Matrix approaches to modeling land carbon and nitrogen cycles

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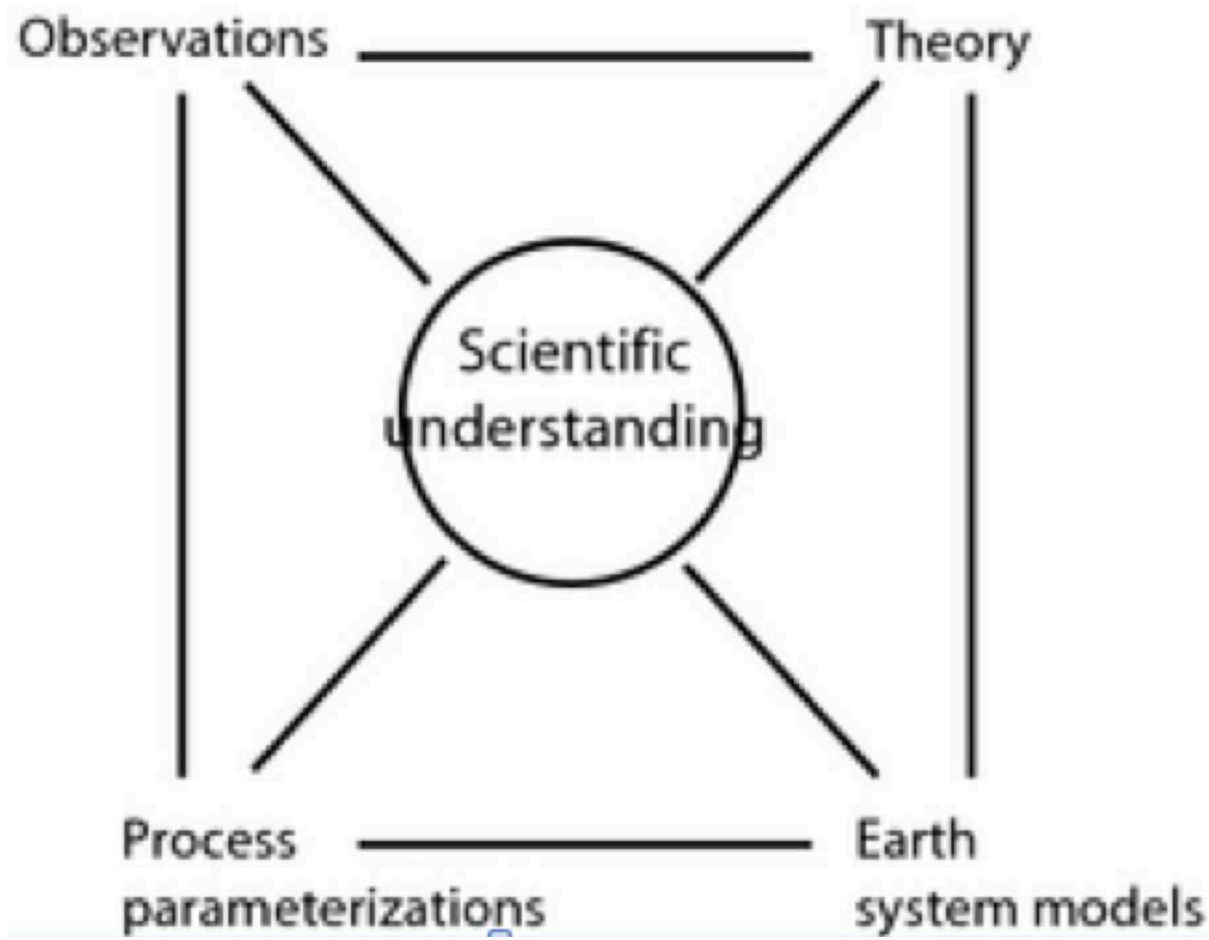
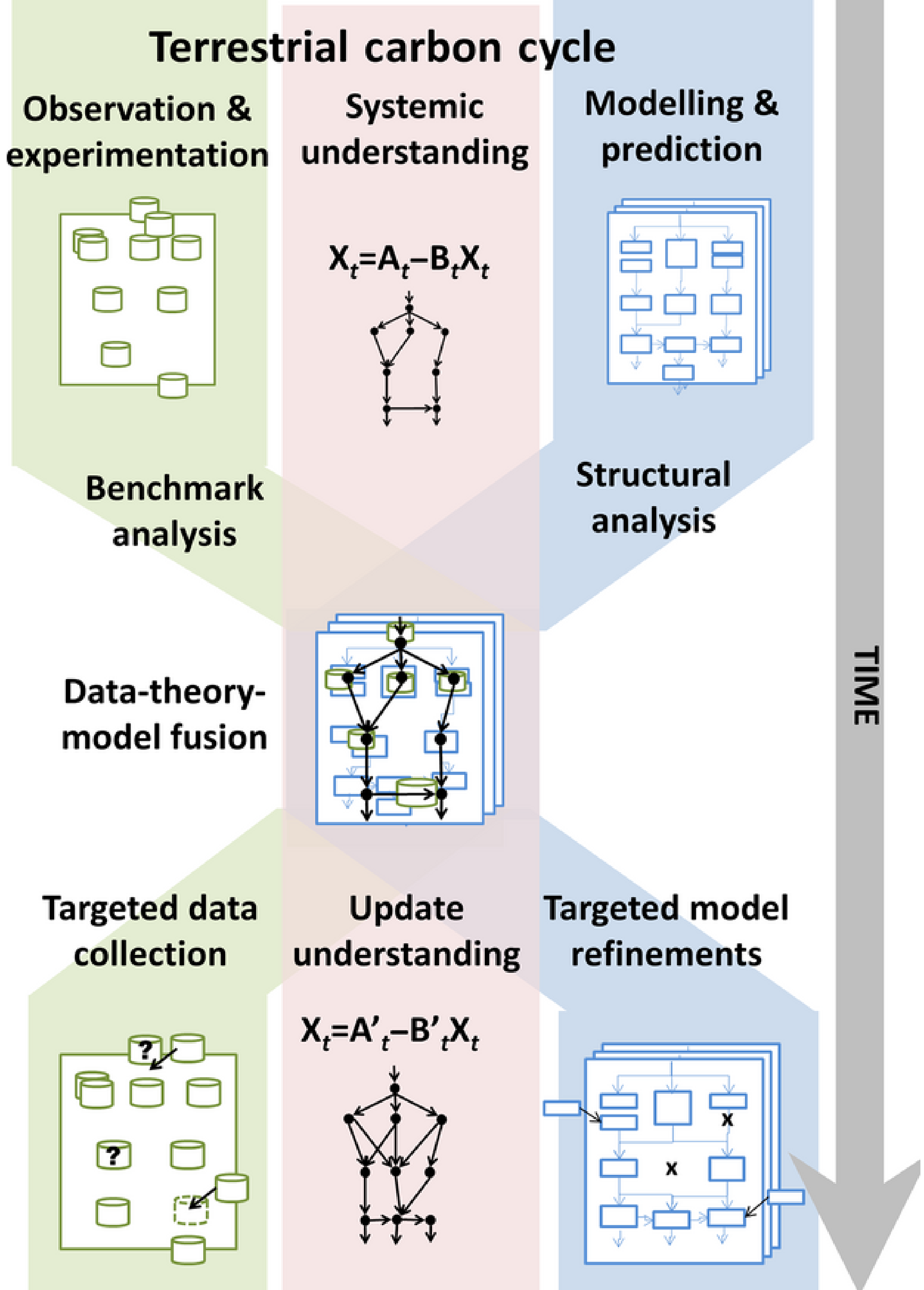
Observation-theory-model-process integrated approach



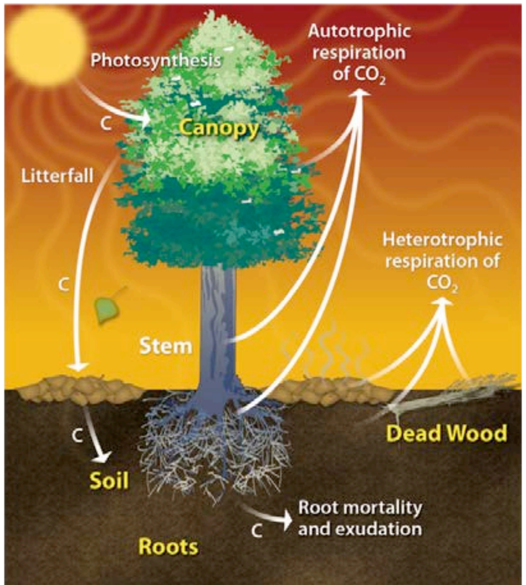
Bonan 2014 *New Phytologist*



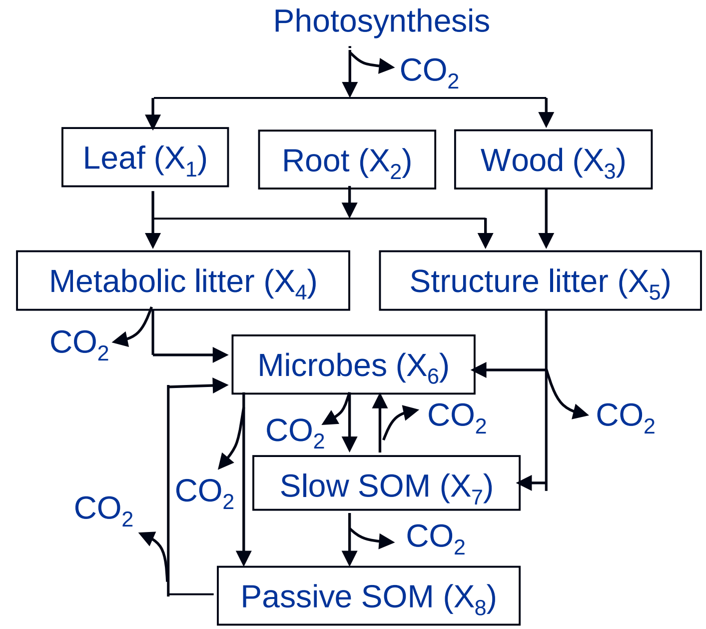
Model-theory-data fusion

Luo, Keenan, Smith 2015 *GCB*

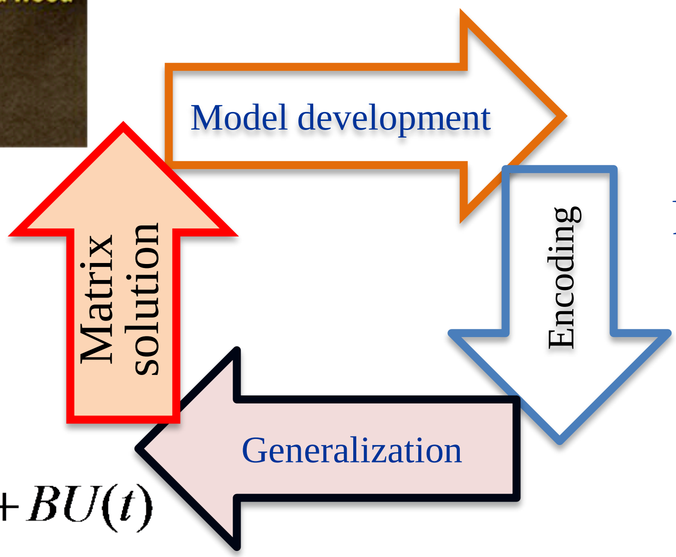
Matrix approach to carbon cycle modeling



A: Basic processes



B: Shared model structure



D: General model

$$\begin{cases} \dfrac{dX(t)}{dt} = A\xi(t)CX(t) + BU(t) \\ X(t=0) = X_0 \end{cases}$$

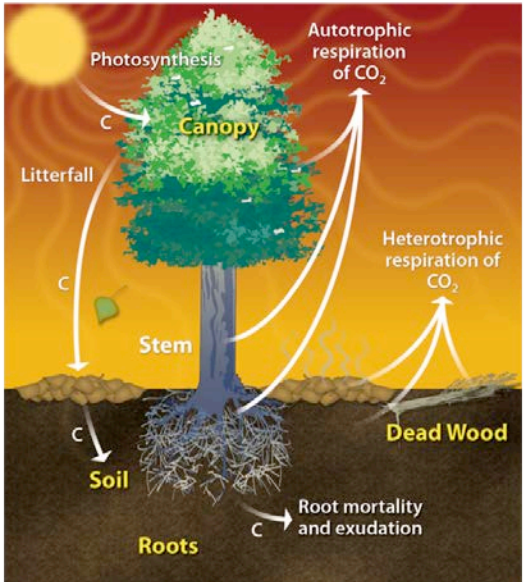
C: Similar algorithm

$$Plant \begin{cases} dX_1(t)/dt = b_1U(t) - \xi c_1 X_1(t) \end{cases}$$

$$a_{75}x_5(t) + c_6 a_{76} x_6(t) - c_7 X_7(t)]$$

$$dX_8(t)/dt = \xi[c_6 a_{86} x_6(t) + c_7 a_{87} x_7(t) - c_8 X_8(t)]$$

Luo et al. 2001 Ecol. Monog.
Luo et al. 2003 GBC
Luo and Weng 2011 TREE
Luo et al. 2015 GCB
Luo et al. 2017 BG



A: Basic processes

Matrix solution

$$\begin{cases} \dfrac{dX(t)}{dt} = \xi ACX(t) + BU(t) \\ X(t=0) = X_0 \end{cases}$$

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Yuanyuan's work

a. Matrix solution

b. Fast spin-up to steady-state solution

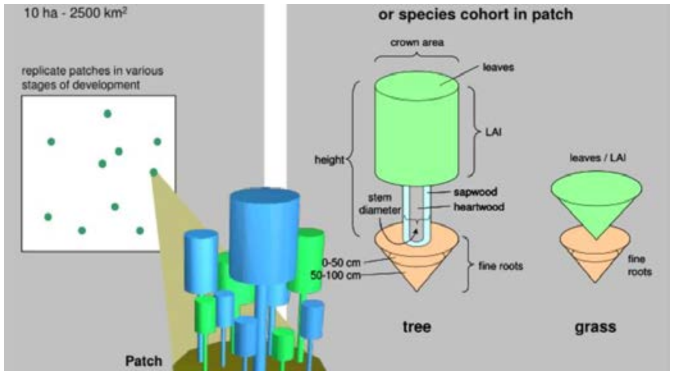
c. Traceability analysis to evaluate model components

d. Data assimilation to improve models

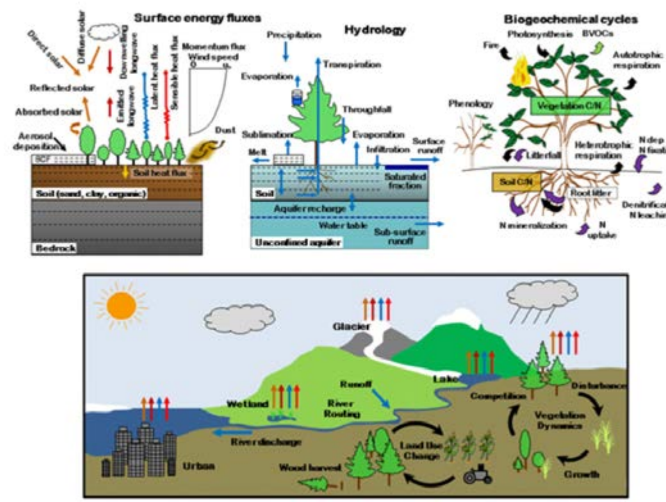
Tractability

Matrix representation

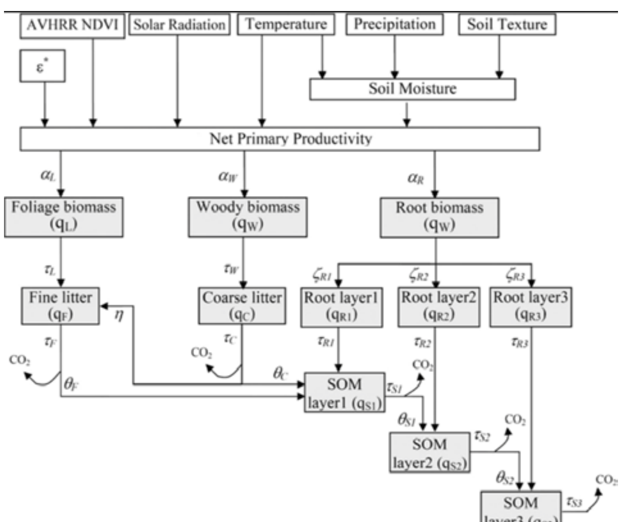
LPJ-GUESS



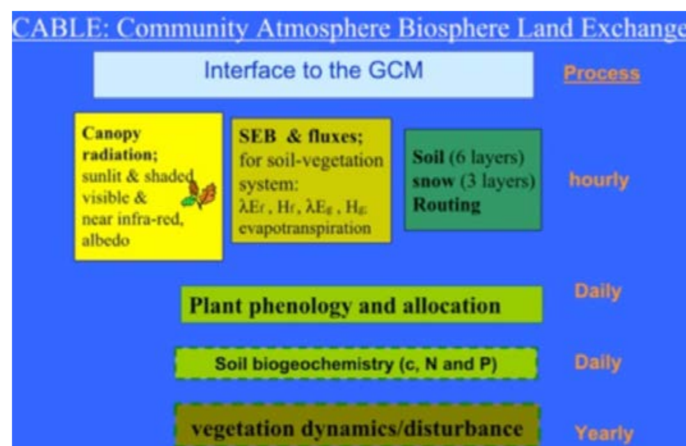
CLM 3.5, 4.0, 4.5

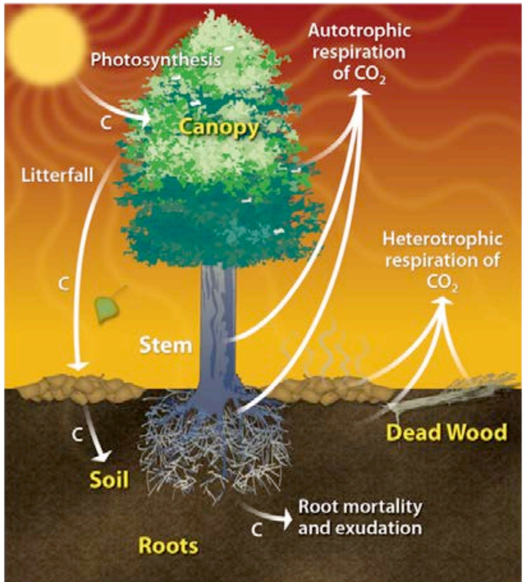


TECO



CABLE





A: Basic processes

Matrix solution

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Applications

1 Diagnostics (Zhou et al. J Climate, Jiang et al. GCB,)
   a. 3D parameter space
   b. Traceability analysis
   c. Variance decomposition

2 Faster spin-up (Huang et al. *In prep.* Xia et al. 2012)

3 Data assimilation (Hararuk et al. 2014, 2015, Shi et al. *In prep.*)

4 Theoretical understanding
   a. Dynamic disequilibrium (Luo and Weng 2011)
   b. Predictability of the terrestrial carbon cycle (Luo et al. 2015)
   c. Transient dynamics (Luo et al. 2017)

3D parameter space

$$\frac{dX}{dt} = I + A\varepsilon kX - \frac{Tri}{dz}X$$

X, C pools

A, transfer matrix

ε, scalar

Tri, tridiagonal matrix

I, external input

k, decomposition rate

Transient dynamics = Capacity - Potential

$$X(t) = \tau_E(t)NPP(t) - X_p(t)$$

Residence time

Production

3rd dimension

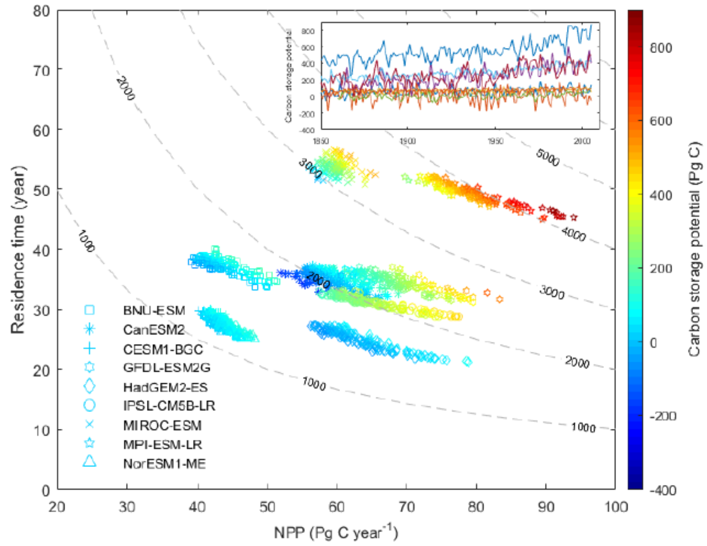
Luo et al. 2017

3D parameter space

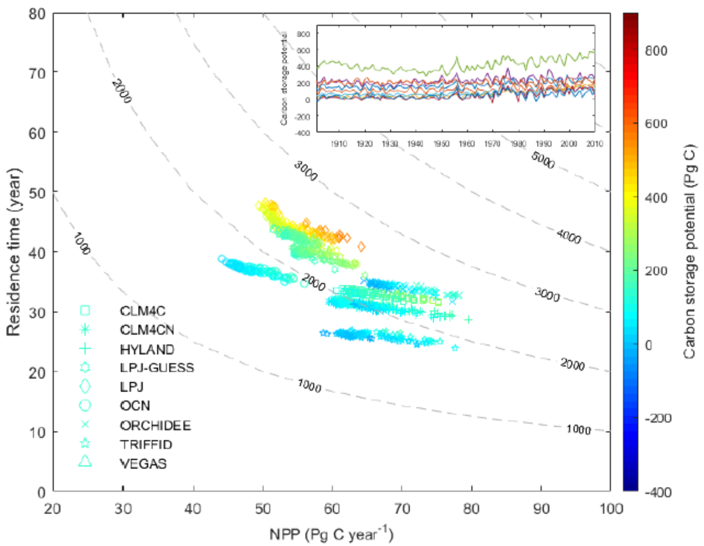
C storage dynamics = Capacity — Potential

$$X(t) = \tau_E(t)NPP(t) - X_p(t)$$

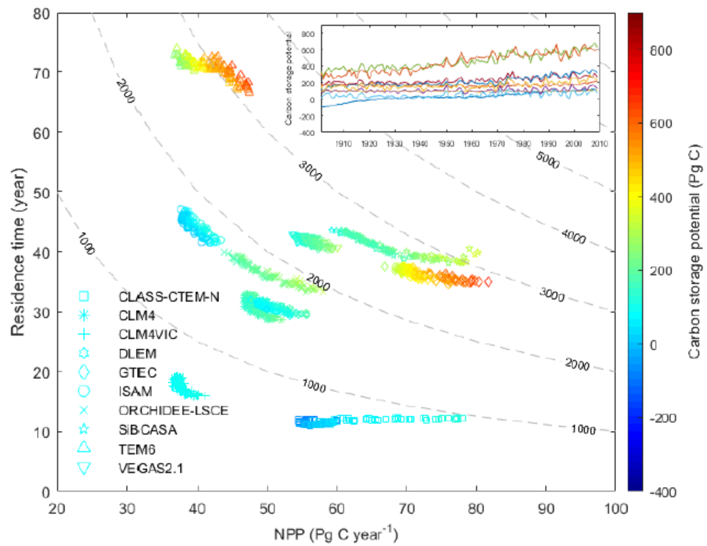
Residence time C input



CMIP5



TRENDY

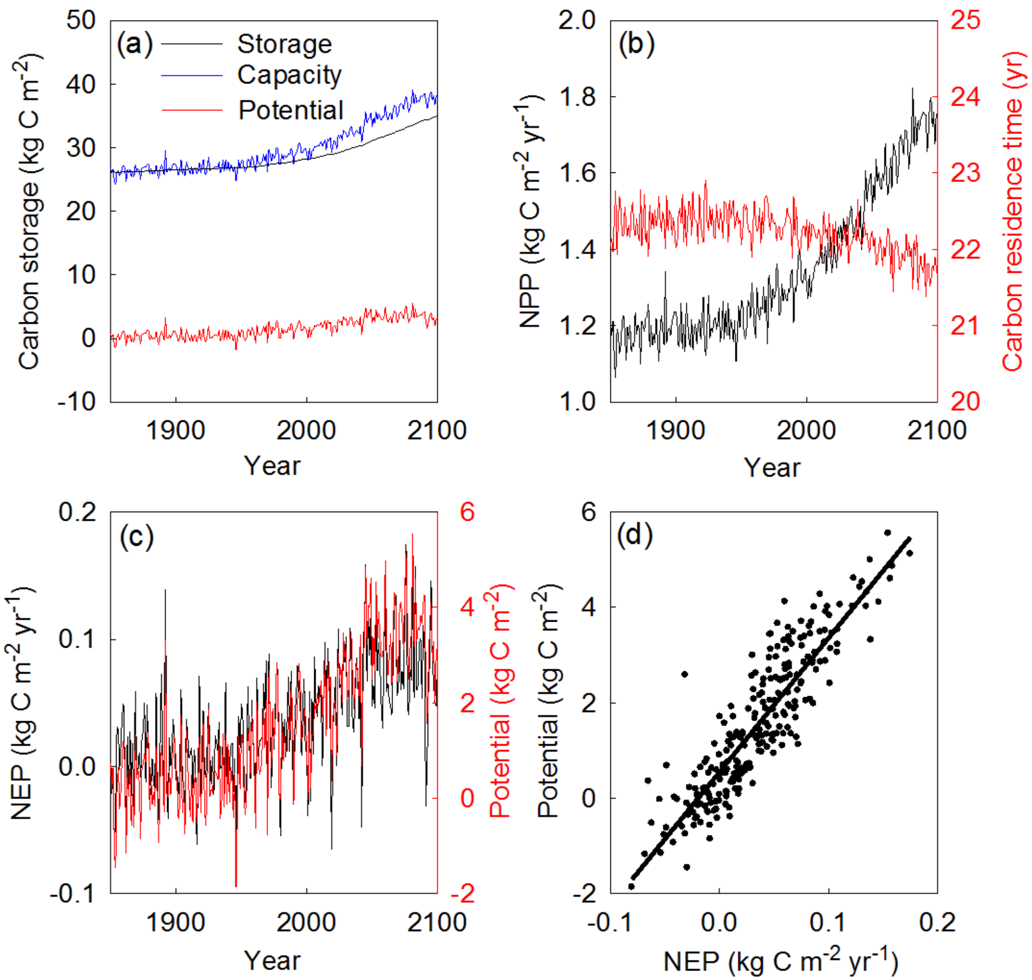


MsTMIP

Zhou et al. *J Climate*

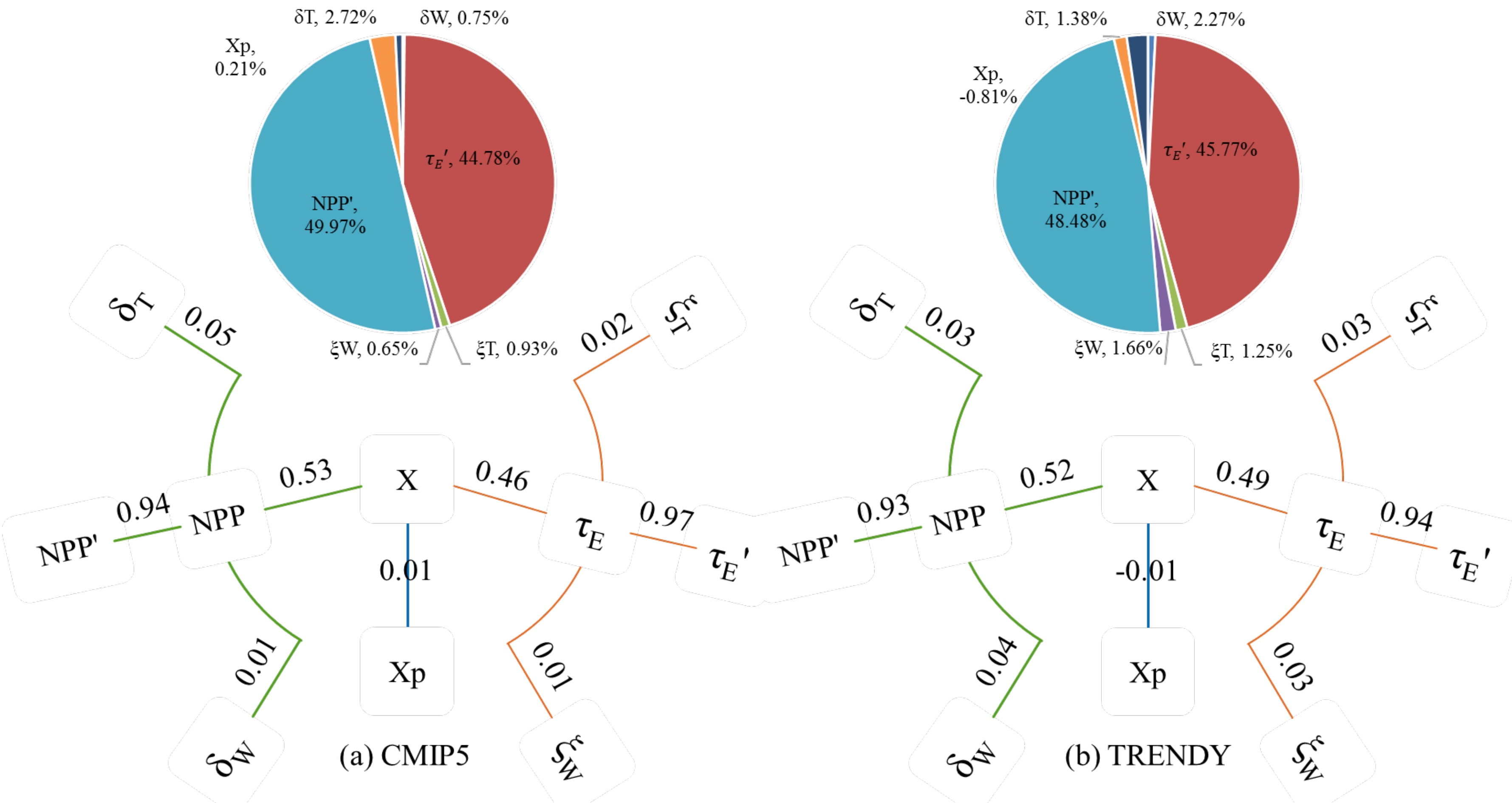
Traceable components

$$X(t) = (A\xi(t)K)^{-1}Bu(t) - (A\xi(t)K)^{-1}X'(t)$$



Luo et al. 2017

Variance decomposition I

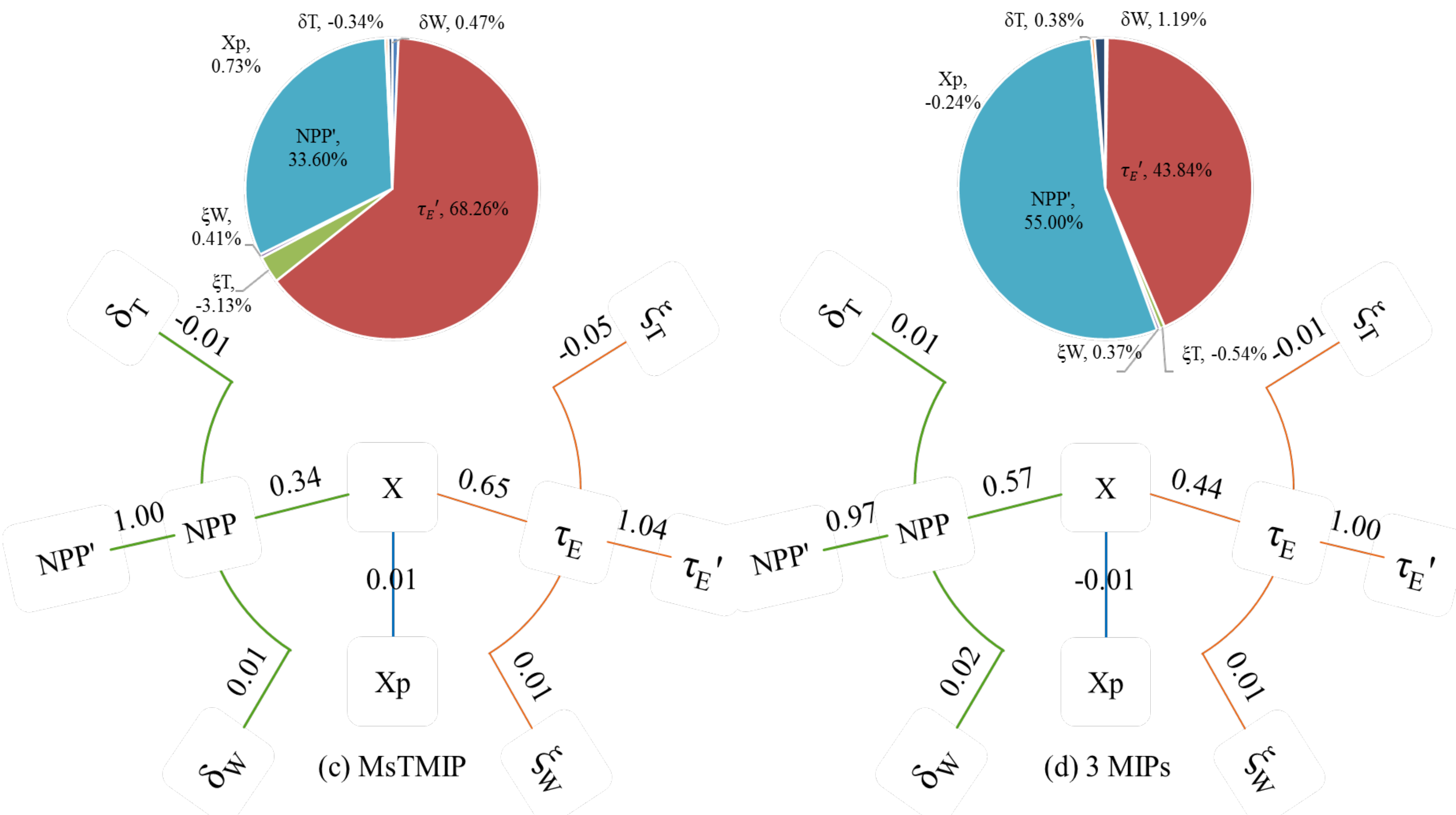


(a) CMIP5

(b) TRENDY

Zhou et al. *J Climate*

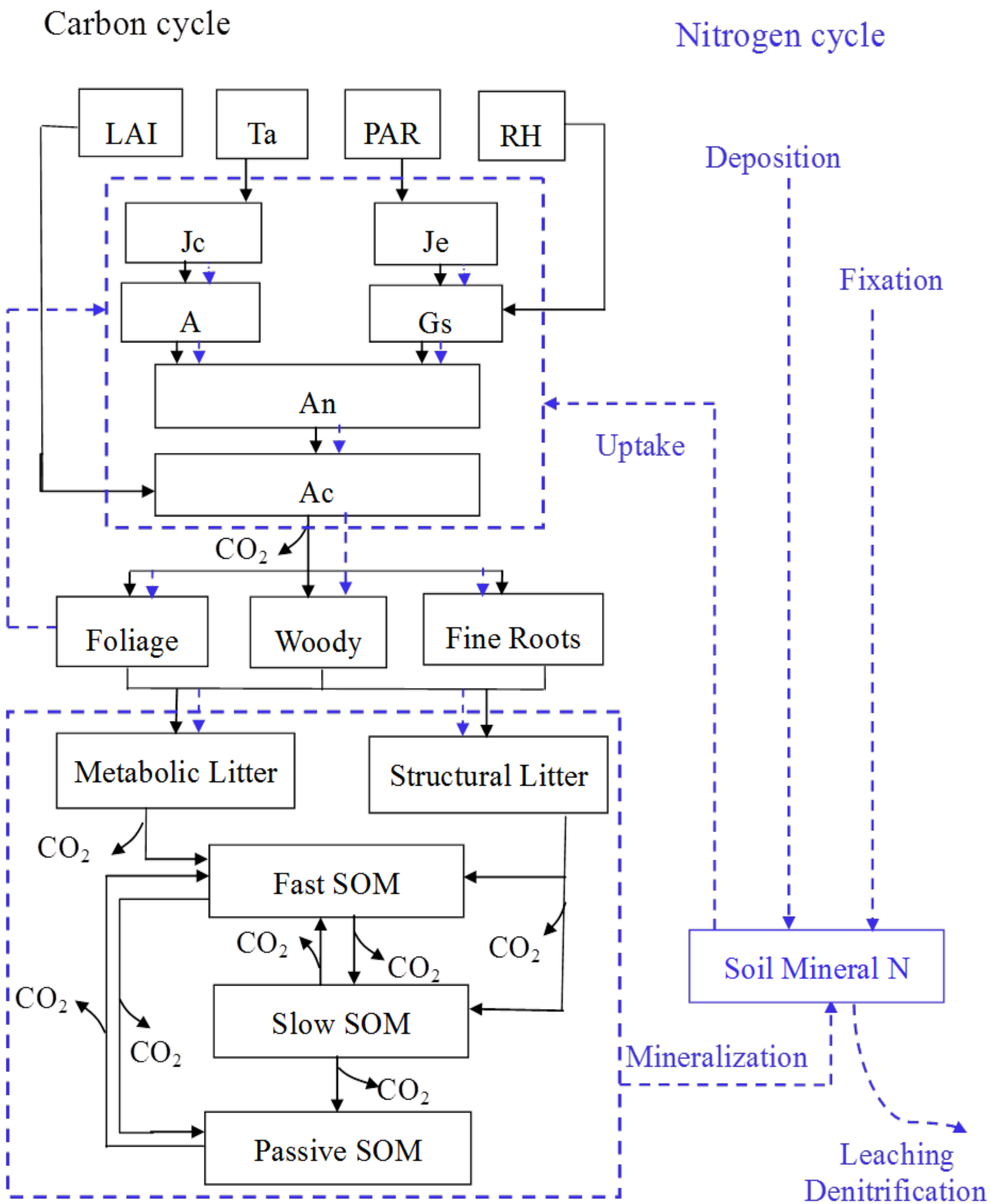
Variance decomposition II



Zhou et al. *J Climate*

Matrix approach to nitrogen Modeling

TECO-CN



Du et al. 2017 JAMES

Coupled carbon-nitrogen matrices

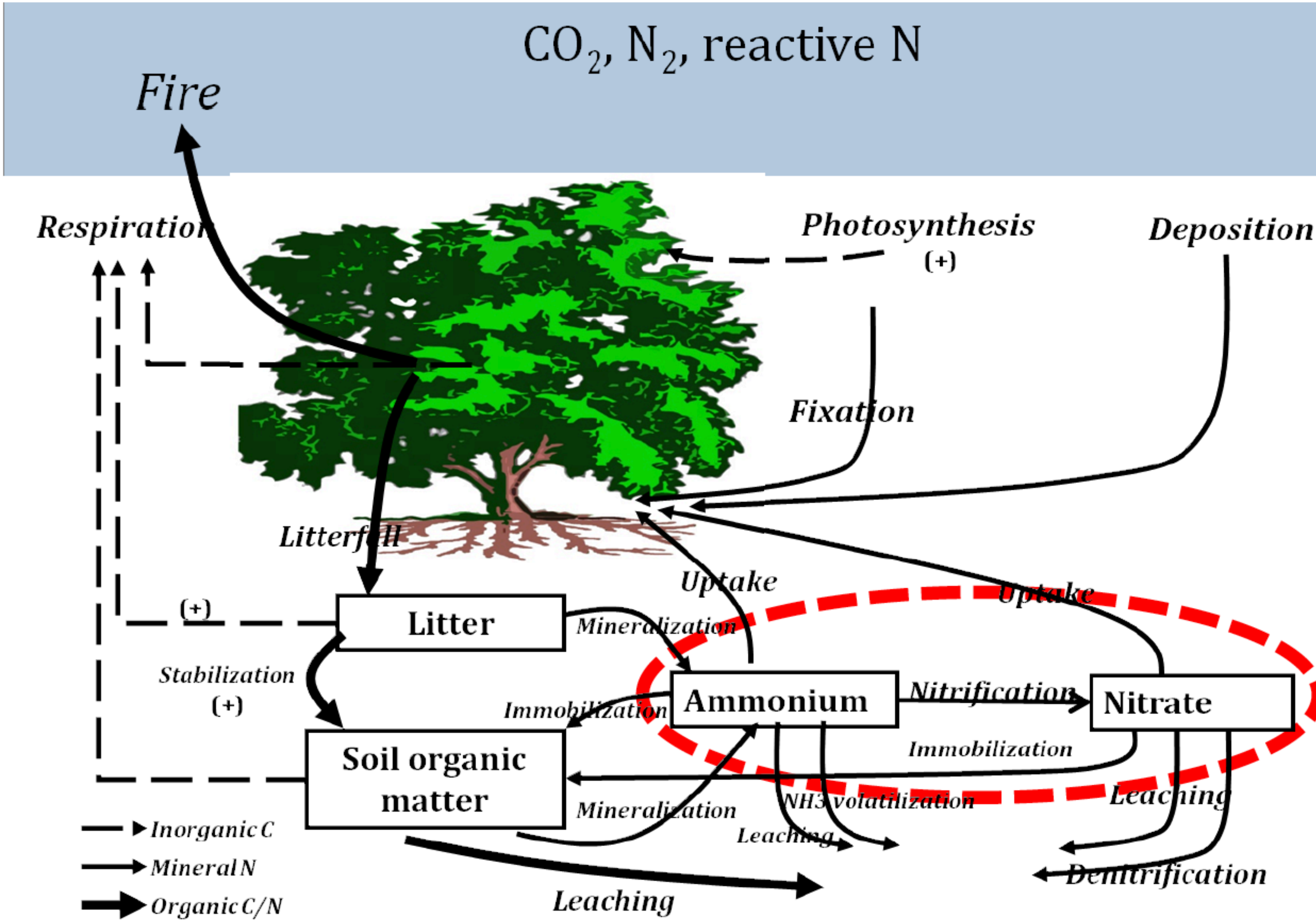
$$\begin{cases} \frac{d}{dt}X(t) = A\xi(t)K(N)X(t) + u(N,t)B \\ \frac{d}{dt}N(t) = A\xi(t)K(N)R^{-1}X(t) + \kappa_u N_{\min}\Pi \\ \frac{d}{dt}N_{\min} = -(\kappa_u + \kappa_L)N_{\min} + A\xi(t)\phi_1^* K(N)R^{-1}X(t) + F(t) \end{cases}$$

$$\begin{cases} X(t=0) = X_0 \\ N(t=0) = N_0 \\ N_{\min}(t=0) = N_{\min,0} \end{cases}$$

Shi et al. 2016 JPE
Du et al. 2017 JAMES

$$R = \begin{pmatrix} \rho_1 & & & & & & & \\ & \rho_2 & & & & & & \\ & & \rho_3 & & & & & \\ & & & \rho_4 & & & & \\ & & & & \rho_5 & & & \\ & & & & & \rho_6 & & \\ & & & & & & \rho_7 & \\ & & & & & & & \rho_8 \end{pmatrix} = diag(\rho)$$

LM3V-N



Gerber et al., 2010; Huang and Gerber, 2015

Matrix Representation of LM3V-N

$$\frac{dX(t)}{dt} = B(t)V(t) - A(t)\xi(t)K(t)X(t)$$

N input: $V(t)$; Scalar: $\xi(t)$; N allocation: $B(t)$; N transfer: $A(t)$; N turnover: $K(t)$

$$X = \begin{pmatrix} leaf \\ root \\ sapwood \\ wood \\ fast\ litter \\ slow\ litter \\ slow\ SOM \\ passive\ SOM \\ amm \\ nitr \end{pmatrix} \quad B = \begin{pmatrix} b\_n1 & 0 \\ b\_n2 & 0 \\ b\_n3 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & i1 \\ 0 & 1 - i1 \end{pmatrix} \quad V = \begin{pmatrix} BNF(t) \\ Dep(t) + Fert(t) \end{pmatrix}$$

Yuanyuan Huang, *unpub*

$$
A=\begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & b_{n1}\frac{k_p}{k_p+k_m+k_l+k_n+k_a} & b_{n1}\frac{k_p}{k_p+k_m+k_l+k_{dn}} \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & b_{n2}\frac{k_p}{k_p+k_m+k_l+k_n+k_a} & b_{n2}\frac{k_p}{k_p+k_m+k_l+k_{dn}} \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & b_{n3}\frac{k_p}{k_p+k_m+k_l+k_n+k_a} & b_{n3}\frac{k_p}{k_p+k_m+k_l+k_{dn}} \\
0 & 0 & -\frac{\alpha3*r3/r4}{\alpha3+g*\mu+\tau} & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
-c_n & -c_n & -\frac{c_n(g*\mu+\tau)}{\alpha3+g*\mu+\tau} & -c_n & 1 & 0 & 0 & 0 & 0 & 0 \\
-(1-c_n) & -(1-c_n) & -\frac{(1-c_n)(g*\mu+\tau)}{\alpha3+g*\mu+\tau} & -(1-c_n) & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -f(Nav)\frac{r6}{r7}rr & 1 & 0 & \frac{k_m}{k_p+k_m+k_l+k_n+k_a} & \frac{k_m}{k_p+k_m+k_l+k_{dn}} \\
0 & 0 & 0 & 0 & 0 & -q\frac{r6}{r8}rr & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & -1 & -1 & -1 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & (1-fg)\frac{k_n}{k_p+k_m+k_l+k_n+k_a} & 1
\end{pmatrix}
$$

$$
\boldsymbol{k}=\begin{pmatrix}
f(s)\alpha1(1-r)+g*\mu+\tau+0.5*f\_d & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & f(s)\alpha2+g*\mu+\tau+0.5*f\_d & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \alpha3+g*\mu+\tau & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & g*\mu+\tau & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & k_{lf}+dom1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & k_{ls}(1+\varepsilon[Nav])+dom2 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & k_{ss}+dom3 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & k_{sp} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & k_p+k_m+k_n+k_l & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & k_p+k_m+k_d+k_l
\end{pmatrix}
$$

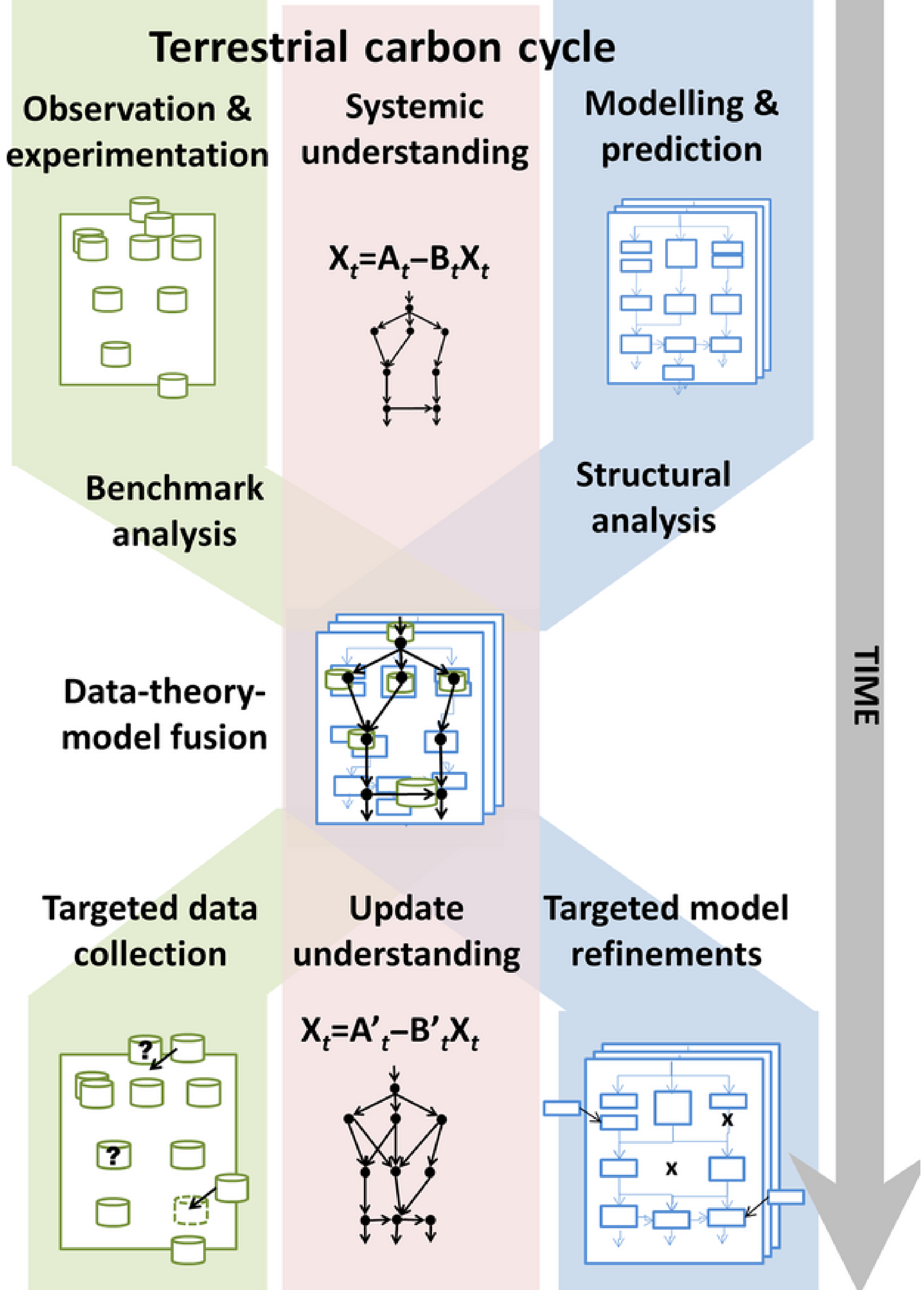
Yuanyuan Huang, *unpub*

Matrix representation of nitrogen model of CLM5

Matrix representation of demographic model

The future

- A library of carbon matrix equations from most of the ESMs
- A library of nitrogen matrix equations from most of the ESMs
- Ensemble modeling with multiple carbon and nitrogen matrices
- Traceable components and their contributions to uncertainty
- Benchmarking traceable components with observations
- Data assimilation to train model components with observations



Vision

To constrain structures and parameters of those well-understood model components

To allow structural variations for those poorly understood components

Luo, Keenan, Smith 2015 *GCB*