A Flexible Transient Subglacial Hydrology Model

February 10, 2016

CESM Land Ice Working Group Meeting

Aleah Sommers[1], Harihar Rajaram[1], Mathieu Morlighem[2]

[1] Department of Civil, Environmental and Architectural Engineering, University of Colorado at Boulder

[2] Department of Earth System Sciences, University of California, Irvine

1 Context

2 Overview of model equations

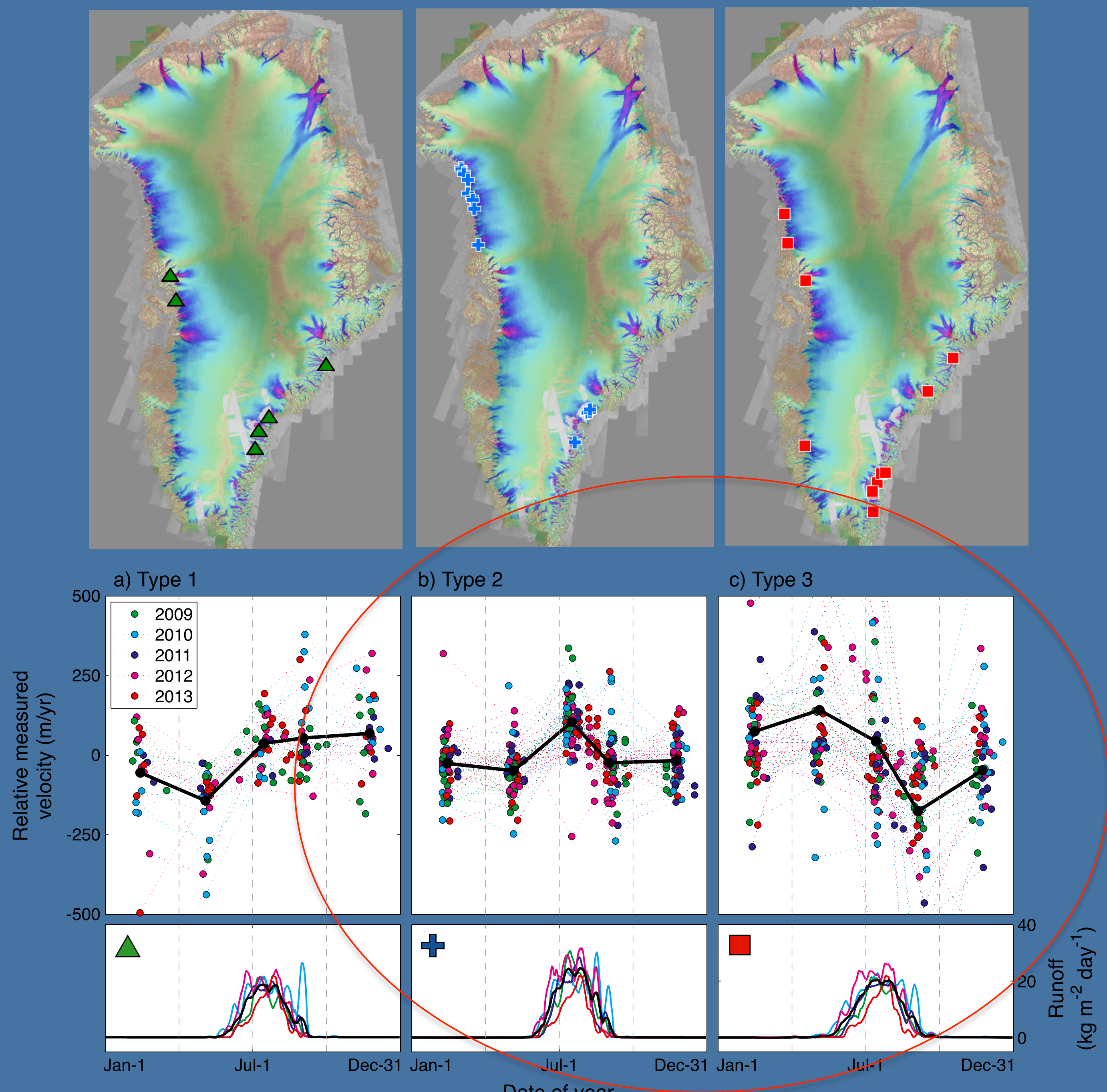
3 Model description

4 Sample results

5 Upcoming plans

Context

- Liquid water reaches the bed via moulins and the englacial system
- High water pressure at the bed causes increased basal sliding
- Geometry of the subglacial system evolves with water input and flow
- Subglacial hydrology may be key to understanding velocity signals and patterns that are not yet well understood



Moon et al. (2014), *Geophys. Res. Let.*

Overview of Model Equations

Continuity Equation (mass balance)

$$\frac{\partial b}{\partial t} + \nabla \cdot \vec{q} = \frac{\dot{m}}{\rho_w} + i_{e \to b}$$

b - subglacial gap height (m)
$\vec{q}$ - gap-integrated basal water flux ($m^2 s^{-1}$)
$\dot{m}$ - melt rate ($m\ s^{-1}$)
ρ_w - density of water ($kg\ m^{-3}$)
$i_{e \to b}$ - rate of input from englacial system to subglacial system ($m\ s^{-1}$)

Basal Gap Dynamics

$$\frac{\partial b}{\partial t} = \frac{\dot{m}}{\rho_i} - A|p_i - p_w|^{n-1}(p_i - p_w)b + \beta u_b$$

ρ_i - bulk density of ice (kg m^{-3})
A - flow law parameter (Pa^{-3} s^{-1})
p_i - ice overburden pressure (Pa)
p_w - water pressure (Pa)
n - flow law exponent (dimensionless, taken as 3)
β - parameter controlling average basal gap increase due to sliding (dimensionless)

Basal Gap Dynamics

$$\frac{\partial b}{\partial t} = \frac{\dot{m}}{\rho_i} - A|p_i - p_w|^{n-1}(p_i - p_w)b + \beta u_b$$

Opening by melt

Creep closure

Opening due to sliding over bumps

ρ_i - bulk density of ice (kg m^{-3})
A - flow law parameter (Pa^{-3} s^{-1})
p_i - ice overburden pressure (Pa)
p_w - water pressure (Pa)
n - flow law exponent (dimensionless, taken as 3)
β - parameter controlling average basal gap increase due to sliding (dimensionless)

Basal Water Flux
(approximate momentum equation)

- Permits either laminar or turbulent flow, depending on Reynolds' number

$$\vec{q} = \frac{-b^3 g}{12\nu(1 + \omega Re)} \nabla h$$

g - gravitational acceleration (m s^{-2})
ν - kinematic viscosity of water at 0 degrees C (m^2 s^{-1})
ω - parameter controlling transition to nonlinear resistance in basal system
Re - Reynolds number, $Re = \frac{|\vec{q}|}{\nu}$ (dimensionless)
h - hydraulic head (m), $h = \frac{p_w}{\rho_w g} + z_b$

Internal Melt Generation
(energy equation)

$$\dot{m} = \frac{1}{L}(G + \vec{u}_b \cdot \vec{\tau}_b - \rho_w g \vec{q} \cdot \nabla h)$$

L - latent heat of fusion for water (J kg^{-1})
G - geothermal flux (W m^{-2})

$\vec{\tau}_b$ - basal traction

Internal Melt Generation
(energy equation)

$$\dot{m} = \frac{1}{L}\left(G + \vec{u}_b \cdot \vec{\tau}_b - \rho_w g \vec{q} \cdot \nabla h\right)$$

Heat due to geothermal flux

Frictional heat from sliding

Heat generated by water flux

L - latent heat of fusion for water (J kg^{-1})
G - geothermal flux (W m^{-2})

$\vec{\tau}_b$ - basal traction

Combine the previous equations to form a nonlinear elliptic PDE in terms of hydraulic head (*h*):

$$\frac{\partial}{\partial x}[\frac{-b^3 g}{12\nu(1+\omega Re)}\frac{\partial h}{\partial x}] + \frac{\partial}{\partial y}[\frac{-b^3 g}{12\nu(1+\omega Re)}\frac{\partial h}{\partial y}] =$$

$$\dot{m}(\frac{1}{\rho_w} - \frac{1}{\rho_i}) + A|p_i - p_w|^{n-1}(p_i - p_w)b - \beta u_b + i_{e\to b}$$

h - hydraulic head (m), $h = \frac{p_w}{\rho_w g} + z_b$

Model Description

- Use a Picard iteration within each time-step to solve the elliptic PDE for the head distribution - handles the nonlinearity of Reynolds' number (*Re*) and flux (*q*), as well as RHS terms that depend on *h*

- Explicitly update the gap height geometry (*b*) from $\partial b / \partial t$ equation

Model Description

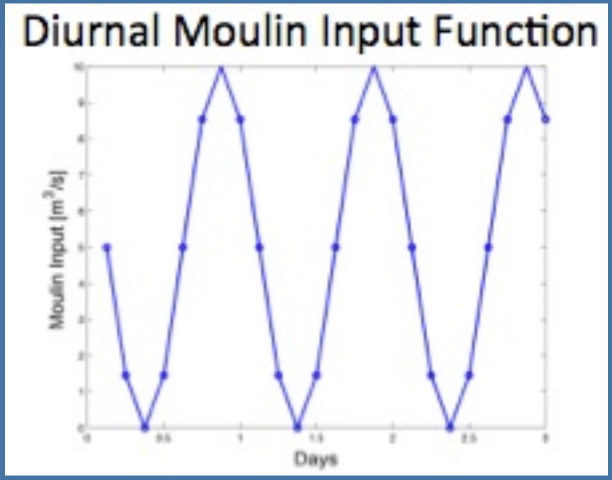
- Currently implemented as a stand-alone model in MATLAB and in the Ice Sheet System Model (ISSM)
- 2D finite element and finite volume formulations
- Channels are not prescribed or treated differently than the rest of the domain; they grow and decay naturally, as does any other gap geometry
- Can handle transient water inputs (seasonal, diurnal cycles)

Sample Test Results

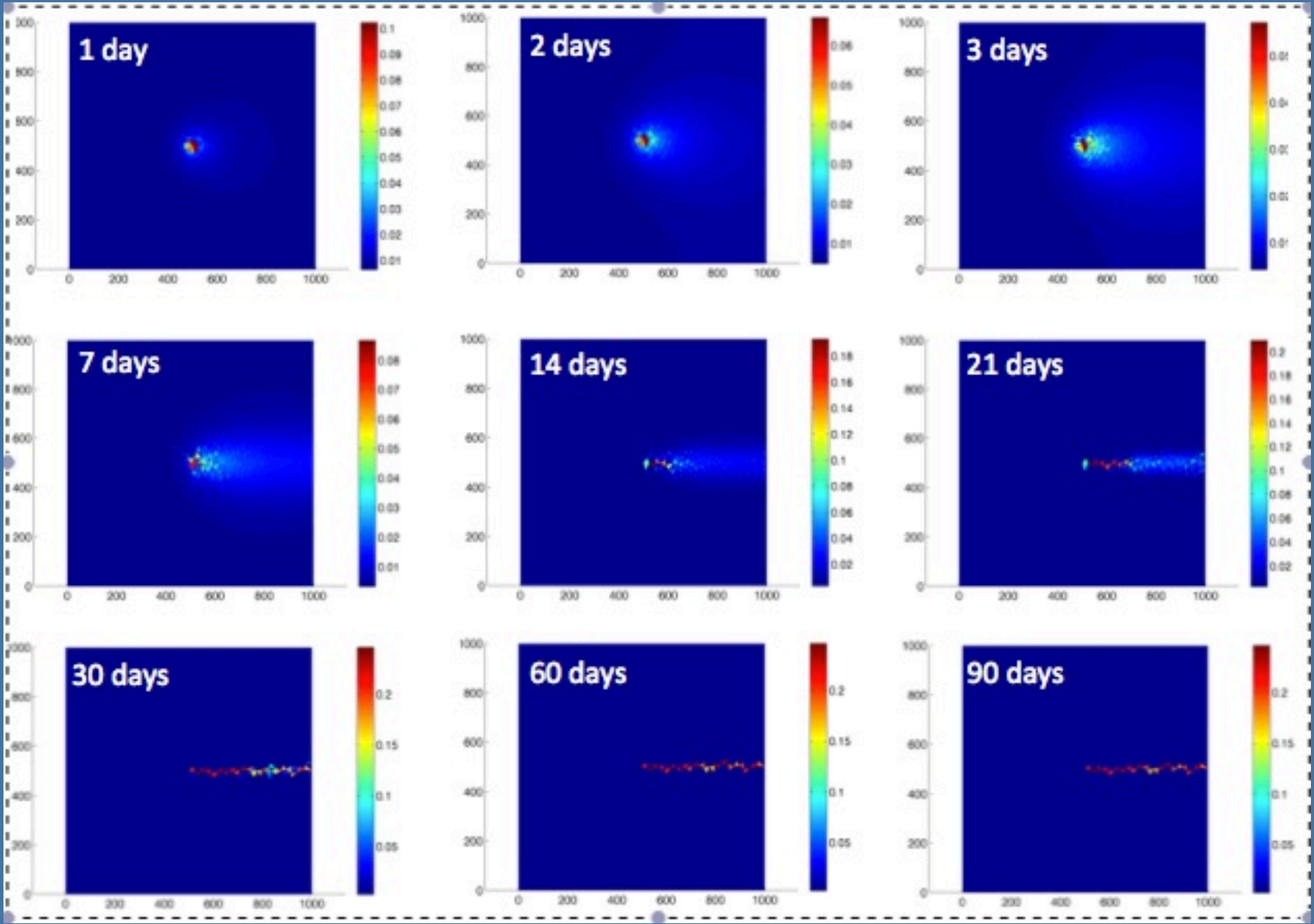
Sample Test Results

Case 1 – Single Moulin

- 1km x 1km square domain
- 300m-thick tilted slab
- Dirichlet boundary condition on right edge (atmospheric pressure), Neumann no-flux condition on top, left, bottom boundaries
- Constant uniform input from englacial system
- Single moulin at center of domain with diurnally-varying input for 90 days, time-step of 3 hours



Subglacial gap height evolution (m)

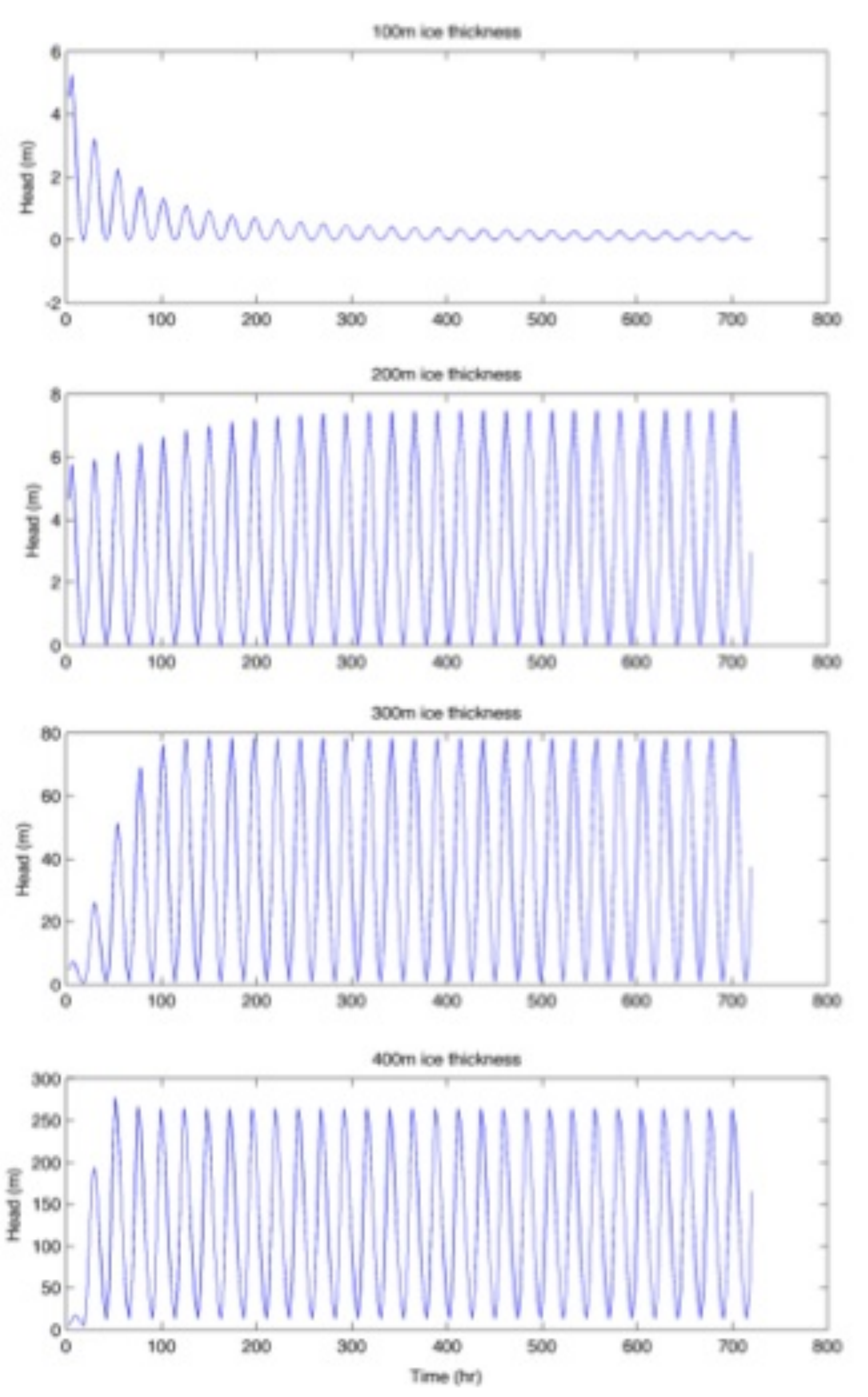


Case 2 - Effect of Ice Thickness

- No englacial input
- Diurnal moulin input at center (non-physical sinusoidal input with maximum 2.5 $m^3 s^{-1}$)

Amplitude of head fluctuations depends on ice thickness (with same input, thicker ice = larger head fluctuations)

Note that this type of sinusoidal moulin input is not physically realistic, but should be a coupled system related to storage.

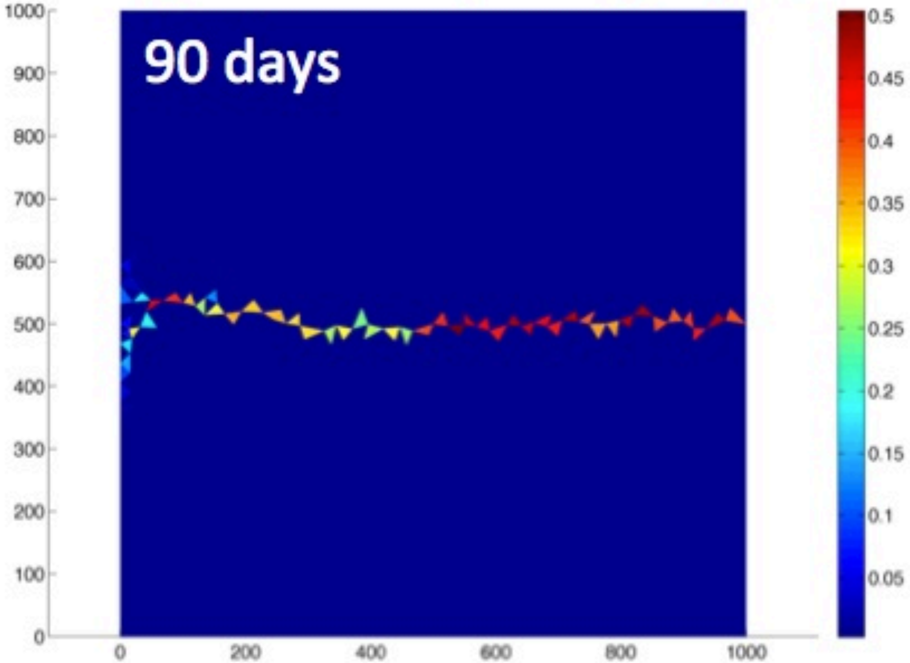


Sample Test Results

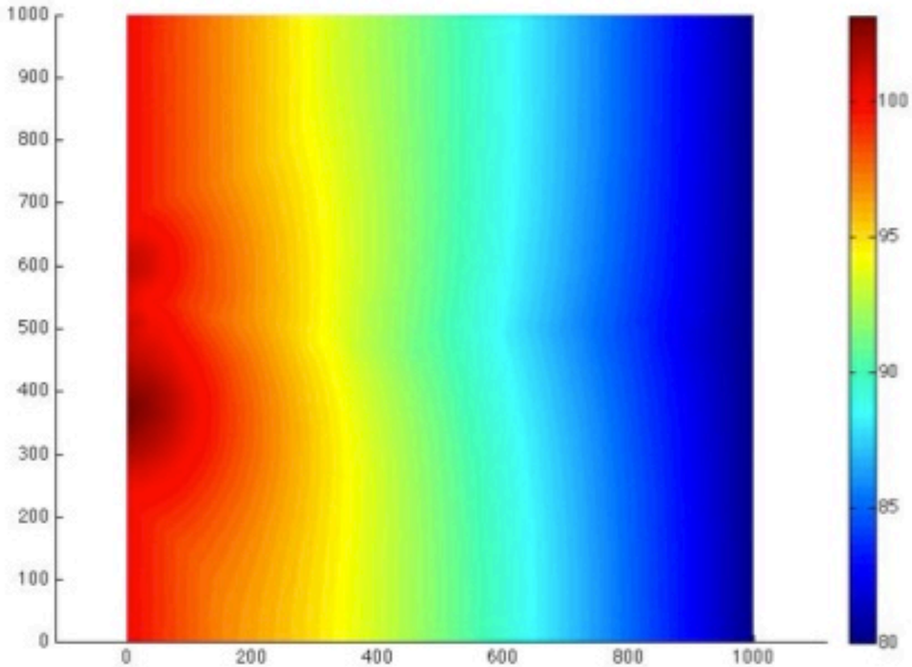
Case 3 – Moulin and Boundary Inflow

- Same as Case 1, but now with diurnally-varying inflow at left boundary (non-zero Neumann condition) from y=400 to 600m
- Useful capability for simulating only part of a drainage domain

Subglacial Gap Height (m)

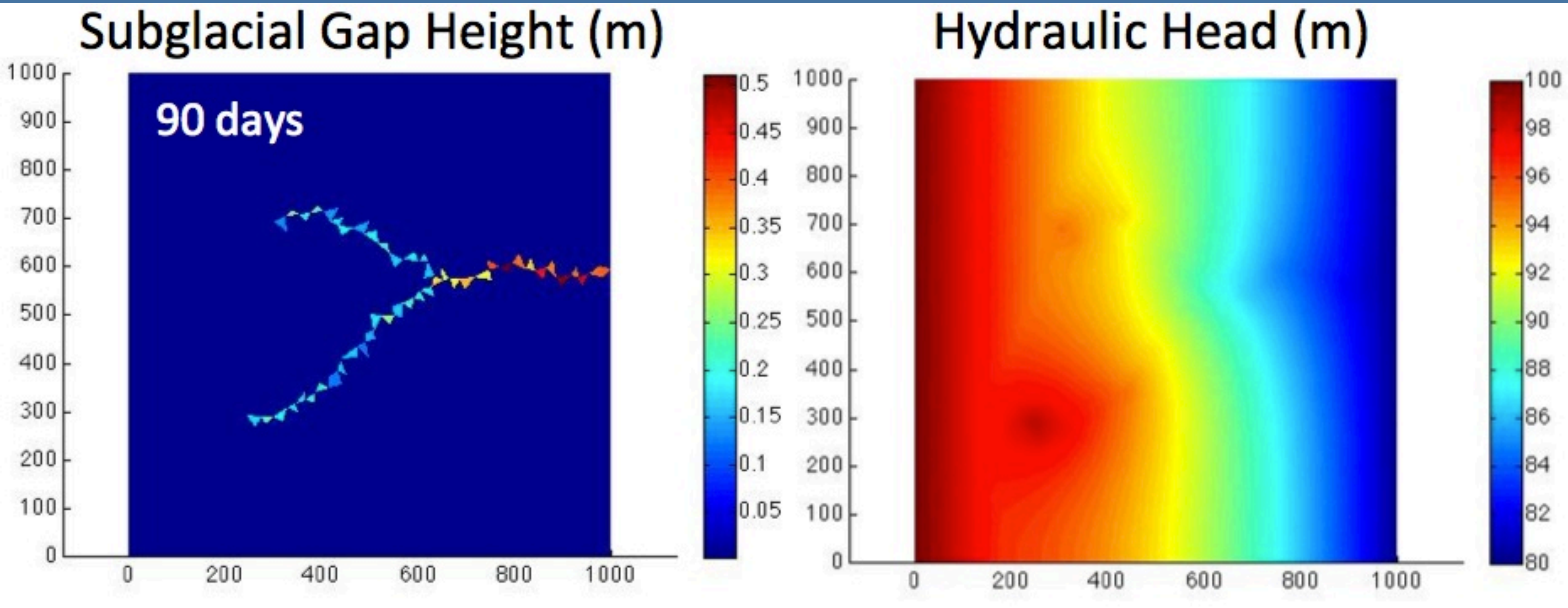


Hydraulic Head (m)



Case 4 - Multiple Moulins

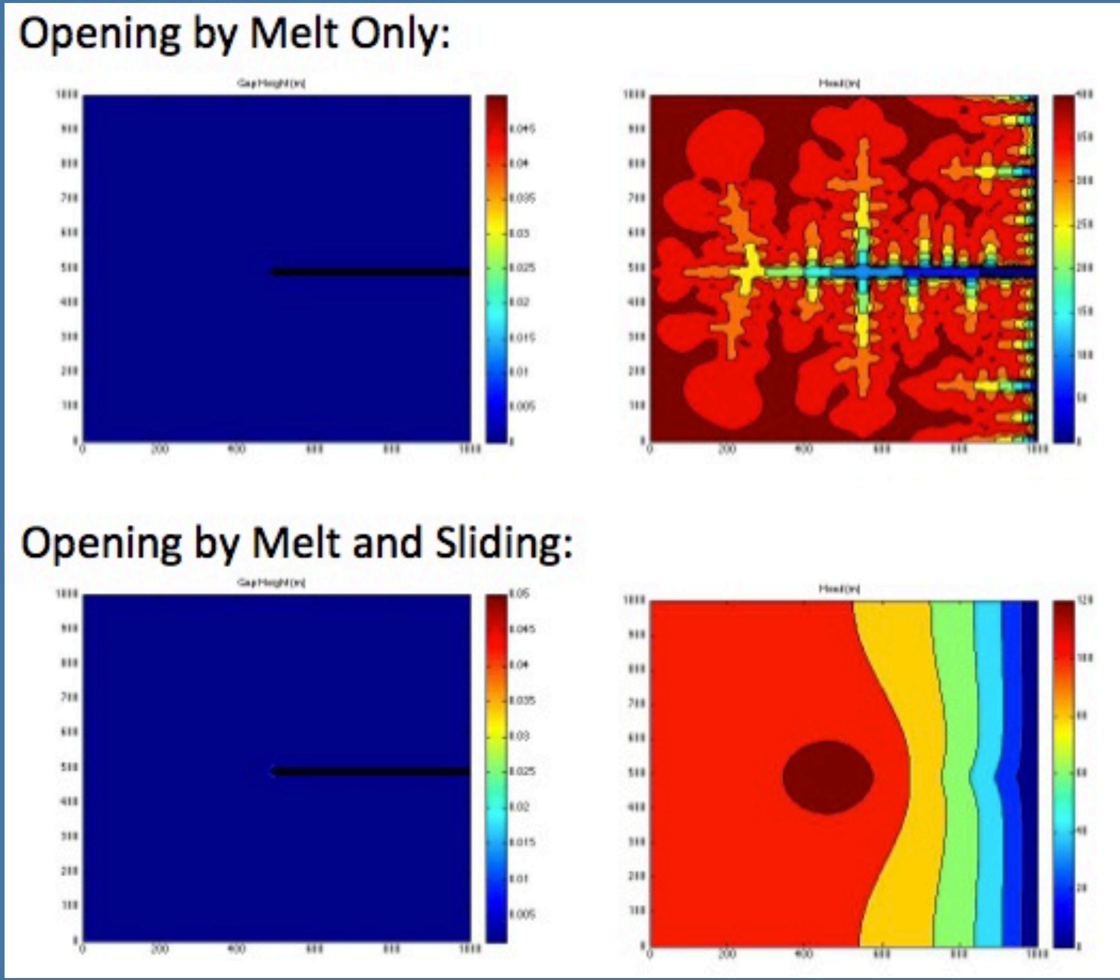
- Same as Case 1, but with 4 moulins with diurnally-varying input at (x=500, y=500), (x=300, y=700), (x=250, y=300), and (x=750, y=600)



Sample Test Results

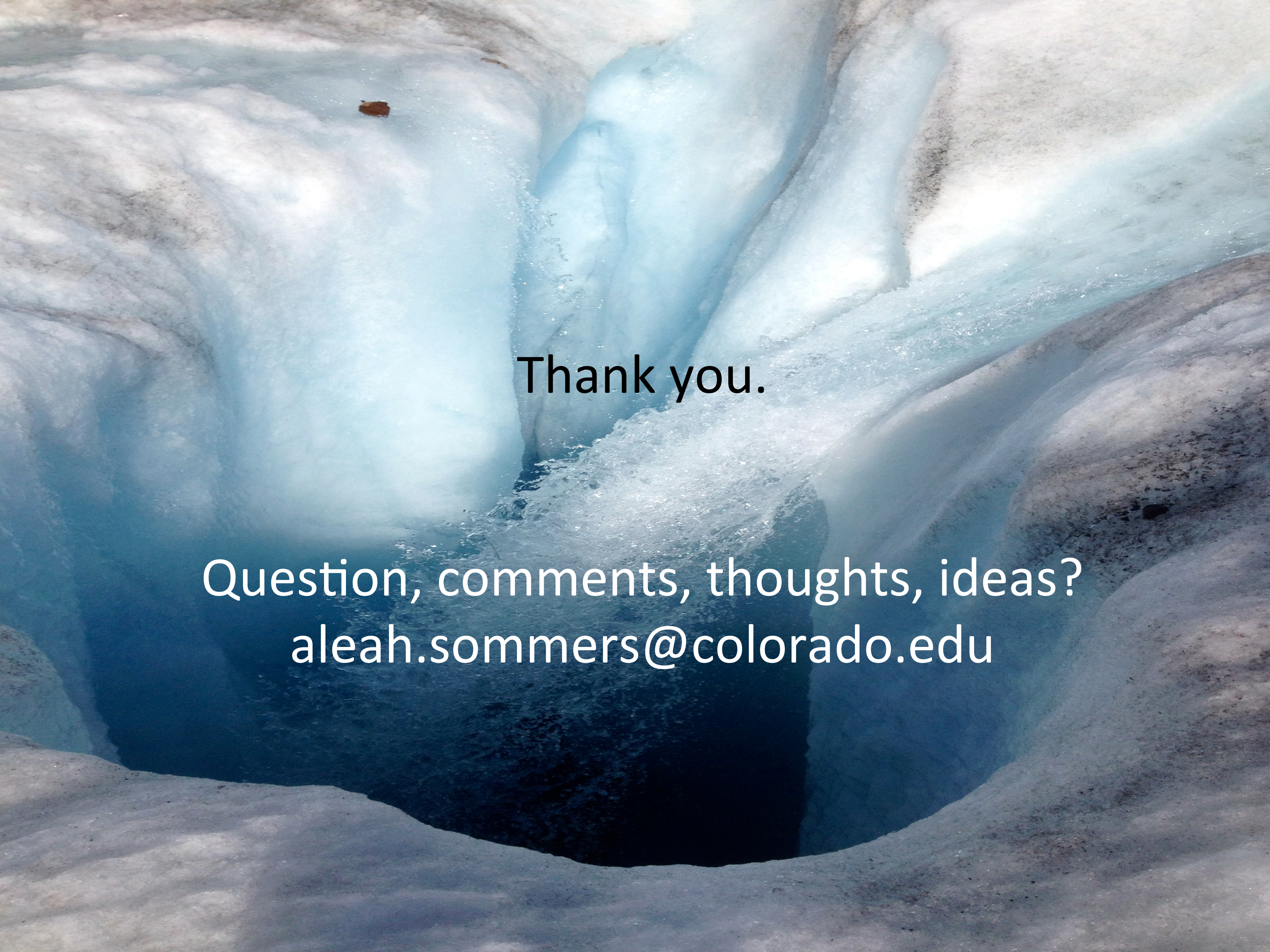
Case 5 – Single Moulin, Steady Input: Effect of Opening by Sliding

- Similar to Case 1, but with steady moulin input at center and thicker ice (500m-thick slab)



Upcoming Plans

- Two-way coupling with ice dynamics: coupling occurs between water pressure and sliding velocity (incorporate into ISSM, CISM, MPAS-LI)
- Numerical experiments on realistic domains with realistic input forcing, with the objective of understanding and explaining the distinctly different seasonal velocity signals observed in outlet glaciers in different regions of the Greenland ice sheet


Thank you.
Question, comments, thoughts, ideas?
aleah.sommers@colorado.edu