Update on Greenland ice-sheet initialization: Optimal control and Bayesian calibration approaches

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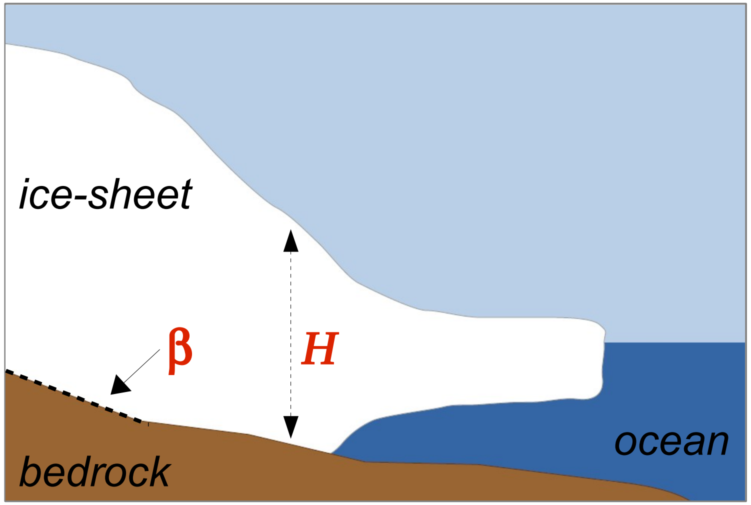
Deterministic Inversion (w/ S. Price and G. Stadler)
Estimation of ice-sheet initial state

Goal: recover initial ice-sheet state and avoid fast initial transients.

How to prescribe ice-sheet mechanical equilibrium:

$$\frac{\partial H}{\partial t} = \underbrace{-\mathrm{div}\,(\mathbf{U}H)}_{\textit{divergence flux}} + \underbrace{\tau_s}_{\textit{Surface Mass Balance}}, \qquad \mathbf{U} = \frac{1}{H}\int_z \mathbf{u}\,dz.$$

At equilibrium: $\boxed{\mathrm{div}\,(\mathbf{U}H) = \tau_s}$



Boundary condition at ice-bedrock interface:

$$(\sigma\mathbf{n} + \beta\mathbf{u})_{||} = \mathbf{0} \quad on \quad \Gamma_\beta$$

Bibliography*:

Arthern, Gudmundsson, J. Glaciology. 2010

Price, Payne, Howat and Smith, PNAS 2011

Brinkerhoff, Meierbachtol, Johnson, Harper, Annals of Glaciology, 2011

Petra, Zhu, Stadler, Hughes, Ghattas, J. Glaciology , 2012.

Morlighem et al. A mass conservation approach for mapping glacier ice thickness, 2013

Deterministic Inversion (w/ S. Price and G. Stadler)

Estimation of ice-sheet initial state

PDE constrained optimization problem

Minimize mismatch between:

- *computed divergence flux and measured SMB*
- *computed and measured surface velocity*
- *computed and reference thickness*

Fulfill (Constraint):

- ***High order nonlinear Stokes equation***

Tune (Control Variables):

- ***Basal friction***
- ***Thickness***

Caveat:

- Temperature field is given

$$\begin{aligned}\mathcal{J}(\beta, H) = \; & \frac{1}{2}\int_{\Gamma} \frac{1}{\sigma_s^2}|\mathrm{div}(\boldsymbol{U}H) - \tau_s|^2\, ds & (\mathcal{J}^{SMB}) \\ & + \frac{1}{2}\int_{\Gamma_{top}} \frac{1}{\sigma_v^2}|\mathbf{u} - \mathbf{u}^{obs}|^2\, ds & (\mathcal{J}^{vel}) \\ & + \frac{1}{2}\int_{\Gamma} \frac{1}{\sigma_H^2}|H - H^{obs}|^2\, ds & (\mathcal{J}^{H}) \\ & + \mathcal{R}(\beta) + \mathcal{R}(H). & \end{aligned}$$

Software Tools:

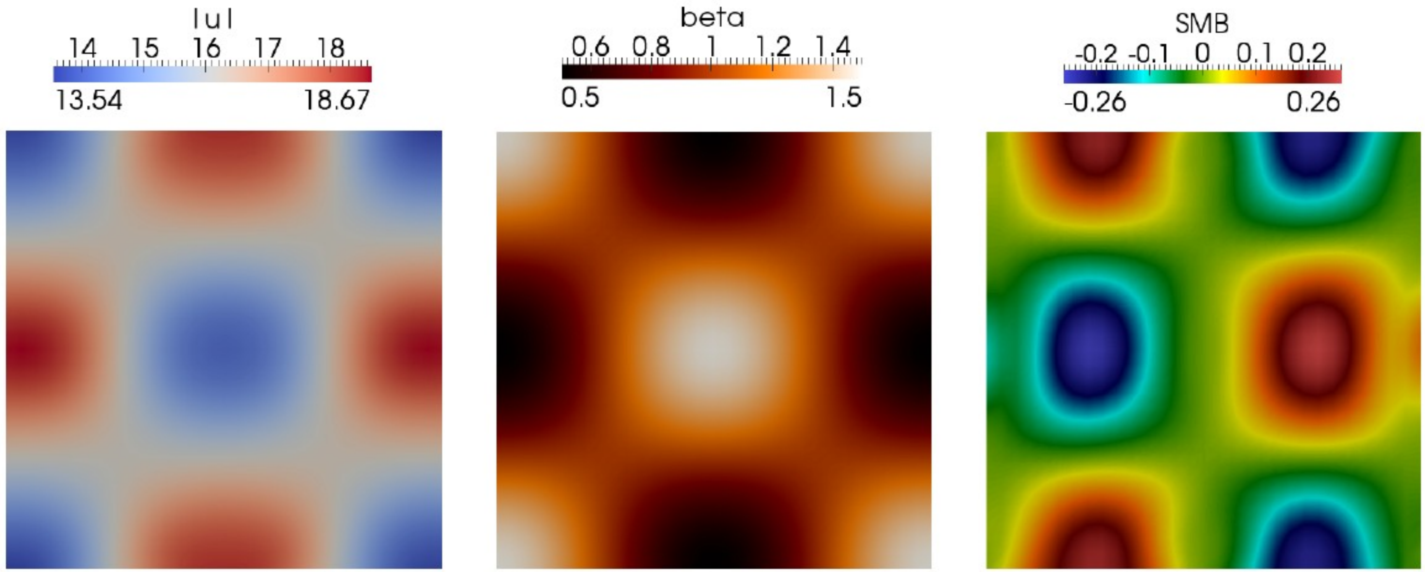
- Assembling: **LifeV**
- Linear solver: **AztecOO & IfPack** (Trilinos).
- Nonlinear solver: **NOX** (Trilinos).
- Gradient Based optimization (LBFGS): **Rol**.

Deterministic Inversion (w/ S. Price and G. Stadler)

Numerical results

Slab example

- Consider ISMIP-HOM like forward simulation:



- Add noise to results of forward simulation to get u^{obs}, τ_s and H^{obs}

- Invert. We consider three cases:

1. Tune **beta** by matching **surf. velocity**. $\mathcal{J}^1(\beta) = \mathcal{J}^{vel} + \mathcal{R}$

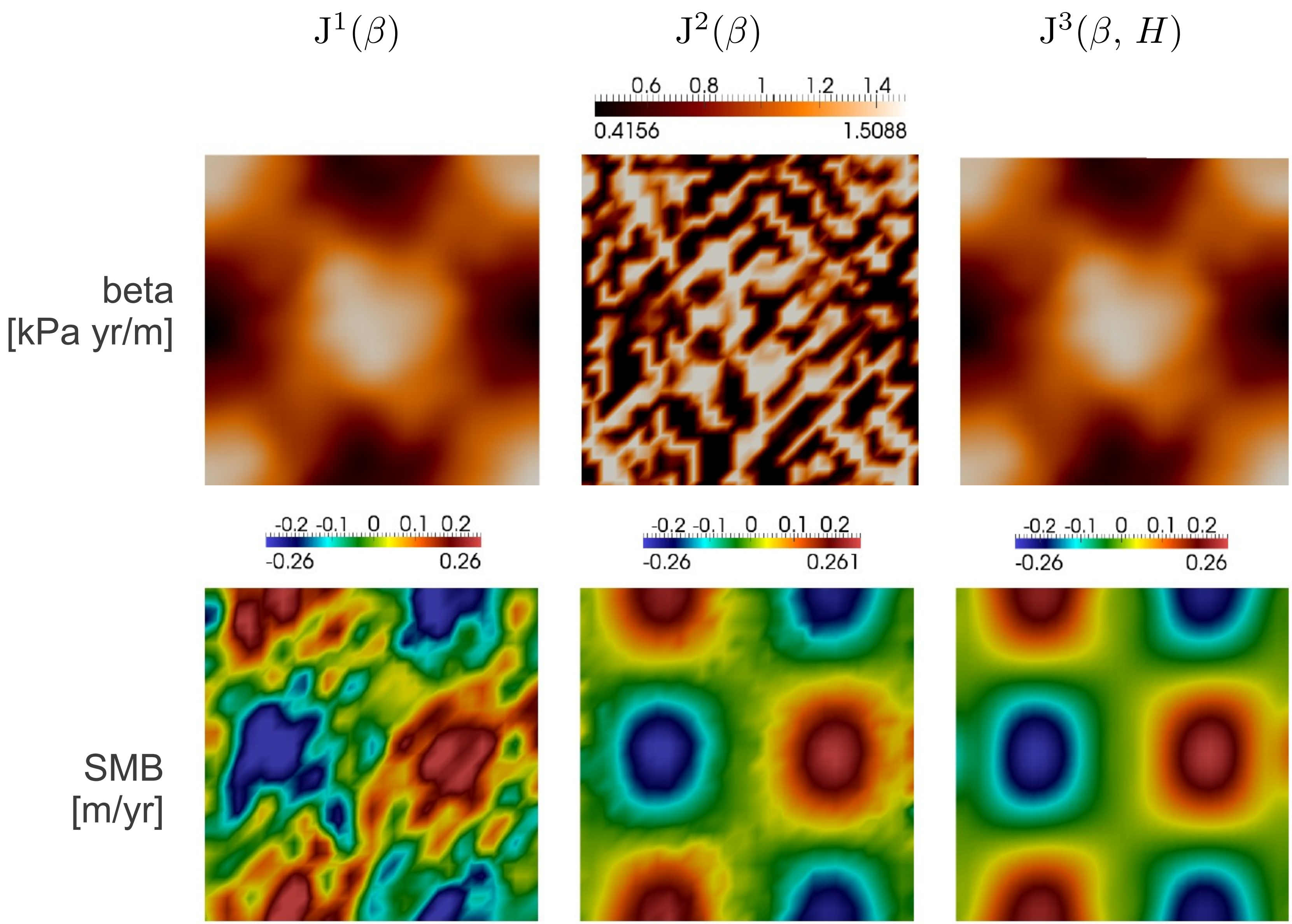
2. Tune **beta** by matching **surf. vel.** and **SMB** $\mathcal{J}^2(\beta) = \mathcal{J}^{vel} + \mathcal{J}^{SMB} + \mathcal{R}$

3. Tune **beta** and **thick.** by matching **surf. vel.** and **SMB** $\mathcal{J}^3(\beta, H) = \mathcal{J}^{vel} + \mathcal{J}^{SMB} + \mathcal{J}^H + \mathcal{R}$

Deterministic Inversion (w/ S. Price and G. Stadler)

Numerical results

Slab example. Optimization results using different merit functionals



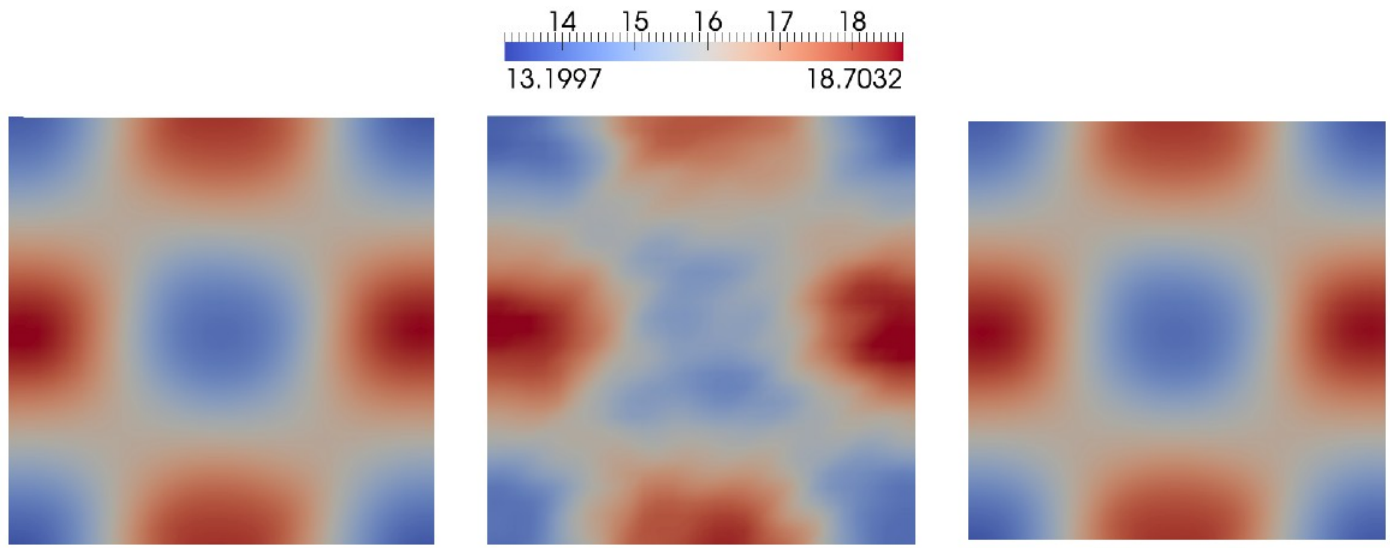
Deterministic Inversion (w/ S. Price and G. Stadler)

Numerical results

Slab example. Optimization results using different merit functionals

$\mathrm{J}^1(\beta)$ $\mathrm{J}^2(\beta)$ $\mathrm{J}^3(\beta,\, H)$

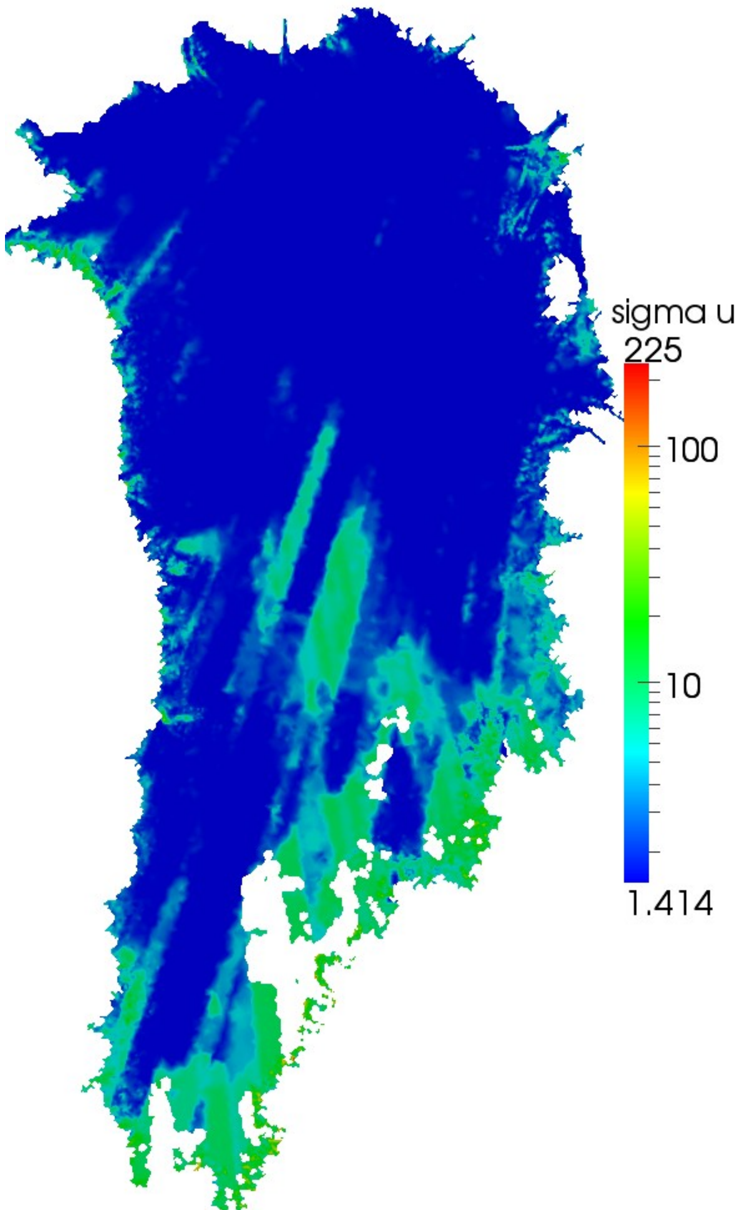
Surf. velocity [m/yr]

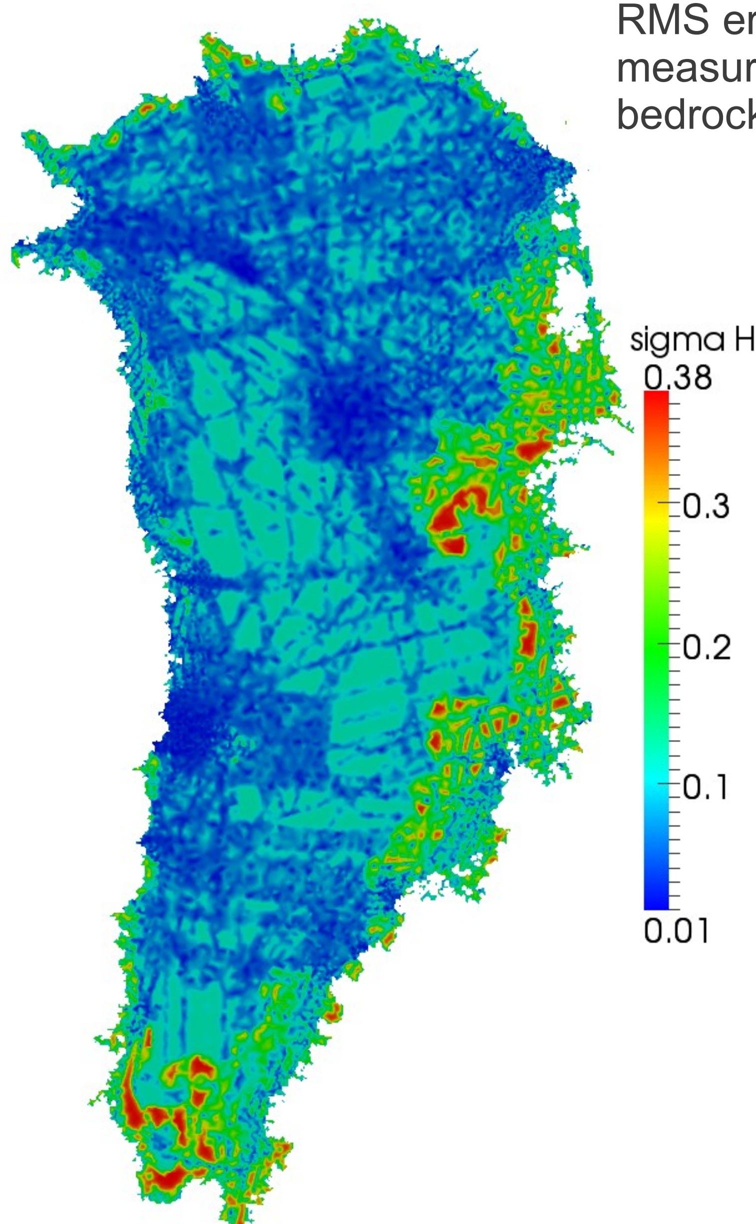


Deterministic Inversion (w/ S. Price and G. Stadler)

Greenland initialization

Errors in data





RMS error of surface velocity measures [m/yr] (left) and bedrock topography [km] (right).

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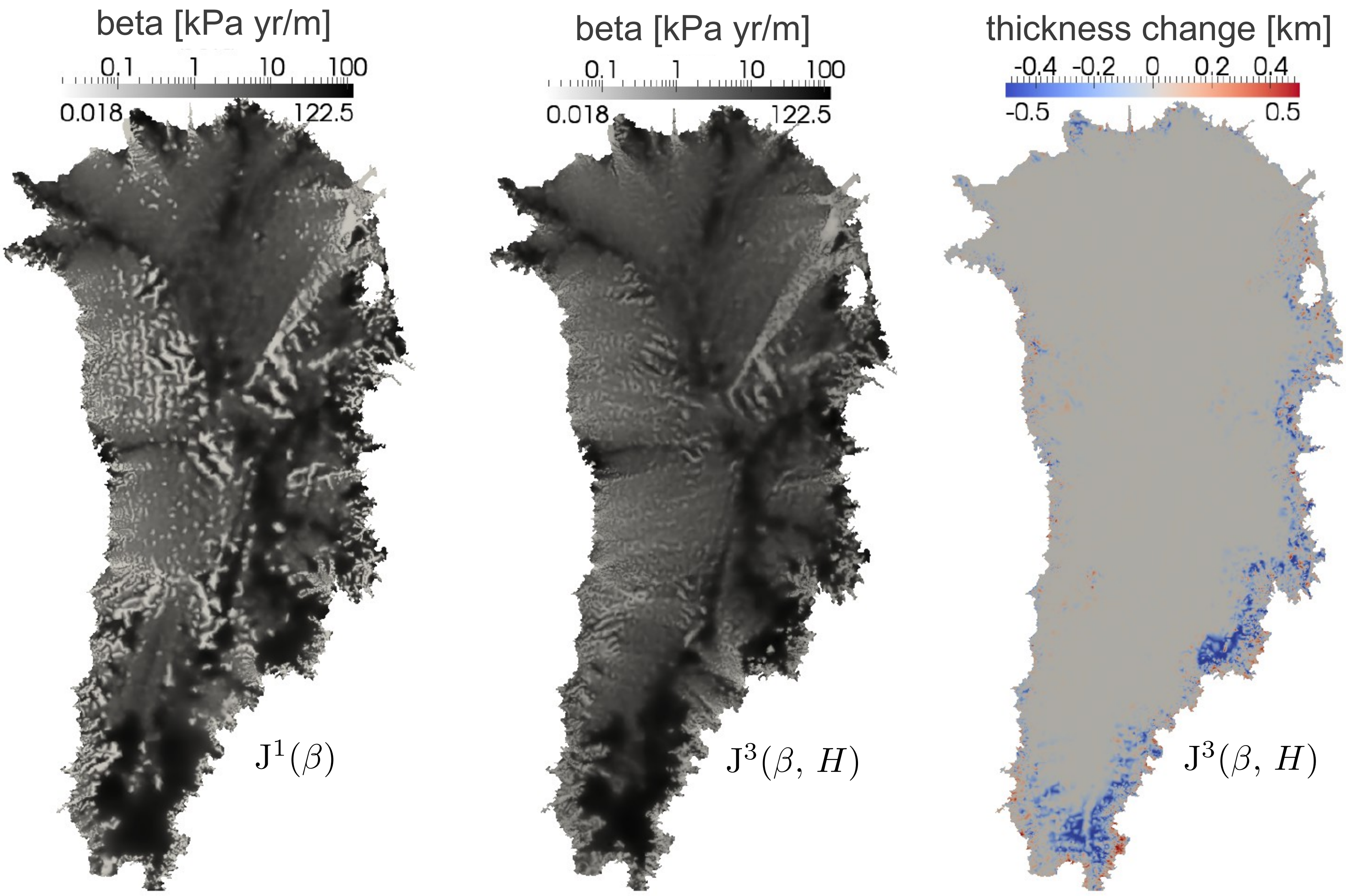
Deterministic Inversion (w/ S. Price and G. Stadler)

Greenland initialization

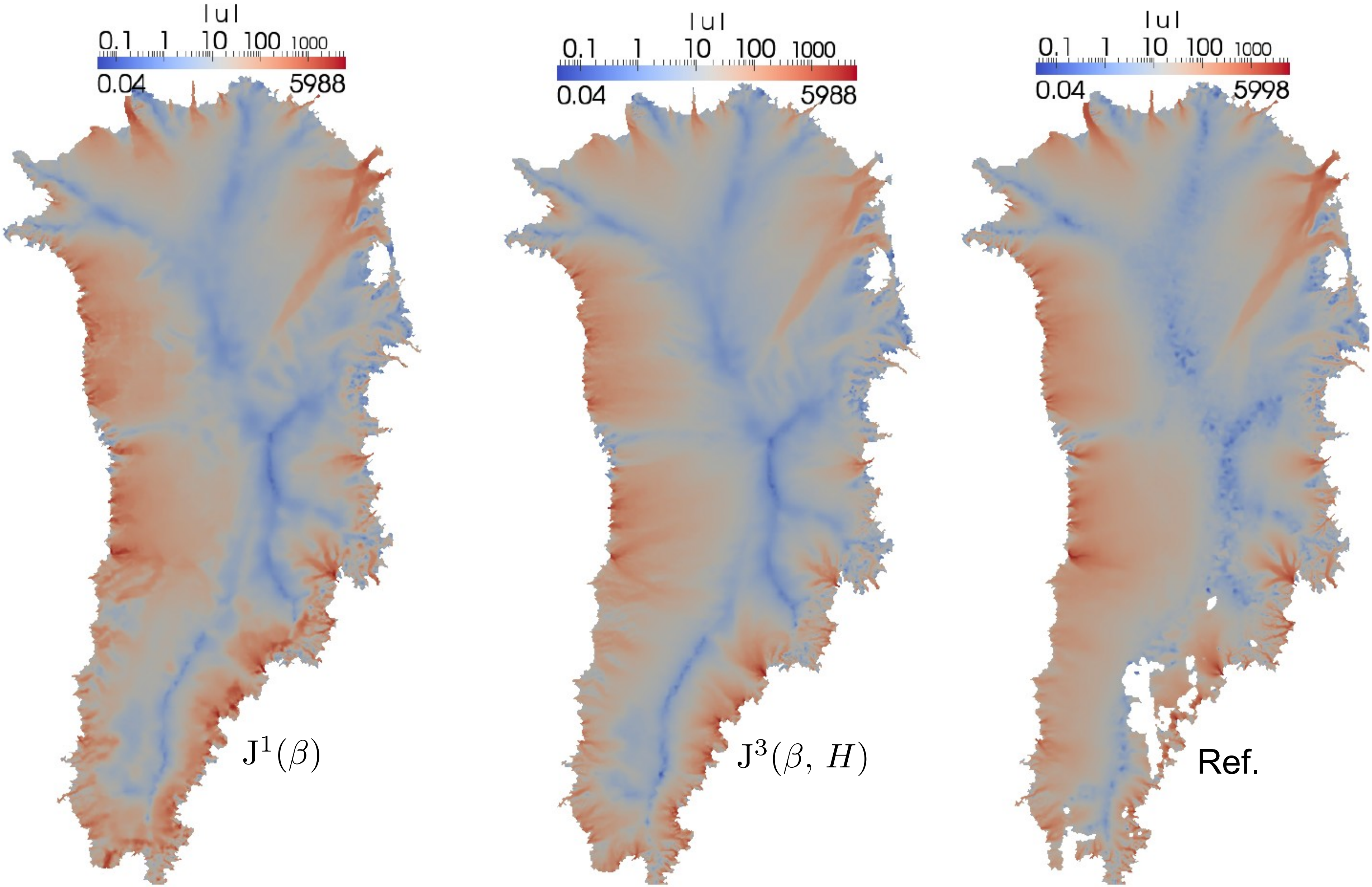
Left, Center: Estimated beta obtained using different cost functionals.
Right: difference between the computed and reference thickness in [km].



Deterministic Inversion (w/ S. Price and G. Stadler)

Greenland initialization

Left, center: computed ***surface velocity*** obtained with different functionals.
Right: reference velocity. Units: [m/yr]

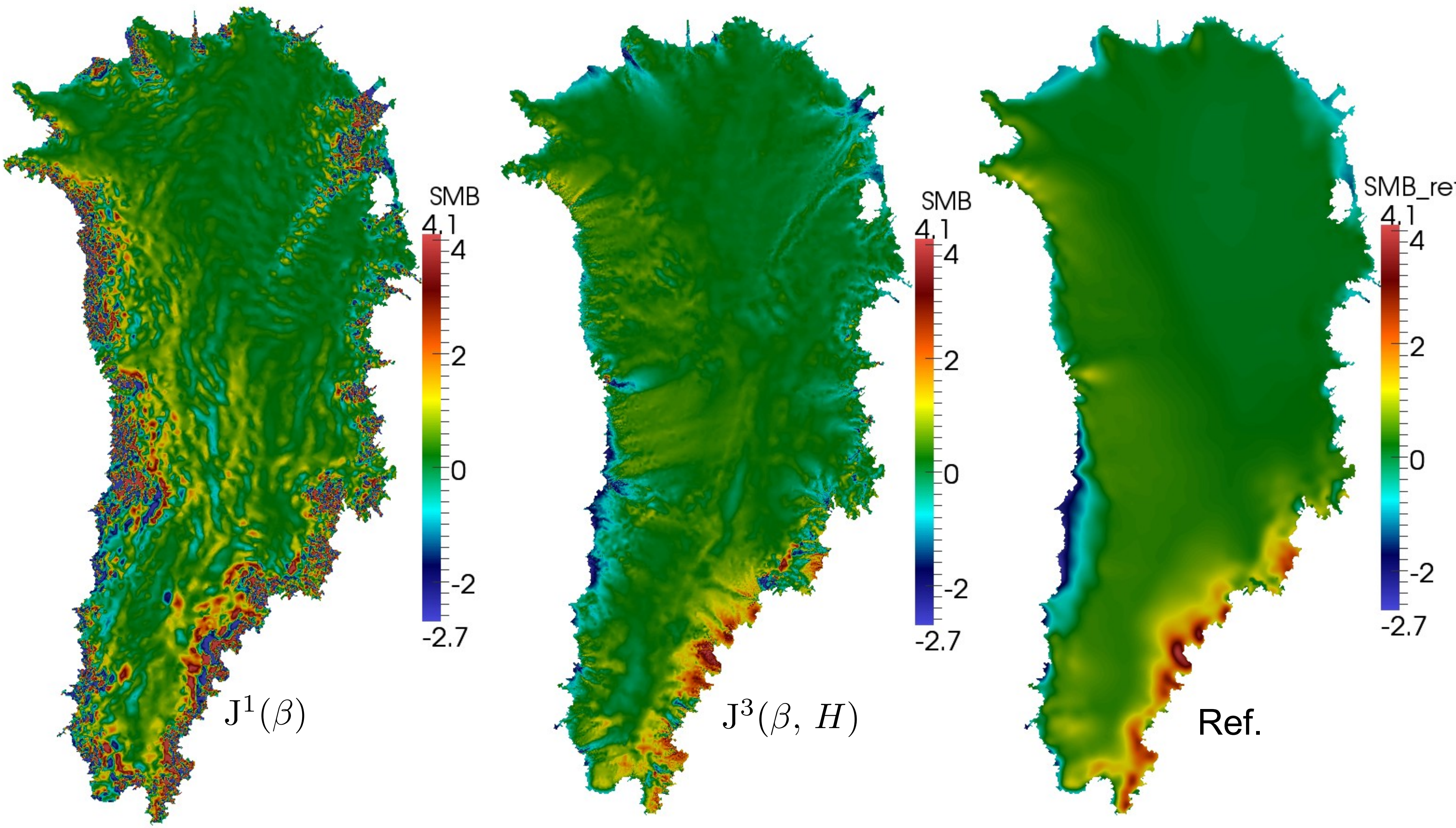


Deterministic Inversion (M. Perego, S. Price and G. Stadler)

Greenland initialization

Left, center: Estimated divergence flow obtained using different functionals.
Right: reference SMB.

SMB, [m/yr]



Bayesian Inversion
(w/ M. Eldred, J. Jakeman, I Kalashnikova, A. Salinger, L. Swiler)

Reduction of parameter space dimension

Difficulty in UQ approach: "***Curse of dimensionality***". The parameter space has O(30,000) parameters (or more).

- Reduce the dimension of the parameter space.

Method of choice: Karhunen-Loeve Expansion (**KLE**).
In our experiment, we reduce the dimension of parameter space to 5.

1. Assume analytic covariance kernel $C(r_1, r_2) = exp\left(-\frac{|r_1 - r_2|^2}{L^2}\right)$.

2. Perform eigenvalue decomposition of C.

3. Take the mean $\bar{\beta}$ to be the deterministic solution and expand β in basis of eigenvector $\{\phi_k\}$ of C, with random variables $\{\xi_k\}$

$$\beta(\omega) = \bar{\beta} + \sum_{k=1}^{K} \sqrt{\lambda_k}\phi_k \xi_k(\omega)$$

*Expansion done on $\log(\beta)$ to avoid negative values for β.

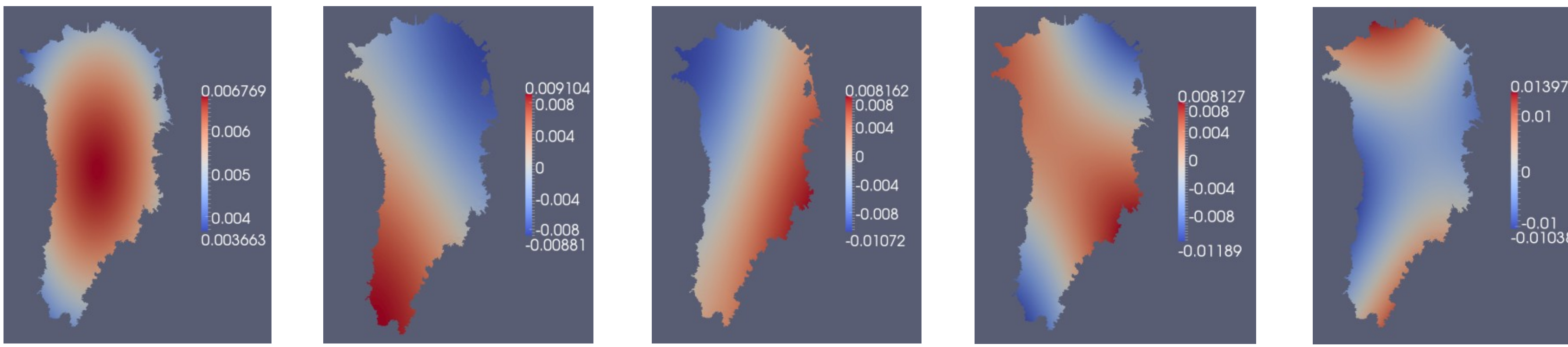
Development(?): parameter reduction based on physical knowledge.
(e.g. include *basal hydrology model*)

Bayesian Inversion

(w/ M. Eldred, J. Jakeman, I Kalashnikova, A. Salinger, L. Swiler)

Reduction of parameter space dimension: Greenland modes

- 5 KLE modes capture 95% of covariance energy (parallel C++/Trilinos code **Anasazi**).



Only spatial correlation has been considered.

Development(?): Build modes using information from the model (e.g. using family of deterministic basal friction coefficients).

Bayesian Inversion

(w/ M. Eldred, J. Jakeman, I Kalashnikova, A. Salinger, L. Swiler)

Compute model surrogate and invert

- Mismatch (**ALBANY**): $\mathcal{J}(\beta) = \dfrac{1}{2}\displaystyle\int_\Gamma \dfrac{1}{\sigma_s^2}\left|\mathrm{div}(\boldsymbol{U}H) - SMB\right|^2 ds.$

- ***Build Surrogate Model.*** Polynomial chaos expansion (**PCE**) was formed for the mismatch over random variables using uniform prior distributions. **DAKOTA**.

- **Inversion/Calibration**. Markov Chain Monte Carlo (**MCMC**) was performed on the PCE with 100K samples **QUESO**.

Development(?): use simple physical model (***e.g. L1L2 or SIA***) as the surrogate model.

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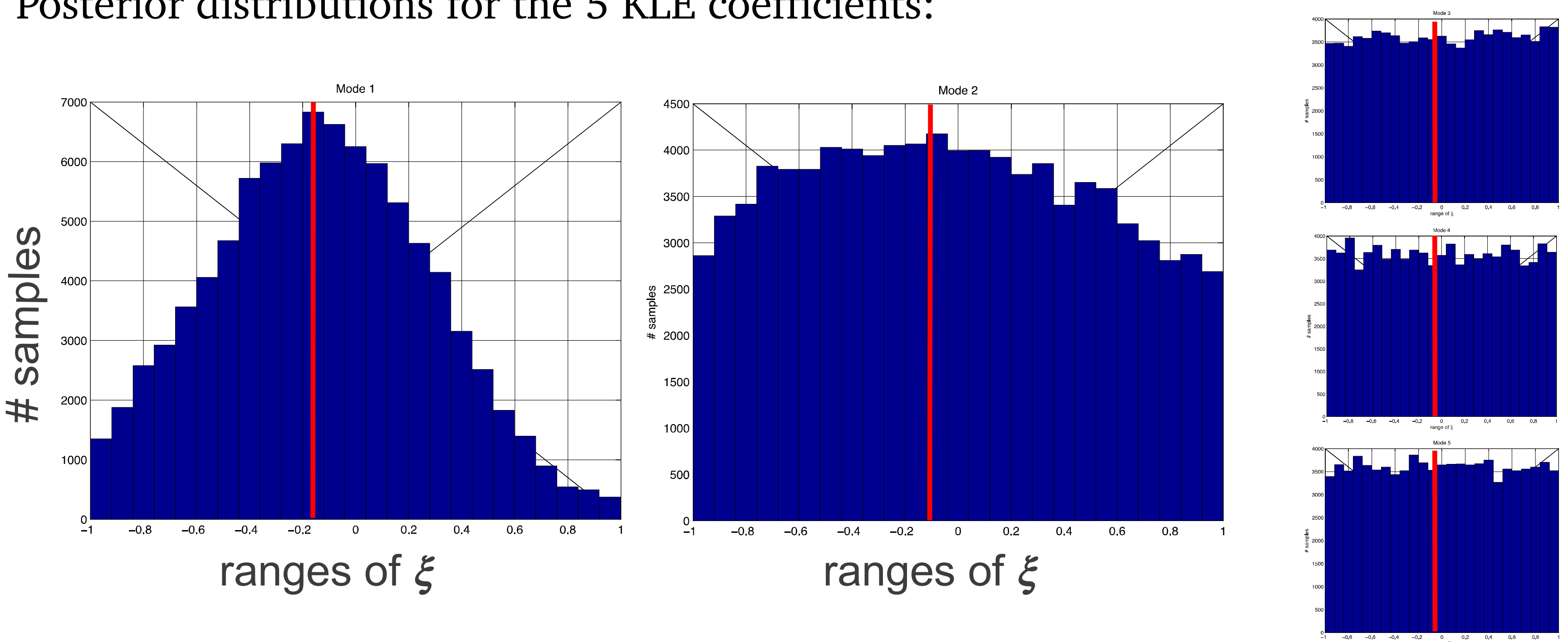
Numerical Results

Posterior distributions for the 5 KLE coefficients:



MAP solution: $\xi = (-0.16, -0.08, 0, 0, 0)$

Bayesian Inversion

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Numerical Results