Parameterization of basal hydrology near grounding lines in a one-dimensional ice sheet model

Gunter Leguy, Xylar Asay-Davis, William Lipscomb

Los Alamos National Laboratory

January 31th, 2014

Introduction

We are exploring how basal physics influences resolution in a 1-D vertically integrated flowline model. Goals:

- Present a new parametrization of the basal shear stress (in the case of a Marine ice sheet):
  - Physically motivated (ocean connection)
  - Transitions smoothly between finite basal friction in the ice sheet and zero basal friction in the ice shelf
- Hope for $\approx$ 1km resolution near the grounding line.

Model equations

Valid in both, the ice sheet and the ice shelf:

$$\begin{aligned} \text{conservation of mass}: &\quad H_t + (uH)_x = a, \\ \text{vertically integrated stress} - \text{balance equation}: &\quad \tau_l + \tau_b + \tau_d = 0. \end{aligned}$$

$$\begin{aligned} \tau_l &= \left[2\bar{A}^{-\frac{1}{n}} H |u_x|^{\frac{1}{n}-1} |u_x| \right]_x, \\ \tau_d &= -\rho_i g H s_x, \\ \tau_b &= \text{basal shear stress}, \end{aligned}$$

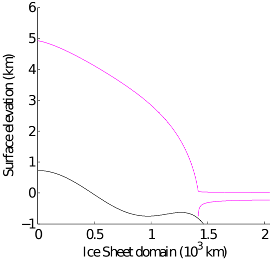
$$s = \begin{cases} H - b & x < x_g \\ \left(1 - \frac{\rho_i}{\rho_w}\right) H & x \geq x_g \end{cases}$$

Los Alamos NATIONAL LABORATORY EST. 1943

Assumption and Boundary conditions

Symmetric ice sheet at the ice divide

$$\left.\begin{aligned} u &= 0 \\ s_x = (H-b)_x &= 0 \end{aligned}\right\} \text{ at } x = 0,$$



At the grounding line, we have the flotation condition and the balance between τ_l and τ_d.

$$\left.\begin{aligned} H = & \ \frac{\rho_w}{\rho_i} b, \\ 2\bar{A}^{-\frac{1}{n}} |u_x|^{\frac{1}{n}-1} u_x = & \ \frac{1}{2}\rho_i g \left(1 - \frac{\rho_i}{\rho_w}\right) H \end{aligned}\right\} \text{ at } x = x_g.$$

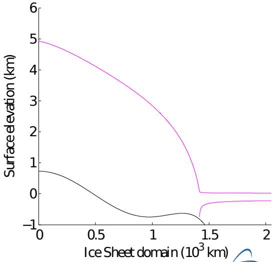
Los Alamos NATIONAL LABORATORY EST.1943

Basal stress

We adopt the formulation from Schoof (2005):

$$\tau_b = -\, C|u|^{\frac{1}{n}-1} u \left(\frac{N^n}{\kappa u + N^n} \right)^{\frac{1}{n}}.$$

- $\kappa = \dfrac{m_{\max}}{\lambda_{\max} A_b}$
- λ_{max} = wavelength of bedrock bumps
- m_{max} = maximum bed obstacle slope
- A_b = average ice temperature at the bed
- Effective pressure: $N \equiv p_i - p_w$
    - $p_i \equiv \rho_i g H$
    - p_w?



Los Alamos NATIONAL LABORATORY EST.1943

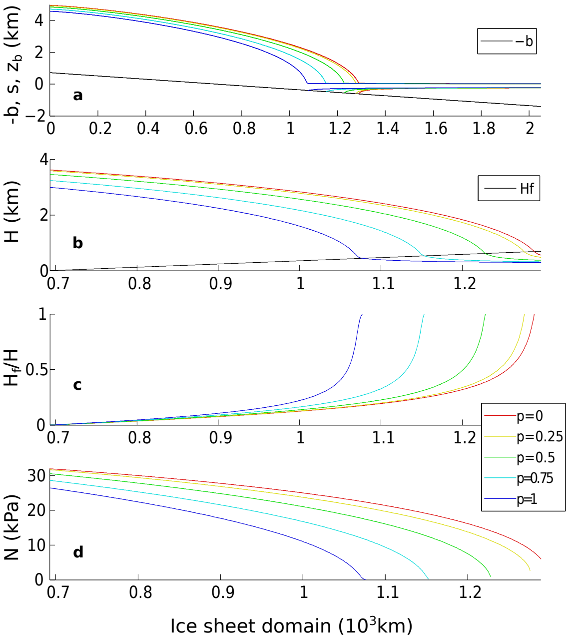
Basal stress (continued)

$$N(p) = \rho_i g H \left(1 - \frac{H_f}{H}\right)^p,$$

where $H_f = max(0, \frac{\rho_w}{\rho_i} b)$, and $p \in [0, 1]$.

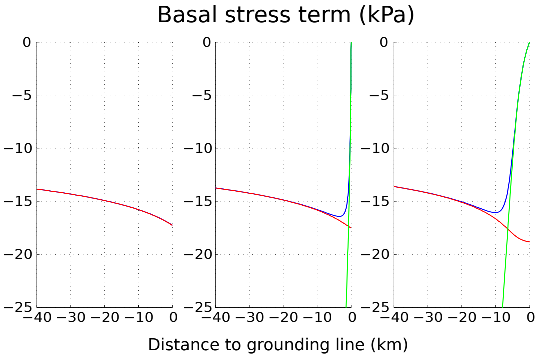
$N(p)$ satisfies the following limits:

- When $p = 0$, $N(p) = \rho_i g H$ (no water-pressure support).
- When $p = 1$, $N(p) = \rho_i g (H - H_f)$ (full water-pressure support from the ocean wherever the ice-sheet base is below sea-level).
- At the grounding line when $p > 0$, $N(p) = 0$ (τ_b is continuous across the grounding line).


-b, s, z_b (km)
−b
a
H (km)
Hf
b
H_f/H
c
p=0
p=0.25
p=0.5
p=0.75
p=1
N (kPa)
d
Ice sheet domain (10^3km)

Basal stress (continued): $\tau_b = -C|u|^{\frac{1}{n}-1}u\left(\frac{N(p)^n}{\kappa u + N(p)^n}\right)^{\frac{1}{n}}$.

- 2 asymptotic limits:
  - if $\kappa u \ll N(p)^n$ and $\tau_b \approx -\frac{\gamma}{\kappa^{1/n}}|u|^{\frac{1}{n}-1}u$ (frozen ice at the bed)
  - if $\kappa u \gg N(p)^n$ and $\tau_b \approx -CN(p)\frac{u}{|u|}$. (hydrological connection)
- Transitions smoothly from non-zero basal stress in the ice sheet to zero basal sliding in the ice shelf



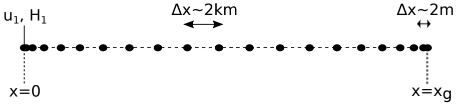
a) p = 0 **b)** p=0.5 **c)** p=1

Los Alamos NATIONAL LABORATORY EST.1943

Numerics

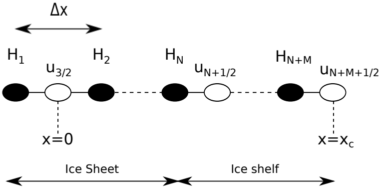
Chebyshev polynomials:

- Spectrally accurate
- GL lies on a grid point
- Used as a benchmark solution for fixed-grid model
- Harder to implement in 3-D models



Fixed-grid

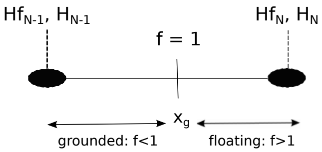
- Suitable for 3-D model
- Constant resolution
- Numerically less accurate
- GL usually falls between two grid points leading to interpolation error
  - Possibility to use GLP



Numerics: Fix grid GLP

In our code we make sure that the grounding line is located in the last grounded cell.

- sub-grid-scale interpolation of the grounding line
- use a GLP similar to PA_GB1 in Gladstone et al. (20120a):
    1. Determine GL using linear interpolation of the function $f \equiv H_f / H$
    2. Compute basal and driving stresses once each assuming that the cell is entirely grounded and then entirely floating.
    3. The stresses are linearly interpolated between their grounded and floating values.



Numerics (continued)

How to compare our results for all p-values?

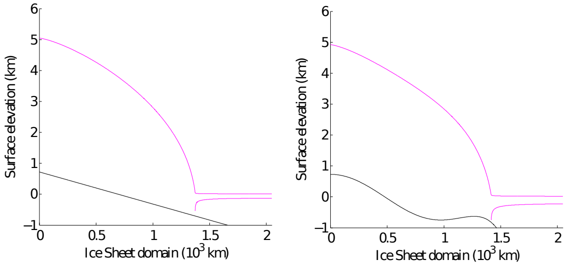
- Qualitative
  - Schoof (2007) boundary layer solution (in the case of frozen bed type basal sliding)
- Quantitative
  - Schoof (2007) boundary layer solution (good approximation for p=0)
  - Chebyshev polynomial numerical scheme

Los Alamos NATIONAL LABORATORY EST. 1943

Numerics (end)

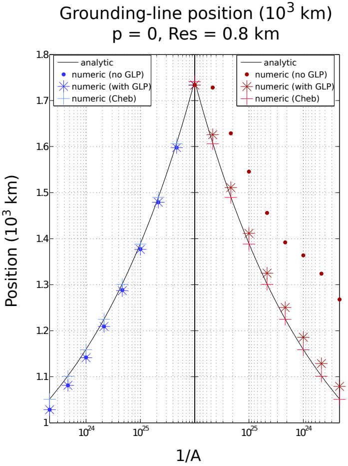
Experimental set-up:

- Fixed-grid resolution comparison: 3.2 km, 1.6 km and 0.8 km.
- Fixed-grid with no GLP and with GLP.
- We will show results for 3 values of p.
- Constant accumulation rate: $a = 0.3$ m/yr.
- MISMIP experiment: neutral equilibrium experiments.
- Two different bed topographies (as in Schoof (2007a)).

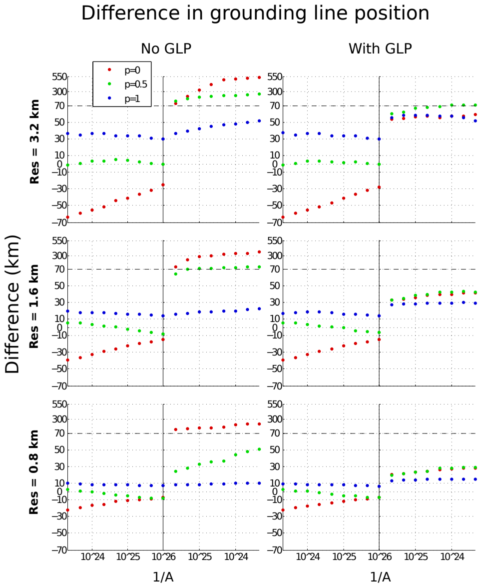


Results linear bed

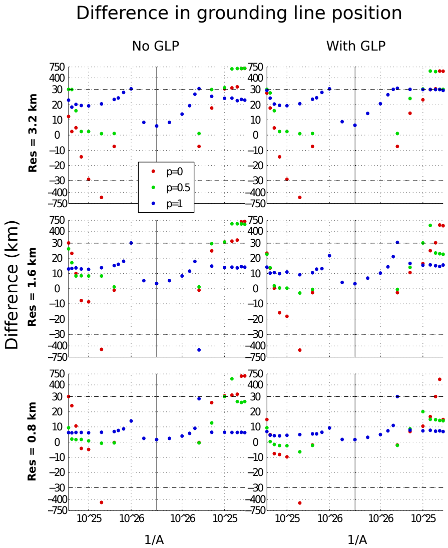
The fixed-grid solution is inaccurate in capturing the retreat.

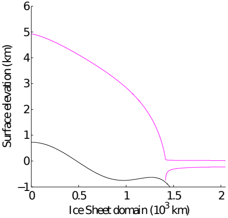


Results linear bed (continued): MISMIP-type experiments 1 & 2

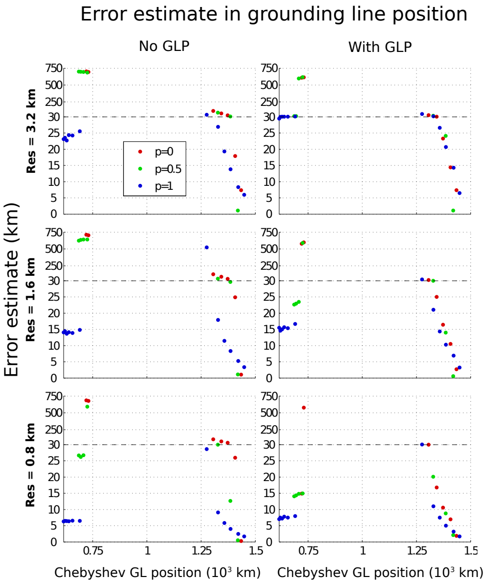


Results poly bed (continued): MISMIP-type experiment 3





Results poly bed (continued): MISMIP-type experiment 3

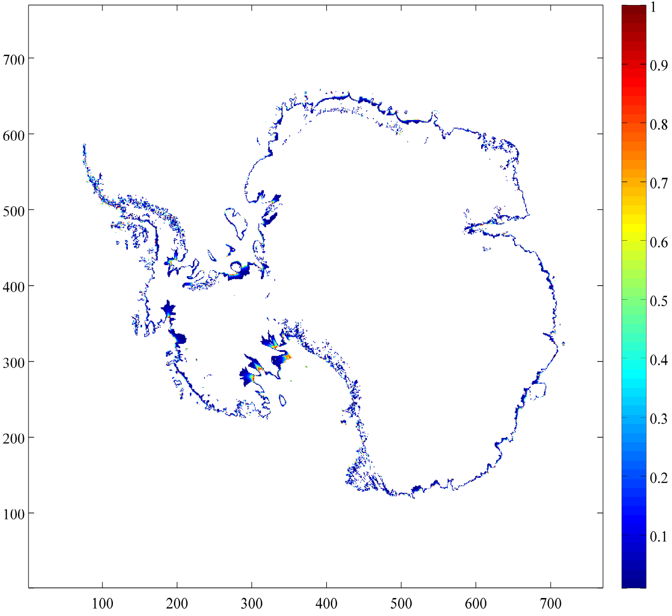


conclusion

- Error estimate decreases substantially as the value of p increases and therefore the required model resolution decreases as p increases.
- Adding a GLP is always beneficial besides sometimes for high values of p.
- p does not play any role in the bulk of the ice sheet. It impacts a relatively small distance (no more than 20 km from the grounding line in our experiments) which is enough to impact the solution.
- Resolution of $\approx$ 1km or coarser is sufficient to capture the solution.

Paper submitted in the cryosphere

Ratio $\kappa u/N(p=1)^n$ for values between 0.01 and 1



THANK YOU