Enlargement of englacial conduits in cold ice – basic theory and some simple experiments

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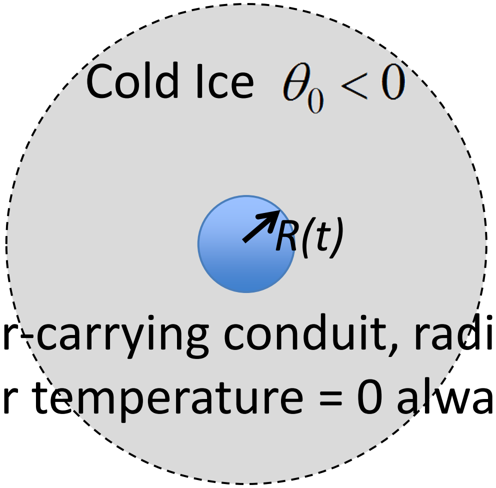
Basic processes enabling establishment of englacial hydrologic network in cold ice –

Fracture propagation, melt enlargement of conduits/passages

(need to work against creep closure and refreezing)

This research focuses only on melt enlargement and refreezing

Simplified model of water-carrying conduit in cold ice - neglecting advective transport in water (along conduit length), assuming water discharge rate is constant, water temperature = 0 degrees)



Cold Ice $\theta_0 < 0$

$R(t)$

water-carrying conduit, radius R
water temperature = 0 always

Heat conduction equation in cold ice (radial)

$$\rho_i C_i \frac{\partial \theta}{\partial t} - \frac{1}{r}\frac{\partial}{\partial r}\left(r k_i \frac{\partial \theta}{\partial r} \right) = 0, \; r > R(t)$$

Initial condition (cold ice)

$$\theta(r, t=0) = \theta_0, \; r > R(t=0)$$

Boundary conditions at ice-water interface *R(t)*…a moving boundary

$\theta(R(t), t) = \theta_{PMP} = 0$ at the phase change interface

Energy Balance condition at interface

$$\rho_w g Q \left| \frac{dh}{dx} \right| = \rho_w L \times 2\pi R \frac{dR}{dt} - 2\pi R k_i \frac{\partial \theta}{\partial r}\bigg|_{r=R(t)}$$

Energy supplied by viscous and turbulent dissipation in flowing water

Energy available to grow conduit

Energy conducted into cold ice (heat loss from water)

….all per unit length of conduit

Energy supplied by viscous and turbulent dissipation in flowing water

$$\rho_w g Q \left|\frac{dh}{dx}\right| = \frac{\rho_w f Q^3}{4\pi^2 R^5}$$

Discharge (flow rate)

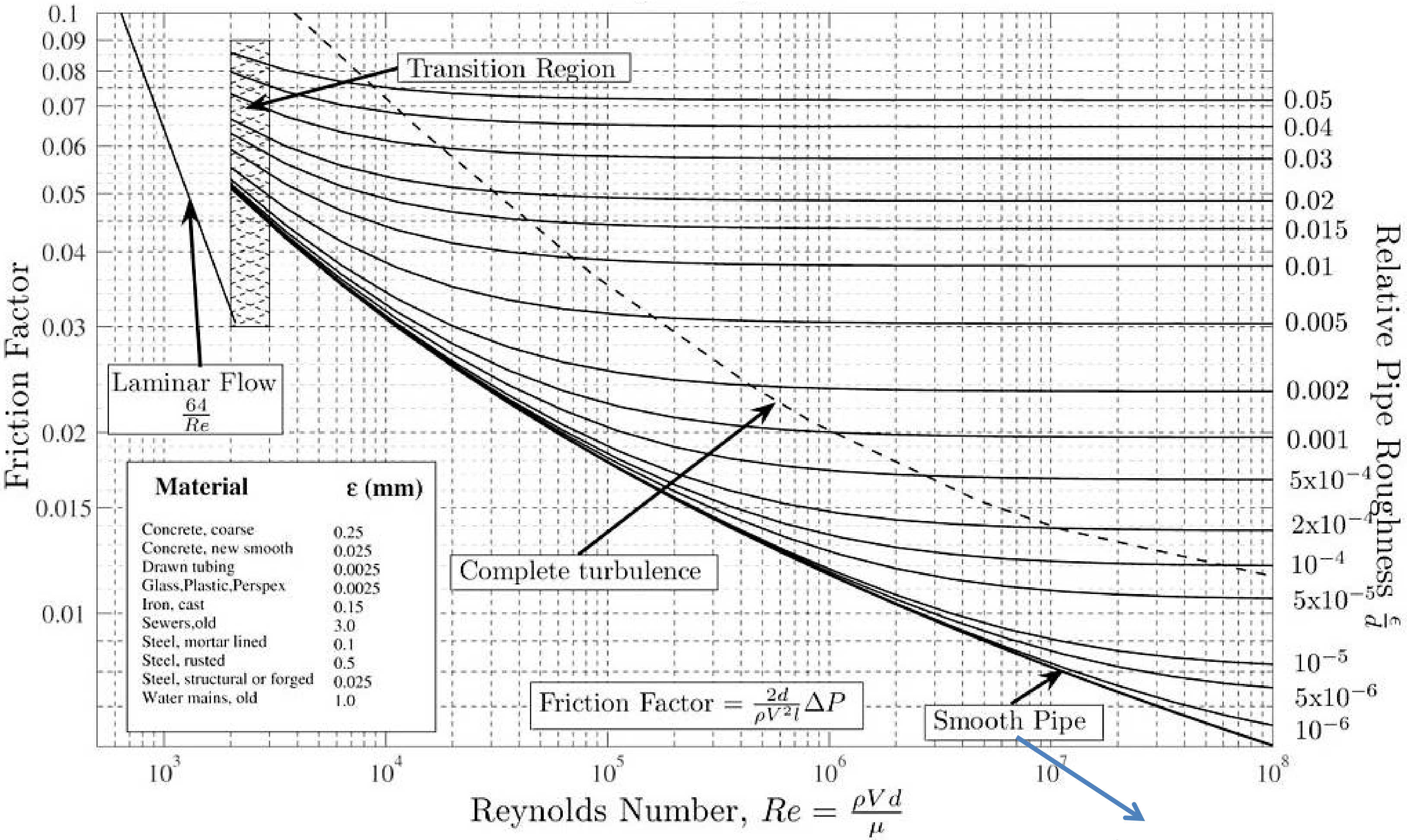
Head loss per unit length

$$\left|\frac{dh}{dx}\right| = \frac{f}{2R}\frac{V^2}{2g} = \frac{f}{2R}\frac{Q^2}{2g(\pi R^2)^2}$$

f = Darcy-Weisbach friction factor

(head loss formula for pipe flow)

Moody Diagram



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Steel, rusted	0.5
Steel, structural or forged	0.025
Water mains, old	1.0

Blasius Resistance Law

Radius evolution equation $\frac{dR}{dt} = \left(\frac{\rho_w f Q^3}{4\pi^2 R^5} + 2\pi R k_i \left. \frac{\partial \theta}{\partial r} \right|_{r=R(t)} \right) / (2\pi R \rho_w L)$

Stagnant water (no flow, no internal energy generation by dissipation)

$$\frac{dR}{dt} = \left(2\pi R k_i \left. \frac{\partial \theta}{\partial r} \right|_{r=R(t)} \right) / (2\pi R \rho_w L)$$

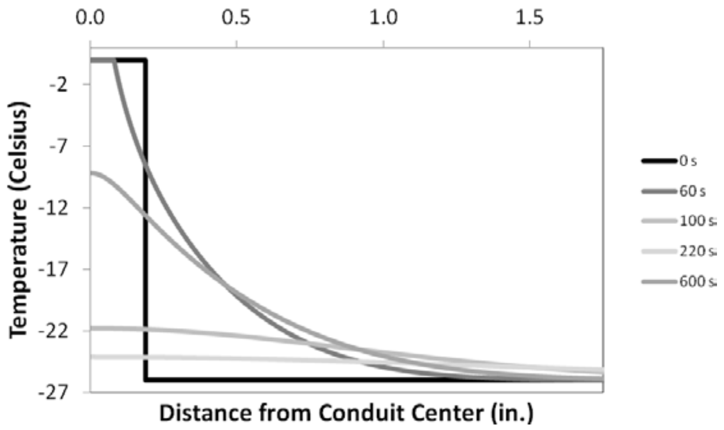
Negative
(warmer inside)

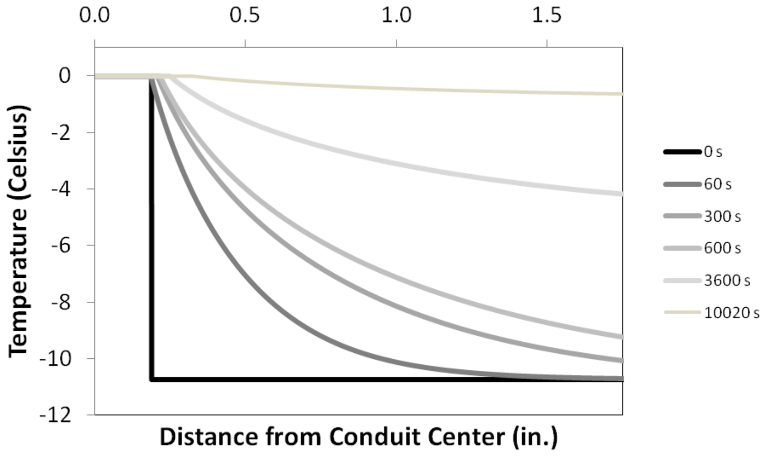
…obviously always get refreezing (energy in water is transferred to ice, causing water to refreeze, conduit radius shrinks)

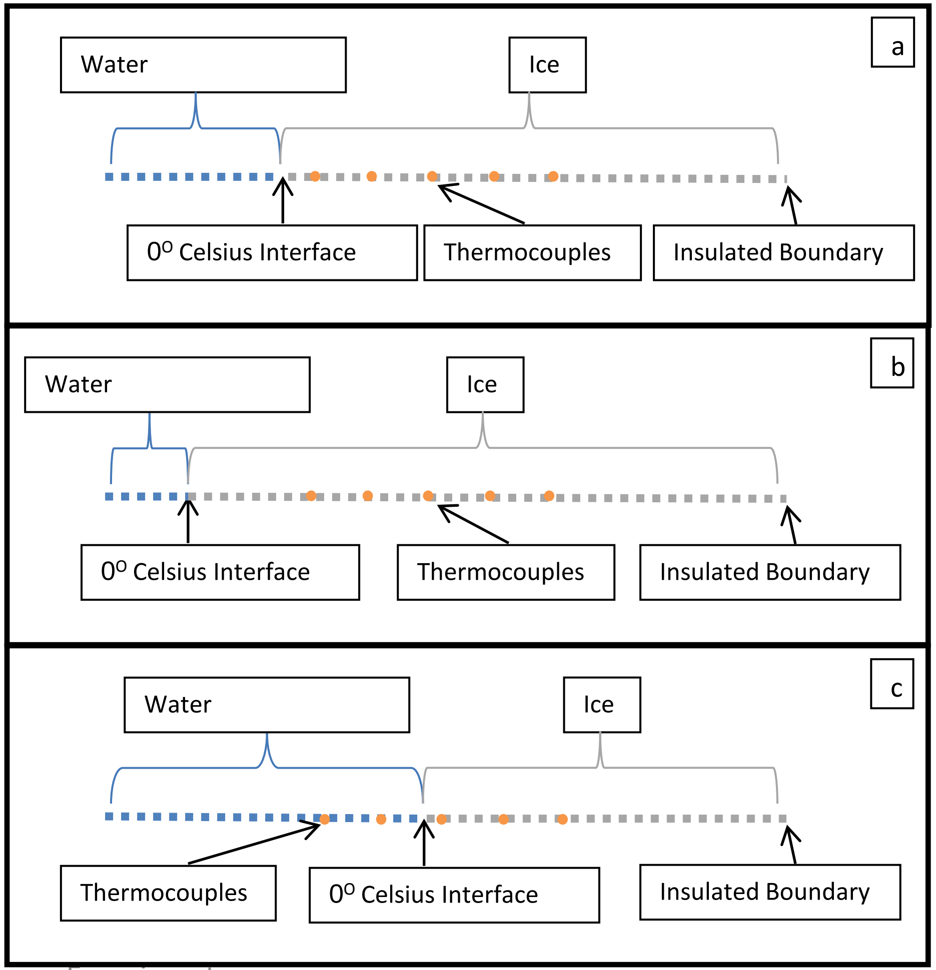
To get conduit to grow, need $\frac{\rho_w f Q^3}{4\pi^2 R^5}$ TO BEAT OUT $2\pi R k_i \left. \frac{\partial \theta}{\partial r} \right|_{r=R(t)}$

i.e. supply more energy to conduit wall by viscous/turbulent dissipation than is extracted by heat loss from water to ice by conduction……in an integrated sense over some time interval (the first term’s integral $\sim \Delta t$, the conductive flux term’s integral $\sim \sqrt{\Delta t}$ at early time) – not entirely satisfying analytical criterion for “critical flow rate” Q to produce growth.

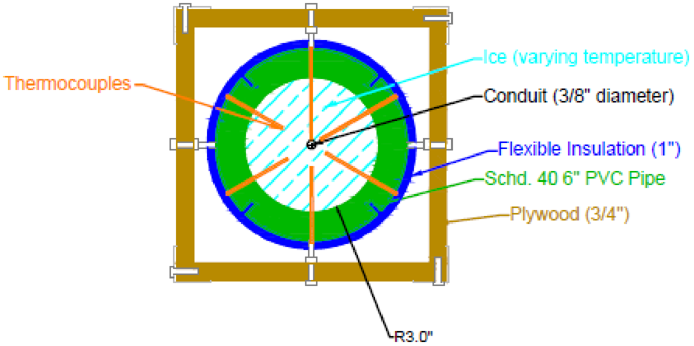
Numerical Model



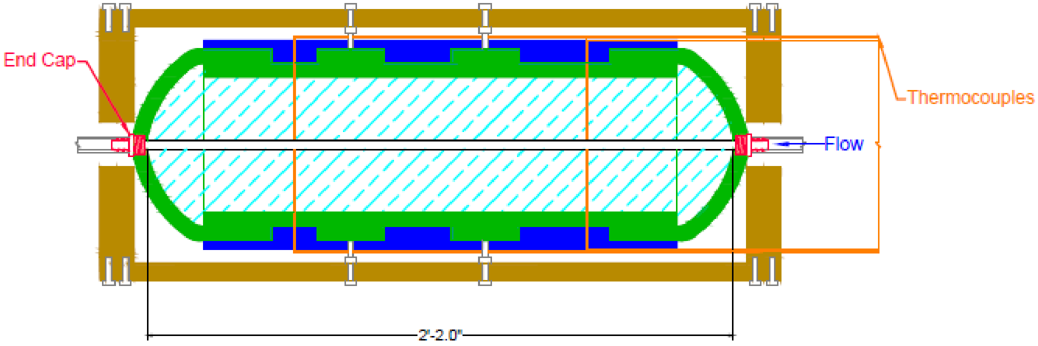




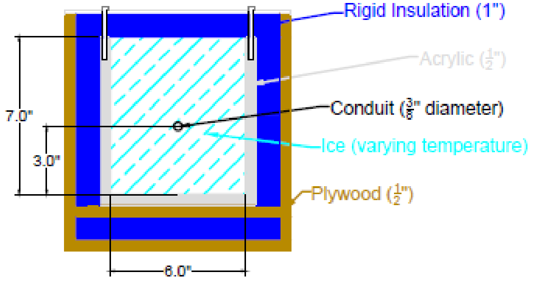
Experimental Schematics

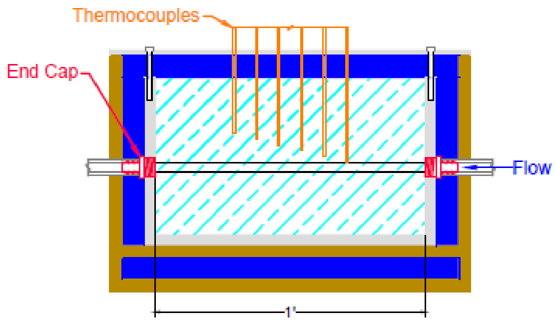


End Cross-Section View



Side Cross-Section View





Ice Sample Container Comparison

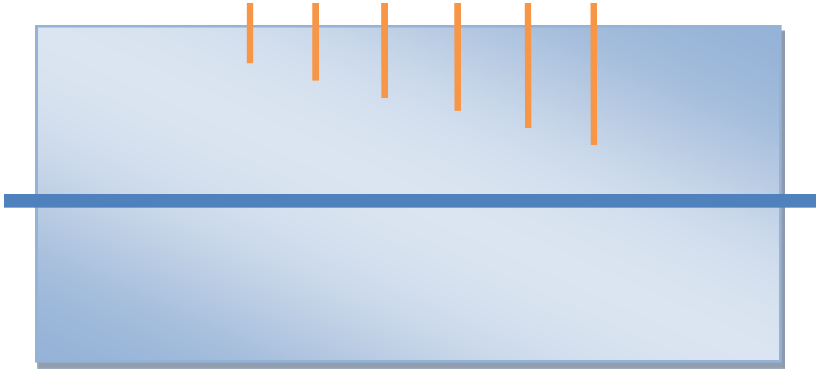
Insulated Box

PVC

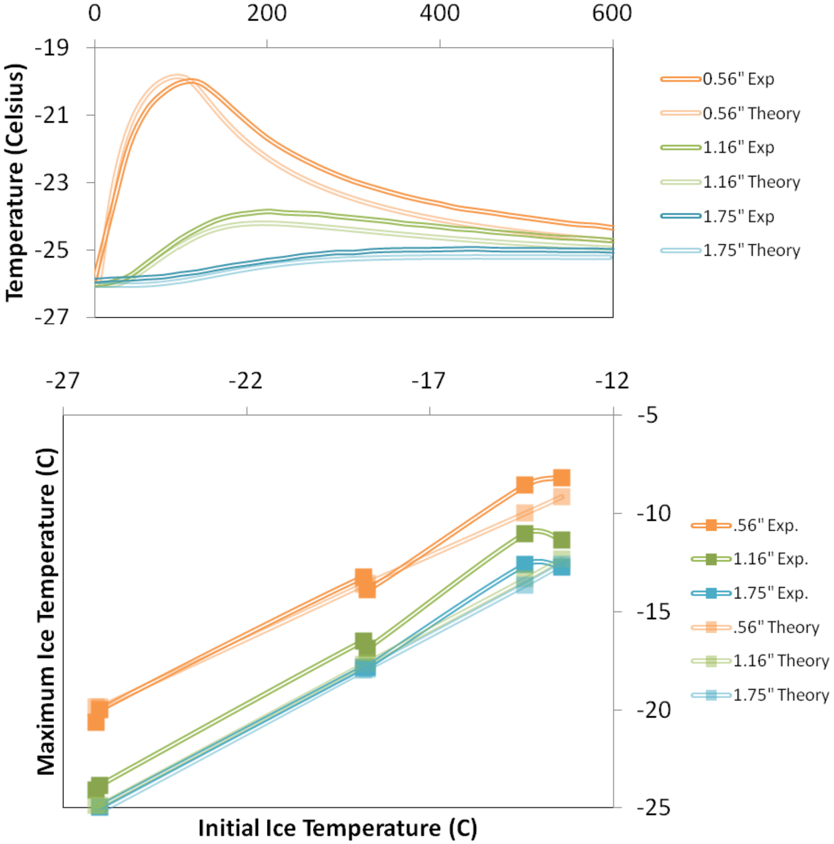
Stagnant Water Tests

- Performed at CU
- Conduit re-augered after each succesful test
  - Multiple tests per ice sample
- Ice Dimensions
  - 6” Tall
  - 6” Wide
  - 12” Long
  - 3/8” Initial conduit radius

- 0 Degrees Celsius Water
  - Poured into each end
- 6 thermocouples recording temperature

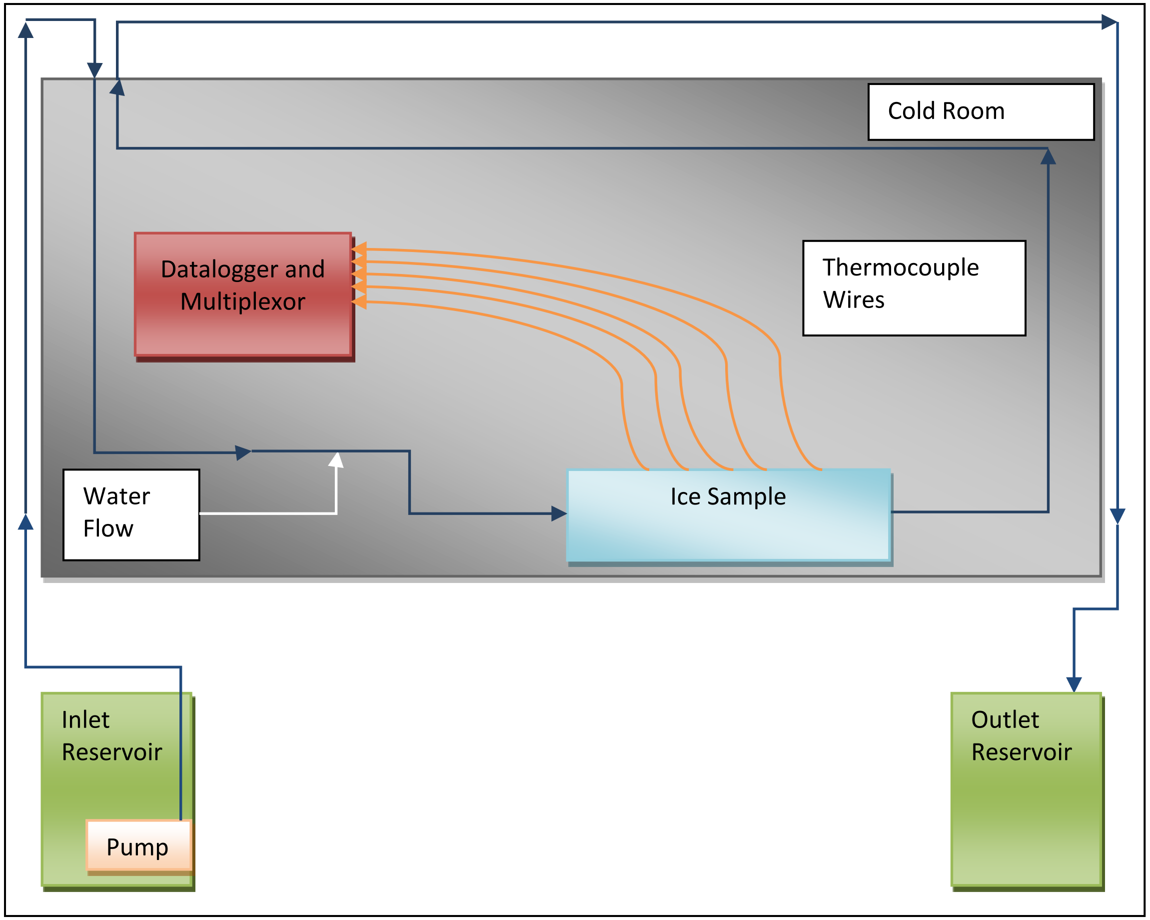


Stagnant Comparison



- Model matches temperature values well
  - Locations furthest from center best
- Differences attributed to not perfectly radial heat transfer (square box)

Low Water Flow Rate Tests

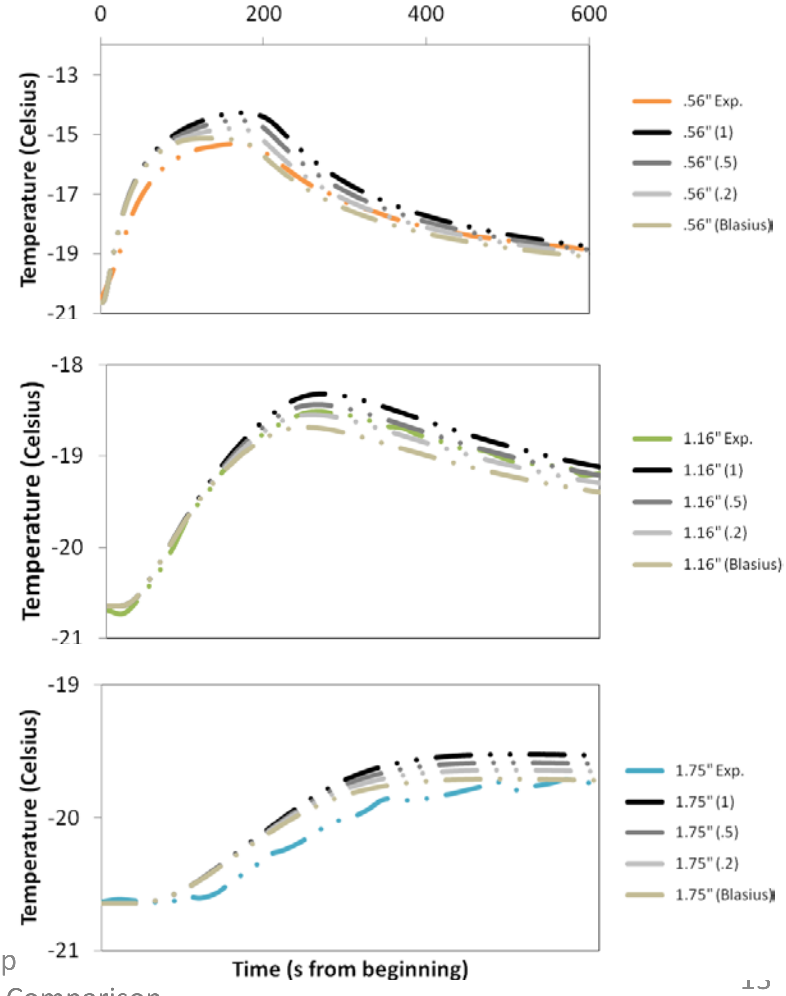


Low Water Flow Rate Comparison

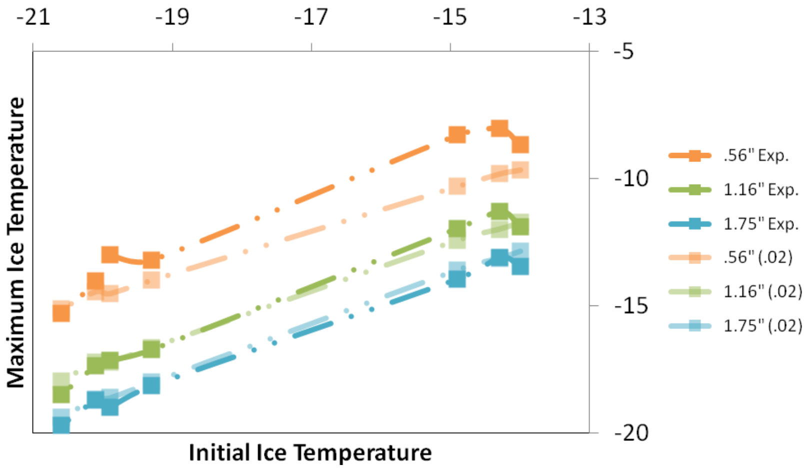
- Ice input as initial constant temperature
- Radial numerical code used

- Blasius smooth pipe approximation fits well
  - Darcy-Weisbach friction factor
- Model best at higher ice temperatures

Low Water Flow Rate Comparison



Low Water Flow Rate Comparison



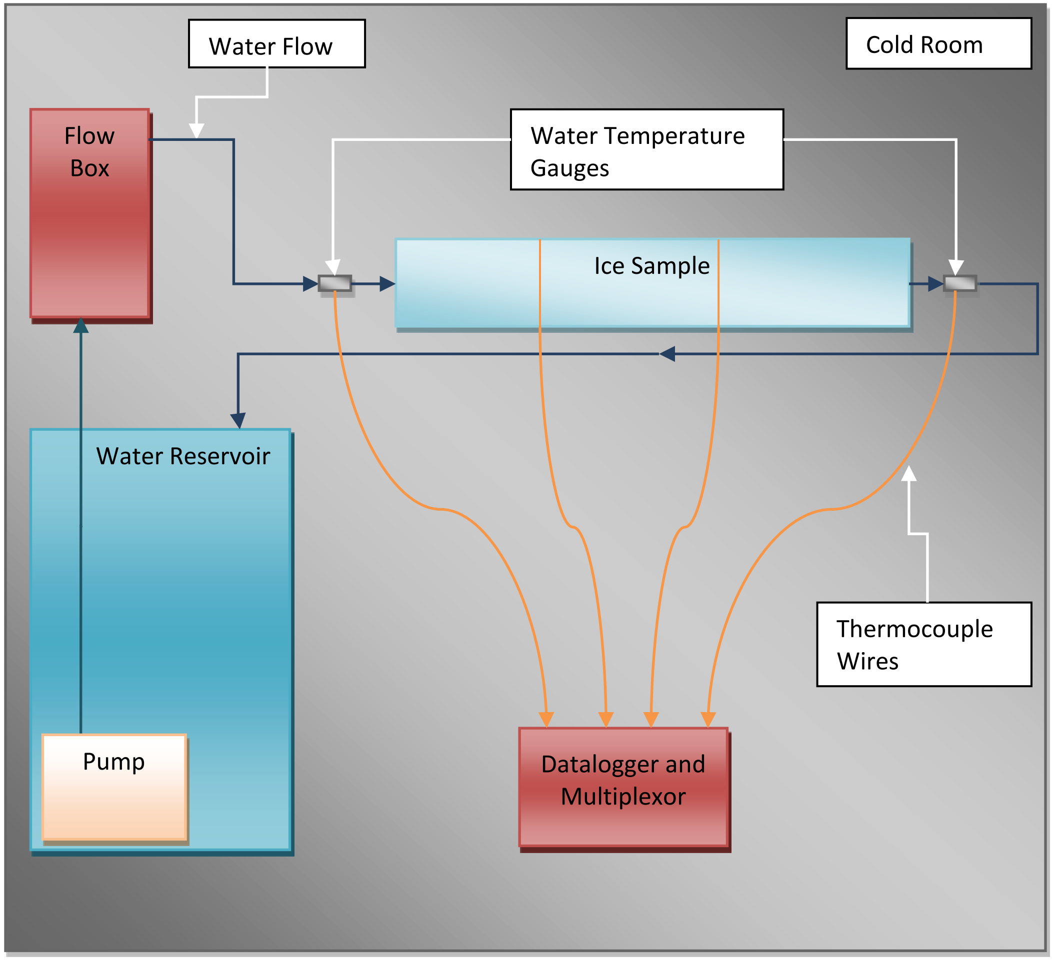
Low Flow					
Initial Ice	**Critical Flow Rate (gpm)**				
Temperature (C)	**Initial**	**Blasius**	**f=.2**	**f=.5**	**f=1**
-14.0	2.05	7.25	3.54	2.61	2.07
-14.3	1.87	7.31	3.56	2.63	2.08
-14.9	2.05	7.42	3.61	2.66	2.11
-19.3	1.87	8.15	3.94	2.90	2.30
-19.9	2.08	8.24	3.98	2.93	2.33
-20.1	3.08	8.27	3.99	2.94	2.33
-20.6	2.57	8.34	4.03	2.97	2.35

- Friction factor of 1 too high
  - Blasius smooth pipe approximation used (f=.02)
- Nearest temperature recording slightly underpredicts temperature

High Water Flow Rate Tests

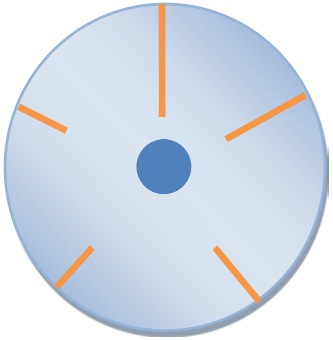
- Performed at NASA GSFC
- Conduit re-augered after each succesful test
  - Multiple tests per ice sample
- Ice Dimensions
  - 6” Diameter
  - 26” Long
  - 3/8” Initial conduit radius
- 0 Degrees Celsius Water
  - Pumped through ice
- 12 thermocouples recording temperature
  - 2 profile locations

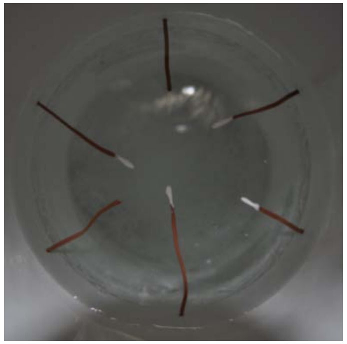
High Water Flow Rate Tests

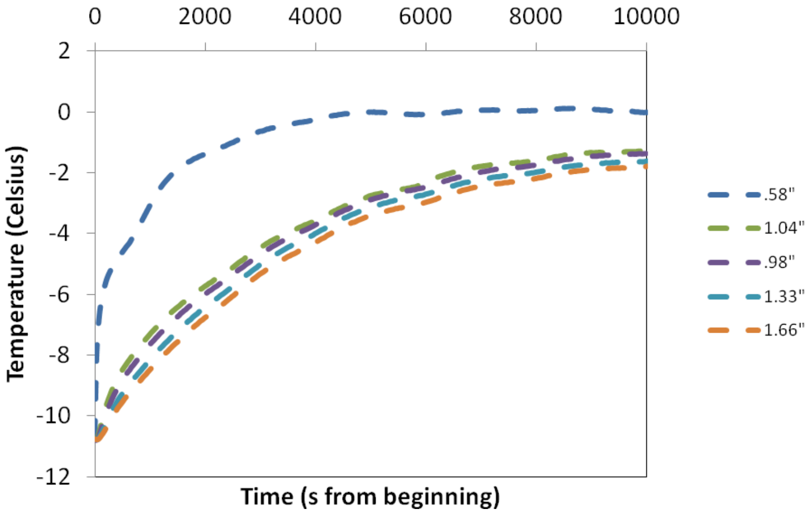


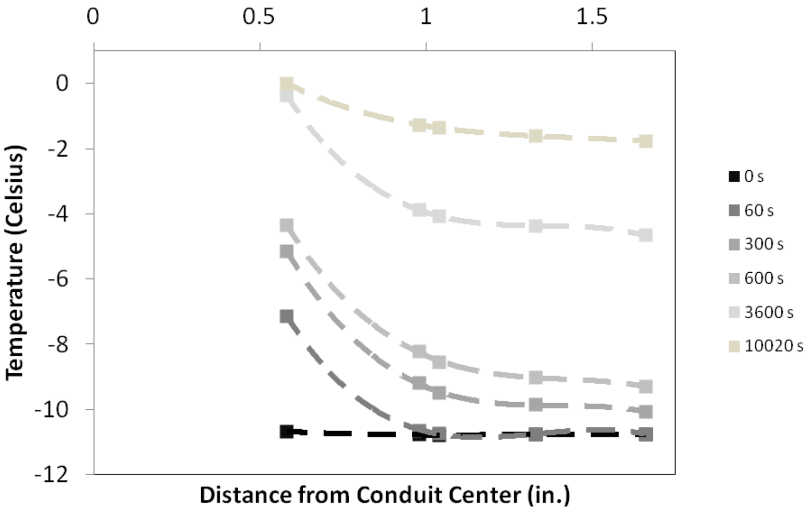
High Water Flow Rate Tests

High Flow
Initial Ice Temperature (Celsius)
-1.66
-4.53
-4.60
-5.43
-5.51
-5.73
-6.03
-9.55
-10.74

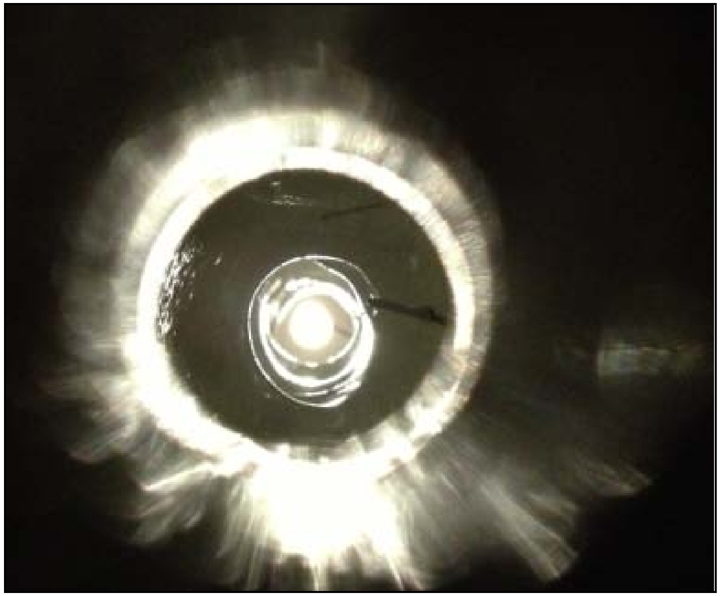




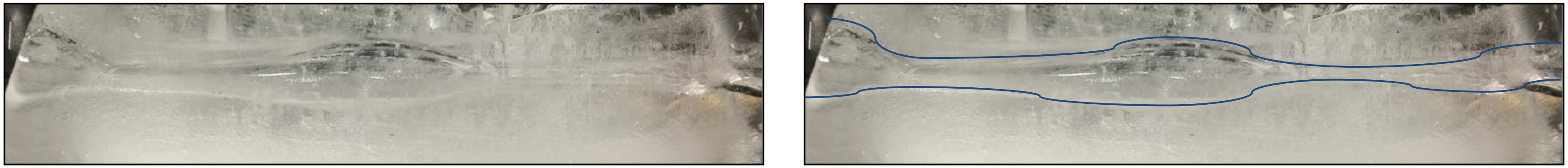




High Water Flow Rate Tests

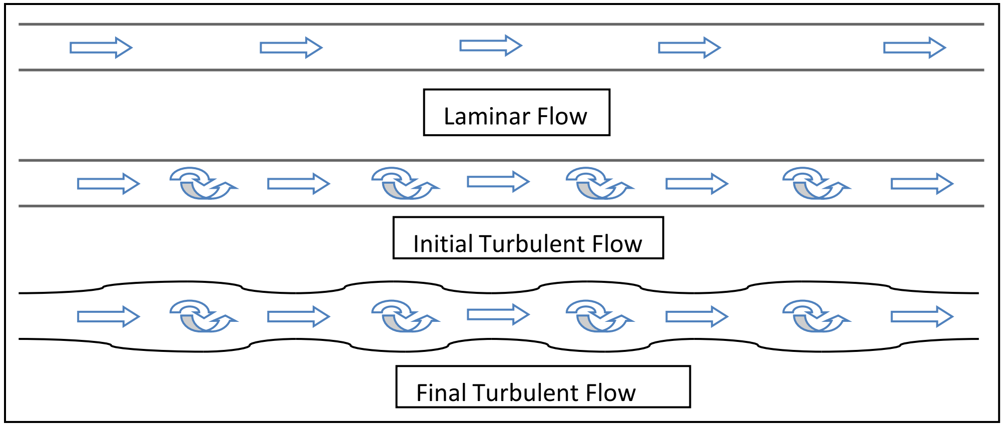


- Final conduit geometry
  - Radius not constant through ice sample length
  - Scalloping due to turbulent flow during conduit expansion



Scalloping

- Turbulent Flow
  - Eddies form and carve dimples
- Scallops turn walls from smooth to (very, very) rough

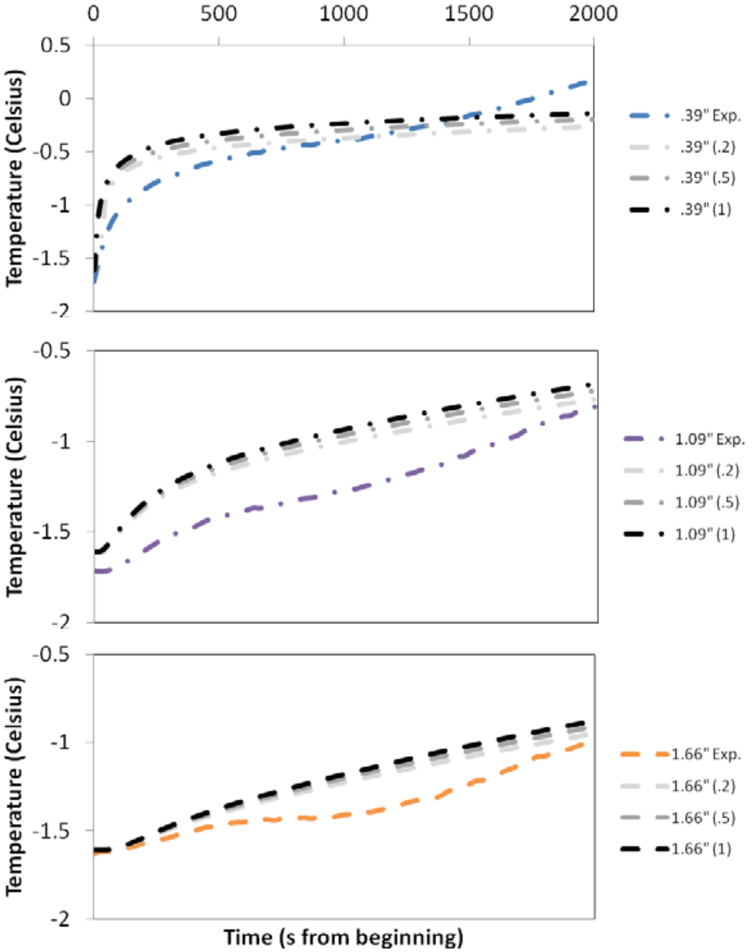


Laminar Flow

Initial Turbulent Flow

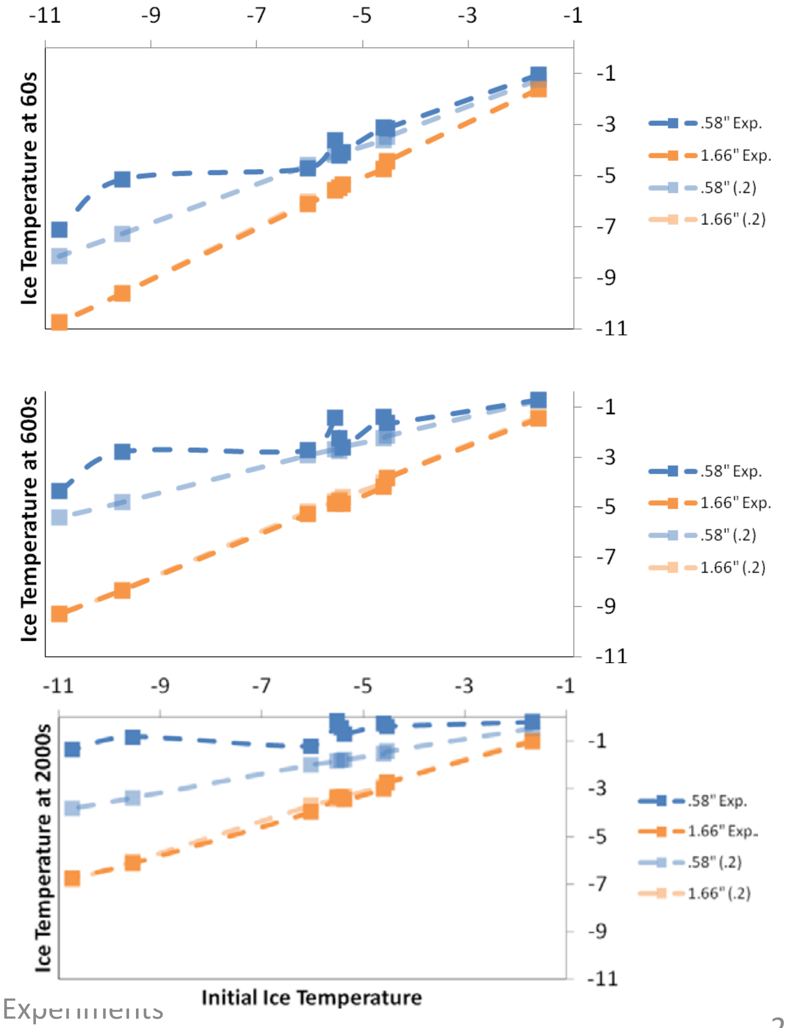
Final Turbulent Flow

High Water Flow Rate Tests



High Water Flow Rate Tests

- Friction factor of 1 too high, Blasius too low
  - f=0.2 best fit
- Nearest temperature recording slightly underpredicts temperature
  - Multidimensionality & scalloping



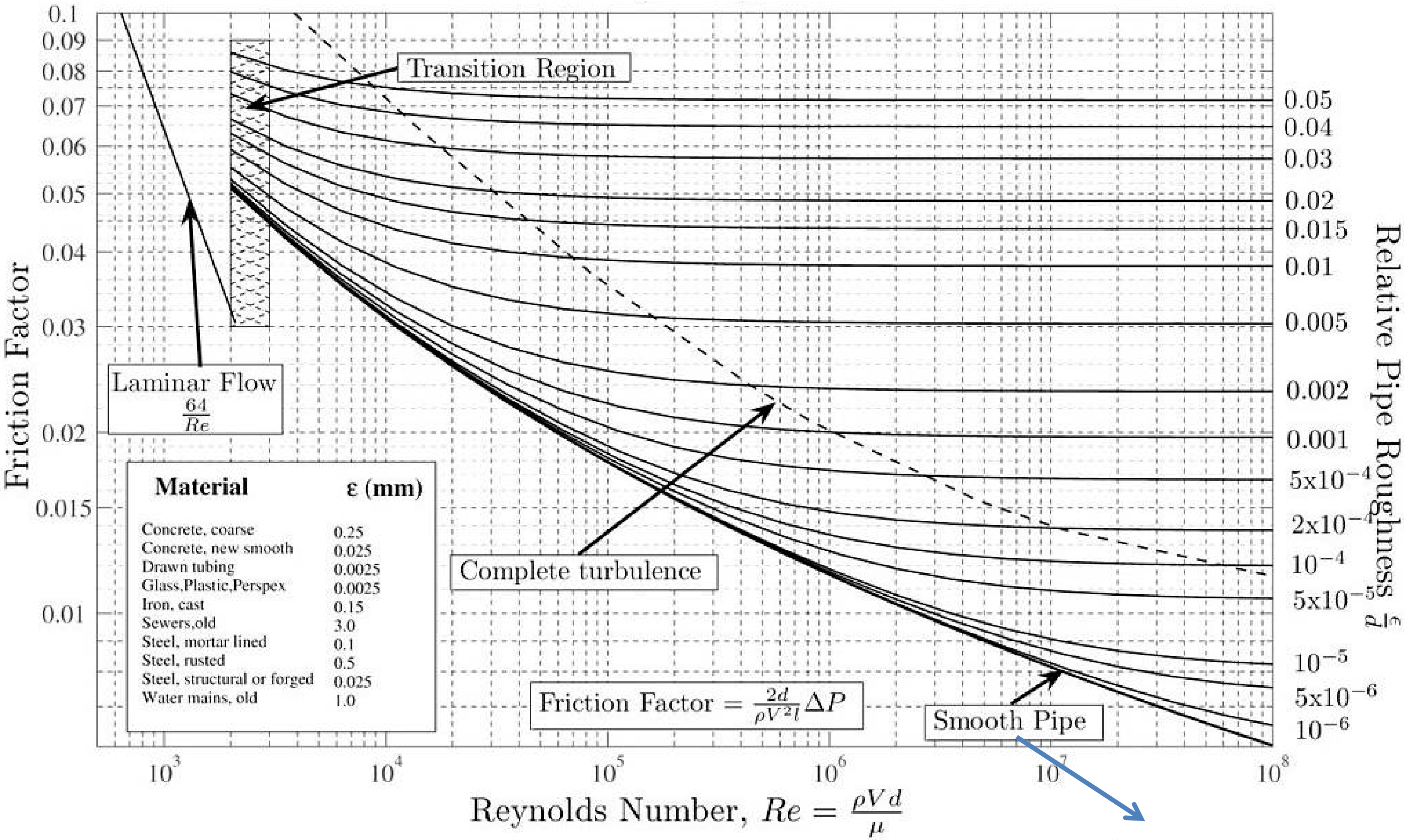
High Water Flow Rate Tests

- Ice input as initial constant temperature
- Radial numerical code used
  - Not significant time for latent heat to reach boundary to affect model
- Friction factor of 0.2 fits best
  - Darcy-Weisbach friction factor

- Model best at low time
  - Conduit expansion not uniform along length
  - Scalloping occurs

High Flow (4 gpm)				
Initial Ice Temperature (C)	**Critical Flow Rate (gpm)**			
	Blasius	**f=.2**	**f=.5**	**f=1**
-1.66	3.34	1.74	1.28	1.02
-4.53	4.81	2.43	1.79	1.42
-4.60	4.84	2.44	1.80	1.43
-5.43	5.14	2.58	1.90	1.51
-5.51	5.17	2.59	1.91	1.52
-5.73	5.24	2.63	1.94	1.54
-6.03	5.34	2.67	1.97	1.56
-9.55	6.31	3.12	2.30	1.82
-10.74	6.58	3.22	2.37	1.88

Moody Diagram



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Blasius Resistance Law

In Summary

Very good agreement between experiment and theory/numerical model

Very interesting insights on friction factor in englacial conduits even at this small laboratory scale

– under refreezing conditions, conduits behave like smooth conduits (Blasius Resistance law provides best fit)

– but under conditions of conduit growth, scalloping causes extremely high “friction factors” (f=0.2 > typical f used even for “rough” pipes)…confirms field observations reported in recent paper by Gulley et al. (2013) on roughness of englacial and subglacial conduits….scalloping is likely an inherent self-organized process resulting from interaction between turbulent eddies and melting (glacial karst) or dissolution (limestone karst)

Future Work: inclusion of creep closure effects