CO

Geophysical data with noise and gaps in coverage

Problem Statement



Radar outstanding, but only along flightlines.

Geophysical data with noise and gaps in coverage

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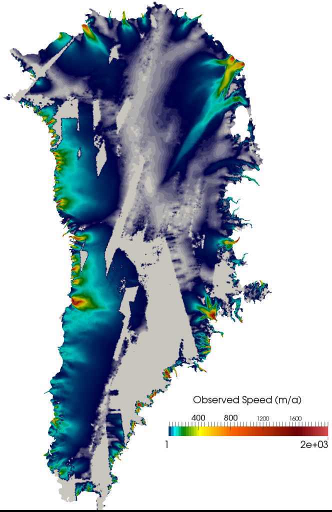

InSAR velocity excellent, but has gaps and noise.

Geophysical data with noise and gaps in coverage

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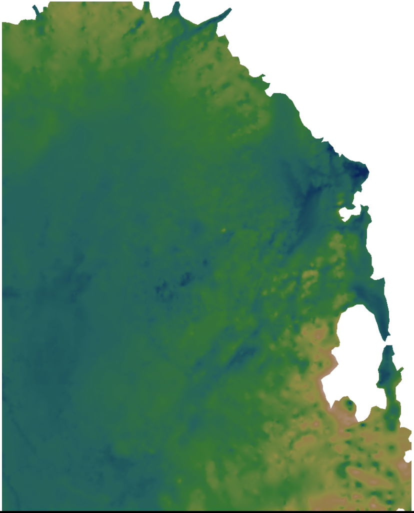

Interpolation of bed produces artefacts.

Geophysical data with noise and gaps in coverage

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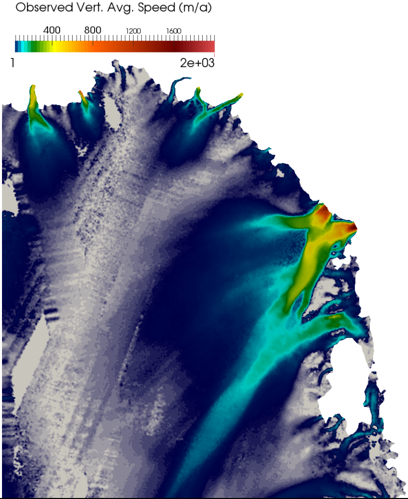

Close inspection of speed reveals noise.

Geophysical data with noise and gaps in coverage

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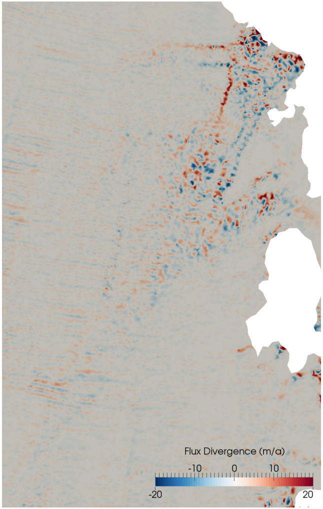
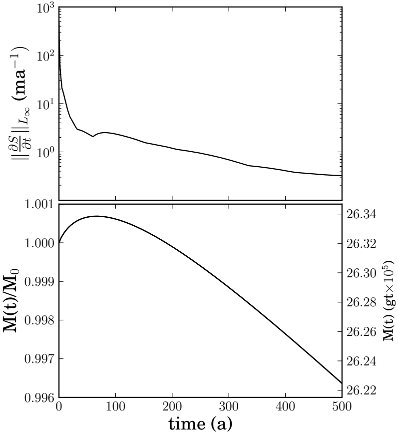


Data are combined to produce flux divergence, $\nabla \cdot (\mathbf{u}H) \neq \dot{a}$

Geophysical data with noise and gaps in coverage

Problem Statement



Prognostic modleing

- Assimilation of surface velocity (traction control variable)
- Steady state temperature
- prognostic run forward

Objective

What do we hope to accomplish?

We seek to *reduce noise and interpolate* geophysical data is a manner is consistent with:

- other observations.
- physics.
- stated errors.
- smoothness requirements.

We like to call this *physics based interpolation*.
The transient portion of prognostic runs should be removed, or greatly reduced.

Optimization cartoon

With (slightly dated) references to popular culture!

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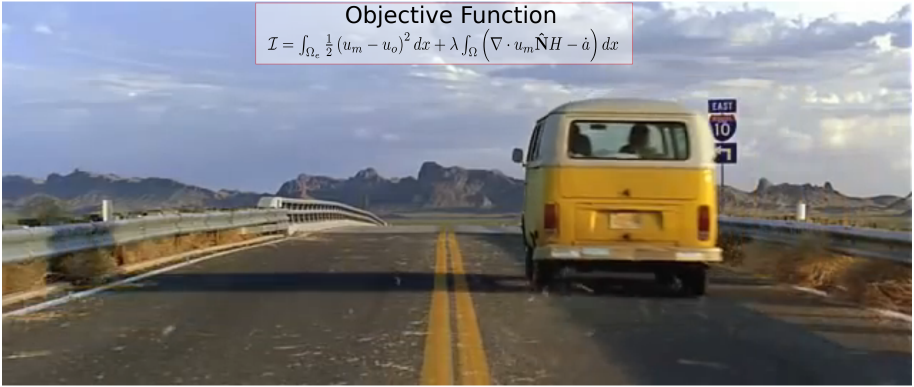

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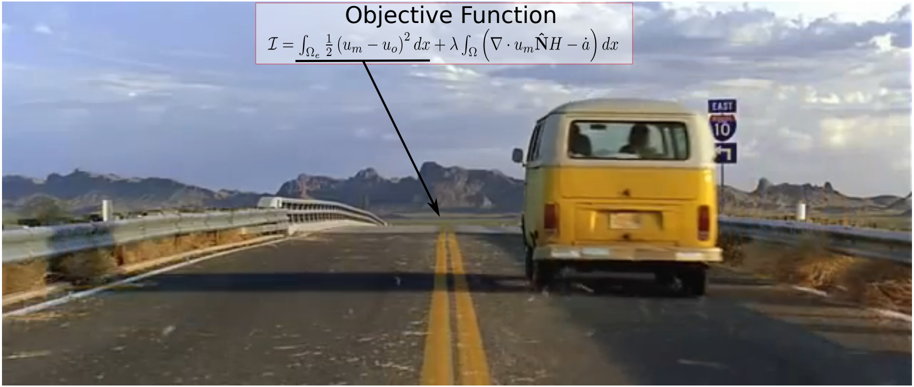

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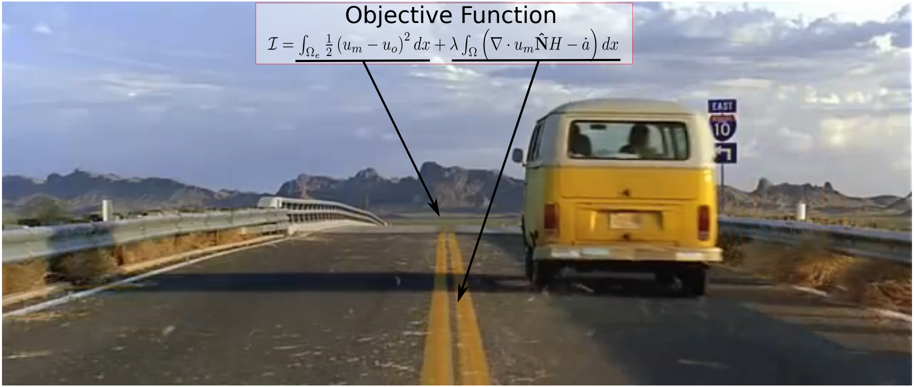

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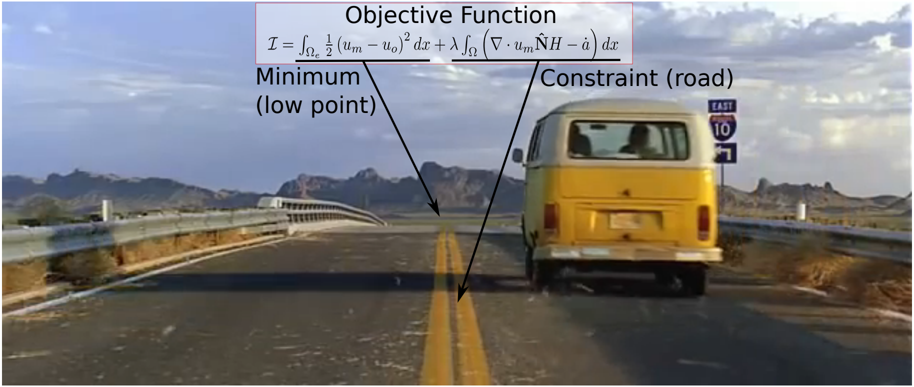
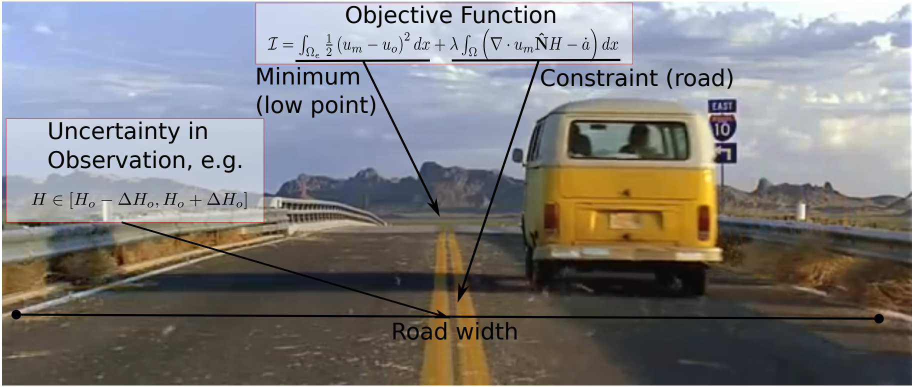


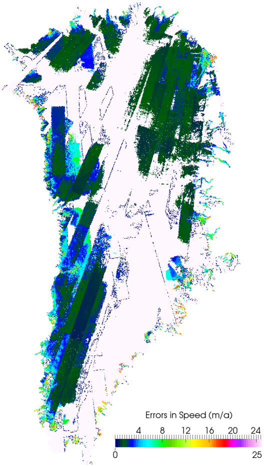
Optimization cartoon

The bounds come from data



A closer look at error bounds

Error bounds enter the constraint



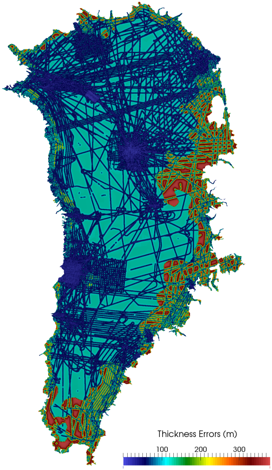
Error bounds

$$
\begin{aligned}
H &\in [H_o - \Delta H_o, H_o + \Delta H_o] \\
u_o &\in [u_o - \Delta u_o, u_o + \Delta u_o] \\
\dot{a} &\in [\dot{a} - \Delta \dot{a}, \dot{a} + \Delta \dot{a}] \\
\hat{\mathbf{N}} &\in \left[\hat{\mathbf{N}} - \Delta \hat{\mathbf{N}}, \dot{a} + \Delta \hat{\mathbf{N}}\right]
\end{aligned}
$$

Speed errors published with InSAR data, note $u_o < .5$ discarded

A closer look at error bounds

Error bounds enter the constraint



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$$

Thickness errors published with Bamber 2013 bed topography, note min. error of 35 m imposed.

A closer look at error bounds

Error bounds enter the constraint

Error bounds

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\end{aligned}
$$

No idea of the errors in apparent mass balance. Guess $\pm$ 10 m.

A closer look at error bounds

Error bounds enter the constraint

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\hat{\mathbf{N}} &\in \left[\hat{\mathbf{N}} - \Delta \hat{\mathbf{N}}, \dot{a} + \Delta \hat{\mathbf{N}}\right]
\end{aligned}
$$

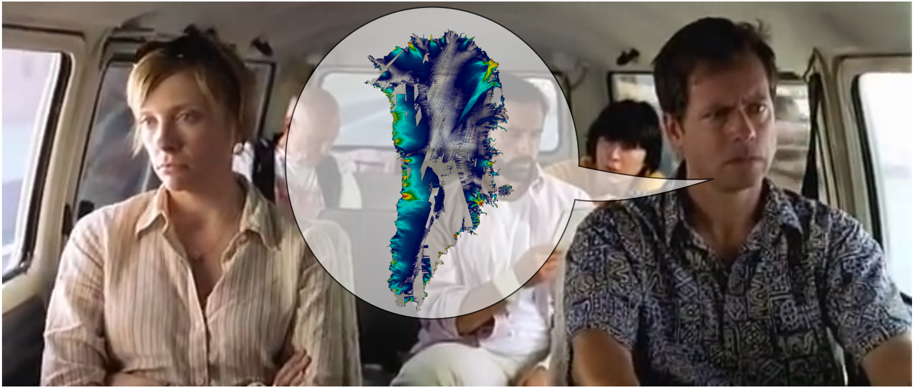
Errors in $\hat{\mathbf{N}}$ estimated to be $\pm 5^\circ$ for fast moving ice and $\pm 1^\circ$ elsewhere.

Optimization cartoon

Inside the BFGS, destination is needed

Optimization cartoon

The destination is the ***data***



Optimization cartoon

Current location is also needed, this is the model output

Optimization cartoon

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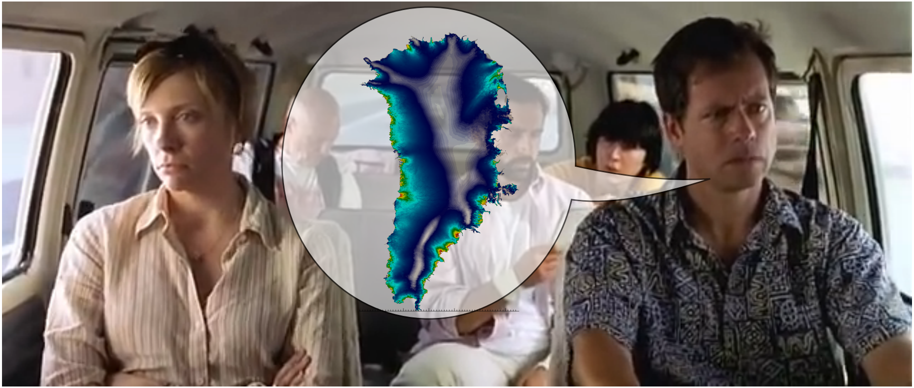

Optimization cartoon

The directions are challenging to understand

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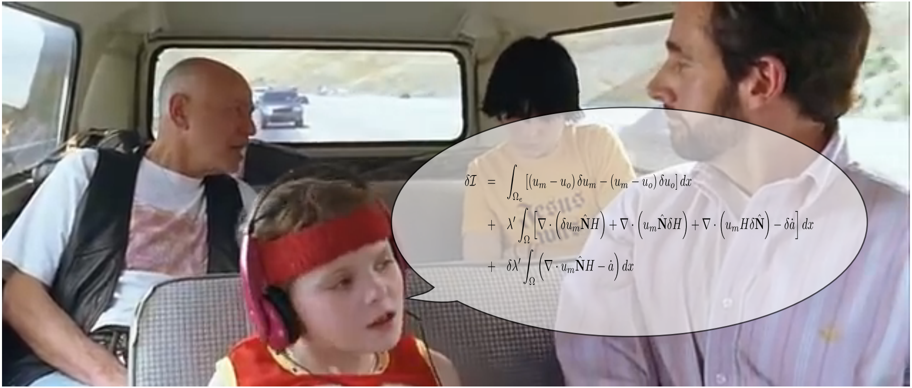

Optimization cartoon

The directions are challenging to understand

Gradients explained

Optimization requires *gradients*

Chain rule variation of objective function

$$
\begin{aligned}
\delta\mathcal{I} &= \delta\mathcal{I}(\delta H, u_o, \dot{a}, \hat{\mathbf{N}}, \lambda') + \delta\mathcal{I}(H, \delta u_o, \dot{a}, \hat{\mathbf{N}}, \lambda') \\
&+ \delta\mathcal{I}(\delta H, u_o, \delta\dot{a}, \hat{\mathbf{N}}, \lambda') + \delta\mathcal{I}(\delta H, u_o, \dot{a}, \delta\hat{\mathbf{N}}, \lambda') \\
&+ \delta\mathcal{I}(\delta H, u_o, \dot{a}, \hat{\mathbf{N}}, \delta\lambda')
\end{aligned}
$$

Gradients explained

Optimization requires *gradients*

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&+ \delta\mathcal{I}(\delta H, u_o, \delta\dot{a}, \hat{\mathbf{N}}, \lambda') + \delta\mathcal{I}(\delta H, u_o, \dot{a}, \delta\hat{\mathbf{N}}, \lambda') \\
&+ \delta\mathcal{I}(\delta H, u_o, \dot{a}, \hat{\mathbf{N}}, \delta\lambda')
\end{aligned}
$$

Find a variation, for example, δH

$$
\delta\mathcal{I}(\delta H, u_o, \dot{a}, \hat{\mathbf{N}}, \lambda) = \int_\Omega \left.\frac{\partial}{\partial \epsilon}\right|_{\epsilon=0} \mathcal{I}(H + \epsilon\delta H, u_o, \dot{a}, \hat{\mathbf{N}}, \lambda)\mathrm{d}x,
$$

Gradients explained

Optimization requires *gradients*

Application of variation throughout

$$
\begin{aligned}
\delta\mathcal{I} &= \int_{\Omega_e} \left[(u_m - u_o)\,\delta u_m - (u_m - u_o)\,\delta u_o \right] dx \\
&+ \lambda' \int_{\Omega} \left[\nabla\cdot\left(\delta u_m \hat{\mathbf{N}} H\right) + \nabla\cdot\left(u_m \hat{\mathbf{N}} \delta H\right) + \nabla\cdot\left(u_m H \delta\hat{\mathbf{N}}\right) - \delta\dot{a} \right] dx \\
&+ \delta\lambda' \int_{\Omega} \left(\nabla\cdot u_m \hat{\mathbf{N}} H - \dot{a} \right) dx
\end{aligned}
$$

Gradients explained

Optimization requires *gradients*

Identification of terms in variation

$$
\begin{aligned}
\delta\mathcal{I} \;=\; & \int_{\Omega_e}\left[\underbrace{(u_m-u_o)\,\delta u_m}_{\text{Adjoint RHS}} - \underbrace{(u_m-u_o)\,\delta u_o}_{g_{u_o}}\right] dx \\
+ \;& \lambda' \int_{\Omega}\left[\underbrace{\nabla\cdot\left(\delta u_m \hat{\mathbf{N}} H\right)}_{\text{Adjoint LHS}} + \underbrace{\nabla\cdot\left(u_m \hat{\mathbf{N}}\delta H\right)}_{g_H} + \underbrace{\nabla\cdot\left(u_m H\delta\hat{\mathbf{N}}\right)}_{g_{\hat{\mathbf{N}}}} - \underbrace{\delta\dot{a}}_{g_{\dot{a}}}\right] dx \\
+ \;& \delta\lambda' \int_{\Omega}\underbrace{\left(\nabla\cdot u_m\hat{\mathbf{N}}H - \dot{a}\right)}_{\text{Forward Model}} dx
\end{aligned}
$$

Optimization cartoon

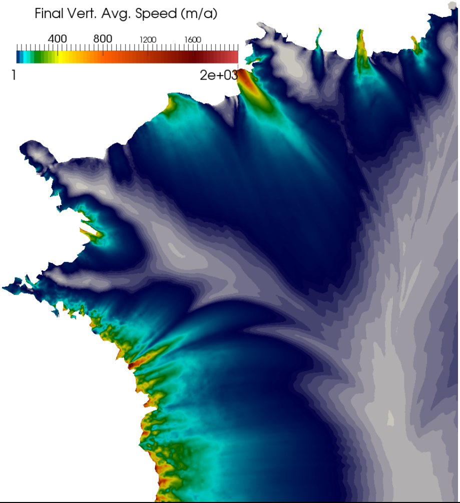
Directions can be simplified

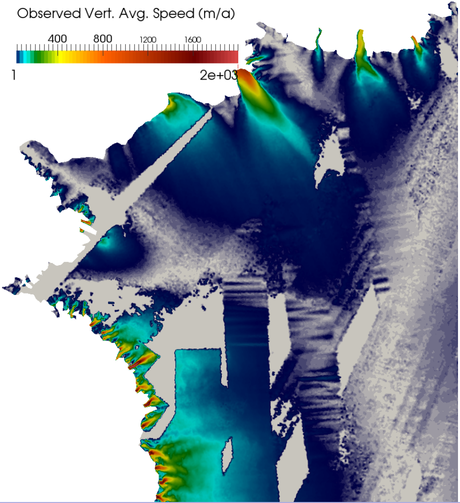
Optimization cartoon

Downhill is good for the BFGS

North West region: speed results

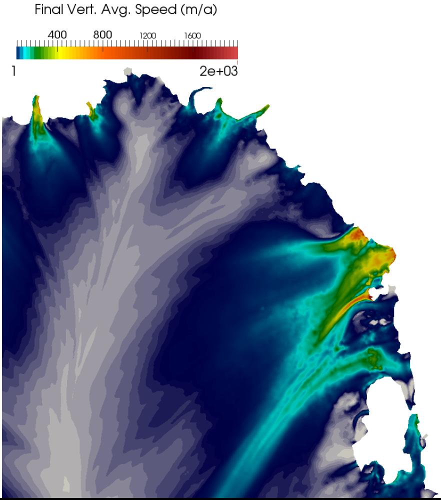
Smoothed and interpolated with physics based PDE-constrained optimization

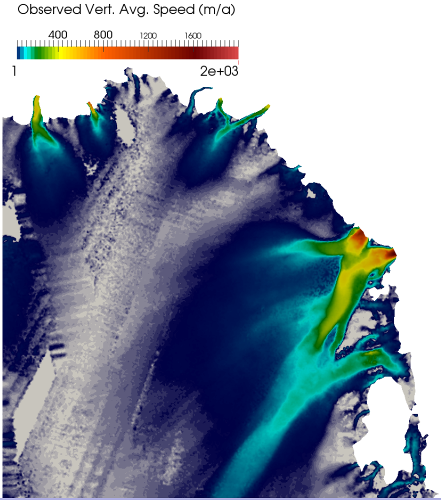




North East region: speed results

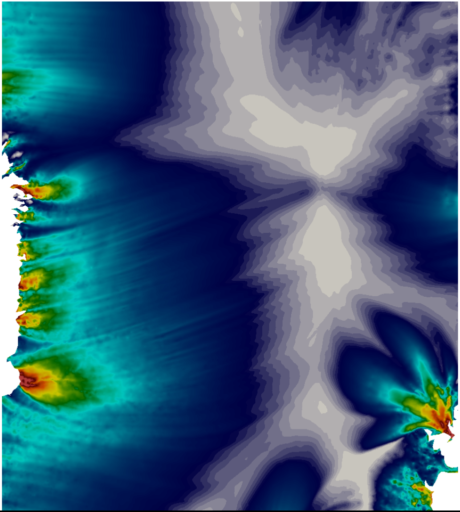
Smoothed and interpolated with physics based PDE-constrained optimization

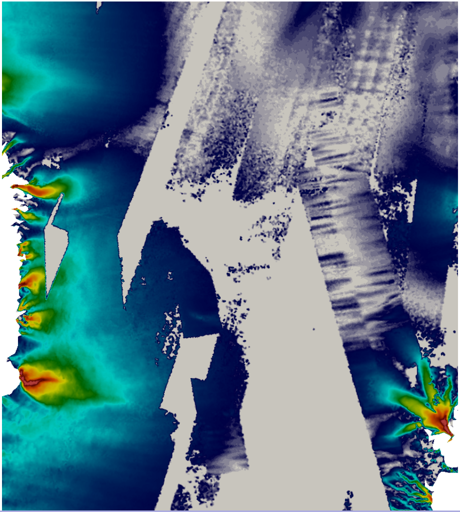




Central region: speed results

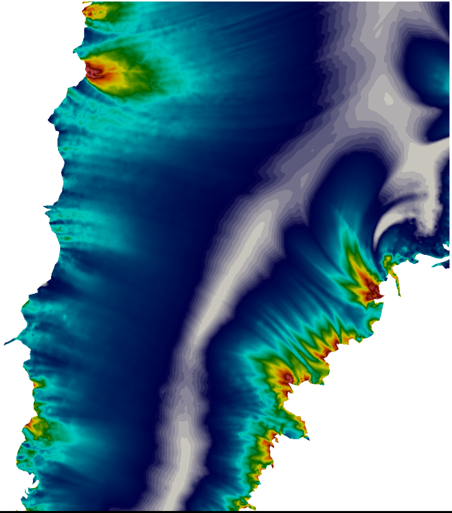
Smoothed and interpolated with physics based PDE-constrained optimization

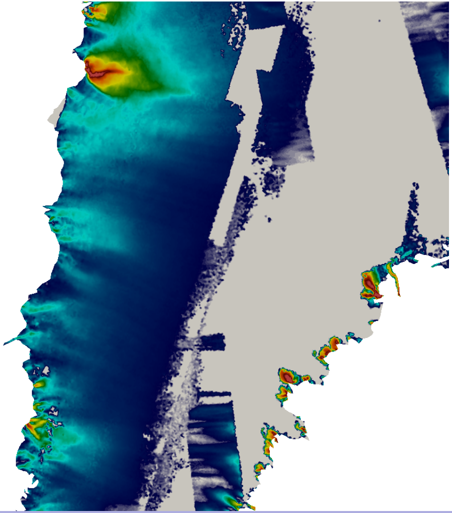




Southern region: speed results

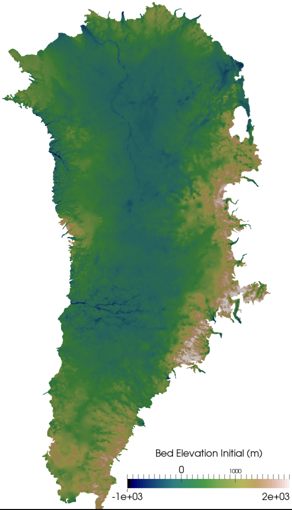
Smoothed and interpolated with physics based PDE-constrained optimization

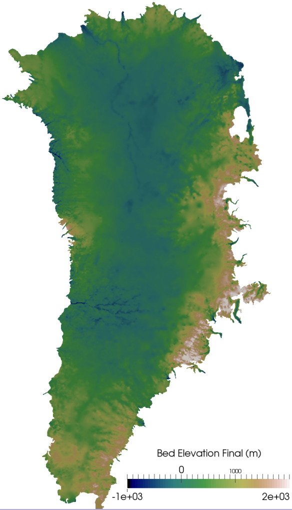




Thickness (bed) results

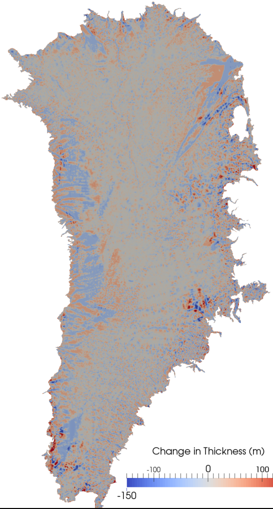
Great interest in this, it conserves mass

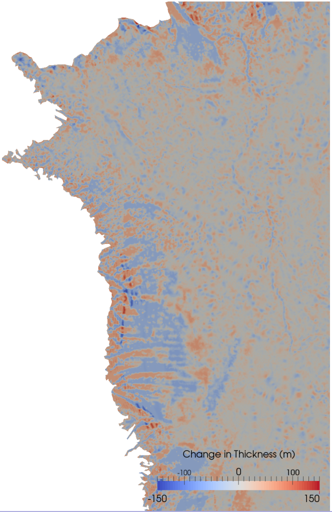




Thickness (bed) results

More interesting to look at changes in thickness



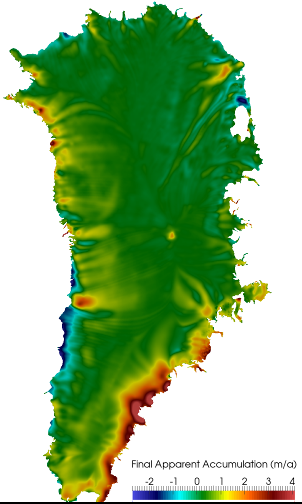


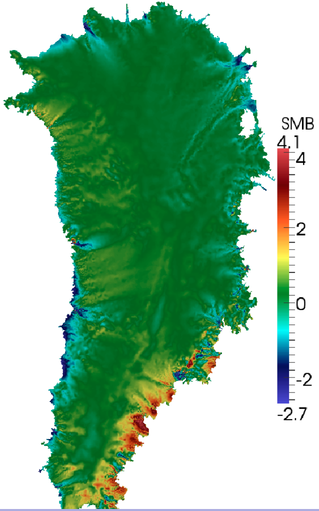
Apparent accumulation results

$\dot{a}' = \dot{a} - \frac{\partial H}{\partial t}$

Model intercomparison (MPAS)

Differences likely due to regularization





Direction results

$\hat{\mathbf{N}} = (n_x, n_y)$, n_y plotted here.

Conclusion

Are the transients gone?

- transients in prognostic runs are lower
- speeds near terminus are not as “smooth” as the data show them to be
- it’s not clear how good is good enough. Current RMSE $\sim$ 60 m/a
- the role of regularization and the objective function need to be explored