

Nonlinear Scale Interactions and Energy Pathways in the Ocean

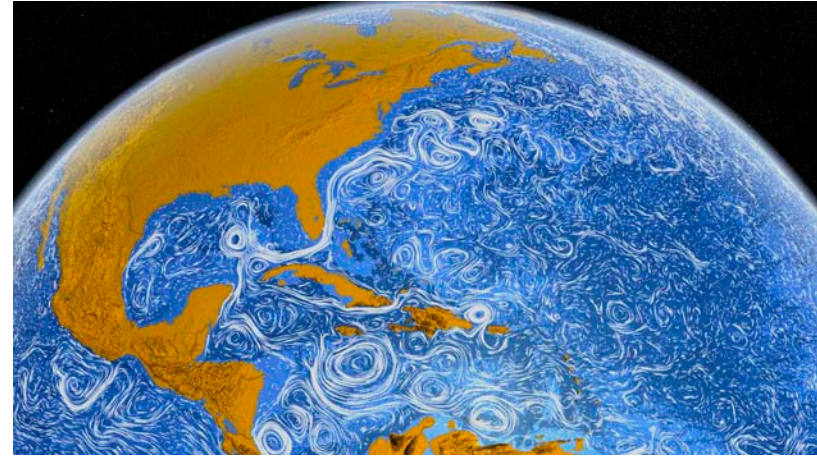
^{1,2}Hussein Aluie , ¹Matthew Hecht

& ³Geoff Vallis

**¹LANL, the ²New Mexico Consortium and ³U of Exeter
with support from LANL's
Institute for Geophysics, Planetary Physics and Signatures
and
Center for Nonlinear Studies**

Ocean Circulation

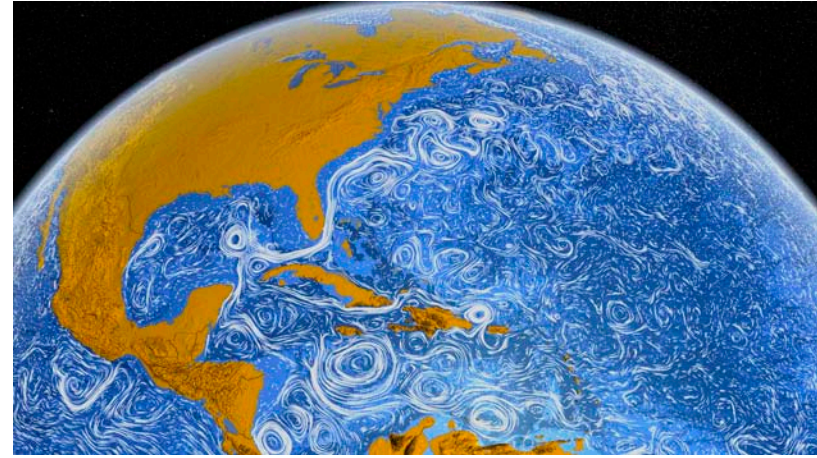
- Multiscale Flow: $O(10^4)$ km -- $O(1)$ mm



Source: NASA

Ocean Circulation

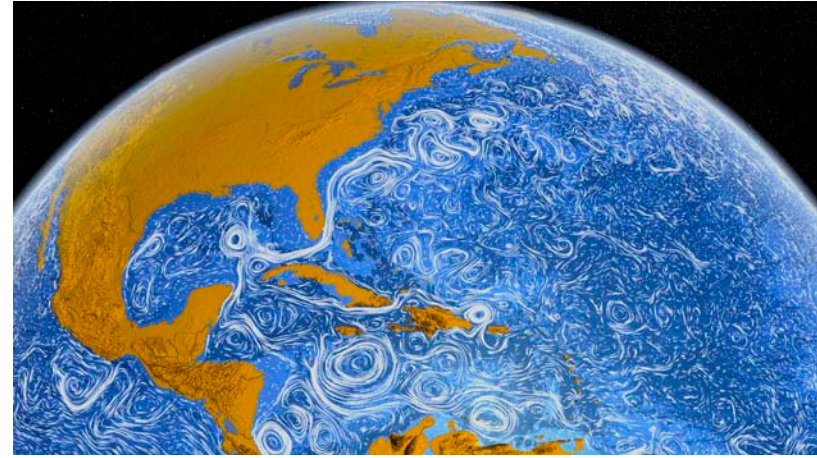
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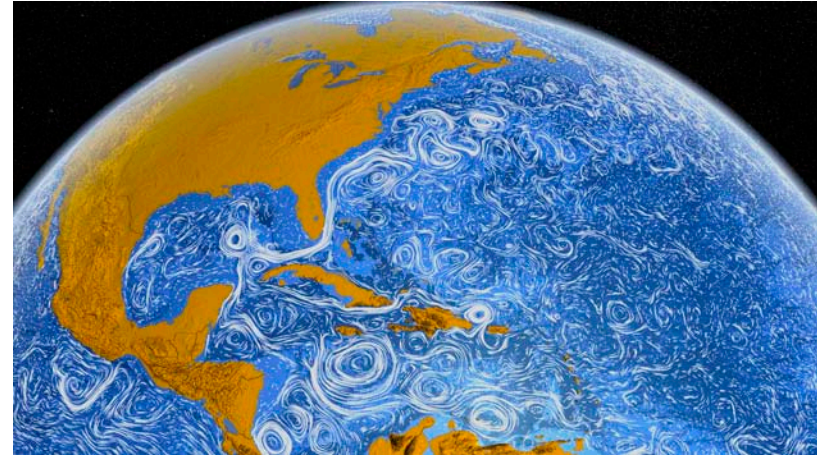
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- Mean Kinetic energy (KE) is concentrated in narrow intense currents.



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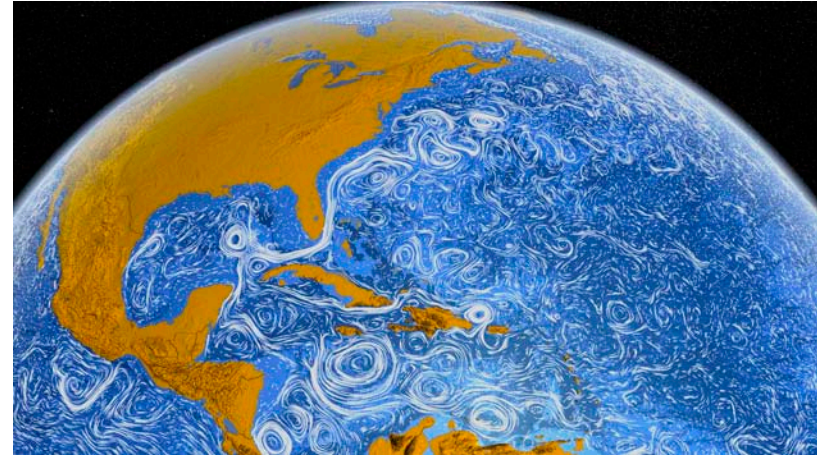
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Ocean Circulation

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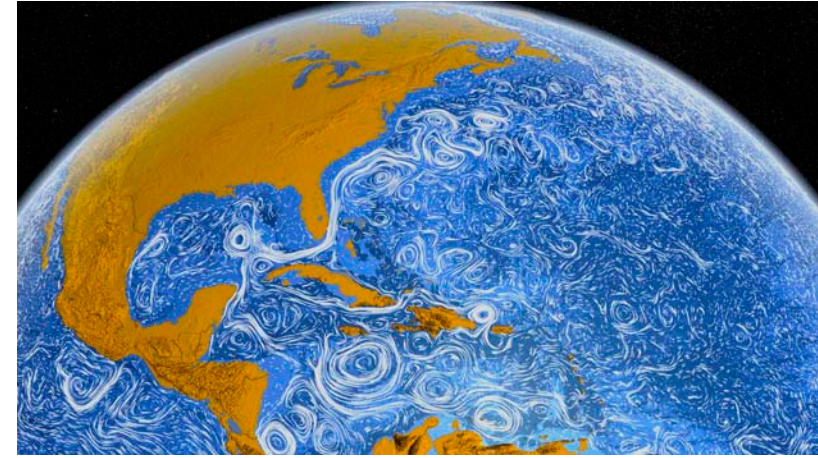
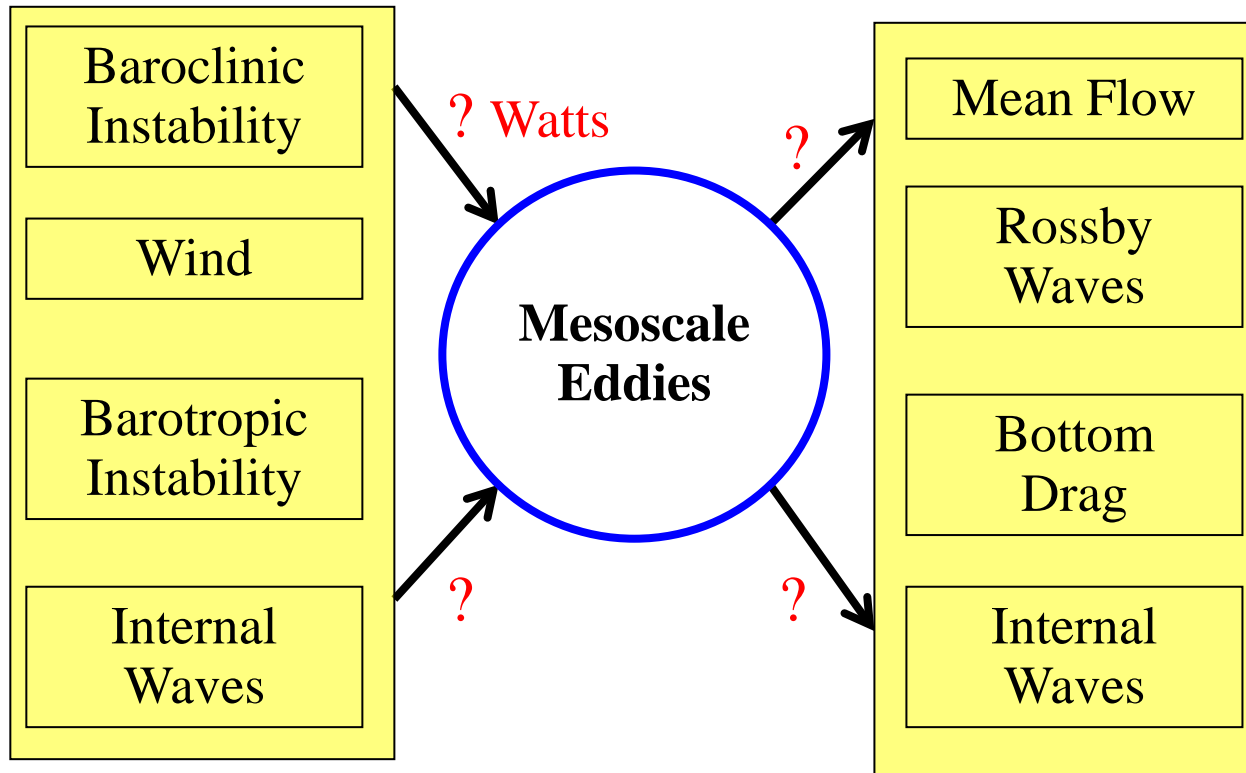
Source: NASA

The Problem

- What is the flow of energy between different spatial scales, different forms?

Ocean Circulation

Nonlinear Processes & Structures

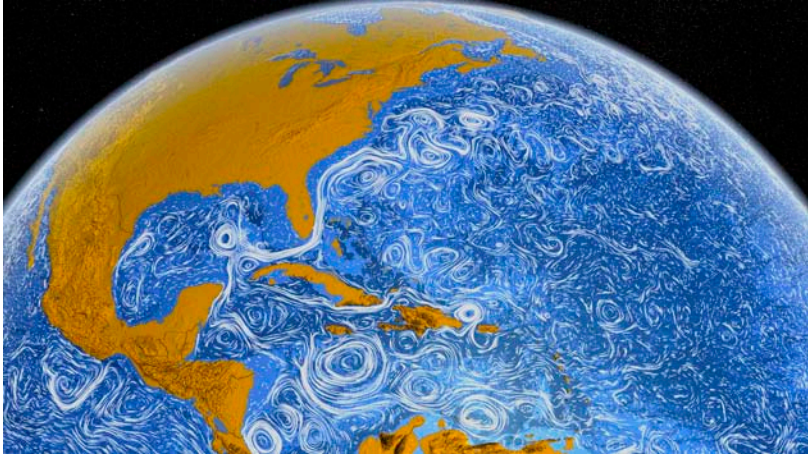


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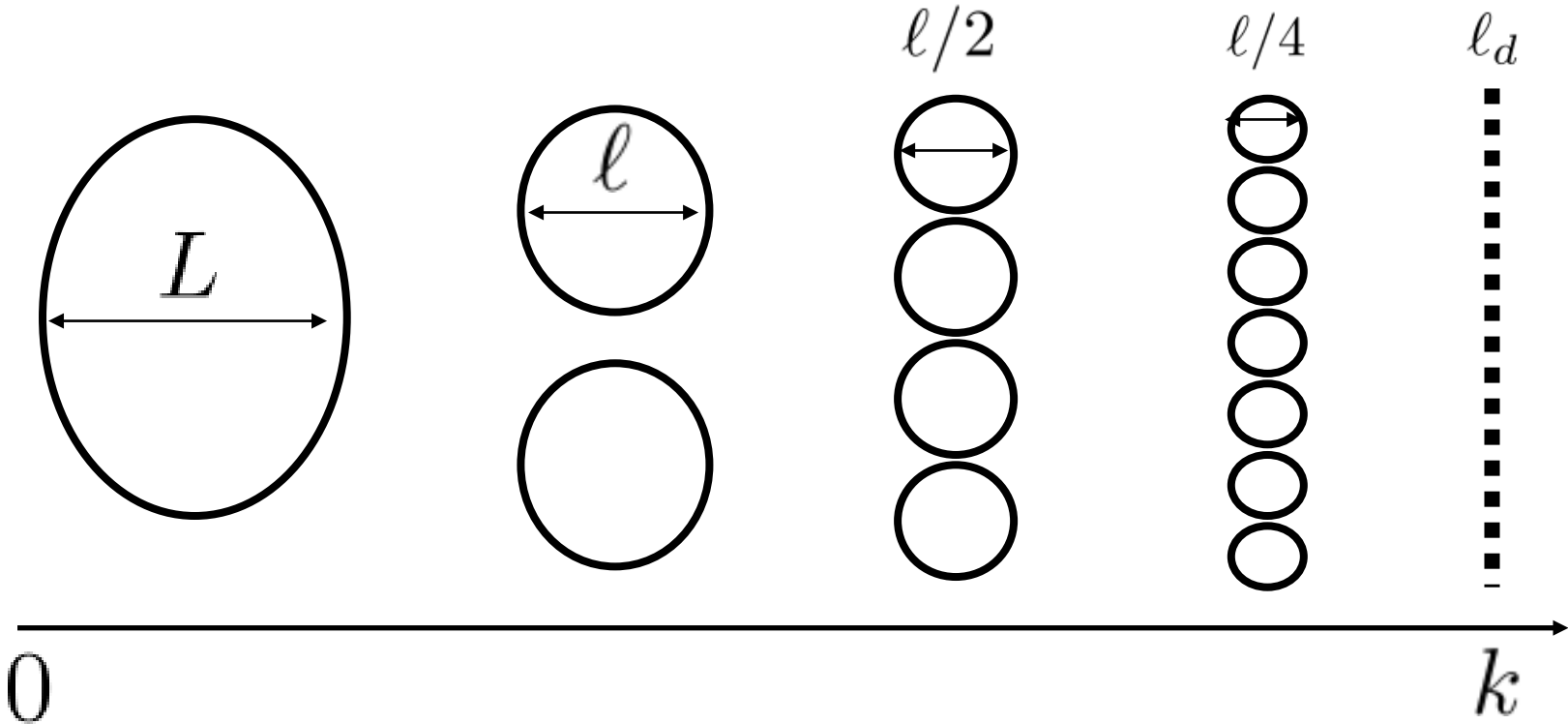
The Problem

- What is the flow of energy between different spatial scales, different forms (how big are these arrows)?

Traditional Approach



Source: NASA

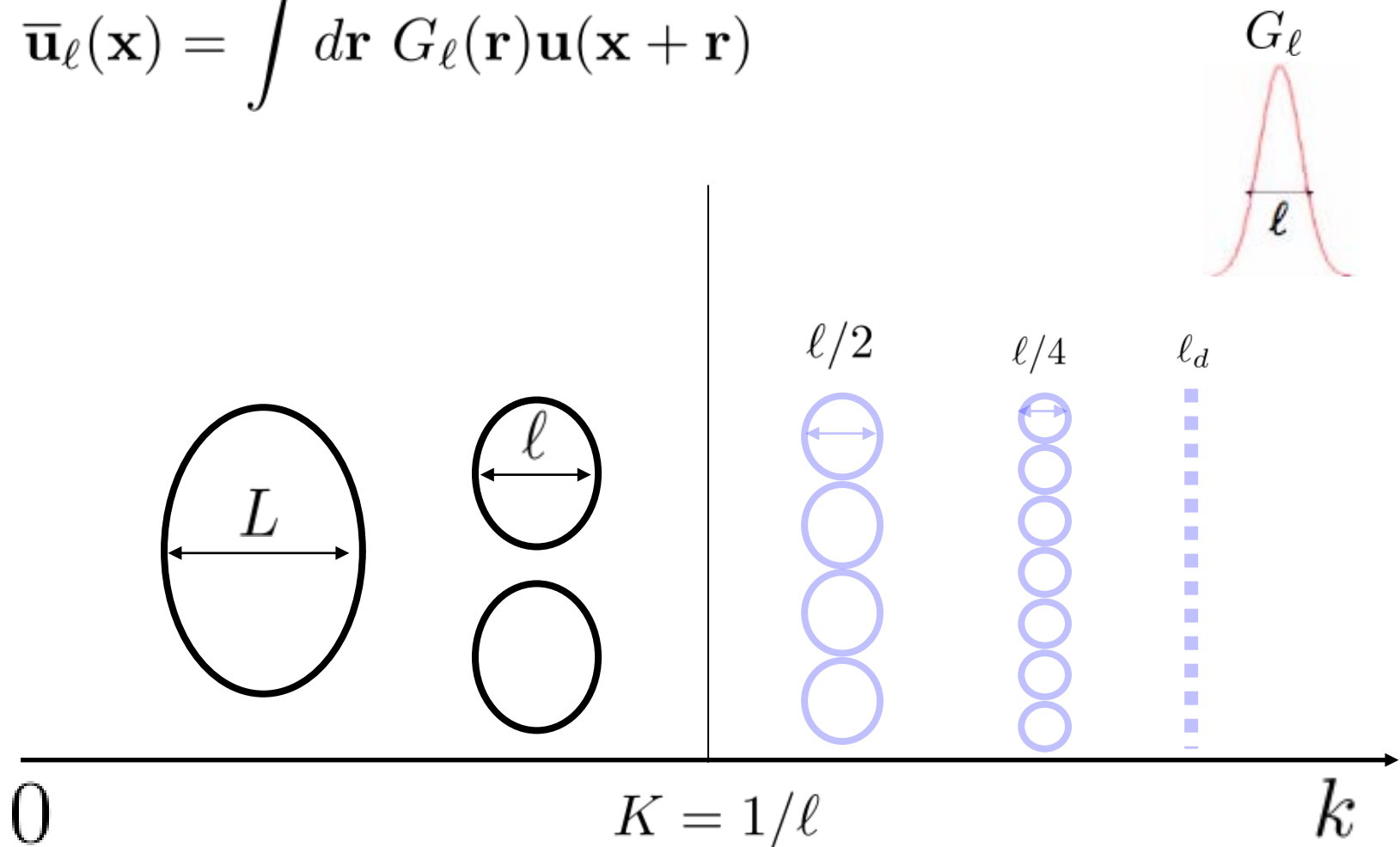


Our Approach

Coarse-graining (Filtering)

Leonard (1974),
Germano (1992), Eyink (1994),
Piomelli et al. (1991), Liu et al (1994),
Chen, Ecke, Eyink (2003),...

$$\bar{\mathbf{u}}_\ell(\mathbf{x}) = \int d\mathbf{r} G_\ell(\mathbf{r}) \mathbf{u}(\mathbf{x} + \mathbf{r})$$

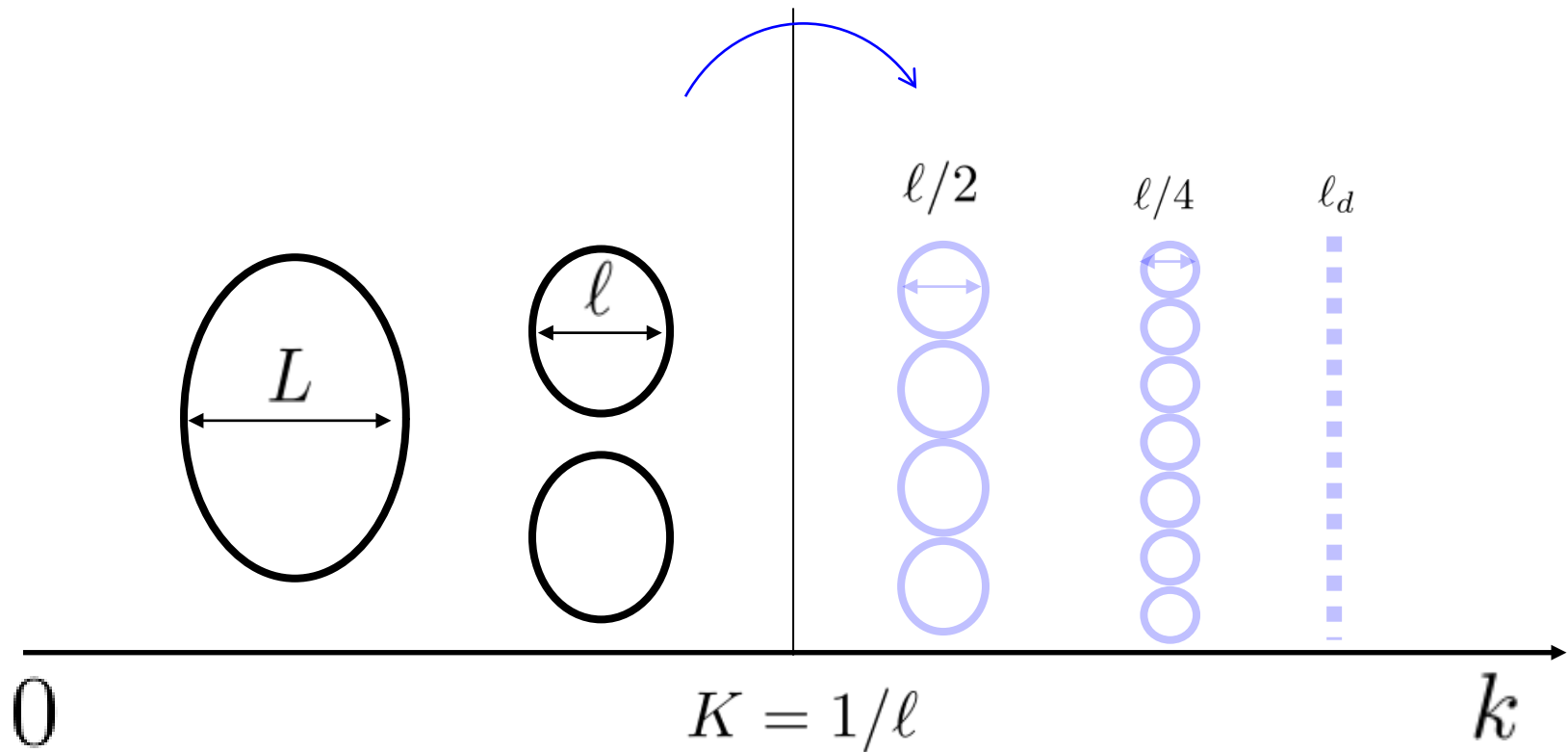


Cascade of Energy

Large-scale energy budget

$$\partial_t \frac{|\bar{\mathbf{u}}|^2}{2} + \nabla \cdot [\dots] = -\Pi_\ell^E - \nu |\nabla \bar{\mathbf{u}}|^2$$

Energy flux



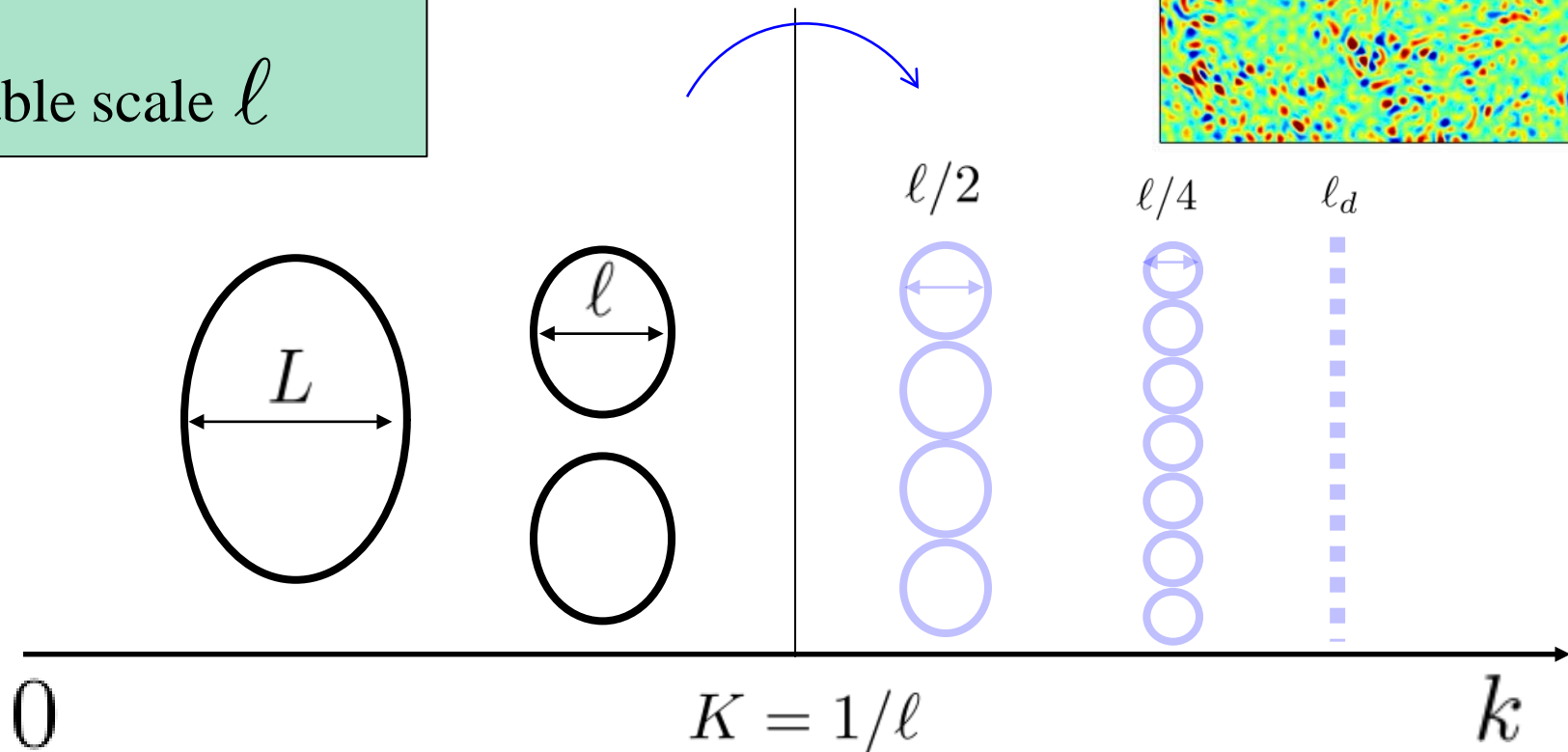
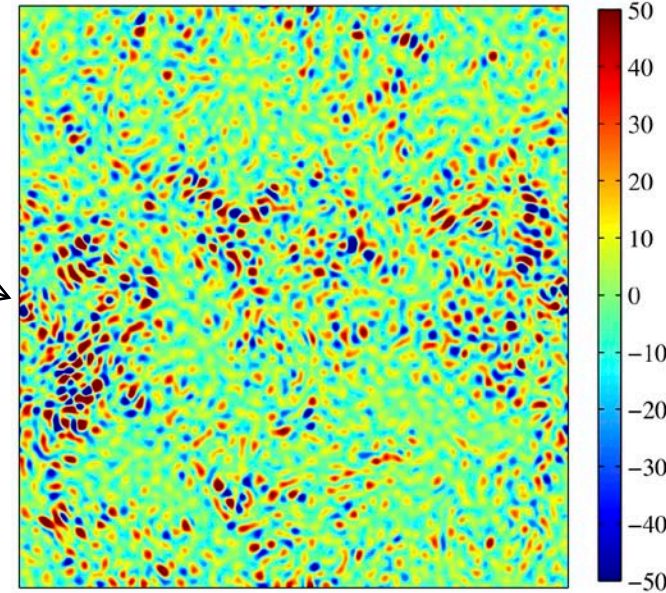
Cascade of Energy

Large-scale energy budget

$$\partial_t \frac{|\bar{\mathbf{u}}|^2}{2} + \nabla \cdot [\dots] = -\Pi_\ell^E - \nu |\nabla \bar{\mathbf{u}}|^2$$

- Every point \mathbf{X} and every instant t
- Variable scale ℓ

Energy flux



Measuring Energy Transfer

Large-scale energy budget

$$\partial_t \frac{|\bar{\mathbf{u}}|^2}{2} + \nabla \cdot [\dots] = -\Pi_\ell^E - \nu |\nabla \mathbf{u}|^2$$

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Subgrid scale (SGS) flux

$$\Pi_\ell^E(\mathbf{x}) = -\partial_j \bar{u}_i [\overline{u_i u_j} - \bar{u}_i \bar{u}_j]$$

Measuring Energy Transfer

Large-scale energy budget

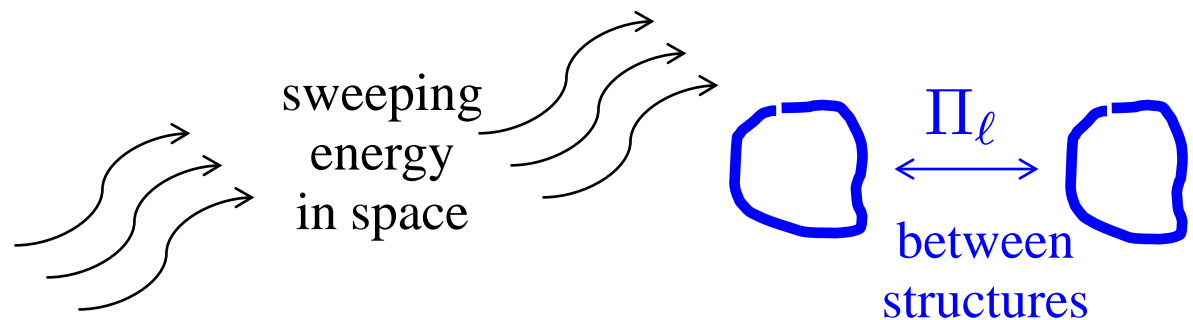
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Subgrid scale (SGS) flux

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Frisch (1995), used by Scott & Wang (2005), Tulloch, Marshall, Hill and Smith (2011)

$$\Pi_\ell(\mathbf{x}) = \bar{u}_i u_j \partial_j (u_i - \bar{u}_i)$$



Measuring Energy Transfer

Large-scale energy budget

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Subgrid scale (SGS) flux

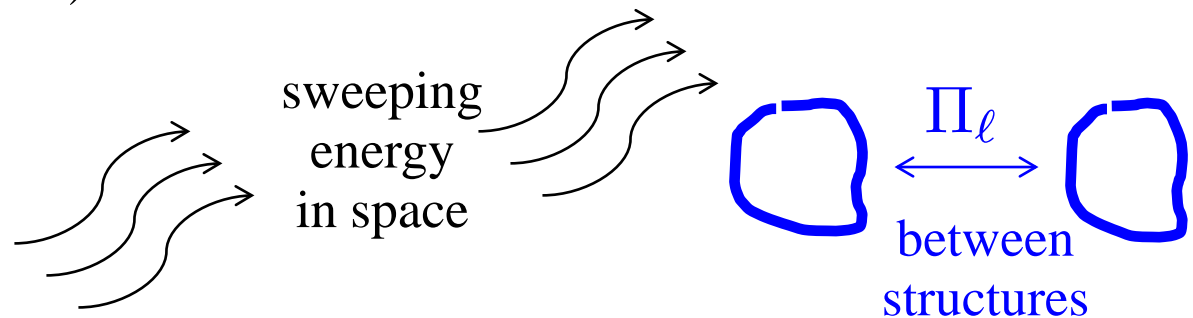
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Lindborg (2006), Mininni, et al. (2008)

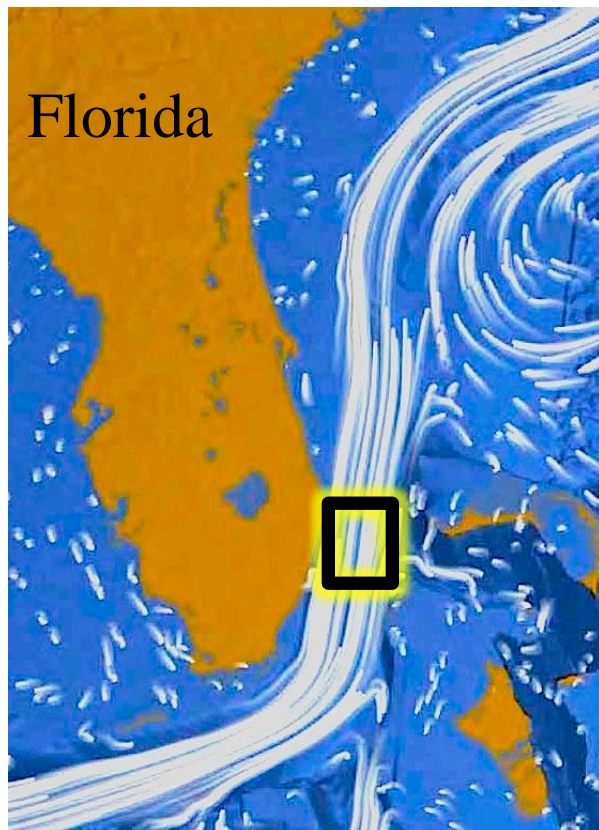
$$\Pi_\ell(\mathbf{x}) = \bar{u}_i \partial_j (\overline{u_j u_i})$$



Measuring Energy Transfer

Any measure of the energy exchange must satisfy:

1. Galilean Invariance
2. Vanish in the absence of subscale fluctuations

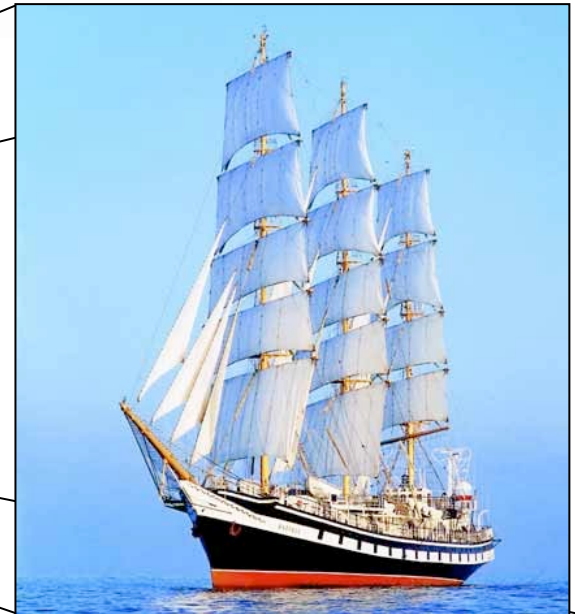


Gulf Stream

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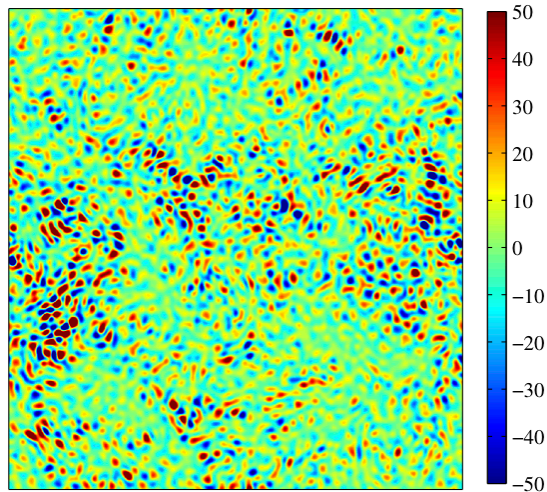
Gulf Stream

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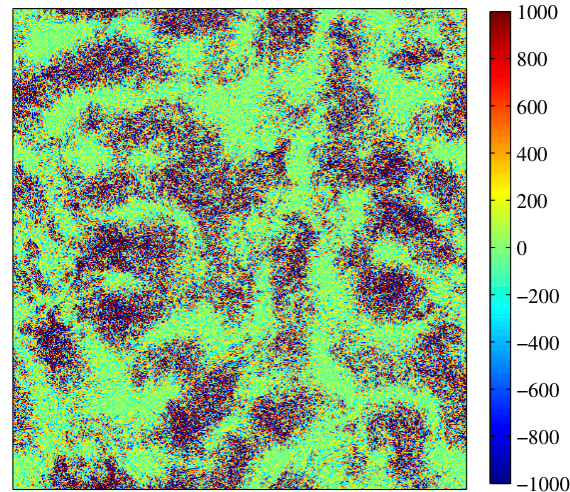
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SGS
definition



Frisch (1995)
definition

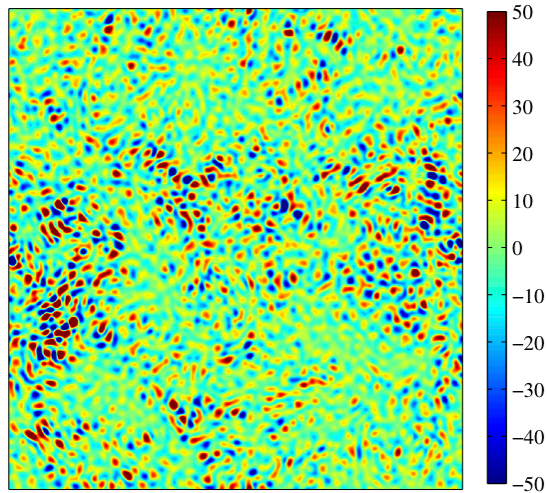


Measuring Energy Transfer

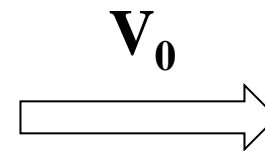
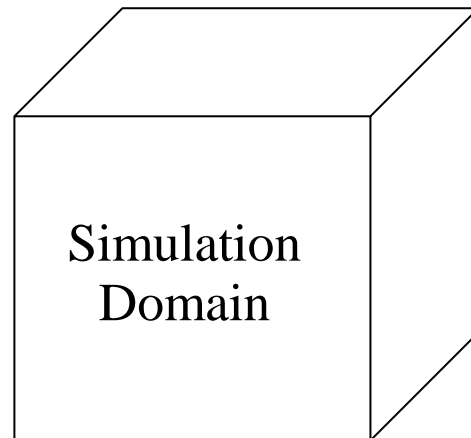
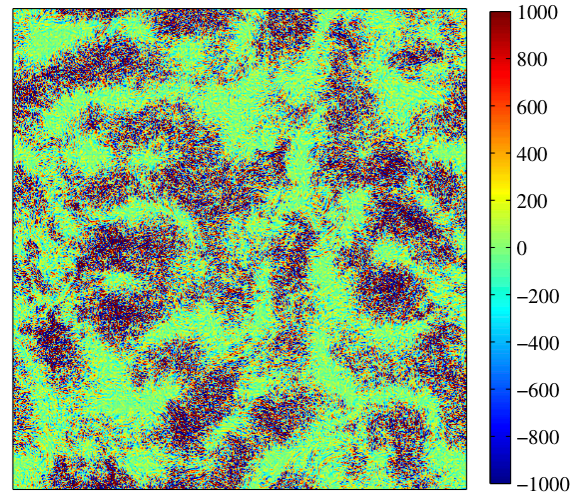
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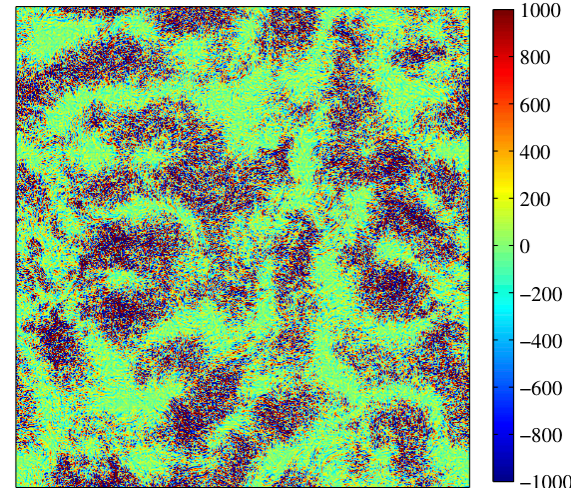
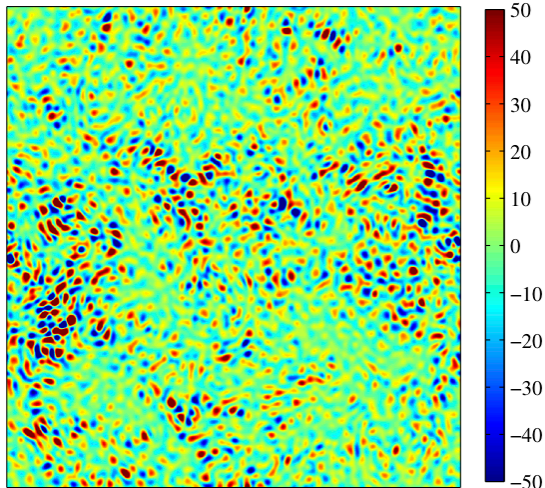


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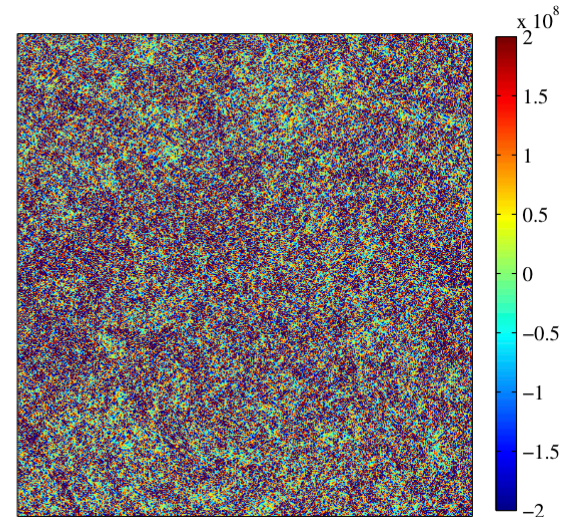
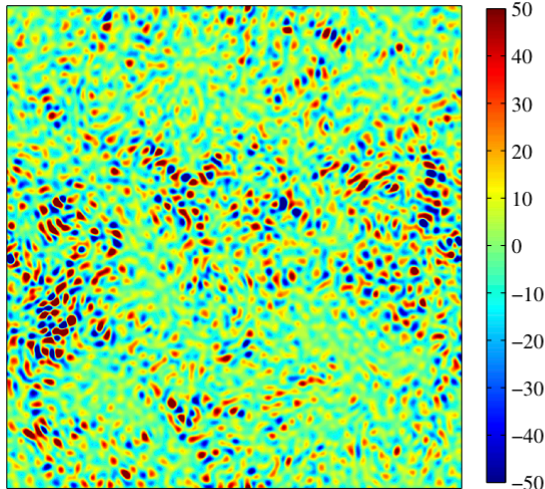
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After
boosting



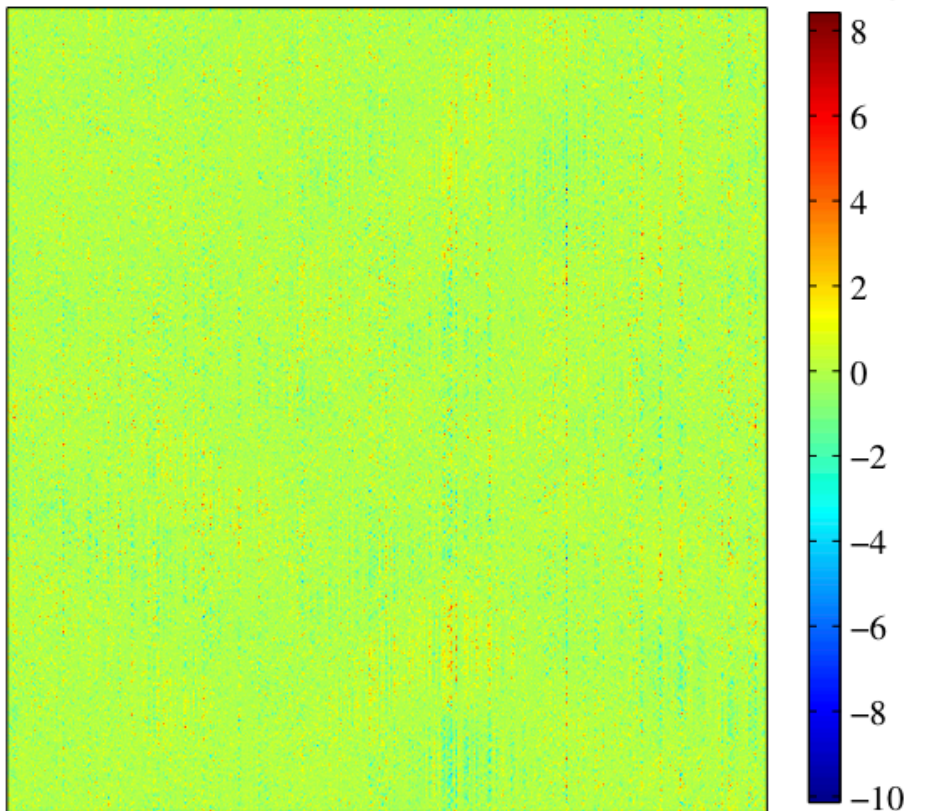
After
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Measuring Energy Transfer

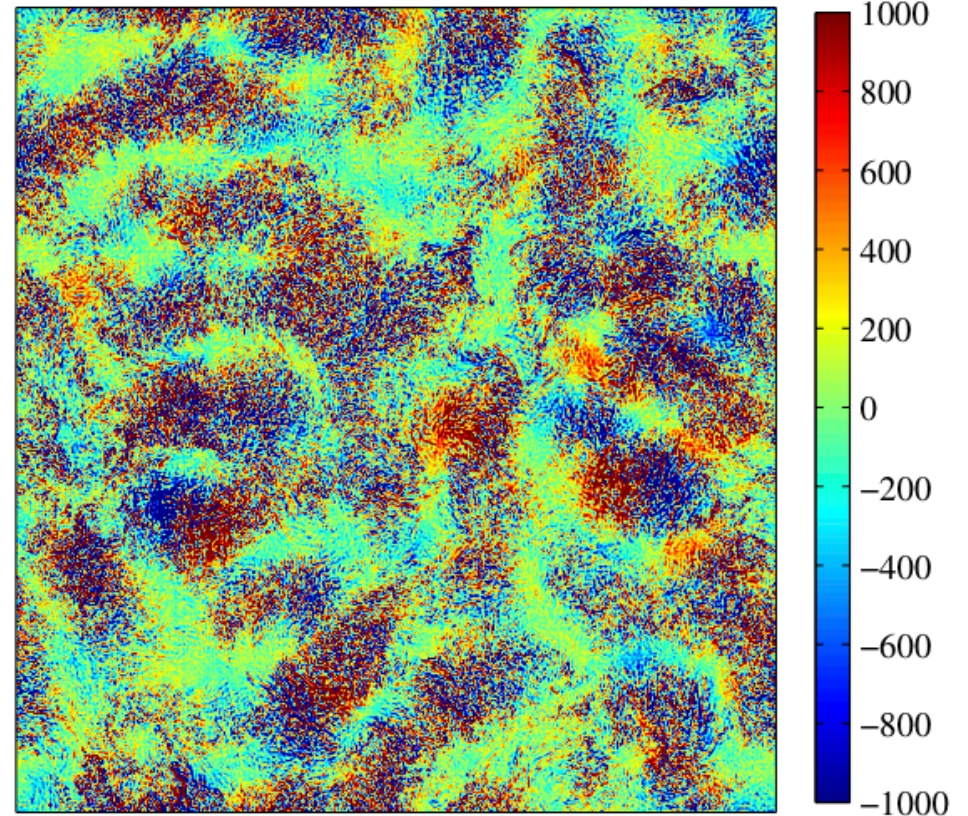
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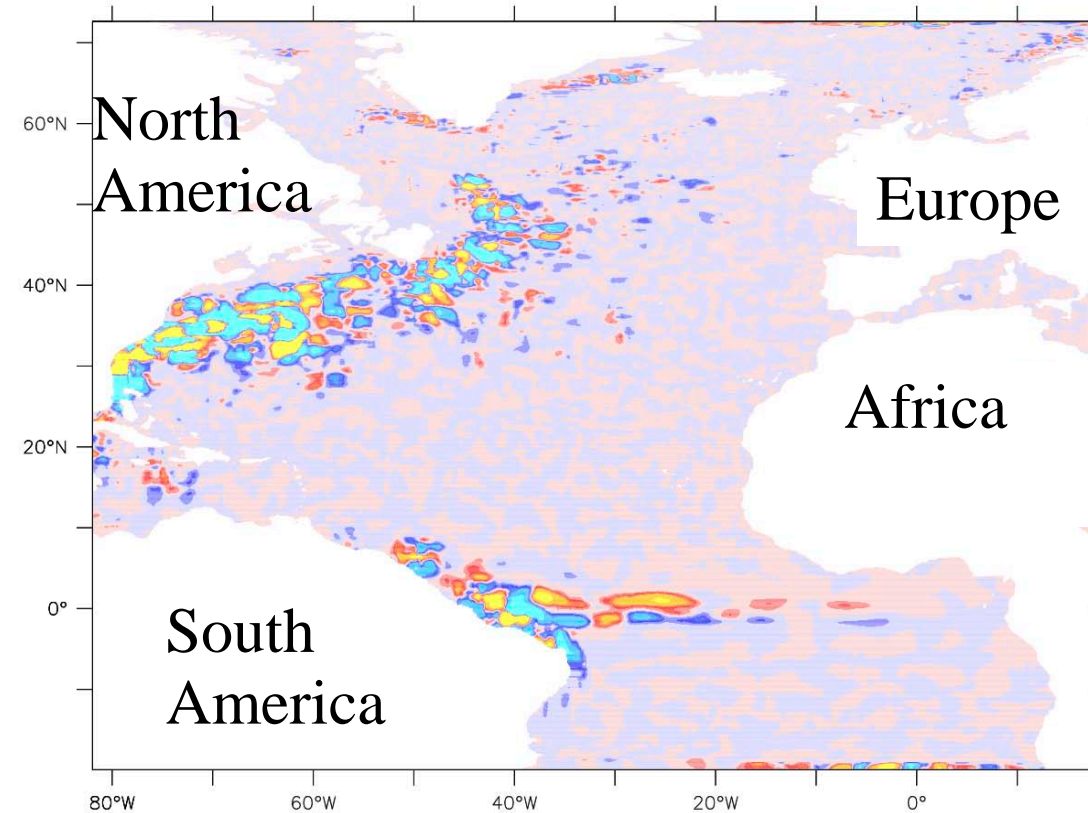
SGS definition



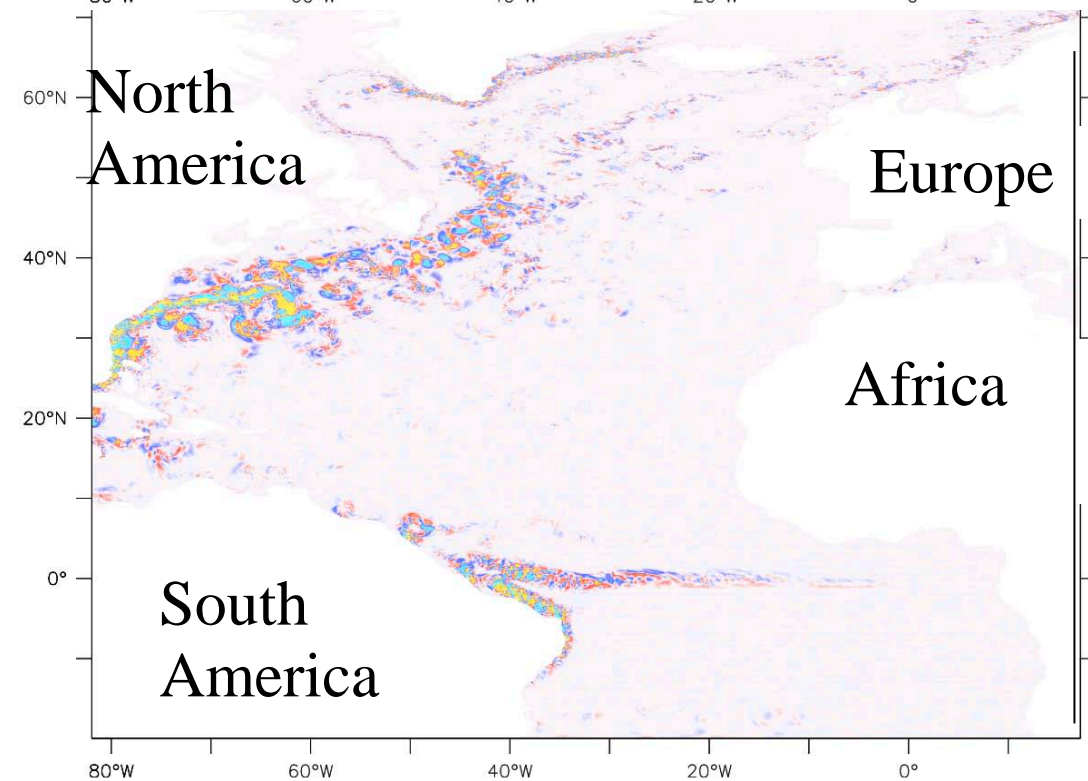
Lindborg (2006) definition

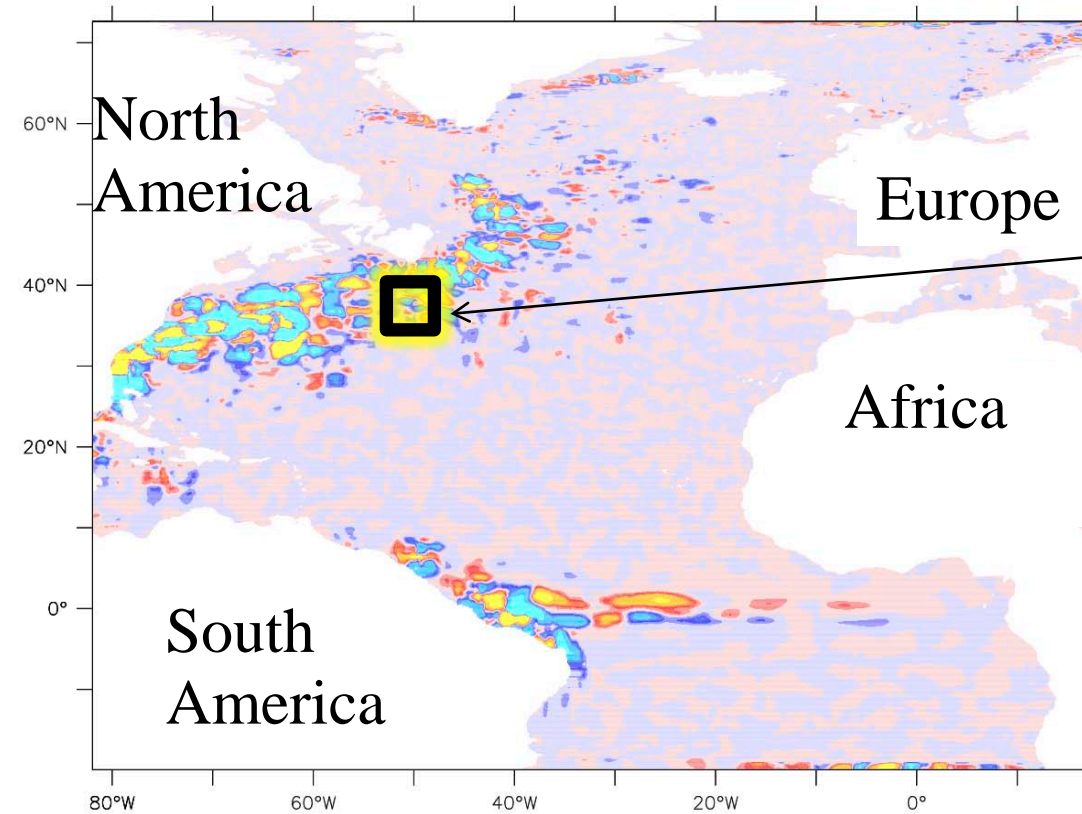


SGS definition $\ell = 200 \text{ km}$



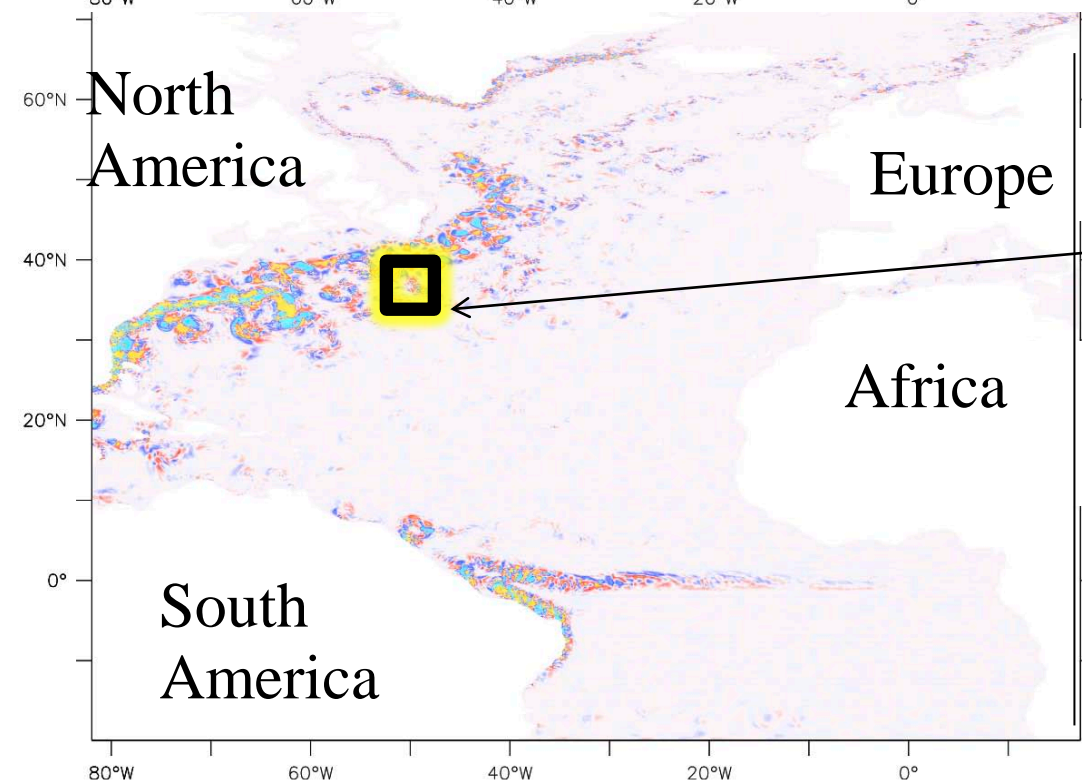
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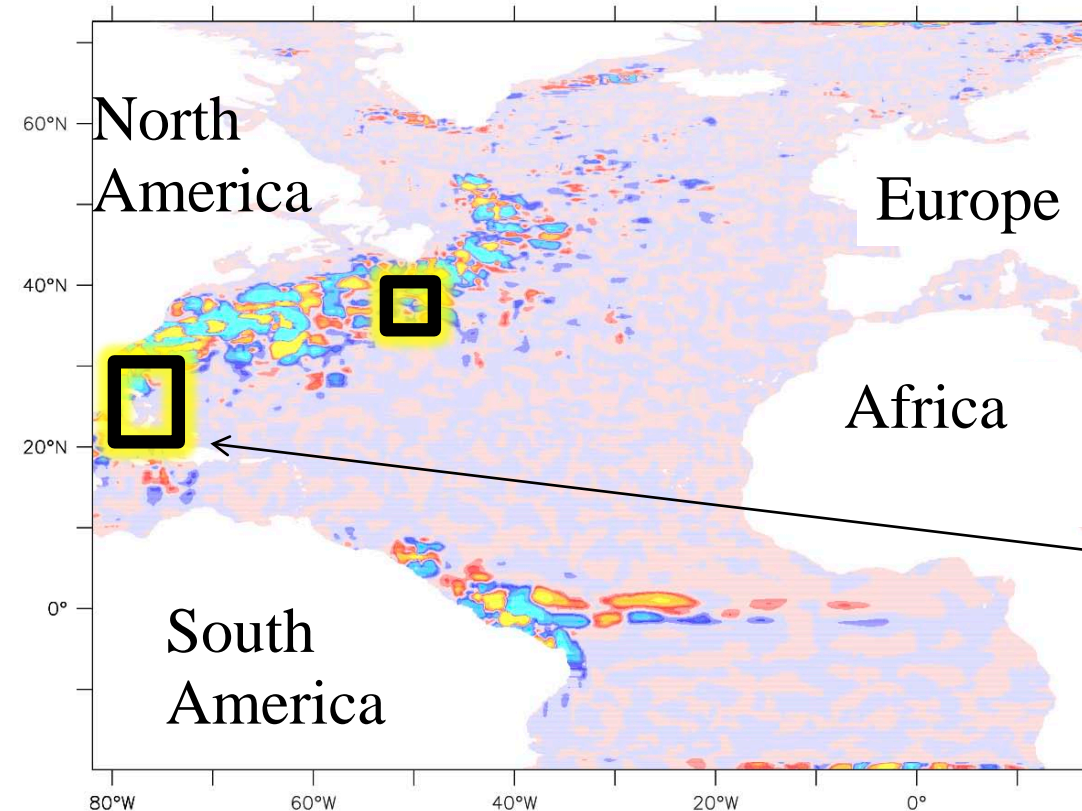
SGS definition $\ell = 200 \text{ km}$

$$\langle \Pi_{\ell}^{\text{SGS}} \rangle = +0.9 \text{ W/km}^2/\text{m}$$



Frisch definition $\ell = 200 \text{ km}$

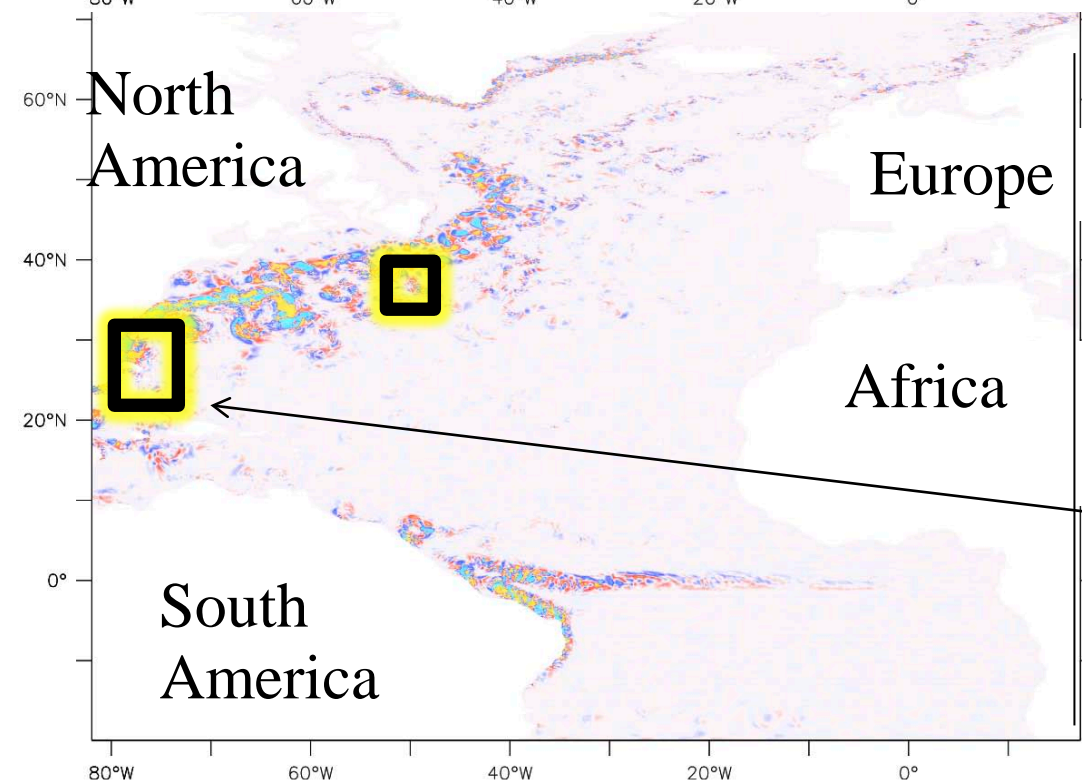
$$\langle \Pi_{\ell}^{\text{Frisch}} \rangle = -22.9 \text{ W/km}^2/\text{m}$$



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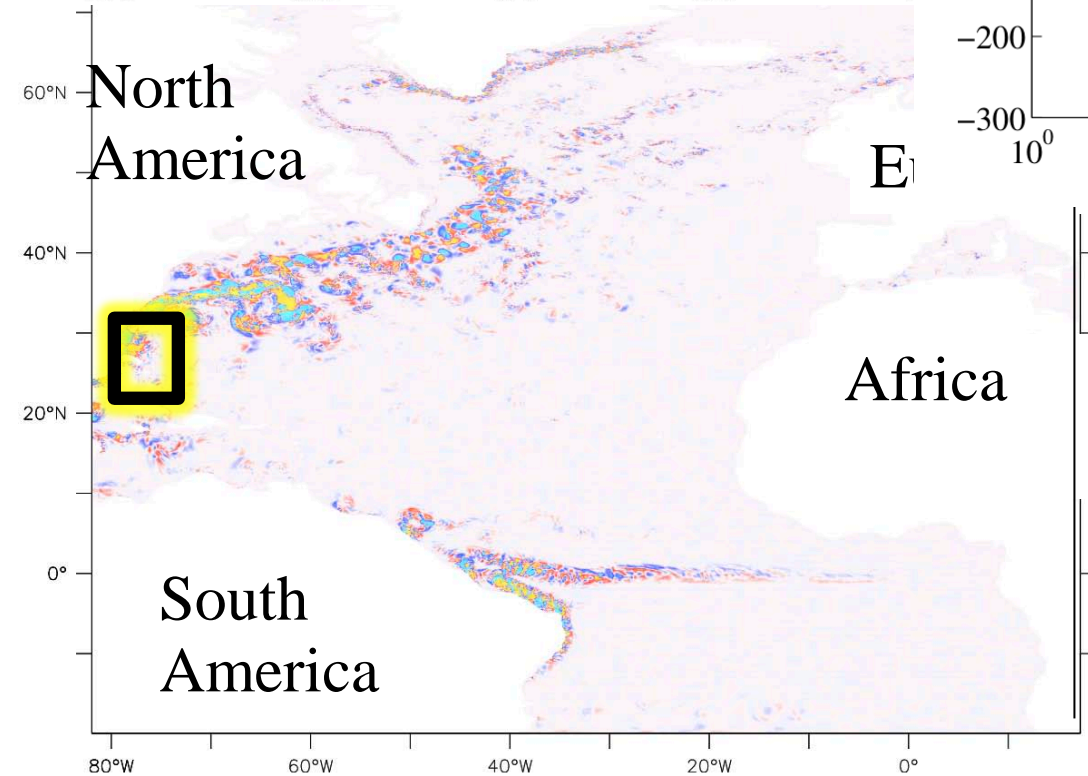
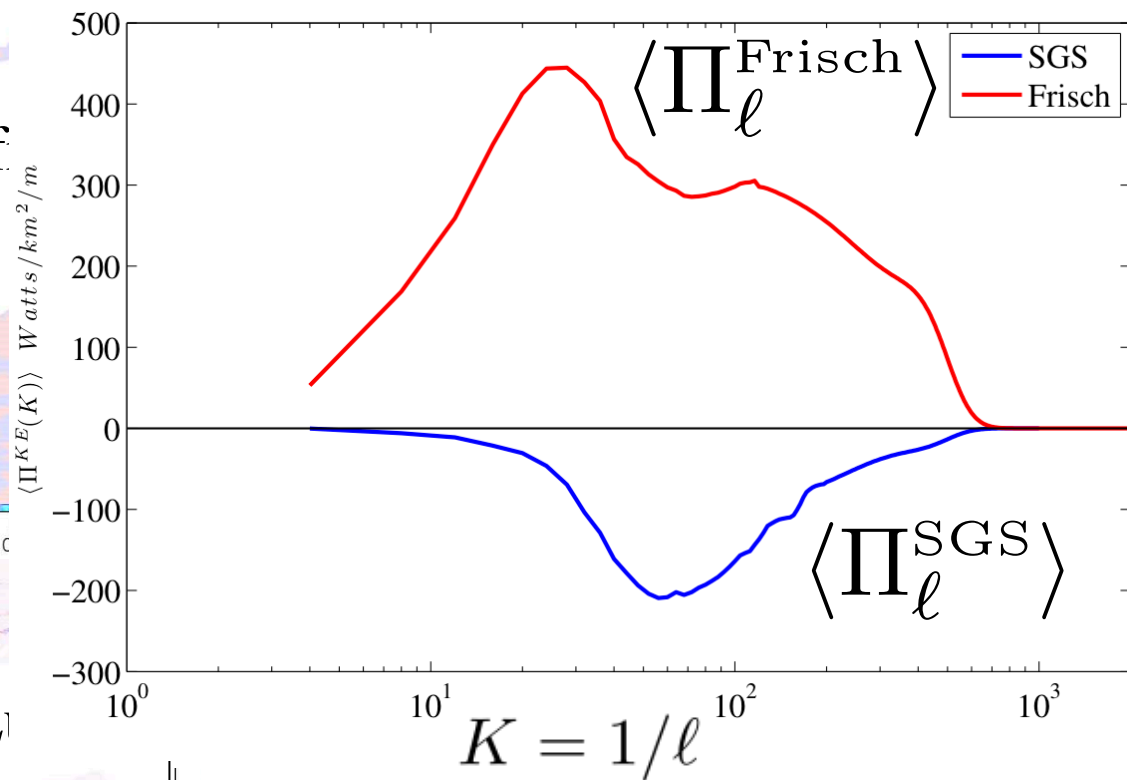
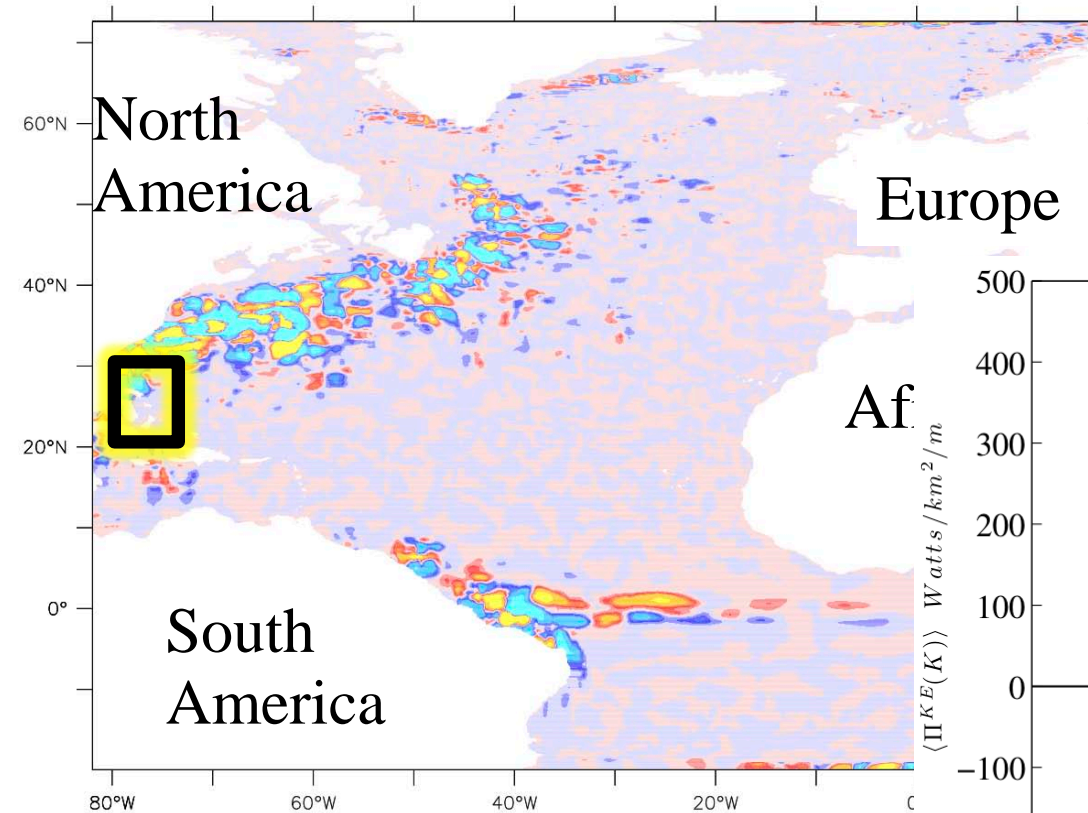
$$= -197 \text{ W/km}^2/\text{m}$$



Frisch definition $\ell = 200 \text{ km}$

$$\langle \Pi_{\ell}^{\text{Frisch}} \rangle = -22.9 \text{ W/km}^2/\text{m}$$

$$= +319 \text{ W/km}^2/\text{m}$$



In summary

- Traditional homogeneous/isotropic turbulence tools break down in the Ocean.
- Spectral Transfer and Flux can be qualitatively wrong.
- Guided by basic physical principles ([Galilean invariance, ...](#)), the SGS approach to coarse-graining provides a robust means to measure energy transfer at different locations in the Ocean.

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- Were ready to publish last summer, before we realized we weren't
 - Since then, have worked through issues of calculating energy transfer on the sphere (scale-dependent commutation of vector operations and filter).