Prognostic Residual-Mean Flow in an Ocean General Circulation Model

A Comparison of GM to a Residual Mean Approach

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The Thickness-Weighted Averaged Equations

Background

- The notion of 3D TWA equations and residual mean flow has been around for a while (DeSzoeke and Bennett 1993).

- Independantly, a number of people have related residual mean flow to Eliassen-Palm vectors, relying in some way on assumptions and approximations.

- Recently Young (2012) showed that the 3D TWA equations are in fact exact and linked to Eliassen-Palm vectors.

- Maddison and Marshall (2013) showed that the EP vectors are columns of the Eliassen-Palm flux tensor (EPFT), and demonstrated that many of the previously derived versions of the EPFT are linked by a gauge degree of freedom.

- It has been suggested that, given the physical and mathematical tractability of the TWA equations, they are well suited for ocean modeling with parameterized eddies.

Summary of this talk

MPAS-O

framework:	TWA	Relation	Conventional
Momentum Advection:	residual velocity	? ⟷	eulerian velocity
Tracer Advection:	residual velocity	? ⟷	residual velocity
Eddy parameterization:	eddy form drag GL 1990	? ⟷	bolus velocity GM 1990

- We implement the thickness-weighted average (TWA) framework in MPAS-O.
- We implemented GM with Ferrari et al. 2010 boundary value tapering scheme.
- We make connections between GM and Greatbatch and Lamb 1990, and
- between the equations solved by conventional OGCMs and the TWA framework.

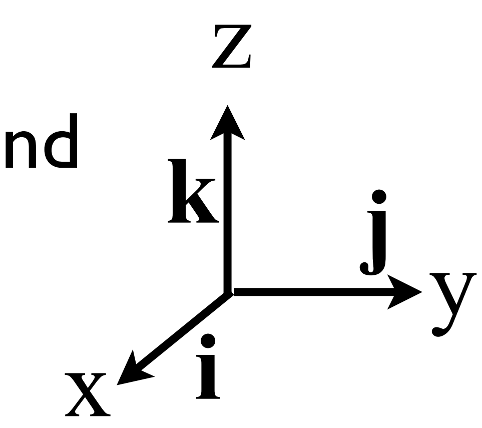
that allow us to use GM to parameterize eddies in the TWA framework.

The Thickness-Weighted Averaged Equations

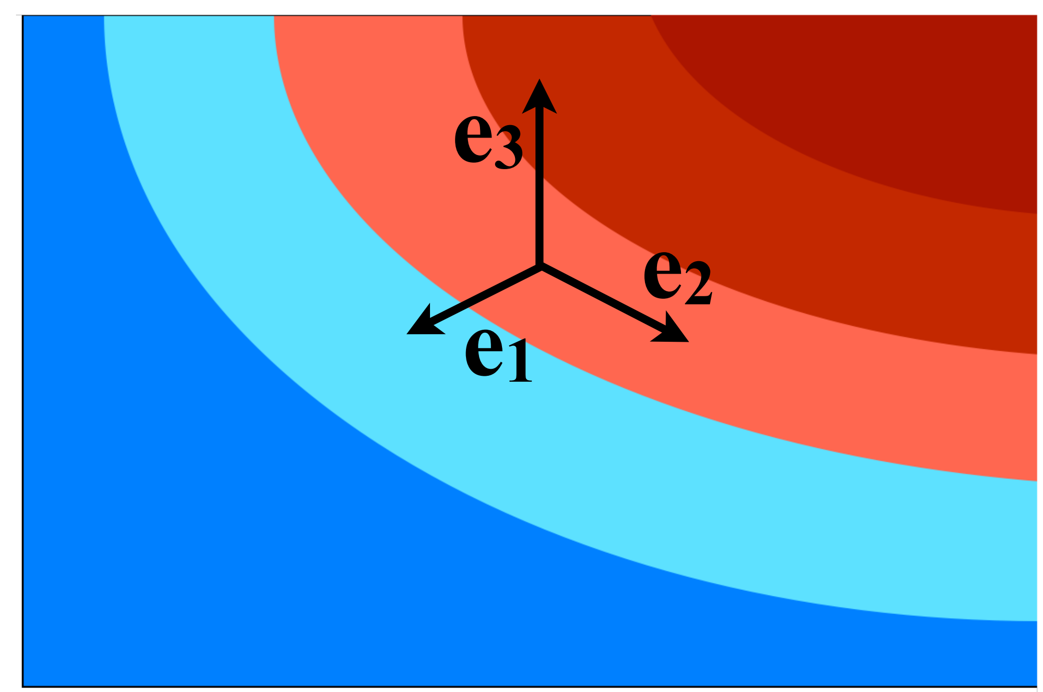
Start with the full Boussinesq equations.

The flow is averaged over micro-scale motions and processes,

- unresolved processes at the micro-scale are parameterized, and
- the flow is stably stratified.



$$
\begin{aligned}
\frac{Du}{Dt} - fv + \frac{\partial p}{\partial x} &= R_x, \\
\frac{Dv}{Dt} + fu + \frac{\partial p}{\partial y} &= R_y, \\
\frac{\partial p}{\partial z} &= b, \\
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} &= 0, \\
\frac{Db}{Dt} &= \varpi
\end{aligned}
$$



$$
\begin{aligned}
\mathbf{e_1} &= \mathbf{i} + \zeta_{\tilde{x}}\mathbf{k} \\
\mathbf{e_2} &= \mathbf{j} + \zeta_{\tilde{y}}\mathbf{k} \\
\mathbf{e_3} &= \sigma\mathbf{k}
\end{aligned}
$$

Transform to buoyancy coordinates ...

The Thickness-Weighted Averaged Equations

Take the thickness-weighted average of an *ensemble* of flows,

$\hat{u} = \dfrac{\overline{u\sigma}}{\overline{\sigma}}$ Ensemble averages at fixed $\tilde{x}, \tilde{y}, b^\sharp$; thickness: $\overline{\sigma} = \left(\dfrac{\partial b^\sharp}{\partial z}\right)^{-1}$

... careful with tensor calculus ... and transform back to cartesian coordinates

$$\frac{D^\sharp \hat{u}}{Dt} - f\hat{v} + \frac{\partial p^\sharp}{\partial x} + \nabla \cdot \mathbf{E}^u = \hat{R}_x,$$
$$\frac{D^\sharp \hat{v}}{Dt} + f\hat{u} + \frac{\partial p^\sharp}{\partial y} + \nabla \cdot \mathbf{E}^v = \hat{R}_y,$$
$$\frac{\partial p^\sharp}{\partial z} = b^\sharp,$$
$$\frac{\partial \hat{u}}{\partial x} + \frac{\partial \hat{v}}{\partial y} + \frac{\partial w^\sharp}{\partial z} = 0,$$
$$\frac{D^\sharp b^\sharp}{Dt} = 0$$

$\sharp$ = "not an averaged quantity", but related to residual quantities

at x, y, z and t

$b^\sharp$ is the buoyancy of the buoyancy surface with average depth z.

$\hat{\mathbf{u}} = \hat{u}\mathbf{i} + \hat{v}\mathbf{j} + w^\sharp\mathbf{k}$ is the advective velocity in $\dfrac{D^\sharp}{Dt}$

The Thickness-Weighted Averaged Equations

Summary: an exact set of TWA Boussinesq equations are obtained

TWA equations ← *Unaveraged* **Boussinesq equations**

$$
\begin{aligned}
\frac{D^\sharp \hat{u}}{Dt} - f\hat{v} + \frac{\partial p^\sharp}{\partial x} + \nabla \cdot \mathbf{E}^u &= \hat{R}_x, \\
\frac{D^\sharp \hat{v}}{Dt} + f\hat{u} + \frac{\partial p^\sharp}{\partial y} + \nabla \cdot \mathbf{E}^v &= \hat{R}_y, \\
\frac{\partial p^\sharp}{\partial z} &= b^\sharp, \\
\frac{\partial \hat{u}}{\partial x} + \frac{\partial \hat{v}}{\partial y} + \frac{\partial w^\sharp}{\partial z} &= 0, \\
\frac{D^\sharp b^\sharp}{Dt} &= 0
\end{aligned}
$$

$$
\begin{aligned}
\frac{Du}{Dt} - fv + \frac{\partial p}{\partial x} &= R_x, \\
\frac{Dv}{Dt} + fu + \frac{\partial p}{\partial y} &= R_y, \\
\frac{\partial p}{\partial z} &= b, \\
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} &= 0, \\
\frac{Db}{Dt} &= \varpi
\end{aligned}
$$

Differences: prognostic variables are reinterpreted, EPFT in the TWA

The Eliassen-Palm flux tensor

All eddy correlations are contained in the EPFT.

The Eliassen-Palm flux vectors are columns of the Eliassen-Palm Flux Tensor:

$$\begin{array}{cccc} & \mathbf{E}^u & \mathbf{E}^v & \\ & \downarrow & \downarrow & \end{array}$$

$$\mathbf{E} = \begin{pmatrix} \widehat{u''u''} + \frac{1}{2\overline{\sigma}}\overline{\zeta'^2} & \widehat{u''v''} & 0 \\ \widehat{u''v''} & \widehat{v''v''} + \frac{1}{2\overline{\sigma}}\overline{\zeta'^2} & 0 \\ \frac{1}{\overline{\sigma}}\overline{\zeta' m'_x} & \frac{1}{\overline{\sigma}}\overline{\zeta' m'_y} & 0 \end{pmatrix},$$

Eddy forces: $\nabla \cdot \mathbf{E}$

Fluctuation around TWA:

$$u'' = u - \hat{u}$$

Fluctuations around EA:

$$\zeta' = \zeta - \overline{\zeta}$$

Components:

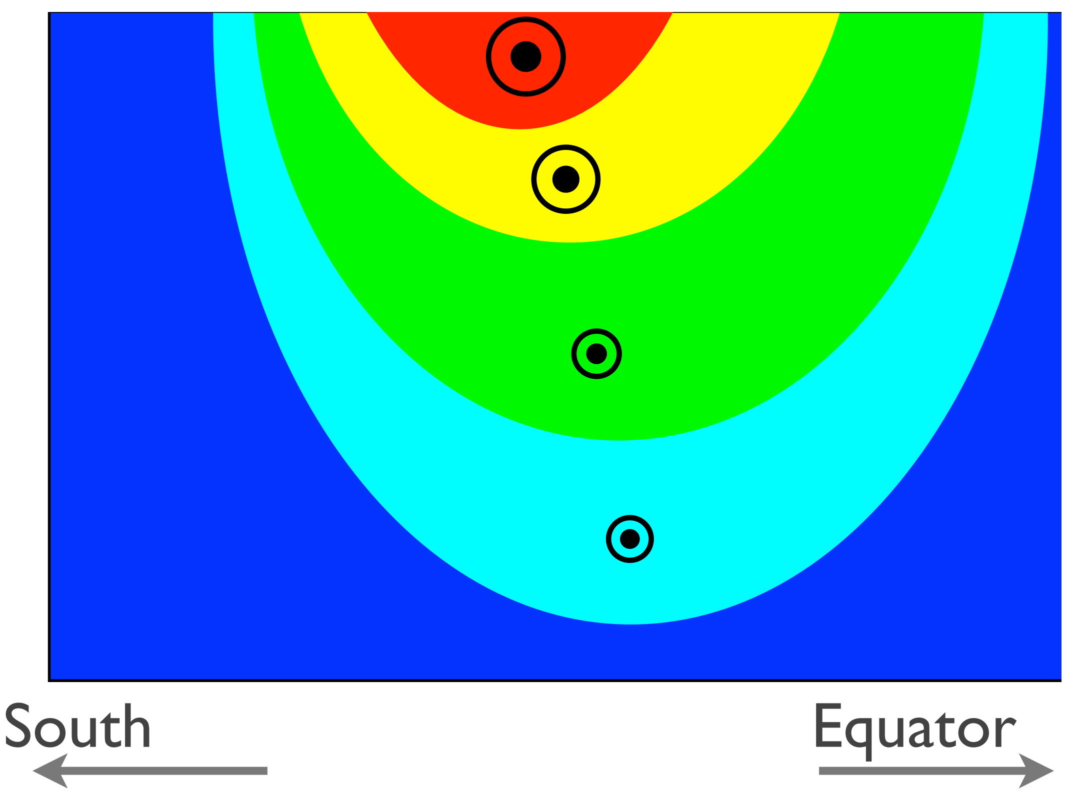
- Reynolds stresses
- eddy potential energy
- eddy form drag, or eddy buoyancy fluxes

Parameterizing the EPFT

Ignore
- Reynolds stresses, and
- eddy potential energy.

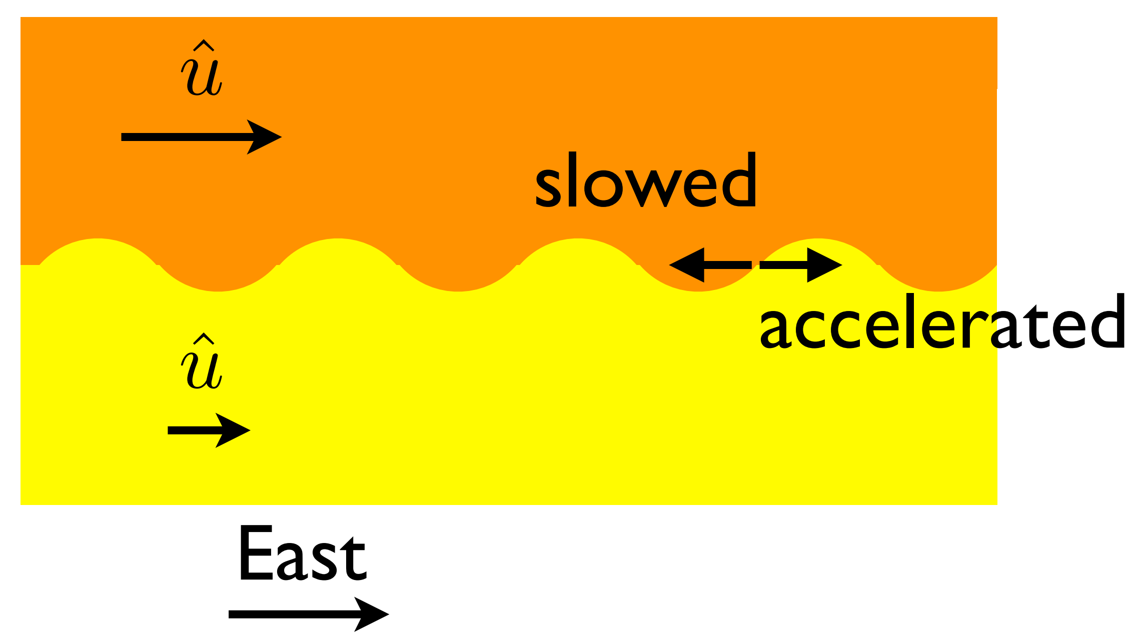
Eddy forces: $\nabla \cdot \mathbf{E}$

$$\mathbf{E} = \begin{pmatrix} \cancel{\widehat{u''u''}} + \cancel{\frac{1}{2\overline{\sigma}}\overline{\zeta'^2}} & \cancel{\widehat{u''v''}} & 0 \\ \cancel{\widehat{u''v''}} & \cancel{\widehat{v''v''}} + \cancel{\frac{1}{2\overline{\sigma}}\overline{\zeta'^2}} & 0 \\ \frac{1}{\overline{\sigma}}\overline{\zeta' m'_x} & \frac{1}{\overline{\sigma}}\overline{\zeta' m'_y} & 0 \end{pmatrix},$$



Greatbatch and Lamb 1990

$$\frac{\overline{\zeta' m'_x}}{\overline{\sigma}} = -\mu \frac{\partial \hat{u}}{\partial z}$$



Parameterizing the EPFT

We can derive GM:

$$\frac{\overline{\zeta' m'_x}}{\overline{\sigma}} = -\mu \frac{\partial \hat{u}}{\partial z}$$

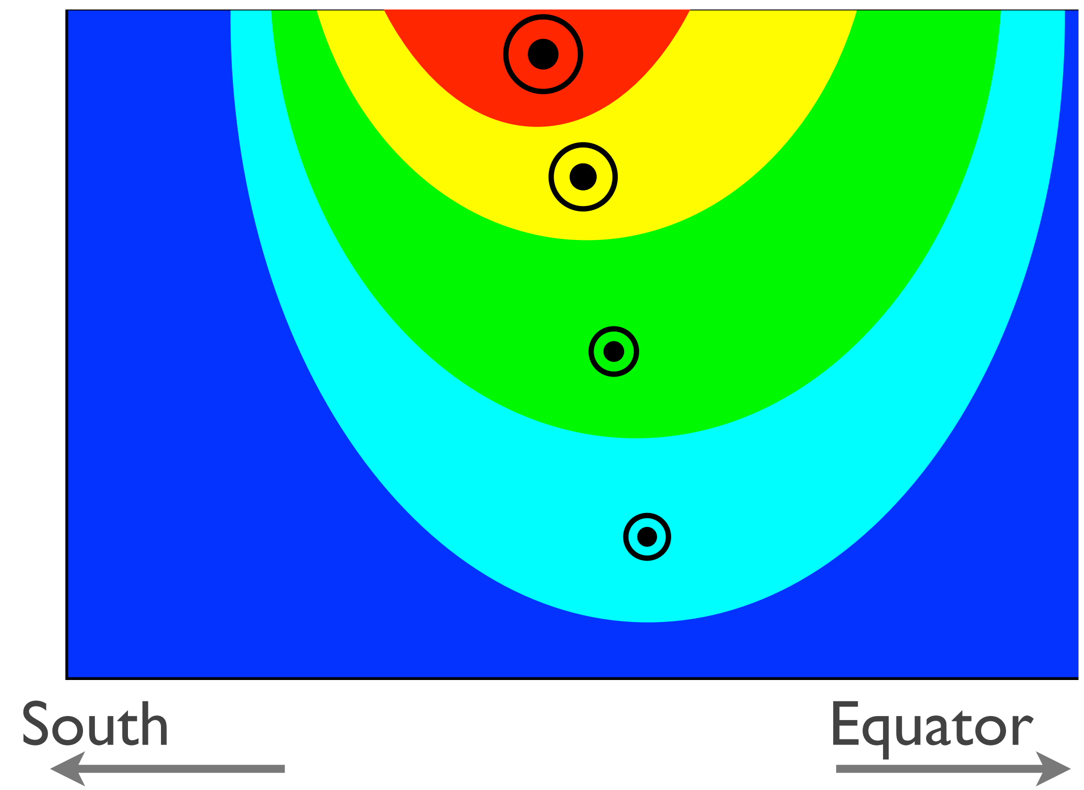
with thermal wind, eddy forces $\nabla \cdot \mathbf{E}$

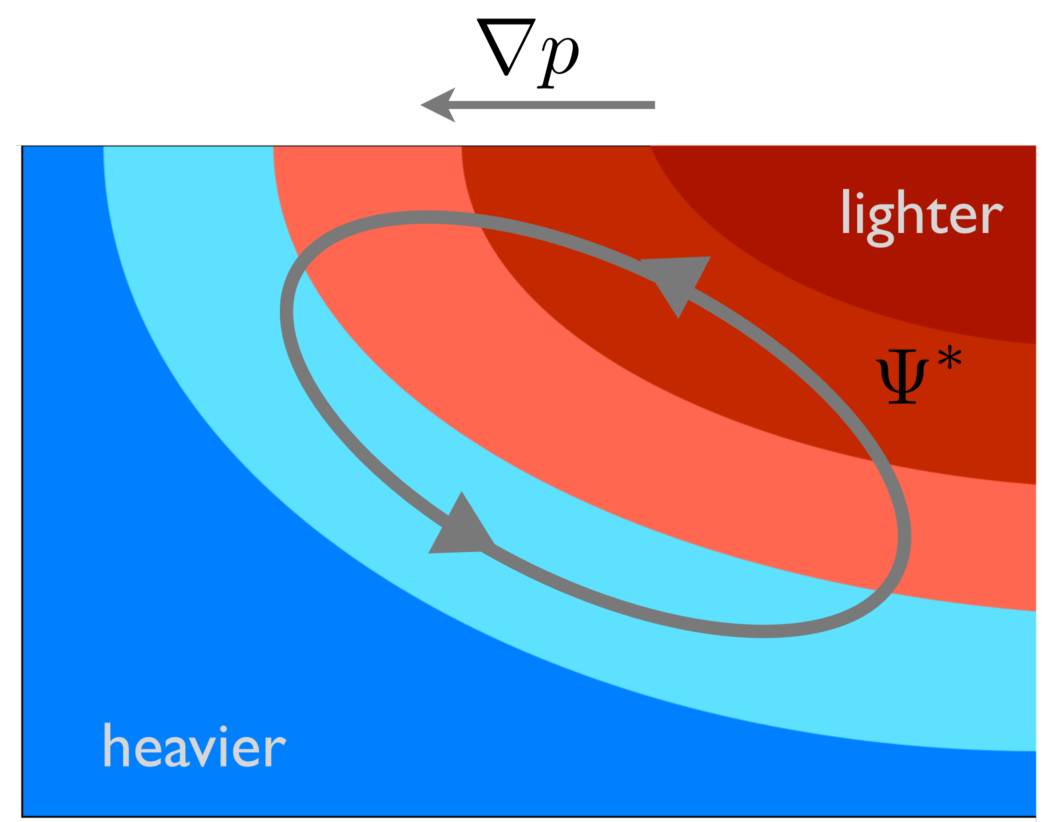
$$\frac{\partial}{\partial z}\left(\frac{1}{\overline{\sigma}}\overline{\zeta' m'_x}\right) = \frac{1}{f}\frac{\partial}{\partial z}\left(\mu \overline{N}^2 \frac{\rho^\sharp_y}{\rho^\sharp_z}\right)$$

scaling form drag and using $\mu = A\dfrac{f^2}{\overline{N}^2}$

$$f\frac{\overline{\sigma' v'}}{\overline{\sigma}} = f v_* \qquad \text{TWA MPAS-O}$$

$$\mathbf{u}_* = -\frac{\partial}{\partial z}(A\,\mathbf{S}) \qquad \text{CNV MPAS-O}$$





Parameterizing the EPFT

From the TWA equations, we derived the GM parameterization.

Vertical eddy viscocity: $\mu = \kappa \frac{f^2}{\overline{N}^2}$

Greatbatch and Lamb 1990: $\mu = A \frac{f^2}{\overline{N}^2}$

$$\Rightarrow \quad A = \kappa$$

From this perspective, the parameterizations by Greatbatch and Lamb 1990 and GM are the same:

➡ A parameterization of eddy form drag.

GM is not used in the TWA framework.

So how are the governing equations in the conventional framework related to the TWA equations?

How are TWA and CNV frameworks related?

$$\hat{u} = \overline{u} + u_*$$

- ignore inertial terms associated with u_*

TWA equations

$$\frac{D^\sharp \hat{u}}{Dt} - f\hat{v} + \frac{\partial p^\sharp}{\partial x} + \nabla \cdot \mathbf{E}^u = \hat{R}_x,$$

$$\frac{D^\sharp \hat{v}}{Dt} + f\hat{u} + \frac{\partial p^\sharp}{\partial y} + \nabla \cdot \mathbf{E}^v = \hat{R}_y,$$

$$\frac{\partial p^\sharp}{\partial z} = b^\sharp,$$

$$\frac{\partial \hat{u}}{\partial x} + \frac{\partial \hat{v}}{\partial y} + \frac{\partial w^\sharp}{\partial z} = 0,$$

$$\frac{D^\sharp b^\sharp}{Dt} = 0$$

Eulerian average Boussinesq equations

$$\frac{D\overline{u}}{Dt} - f\overline{v} + \frac{\partial p^\sharp}{\partial x} = R_x,$$

$$\frac{D\overline{v}}{Dt} + f\overline{u} + \frac{\partial p^\sharp}{\partial y} = R_y,$$

$$\frac{\partial p^\sharp}{\partial z} = b^\sharp,$$

$$\frac{\partial \hat{u}}{\partial x} + \frac{\partial \hat{v}}{\partial y} + \frac{\partial \hat{w}^\sharp}{\partial z} = 0,$$

$$\frac{D^\sharp b^\sharp}{Dt} = 0,$$

$$\frac{D^\sharp \overline{\phi}}{Dt} = R_\phi.$$

But the conventional framework still has eddy form drag forces.
This becomes clear if we look at Ertel Potential Vorticity ...

How are TWA and CNV frameworks related?

Ertel Potential Vorticity: $$\Pi^\sharp = \frac{f + \frac{\partial \hat{v}}{\partial \tilde{x}} - \frac{\partial \hat{u}}{\partial \tilde{y}}}{\overline{\sigma}}$$

TWA framework

$$\frac{D^\sharp \Pi^\sharp}{Dt} = -\nabla \cdot \mathbf{F}^\sharp - \nabla \cdot \Gamma^\sharp \xrightarrow{\text{parameterize eddy forces, adiabatic interior}} \frac{D^\sharp \Pi^\sharp}{Dt} = \nabla \cdot \left(f \mathbf{u}_* \overline{N}^2 \right)$$

vertical component of the curl of the eddy forces

Conventional framework

$$\frac{D^\sharp \Pi^\sharp}{Dt} = \frac{D^\sharp}{Dt}(\omega_* \overline{\sigma}^{-1}) + \nabla \cdot (\mathbf{u}_* \Pi^\sharp) - \nabla \cdot (\mathbf{u}_* \omega_* \overline{\sigma}^{-1}) + \nabla \cdot \frac{\omega_*}{\overline{\sigma}} \overline{\mathbf{e}}_\mathbf{3} - \nabla \cdot \Gamma,$$

parameterize eddies with GM,
adiabatic interior,
small inertial terms $\mathbf{u}_*$

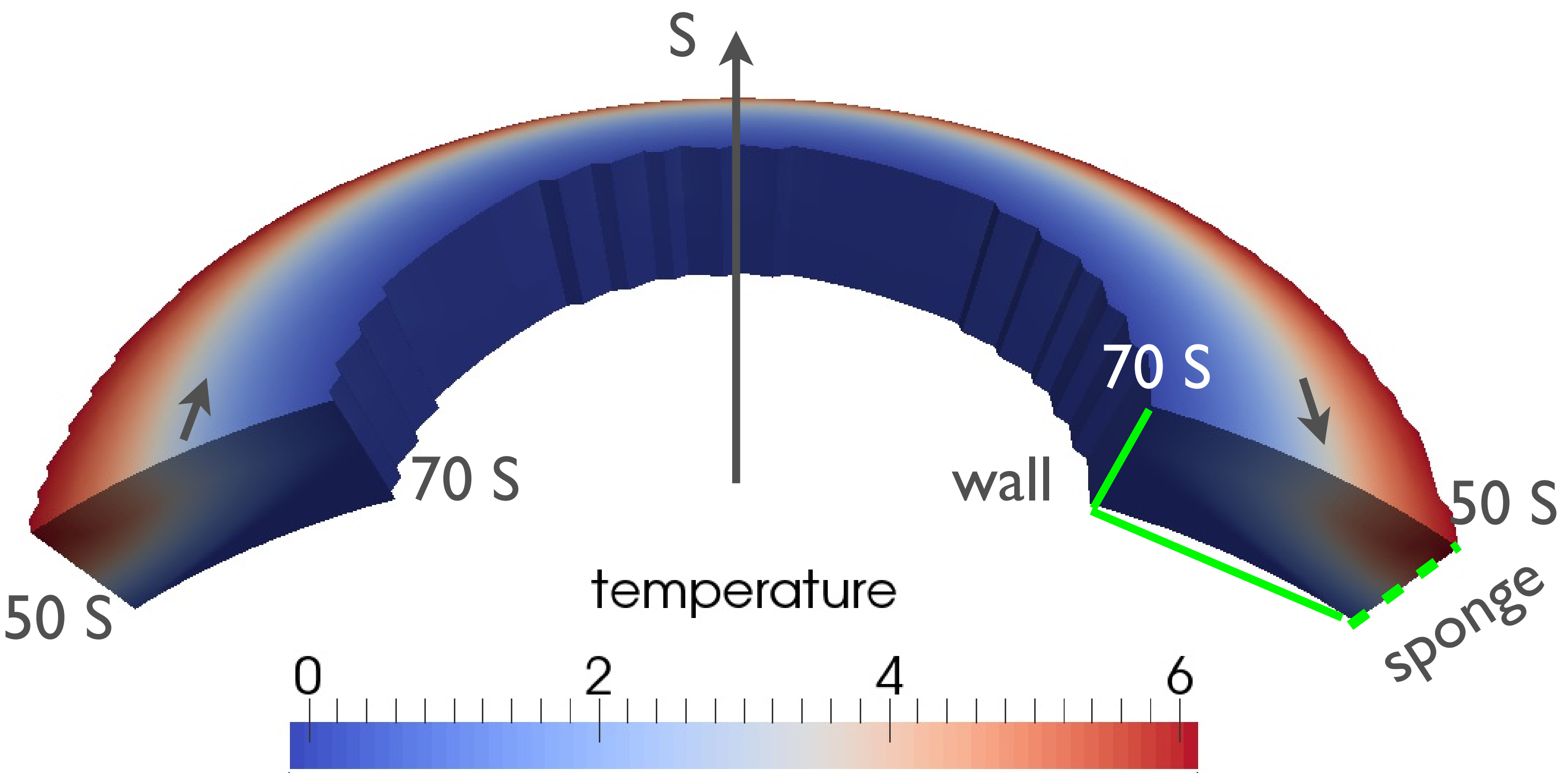
$$\longrightarrow \frac{D^\sharp \Pi^\sharp}{Dt} = \nabla \cdot \left(f \mathbf{u}_* \overline{N}^2 \right)$$

eddy forces

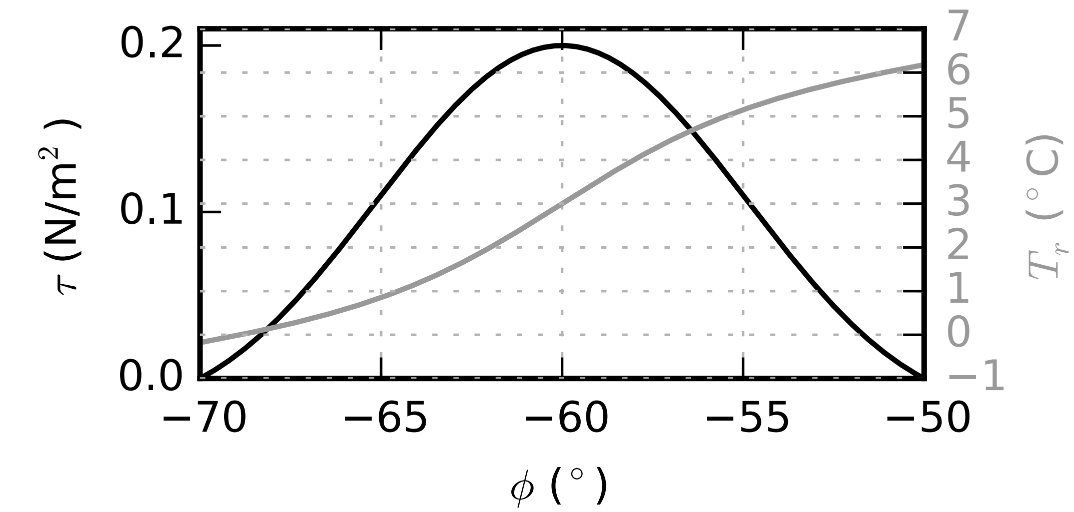
imposing advection by bolus velocity has an eddy force effect

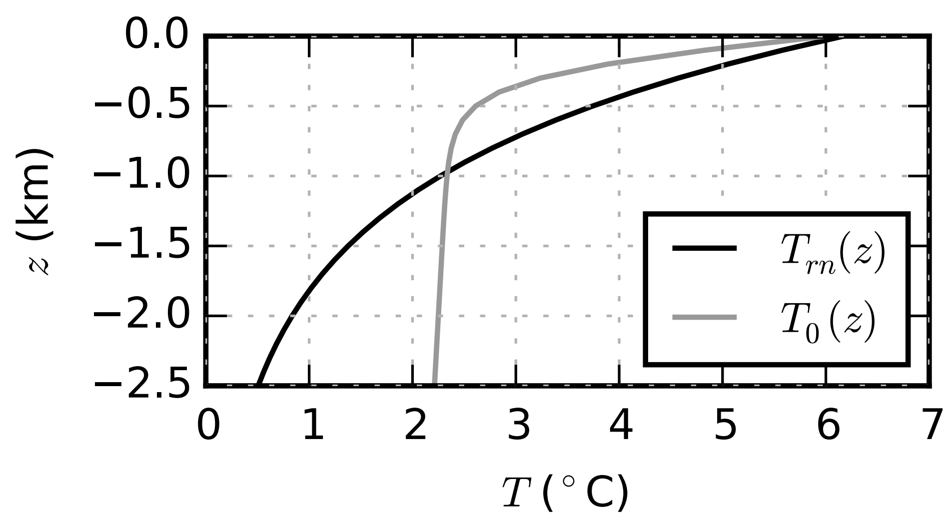
boils down to: ignore inertial terms associated with $\mathbf{u}_*$

Idealized Southern Ocean Testbed



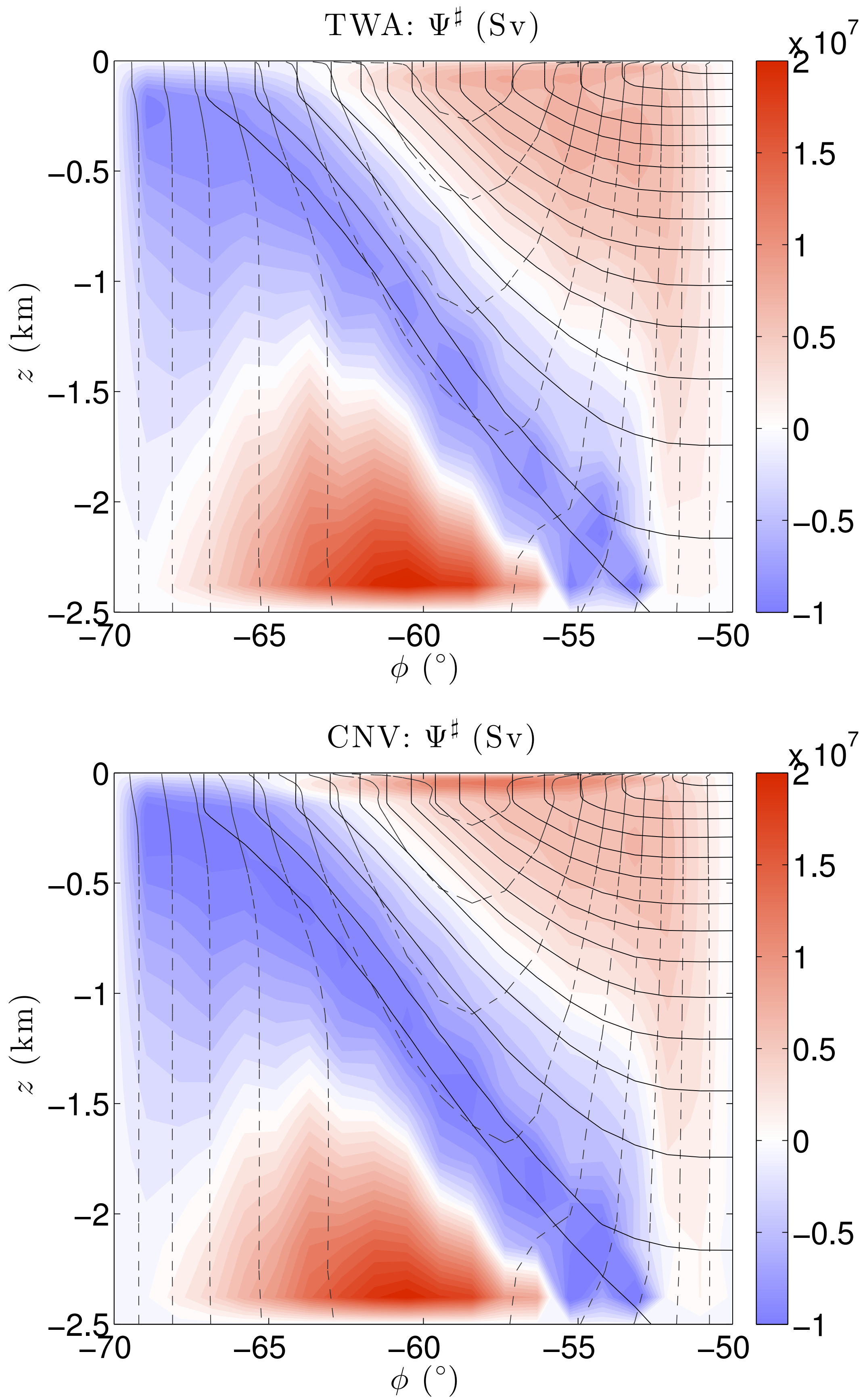
Southern Ocean Surface Climatology



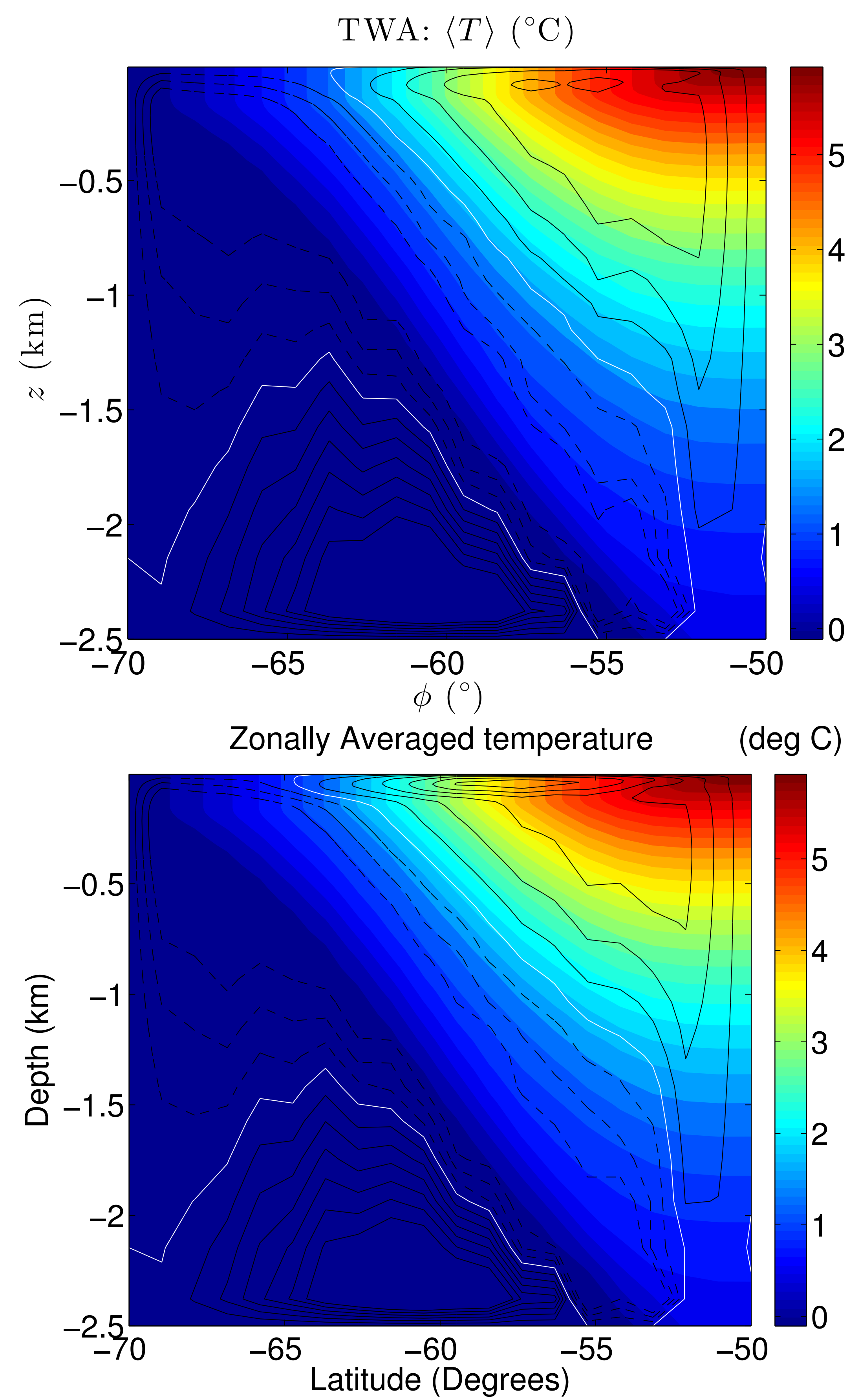


- Prescribed wind stress and temperatures at the surface and north wall.
- Uniform depth of 2.5 km.
- 120 km resolution, 40 layers.
- GM kappa = 1200 m^2/s
- We run the simulations for 300 years.
- We will look at zonally averaged quantities.

Overturning Circulation



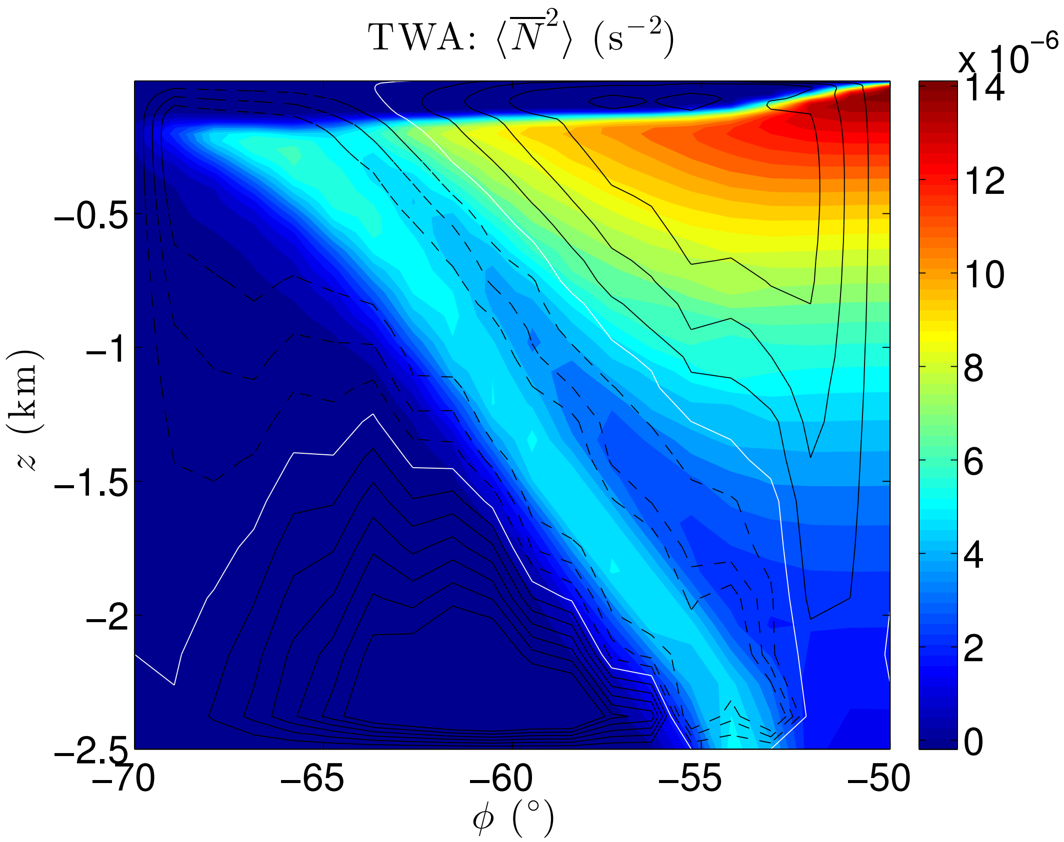
Temperature Field

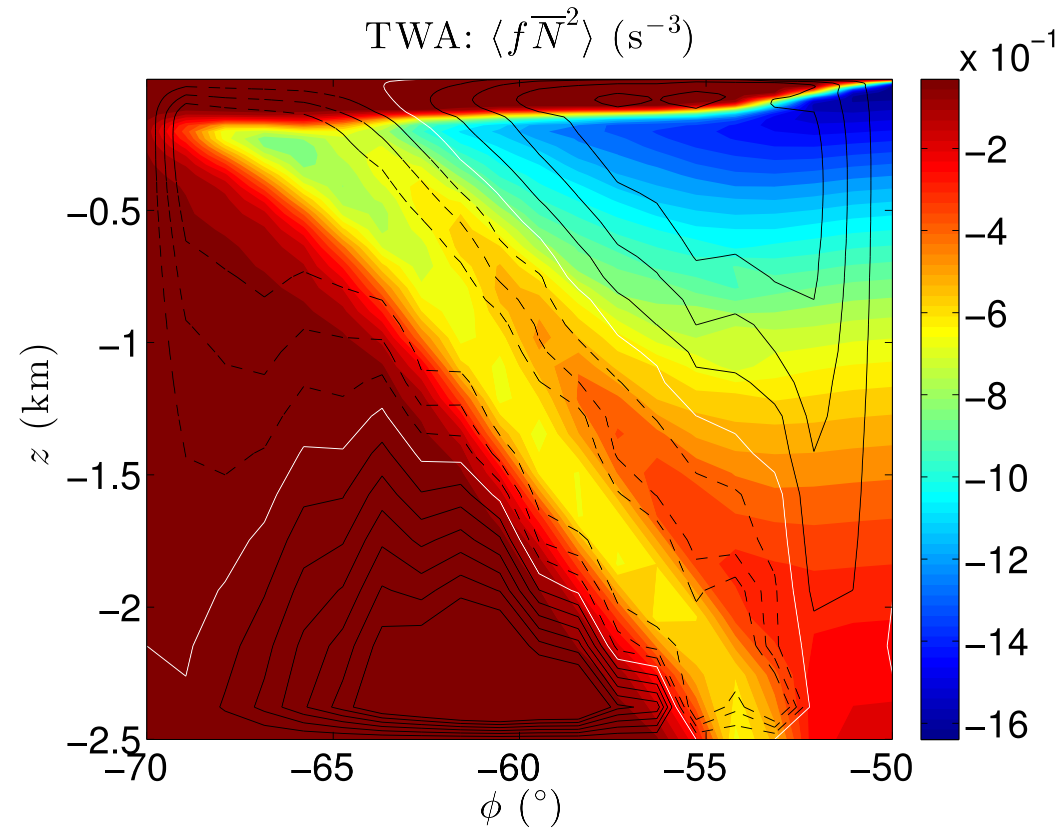


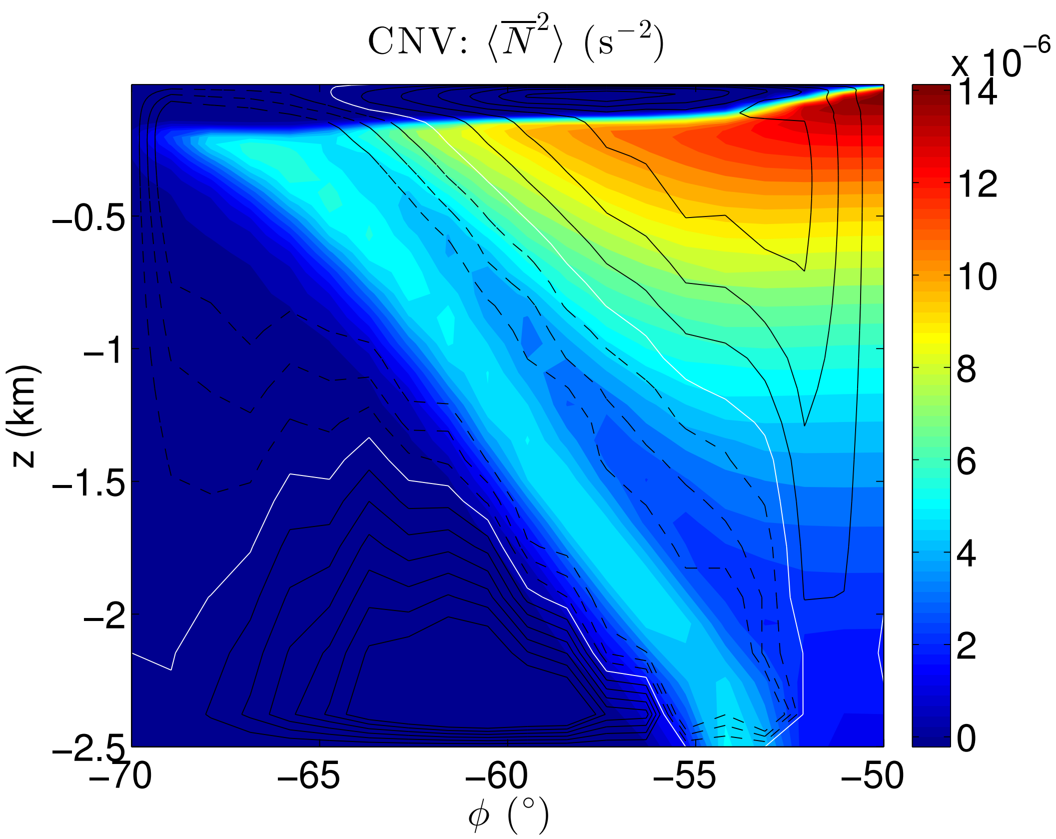
Stratification

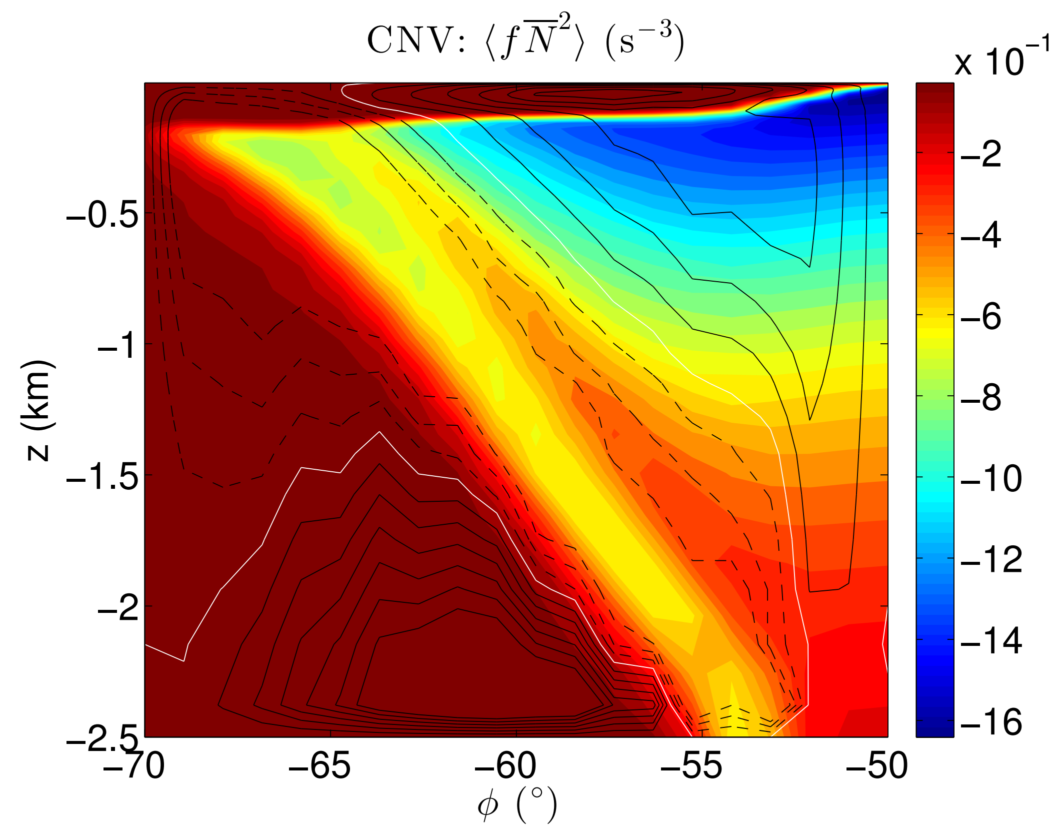
EPV

$$\Pi^\sharp = \frac{f + \frac{\partial \hat{v}}{\partial \tilde{x}} - \frac{\partial \hat{u}}{\partial \tilde{y}}}{\overline{\sigma}}$$

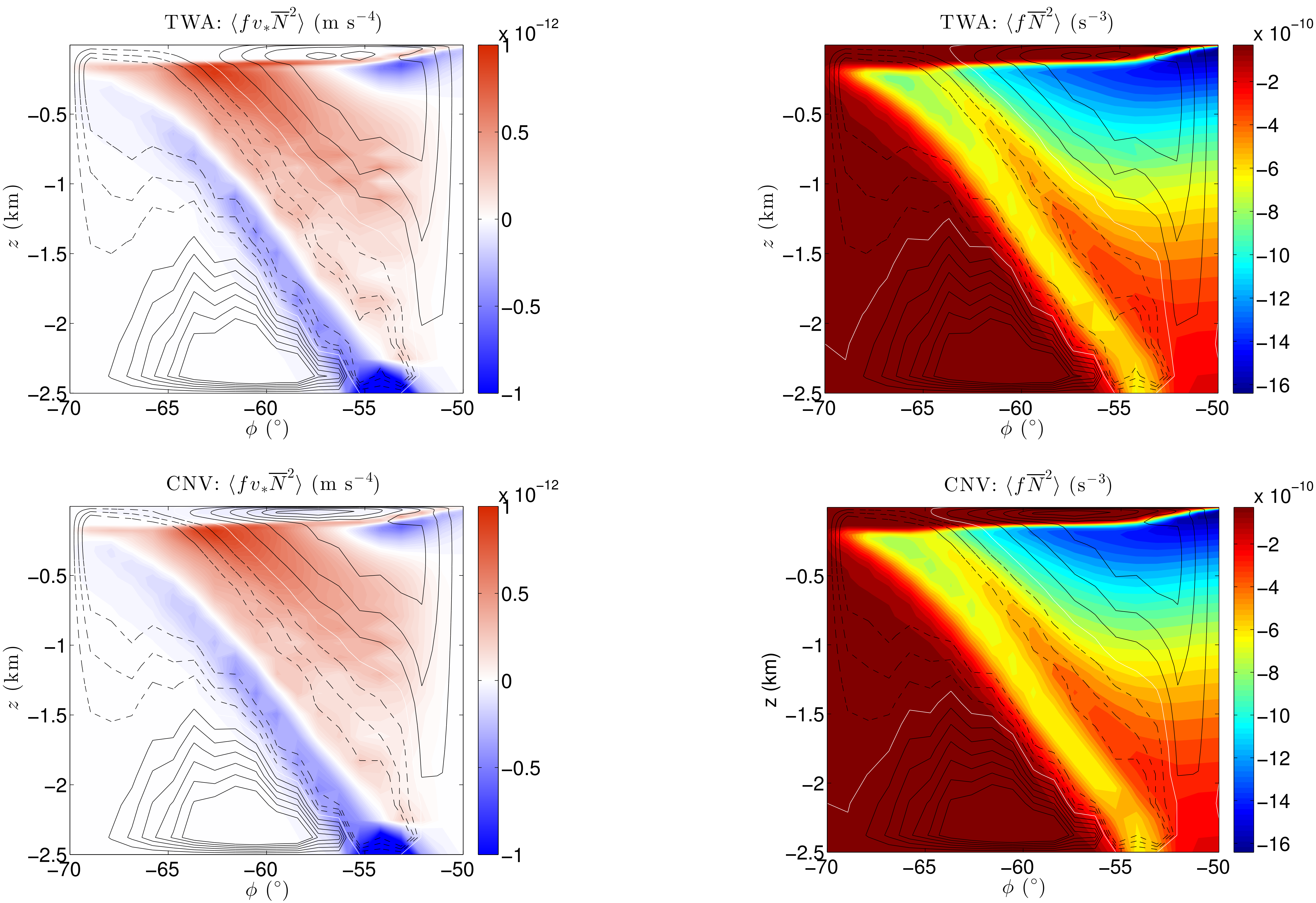




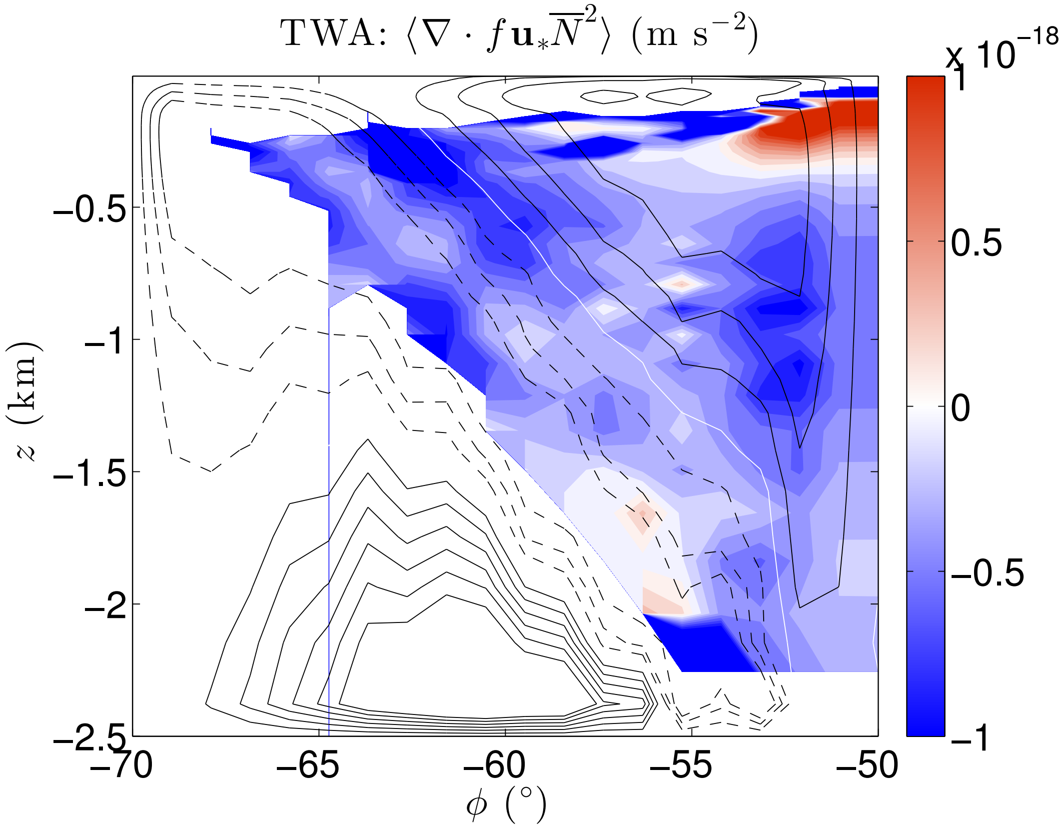


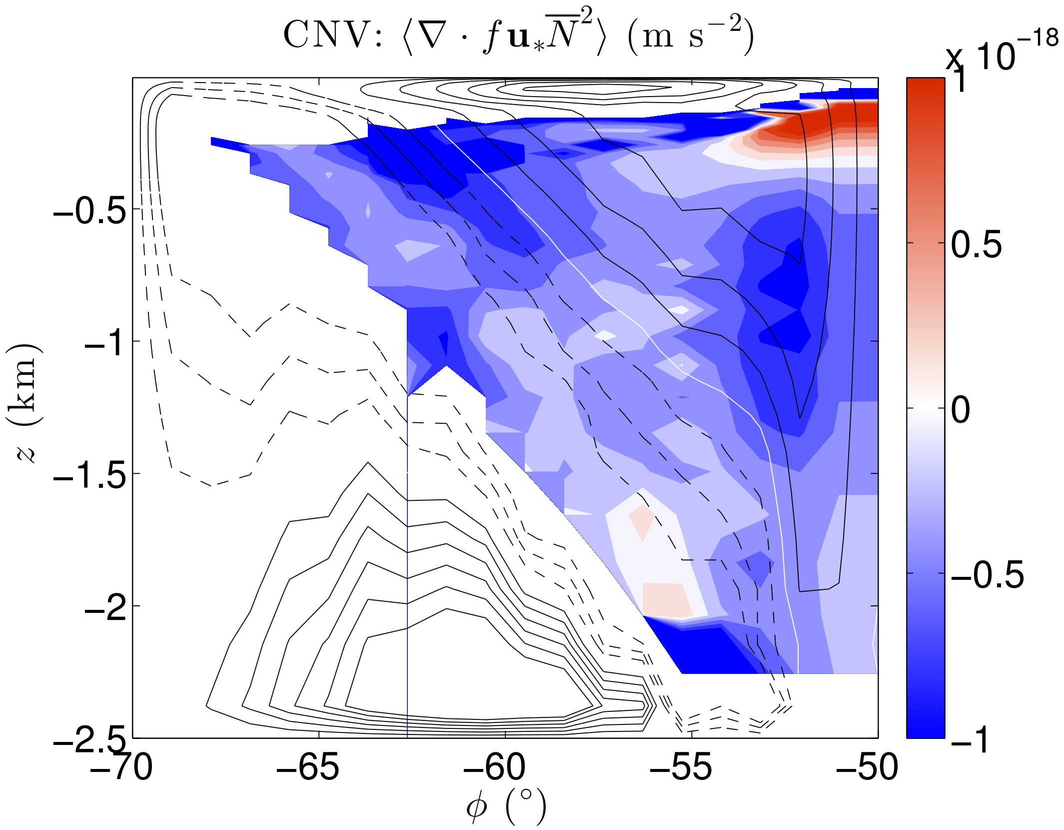


EPV flux by eddy forces $\frac{D^\sharp \Pi^\sharp}{Dt} = \nabla \cdot \left(f \mathbf{u}_* \overline{N}^2 \right)$ EPV $\Pi^\sharp = \frac{f + \frac{\partial \hat{v}}{\partial \tilde{x}} - \frac{\partial \hat{u}}{\partial \tilde{y}}}{\overline{\sigma}}$



EPV tendency by forces

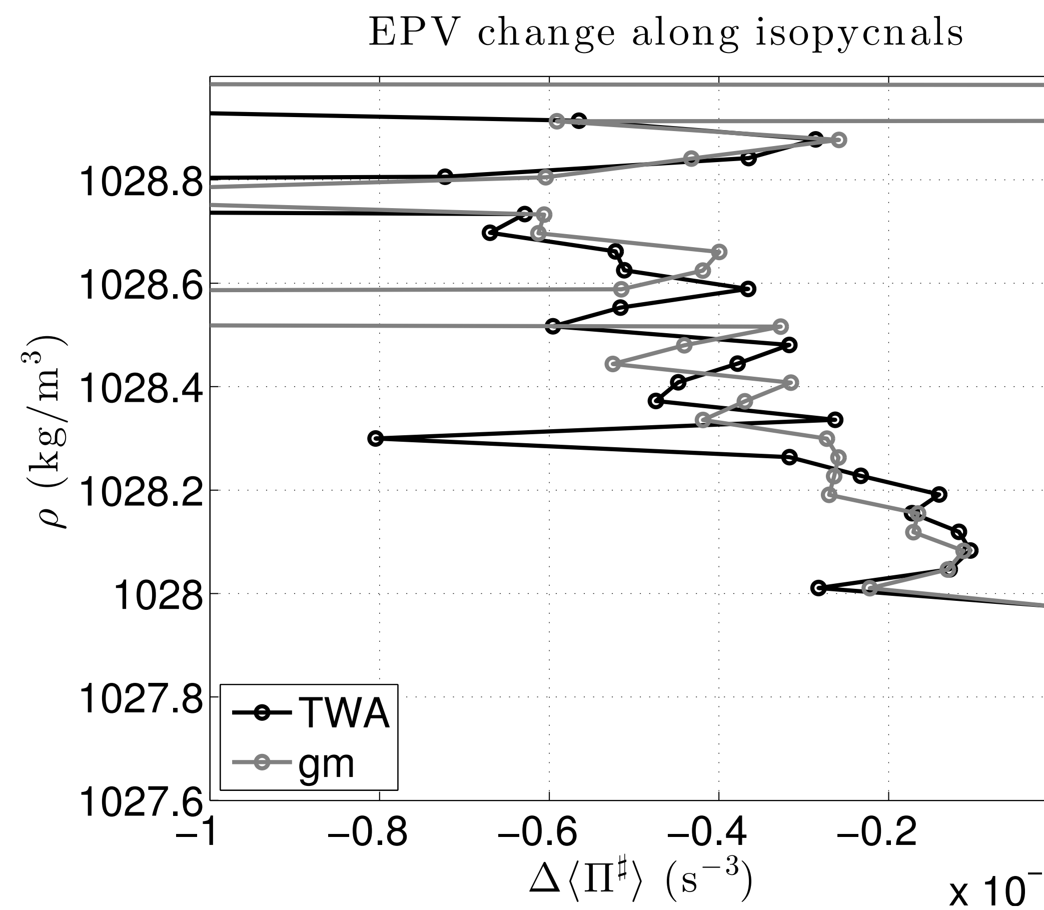




$$\frac{D^\sharp \Pi^\sharp}{Dt} = \nabla \cdot \left(f \mathbf{u}_* \overline{N}^2 \right)$$

Along isopycnals, track changes in EPV induced by eddy forces:

- Starting at -55 degrees South
- ending at ~300 m depth.



Good match in changes in EPV induced by eddy forces.

Summary

MPAS-O

2 frameworks:	TWA	Relation	Conventional
Momentum Advection:	residual velocity	↔	eulerian velocity
Tracer Advection:	residual velocity	↔	residual velocity
Eddy parameterization:	eddy form drag GL 1990	↔	bolus velocity GM 1990

- We made connections between the TWA framework and the conventional framework:
  - we showed how the GM parameterization results from parameterizing eddy form stress in the EPFT,
  - we showed how the conventional framework results from simplifications to the TWA framework.
- From this perspective, the GM parameterization is a model for eddy interfacial form drag in which horizontal momentum is transferred vertically.
- *We have a roadmap to develop eddy parameterizations (GM, etc).*

Auxiliary slides

Parameterizing the EPFT

Ignore

- Reynolds stresses, and
- eddy potential energy.

$$\mathbf{E} = \begin{pmatrix} \cancel{\widehat{u''u''}} + \cancel{\frac{1}{2\overline{\sigma}}\overline{\zeta'^2}} & \cancel{\widehat{u''v''}} & 0 \\ \cancel{\widehat{u''v''}} & \cancel{\widehat{v''v''}} + \cancel{\frac{1}{2\overline{\sigma}}\overline{\zeta'^2}} & 0 \\ \frac{1}{\overline{\sigma}}\overline{\zeta' m'_x} & \frac{1}{\overline{\sigma}}\overline{\zeta' m'_y} & 0 \end{pmatrix},$$

Eddy forces: $\nabla \cdot \mathbf{E}$

Parameterize eddy form drag as downward flux of horizontal momentum:

$$\frac{\overline{\zeta' m'_x}}{\overline{\sigma}} = -\mu \frac{\partial \hat{u}}{\partial z},$$
$$\frac{\overline{\zeta' m'_y}}{\overline{\sigma}} = -\mu \frac{\partial \hat{v}}{\partial z}.$$

$$\frac{\partial}{\partial z}\left(\frac{1}{\overline{\sigma}}\overline{\zeta' m'_x}\right) = \frac{1}{f}\frac{\partial}{\partial z}\left(\mu \overline{N}^2 \frac{\overline{\rho}_y}{\overline{\rho}_z}\right),$$
$$\frac{\partial}{\partial z}\left(\frac{1}{\overline{\sigma}}\overline{\zeta' m'_y}\right) = -\frac{1}{f}\frac{\partial}{\partial z}\left(\mu \overline{N}^2 \frac{\overline{\rho}_x}{\overline{\rho}_z}\right).$$

eddy forces

Greatbatch and Lamb 1990:

$$\mu = A\frac{f^2}{\overline{N}^2}$$

Parameterizing the EPFT

Alternatively, assume geostrophic balance:

$$-f\hat{v} + \frac{\partial p^\sharp}{\partial x} + \frac{\partial}{\partial z}\left(\frac{1}{\overline{\sigma}}\overline{\zeta' m'_x}\right) = 0,$$

$$f\hat{u} + \frac{\partial p^\sharp}{\partial y} + \frac{\partial}{\partial z}\left(\frac{1}{\overline{\sigma}}\overline{\zeta' m'_y}\right) = 0.$$

Decomposing the residual velocities as $\hat{u} = \overline{u} + \frac{\overline{\sigma' u'}}{\overline{\sigma}}$

the eddy forces are:

$$\frac{\partial}{\partial z}\left(\frac{1}{\overline{\sigma}}\overline{\zeta' m'_x}\right) = f\frac{\overline{\sigma' v'}}{\overline{\sigma}} \equiv f v_*,$$

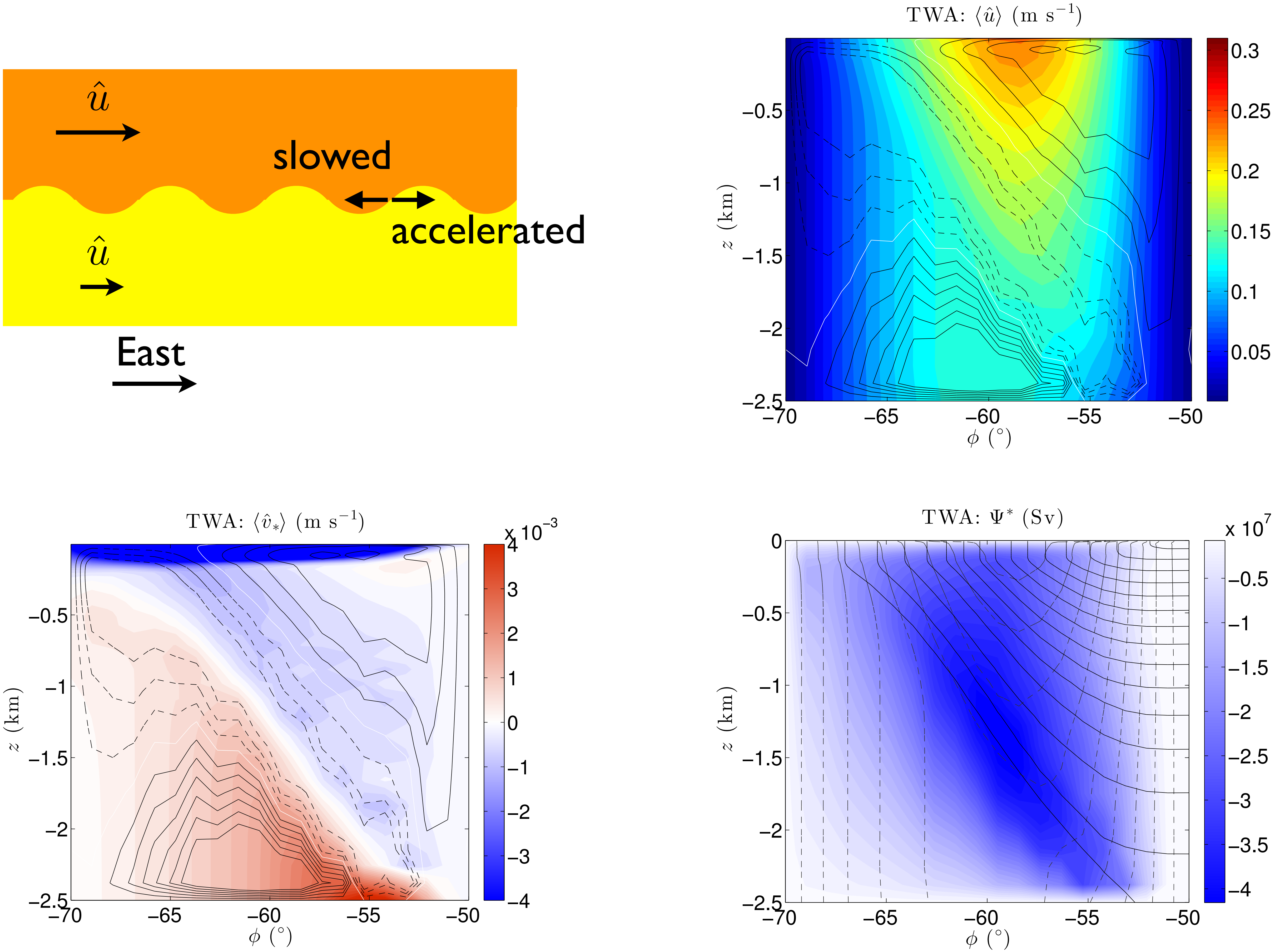
$$\frac{\partial}{\partial z}\left(\frac{1}{\overline{\sigma}}\overline{\zeta' m'_y}\right) = -f\frac{\overline{\sigma' u'}}{\overline{\sigma}} \equiv -f u_*.$$

compare to:

$$= \frac{1}{f}\frac{\partial}{\partial z}\left(\mu \overline{N}^2 \frac{\overline{\rho}_y}{\overline{\rho}_z}\right),$$

$$= -\frac{1}{f}\frac{\partial}{\partial z}\left(\mu \overline{N}^2 \frac{\overline{\rho}_x}{\overline{\rho}_z}\right).$$

combine with $\mu = A\frac{f^2}{\overline{N}^2}$ $\Rightarrow$ $\mathbf{u}_* = -\frac{\partial}{\partial z}\left(A\,\mathbf{S}\right)$



$\hat{u}$
slowed
accelerated
$\hat{u}$
East
TWA: $\langle \hat{u} \rangle$ (m s^{-1})
z (km)
ϕ (°)
TWA: $\langle \hat{v}_* \rangle$ (m s^{-1})
x 10^{-3}
z (km)
ϕ (°)
TWA: Ψ^* (Sv)
x 10^{7}
z (km)
ϕ (°)

Developments and implementations in MPAS-O

- We have GM implemented, with Ferrari et al. 2010 boundary value tapering scheme.
- Implementation of the exact TWA prognostic equations is accomplished by a reinterpretation of the variables in the MPAS-O dynamical core, with
  - no modifications to the dynamical core,
  - no modifications to parameterizations,
  - addition of a tendency term in the momentum equation.
- It remains to be seen what subtleties exist in implementing the TWA framework for realistic ocean climate simulations, e.g.
  - bulk forcing
  - coupling and boundary conditions, etc.

Future work

• Implement a "prognostic kappa" parameterization of the EPFT, based on prognostic eddy energy.

• High resolution (~ 5km) Idealized Southern Ocean with topographic features.
▸ to inform and test the parameterization,
▸ to investigate eddy-mean flow interactions via EPFT around topographic features such as a ridge and a shelf break.