



# Towards a robust representation of nutrient control of the land carbon cycle: Formulation and numerics

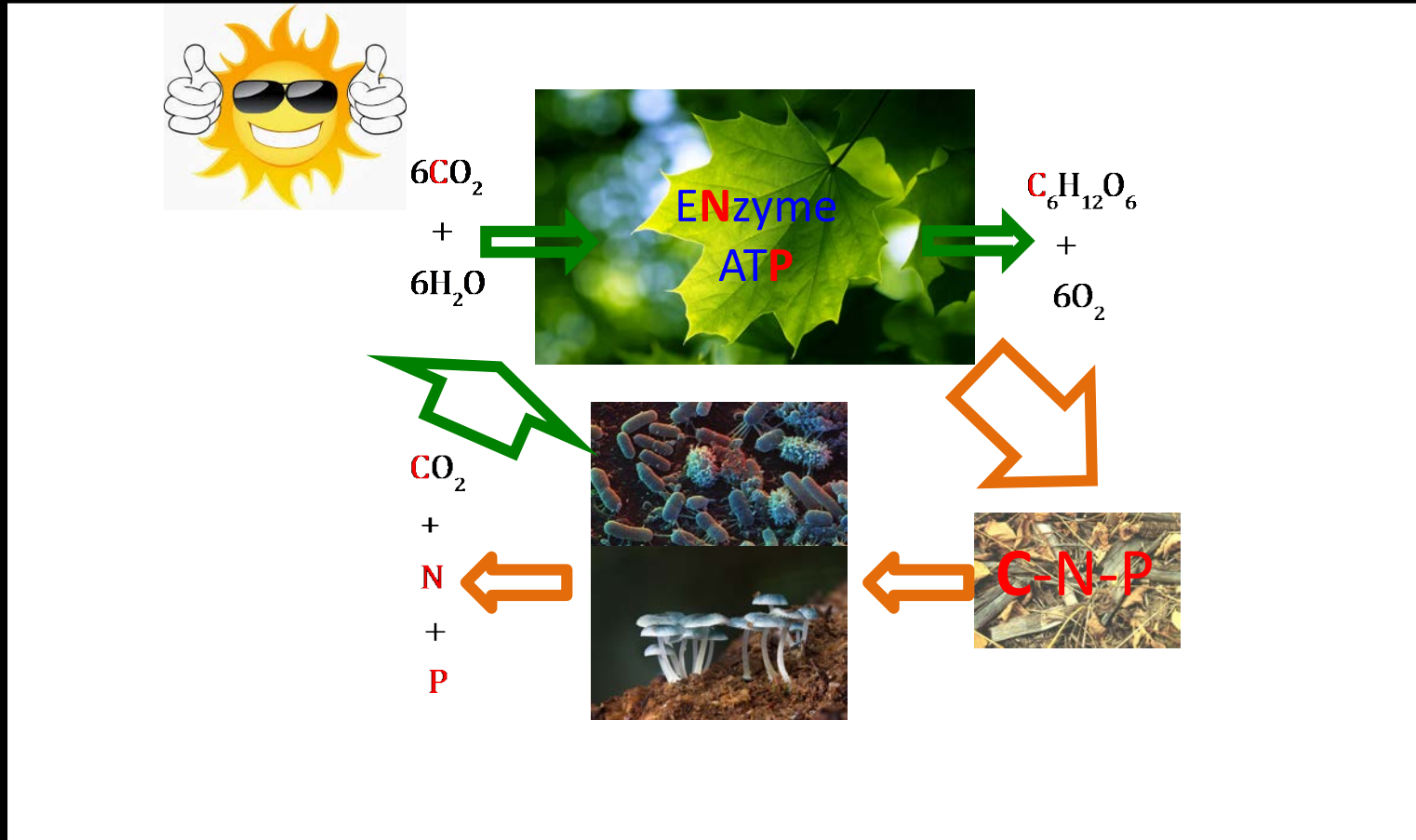
Jinyun Tang, Bill Riley and Qing Zhu



U.S. DEPARTMENT OF  
**ENERGY**

Office of  
Science

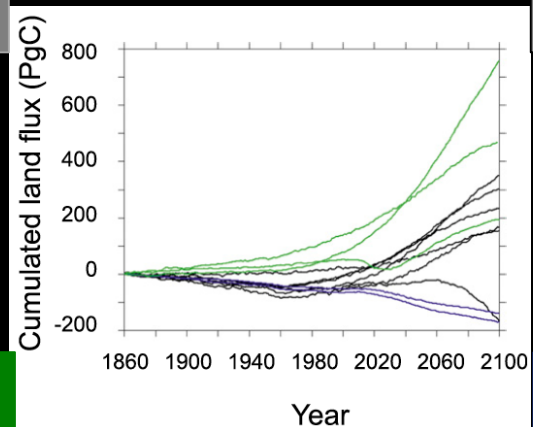
# Theme of biogeochemistry



# Not easy to do it well

Parameterization

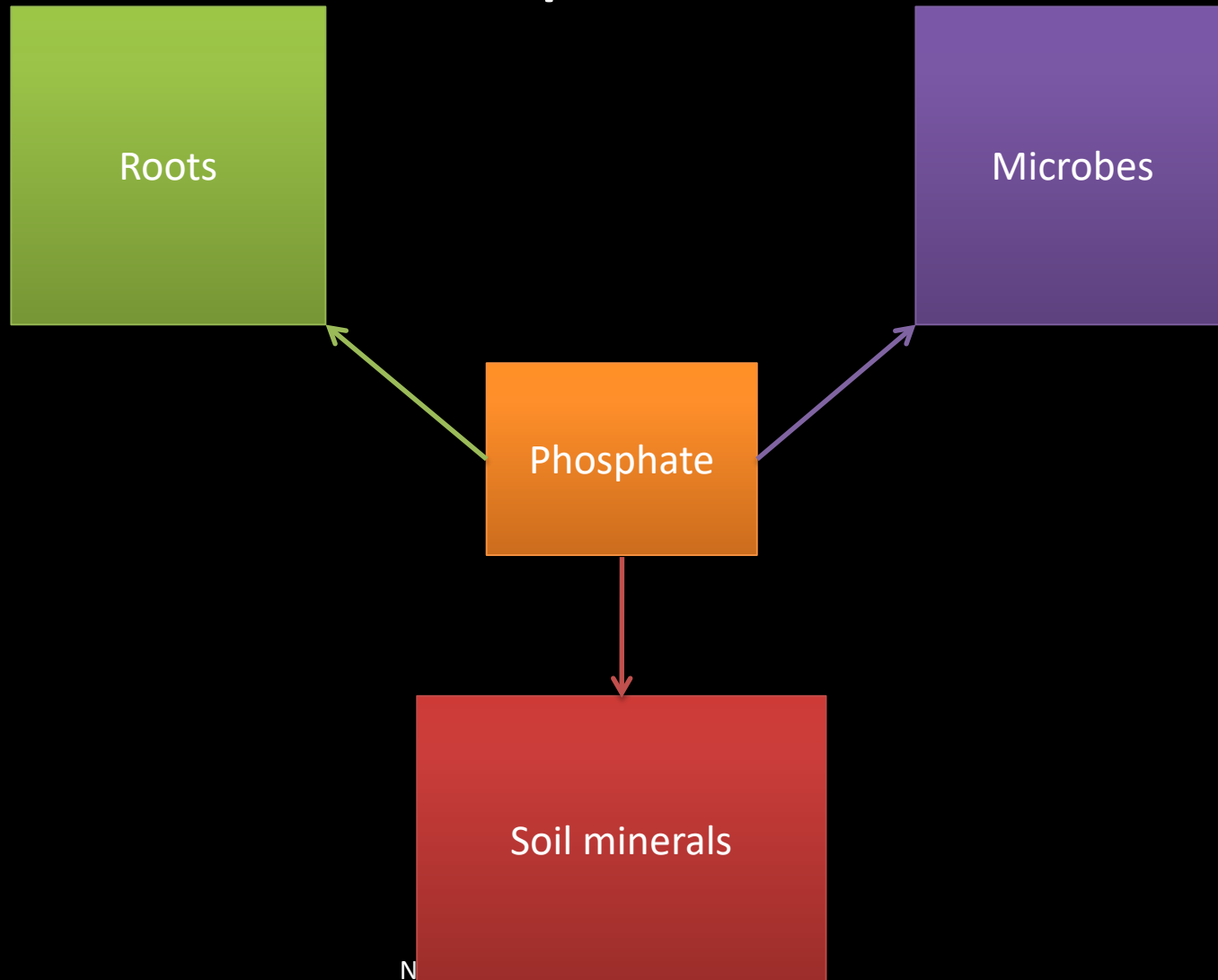
Initial and  
boundary forcing



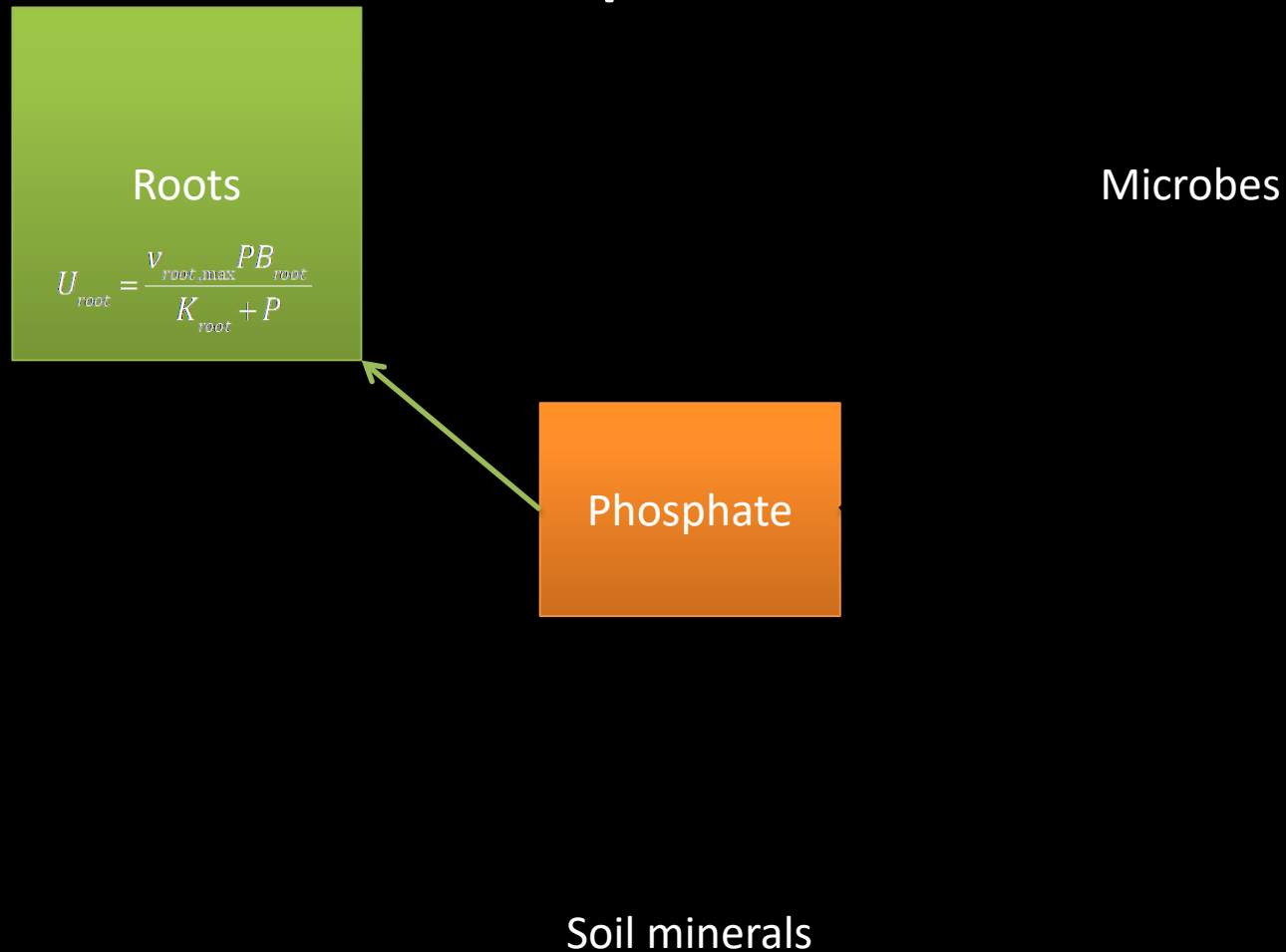
Mechanistic  
formulation

Numerical  
encoding

# The plant-soil-microbe nutrient competition



# The plant-soil-microbe nutrient competition



# The plant-soil-microbe nutrient competition

Roots

Microbes

$$U_{mic} = \frac{v_{mic,max} P B_{mic}}{K_{mic} + P}$$

Phosphate

Soil minerals

# The plant-soil-microbe nutrient competition

Roots

Microbes

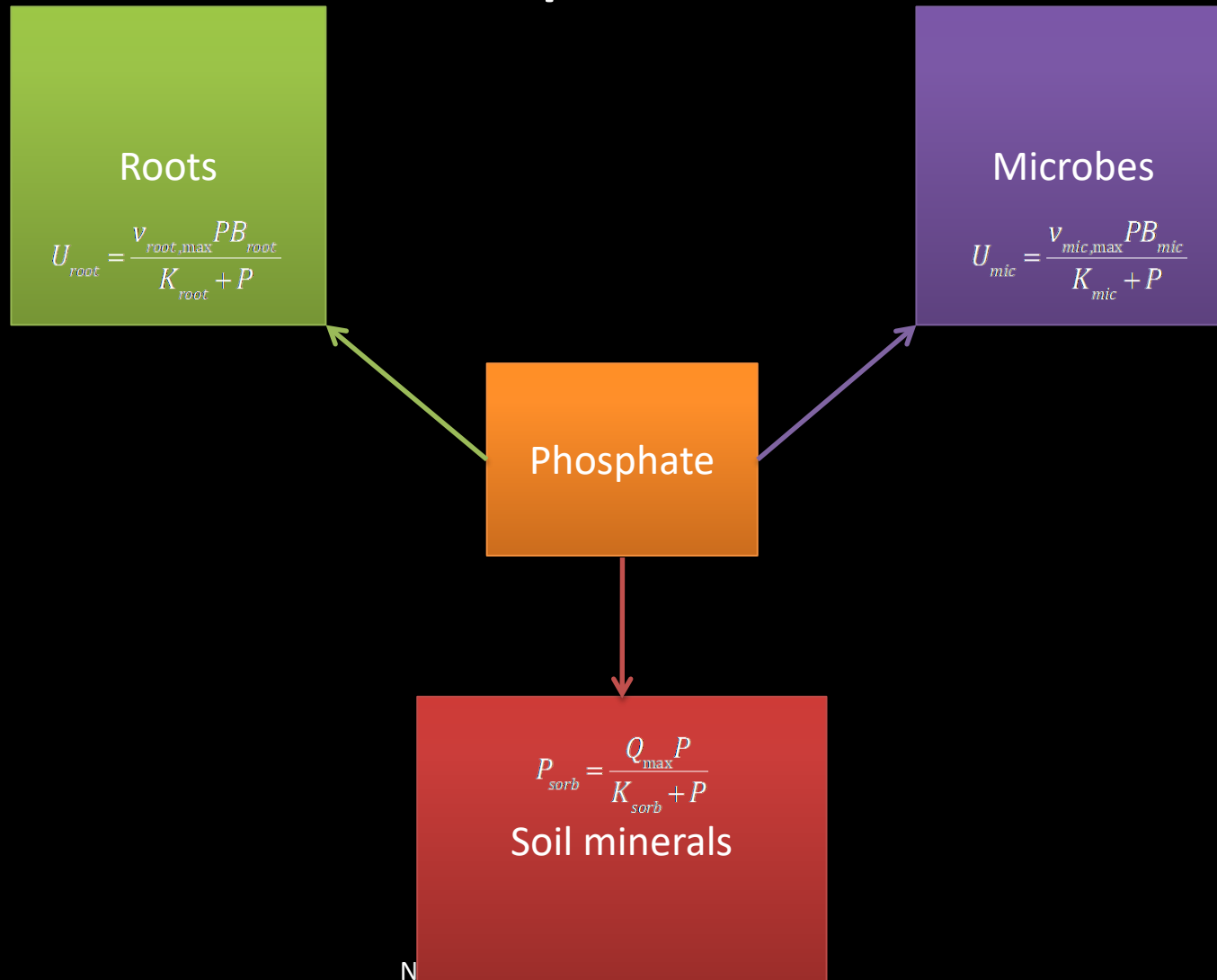
Phosphate



$$P_{sorb} = \frac{Q_{max} P}{K_{sorb} + P}$$

Soil minerals

# The plant-soil-microbe nutrient competition





# The competition conundrum

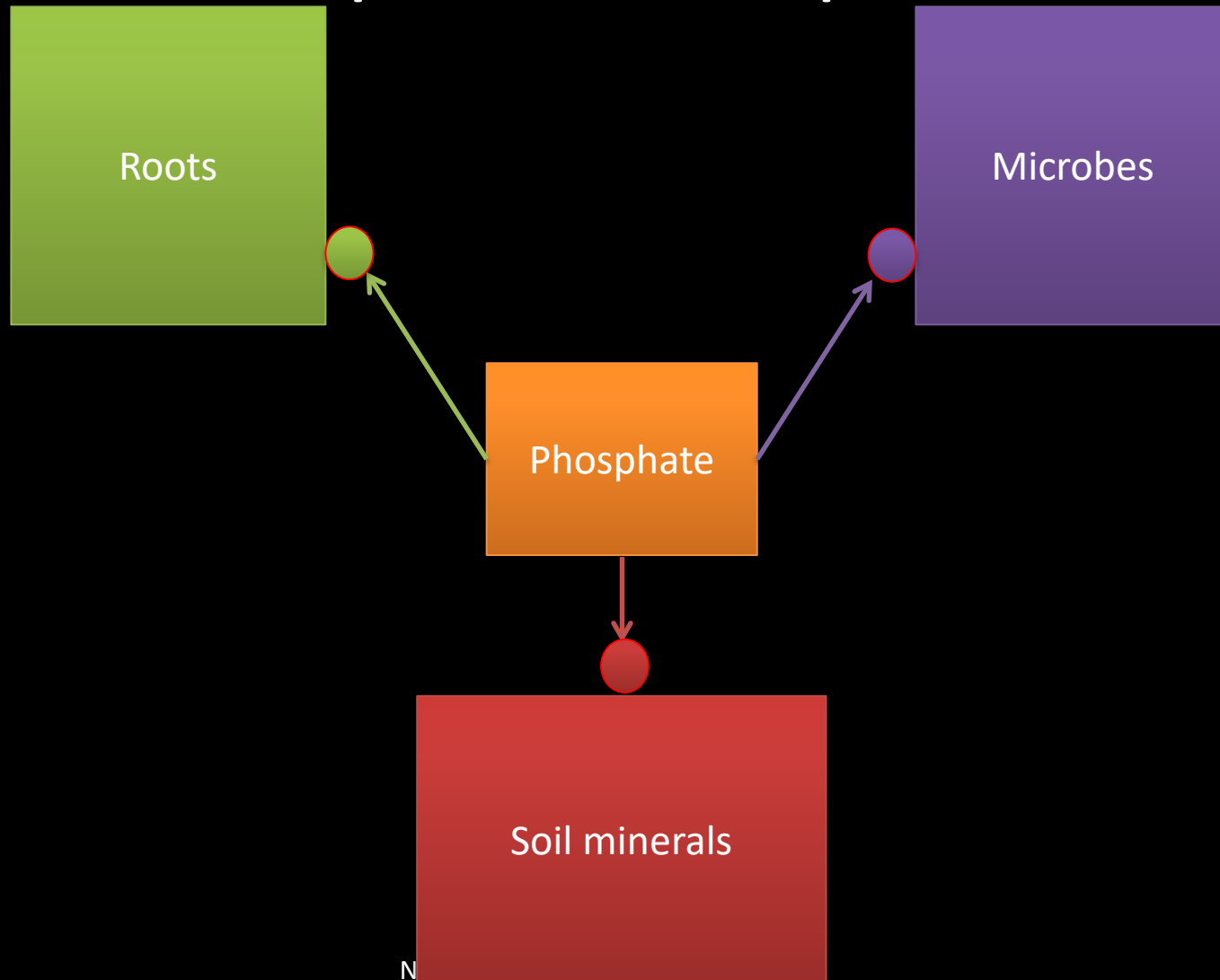
$$\begin{aligned}\frac{dP}{dt} &= -U_{root} - U_{mic} \\ &= -\frac{v_{root,max} PB_{root}}{K_{root} + P} - \frac{v_{mic,max} PB_{mic}}{K_{root} + P}\end{aligned}$$

# The competition conundrum

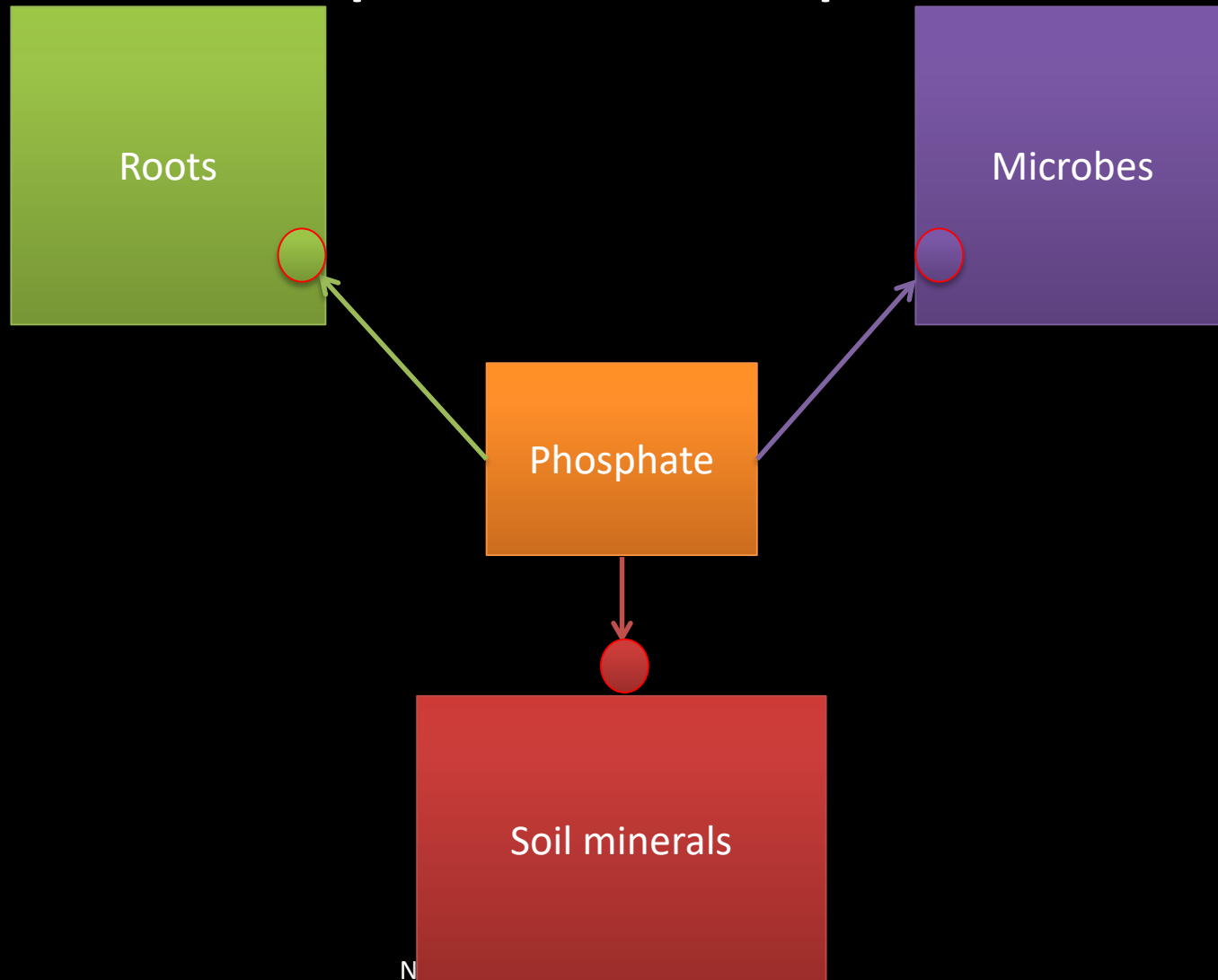
$$\begin{aligned}\frac{dP}{dt} &= -U_{root} - U_{mic} \\ &= -\frac{v_{root,max} PB_{root}}{K_{root} + P} - \frac{v_{mic,max} PB_{mic}}{K_{root} + P}\end{aligned}$$

$$P_{sorb} = \frac{Q_{max} P}{K_{sorb} + P} \quad ?$$

# Two-step plant-soil-microbe nutrient competition: step one



# Two-step plant-soil-microbe nutrient competition: step two



# The Equilibrium Chemistry Approximation Theory for nutrient competition

Step one

$$P_T = P + P_{sorb} + (PB_{root}) + (PB_{mic})$$

$$B_{root} + (PB_{root}) = B_{root,T}$$

$$B_{mic} + (PB_{mic}) = B_{mic,T}$$

$$Q + P_{sorb} = Q_{max}$$

$$(PB_{root})K_{root} = B_{root} \cdot P$$

$$(PB_{mic})K_{mic} = B_{mic} \cdot P$$

$$P_{sorb}K_{sorb} = P \cdot Q$$

Step two

$$U_{root} = v_{root,max} (PB_{root})$$

$$U_{mic} = v_{mic,max} (PB_{mic})$$

# The Equilibrium Chemistry Approximation Theory for nutrient competition

$$U_{root} = \frac{v_{max,root} P_T B_{root,T} / K_{root}}{1 + P_T / K_{root} + B_{root} / K_{root} + B_{mic} / K_{mic} + Q_{max} / K_{sorb}}$$

$$U_{mic} = \frac{v_{max,mic} P_T B_{mic,T} / K_{mic}}{1 + P_T / K_{mic} + B_{mic} / K_{mic} + B_{root} / K_{root} + Q_{max} / K_{sorb}}$$

Tang and Riley, 2013

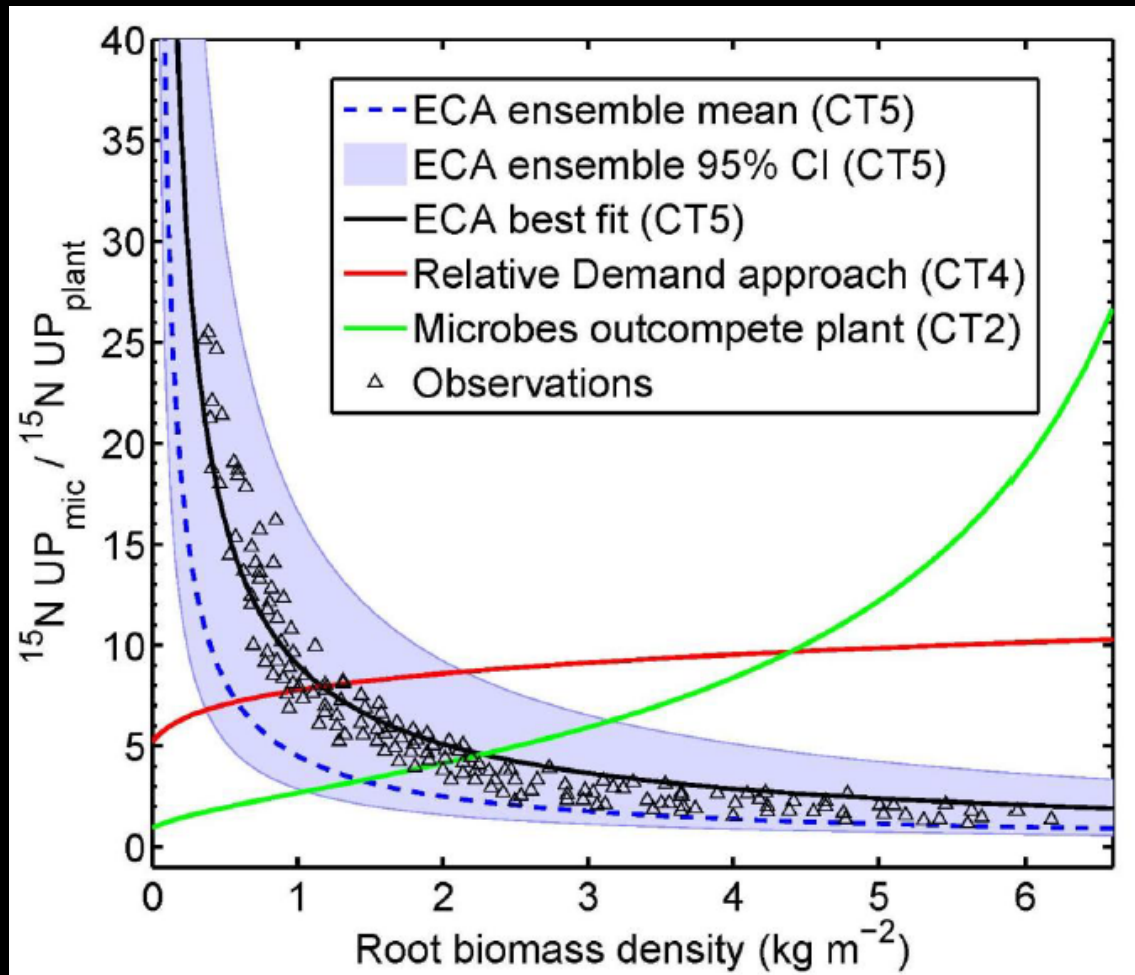
# The Equilibrium Chemistry Approximation Theory for nutrient competition

$$U_{root} = \frac{v_{max,root} P_T B_{root,T} / K_{root}}{1 + P_T / K_{root} + B_{root} / K_{root} + B_{mic} / K_{mic} + Q_{max} / K_{sorb}}$$

$$U_{mic} = \frac{v_{max,mic} P_T B_{mic,T} / K_{mic}}{1 + P_T / K_{mic} + B_{mic} / K_{mic} + B_{root} / K_{root} + Q_{max} / K_{sorb}}$$

Tang and Riley, 2013

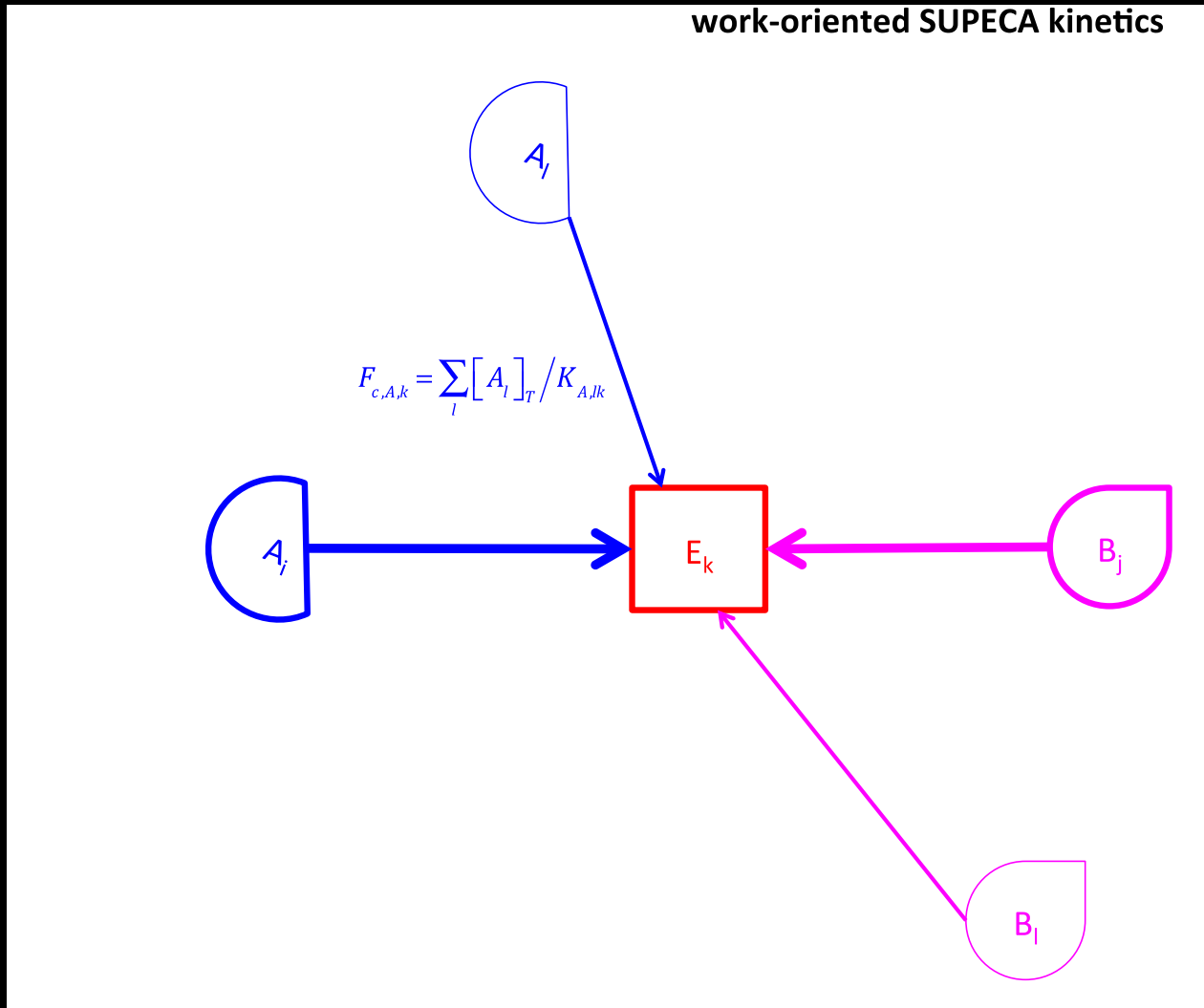
# ECA is quite good: nitrogen competition



Zhu et al., 2016

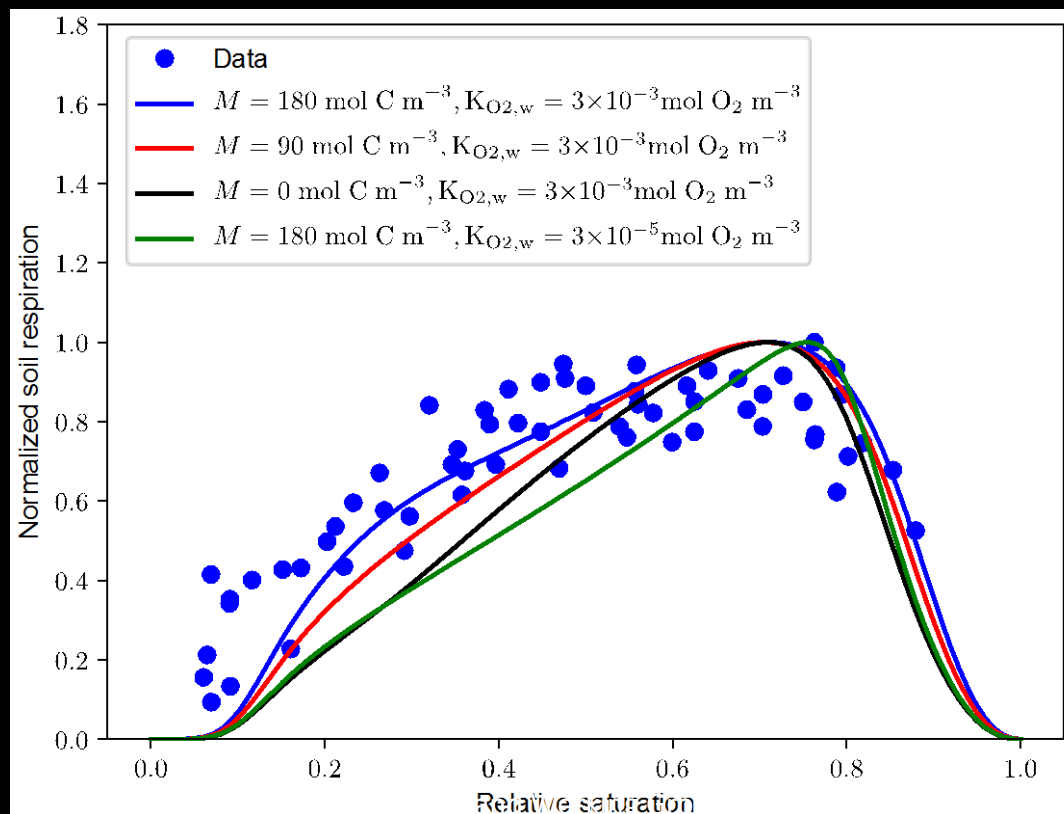
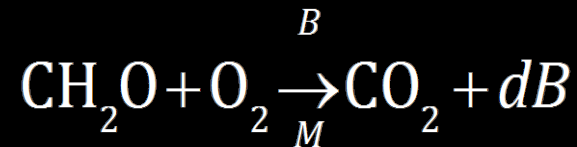


# SUPECA: extended ECA for more realistic substrate-competitor networks



Tang and Riley, 2017

# Application for aerobic decomposition



Tang and Riley, 2017

Mechanistic  
formulation



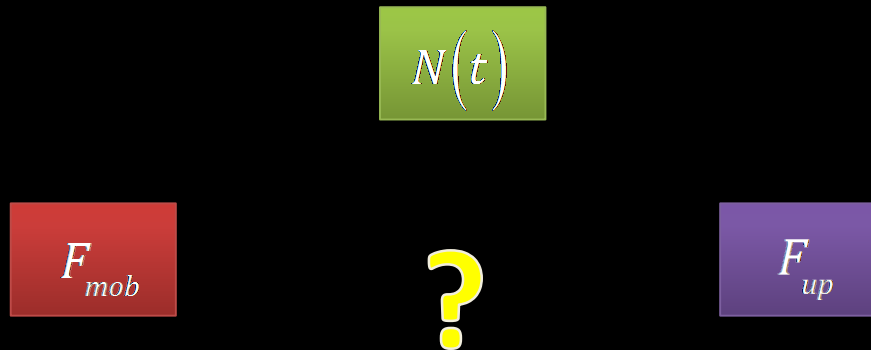
Numerical  
encoding

# The substrate limitation problem

$$N(t + \Delta t) = N(t) + \Delta t (F_{mob} - F_{up}) \geq 0$$

# The substrate limitation problem

$$N(t + \Delta t) = N(t) + \Delta t (F_{mob} - F_{up}) \geq 0$$

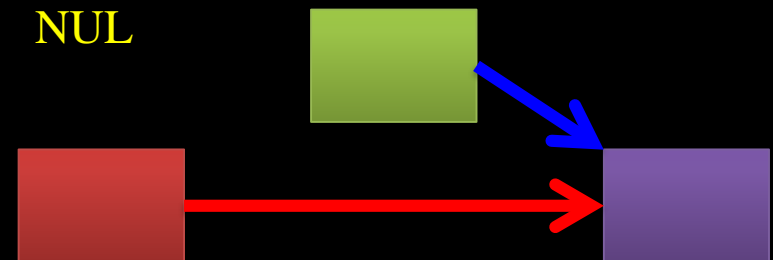
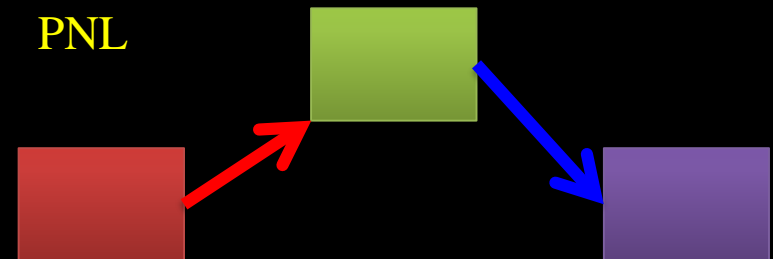
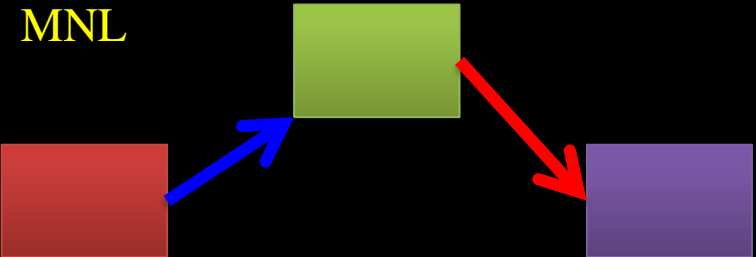


# Three different ways

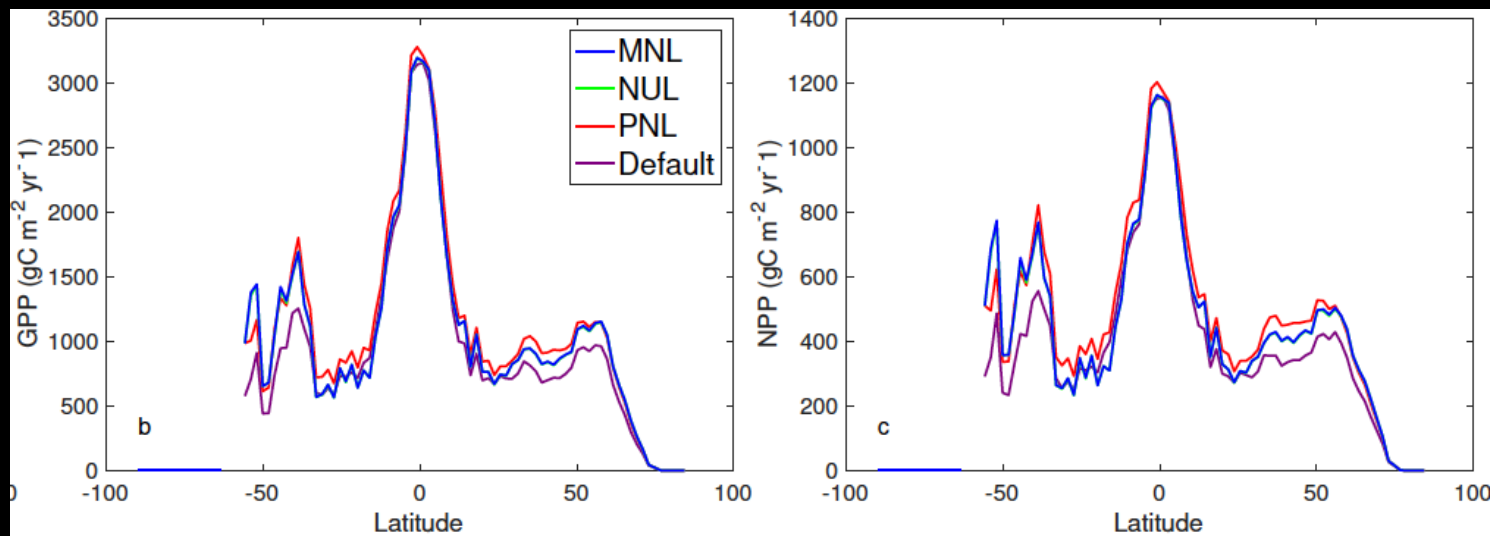
$$\bar{F}_{up} = \min \left\{ \frac{N(t)/\Delta t}{F_{up}}, 1 \right\} F_{up}$$

$$\bar{F}_{up} = \min \left\{ \frac{N(t)/\Delta t + F_{mob}}{F_{up}}, 1 \right\} F_{up}$$

$$\bar{F}_{up} = \min \left\{ \frac{N(t)/\Delta t}{F_{up} - F_{mob}}, 1 \right\} F_{up}$$

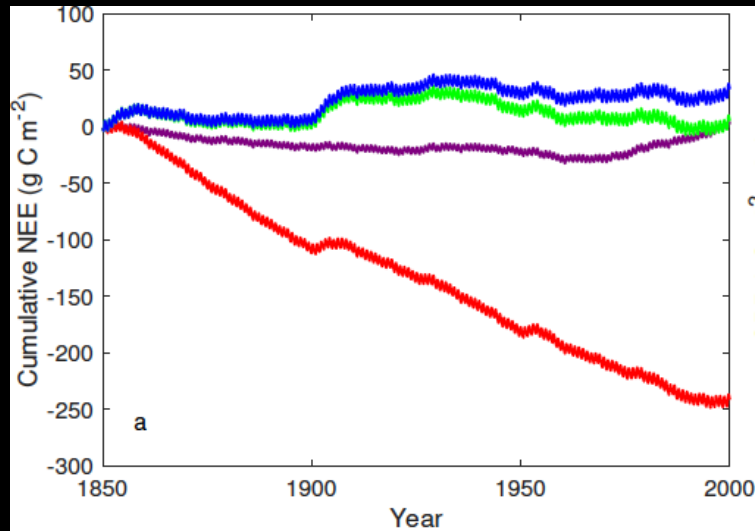


# Small difference in fast response variables

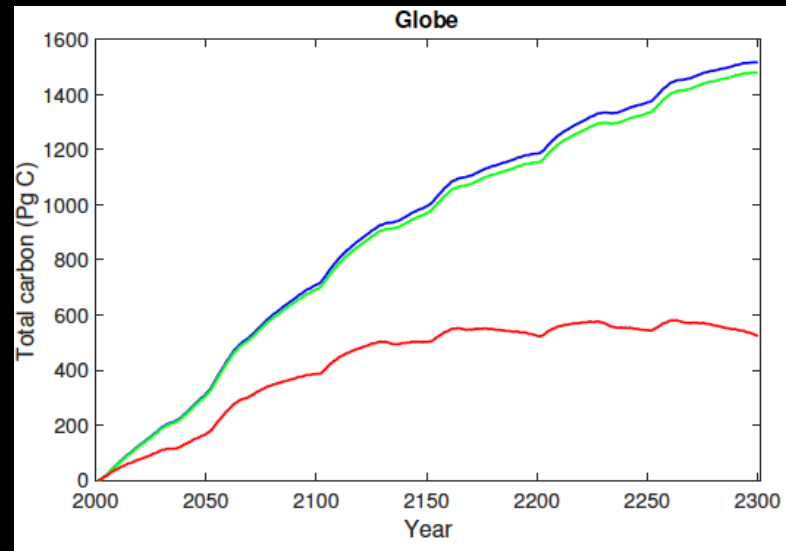


10-year mean for 1991-2000 historical run

# Large differences for slow response variables



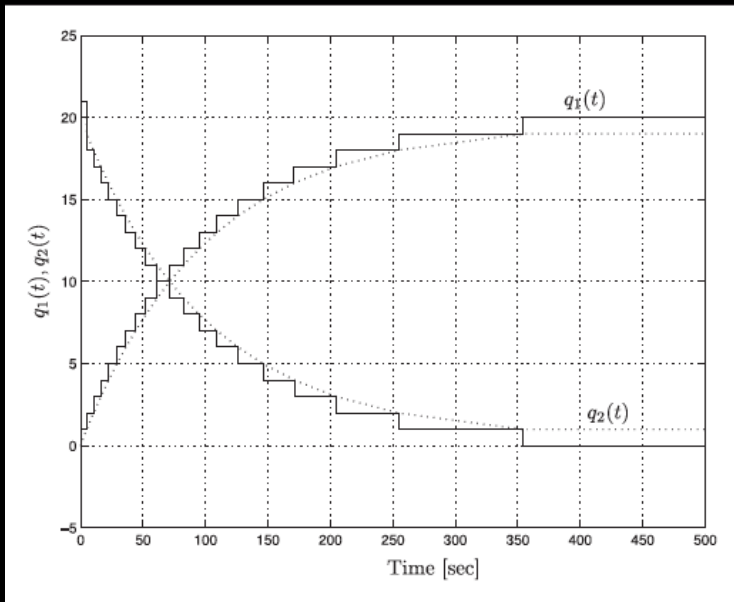
Historical run



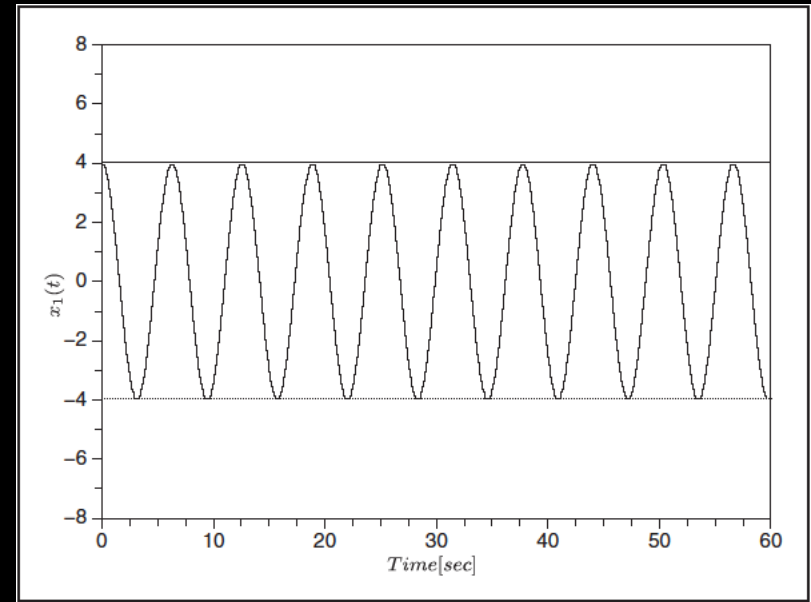
RCP4.5 atmospheric  $\text{CO}_2$  forcing



# Poor numerical implementation could be fatal: one more example



Wrong



Right

Migoni et al., 2012

# Summary

- It's critical to
  - get mechanistic formulations consistent with the processes
  - get the numeric encoding consistent with the mechanistic formulations
- Existing uncertainty analysis of parameterization and initial boundary conditions could be severely biased.

