Stochastic Parameterizations

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February 28, 2017

Subgrid-scale parameterization

$$\partial_t \boldsymbol{u} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} = -\frac{1}{\rho_0} \nabla p + \nu \nabla^2 \boldsymbol{u}$$

Can't resolve solutions ⇒ situation is hopeless?

Numerical model *is* able to resolve some scales.

We need a model for the dynamics of the scales that are *resolvable* by our model.

$$\partial_t \overline{\boldsymbol{u}} + \overline{\boldsymbol{u}} \cdot \nabla \overline{\boldsymbol{u}} = -\frac{1}{\rho_0} \nabla \overline{p} + S + \nu \nabla^2 \overline{\boldsymbol{u}},$$

$$S = \overline{\boldsymbol{u}} \cdot \nabla \overline{\boldsymbol{u}} - \overline{\boldsymbol{u} \cdot \nabla \boldsymbol{u}}$$

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(Subgrid-scale) Parameterization is modeling S (or similar terms in tracer equations).

The low-pass spatial filter $\overline{(\cdot)}$ is *not* an ensemble mean; it is *not* a zonal mean; it is *not* a time average. It *is* a local spatial average.

Define $\boldsymbol{u}' = \boldsymbol{u} - \overline{\boldsymbol{u}}$, the 'eddies.' It includes *all* small-scale features, not just coherent eddies.

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We don't know $\boldsymbol{u}'$ precisely, but we know what a *realistic* $\boldsymbol{u}'$ would look like.

A deterministic parameterization says that for all plausible values of $\boldsymbol{u}'$, S depends only on $\overline{\boldsymbol{u}}$.

For example, **if**

- $\boldsymbol{u}'$ is much smaller scale than the scale of the spatial average, **and**
- $\boldsymbol{u}'$ is statistically homogeneous/stationary over the scale of the spatial average

then tracer flux should be downgradient (turbulent diffusion).

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Mesoscale eddies have diameter $\sim$ 150 km; not resolvable with a 1° model and also not smaller than the averaging scale.

Ran two 16-level QG simulations, deformation radii 26 & 29 km; collected statistics of locally-averaged meridional buoyancy flux.

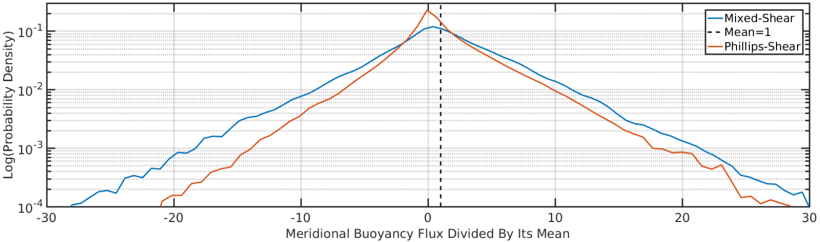

Figure: Flux is averaged over 85 km squares and divided by its mean. If scale separation was sufficient for turbulent diffusion, we would see small Gaussian bumps centered at 1.

In the foregoing example, ‘plausible’ values of $\boldsymbol{u}'$ led to a significant range of plausible values of S.

Stochastic parameterization provides a collection of plausible values for S

Stochastic parameterizations have improved ensemble forecasting and data assimilation in weather models. J. Berner (and *many* coauthors) has a nice recent review in BAMS.

There are currently 2 targets for ocean stochastic parameterizations:

- Eddy-permitting models: In these models mesoscale eddies are a bit too weak; noise can energize them.
- Non-eddy-permitting (coarse) models: improving ocean-intrinsic variability of large scales.

Next slides review these stochastic parameterizations.

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Eddy Permitting Stochastic

No global GCM implementations yet (to my knowledge)

- Kitsios, Frederiksen, and Zidikheri (e.g. OM 2013). Wonderfully thorough (but dense) investigation in QG.
- Jansen & Held OM 2014. Focus on re-introducing energy lost to biharmonic viscosity; similar spirit to some atmospheric backscatter.
- Porta Mana & Zanna OM 2014; Zanna et al. OM 2017; Antsey & Zanna OM 2017: deterministic & stochastic non-Newtonian.

There are other eddy-permitting parameterizations! This talk is only about stochastic ones.

Eddy-permitting models have ocean-intrinsic variability, just too little. Deterministic parameterizations might be OK here.

Coarse Stochastic

Coarse GCMs typically have only atmospheric-forced variability; need for stochastic parameterization is greater.

- Brankart OM 2013: Nonlinear equation of state $\rho = \rho(S, T, z)$ implies $\overline{\rho} \neq \overline{\rho}(\overline{S}, \overline{T}, z)$.
- Andrejczuk et al. MWR 2016: multiply deterministic parameterizations (& air-sea flux) by random numbers
- Williams et al. JClim 2016: stochastic heat source/sink in the ocean

Results left room for improvement…

Do we really need stochastic parameterizations in coarse models?

I ran MITgcm at 8 km resolution in a 3200 km square box, forced by steady asymmetric double-gyre winds and with SST restoring to a half-period sine from 22 C in the south to 2C in the north.

I also ran models at 40 km resolution with either constant-κ or Visbeck-style GM parameterizations.

Typically the coarse models have no variability; sometimes their SST front will oscillate. (Also the mean state is wrong.)

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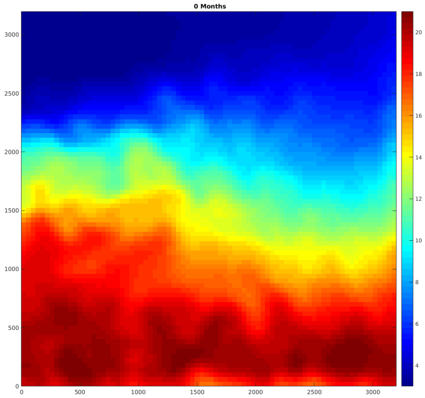


Large-scale part of SST

- Standard deviation of SST variability is up to 1 C.
- Standard deviation of temperature variability up to 2 C in the main thermocline of the subtropical gyre.
- Most variability on time scales longer than seasonal (13 weeks); no observed interdecadal oscillations.

Simplest stochastic Gent-McWilliams: multiply κ by $1 + \xi$, random.

Will not generate variability in a region with flat isopycnals.

Real eddies *can* generate variability even when isopycnals are flat, by randomly converging or diverging along-isopycnal transport.

W. Kleiber & I are working on a stochastic GM that parameterizes the GM skew flux of density (Griffies, 1998), with the property

$$\mathbb{E}[\overline{\boldsymbol{u}'_\perp \rho'}] = -\kappa \nabla_\perp \rho$$

i.e. statistical mean equals a GM parameterization.

Using GM framework the parameterization remains adiabatic; on average removes PE.

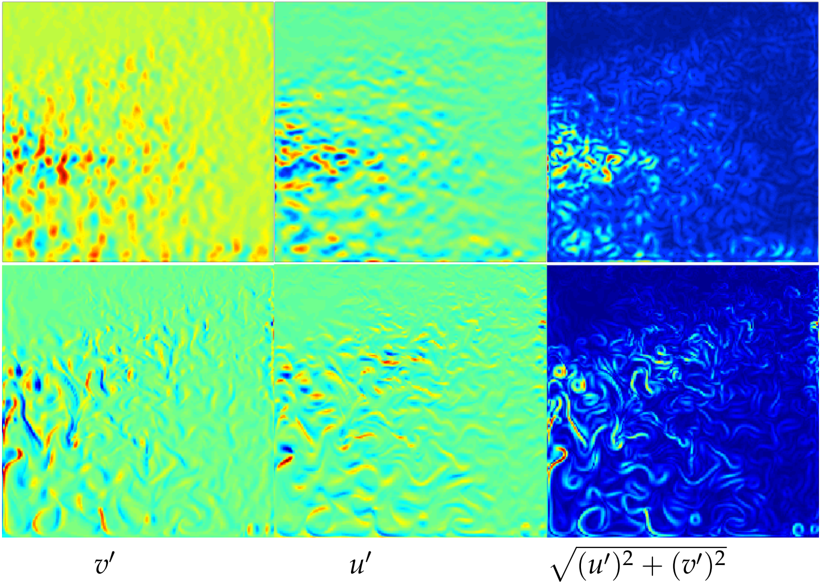
Generates eddy-induced variability even when $\nabla_\perp \rho = 0$.

We generate a synthetic eddy field $\boldsymbol{u}'$ & ρ' on a finer grid, then use it to construct $\overline{\boldsymbol{u}'\rho'}$ directly.

The stochastic eddy model is *much cheaper* than an eddy-resolving simulation.

The approach leads to realistic non-Gaussian flux statistics.

The stochastic eddy model is of independent interest & has other potential uses, e.g. representation error modeling in data assimilation.


v'
u'
$\sqrt{(u')^2 + (v')^2}$

Future Plans

1. Tune up a stochastic eddy model in the box-ocean above. Easier to learn & iterate in this simple setting.
2. Test stochastic GM based on the eddy model in box-ocean.
3. Tune up a stochastic eddy model for the global ocean, based in large part on eddy-resolving POP simulations because observational data is inadequate. Use obs where possible though.
4. Test stochastic GM based on the eddy model in a global 1° MOM6.