

Sea Level Diagnosis:  
Efficient methods for diagnosing  
effective sea level rise in CESM2

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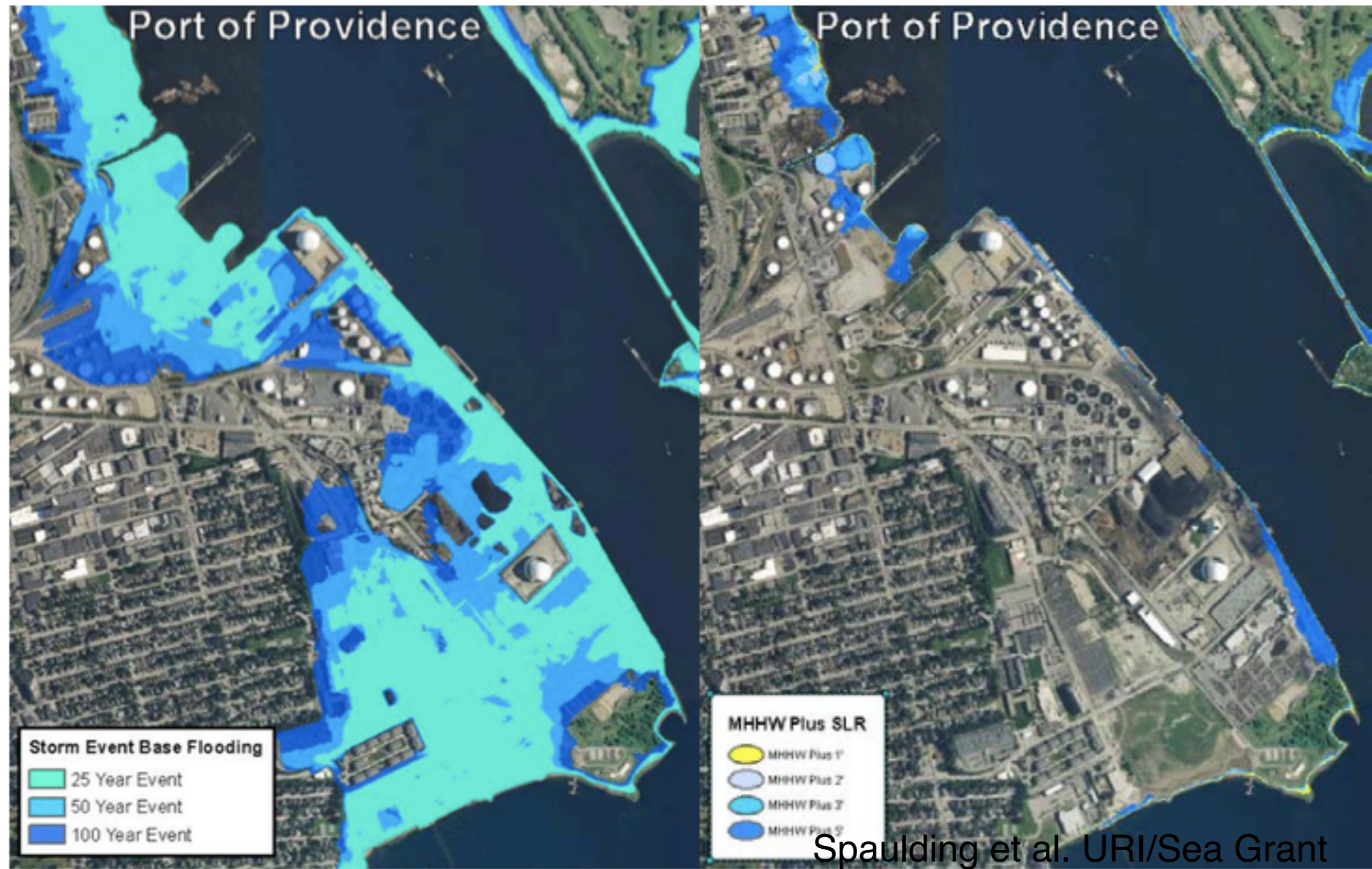


# Pieces—distributed across working groups...

- Regional sea level is composed of a number of different effects:
  - Thermal
  - Land Ice/Glacier Melt
  - Solid Earth Changes
  - Hydrology Changes
  - Dynamics



# Providence has provided.



**Figure 6** Flooding maps for the Port of Providence for the 25, 50, and 100 yr return period (left) and for 1, 2, 3, and 5 ft of sea level rise, relative to Mean High High Water(right).

Spaulding et al, URI/Sea Grant, 2016



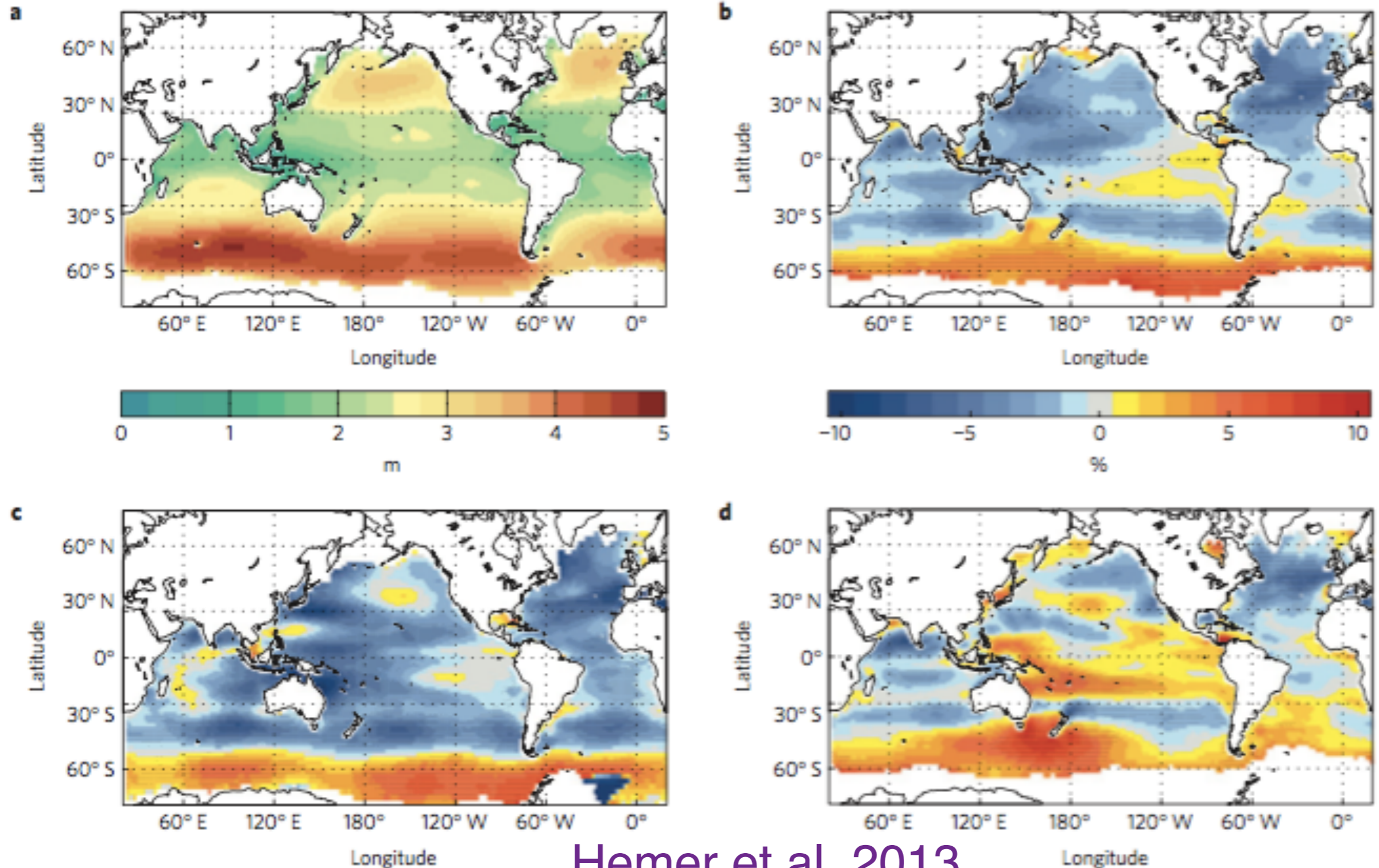
# What about storm surges, tides & waves?



**FIGURE 1.3** High surf during a high tide of nearly 2.7 m removed the front lawn of the Pacific Sands Resort at Neskowin, Oregon, on January 9, 2008. SOURCE: Courtesy of Armand Thibault.



# Climate Change of Waves: COWCLIP (Tomorrow's talk)



Hemer et al, 2013

**Figure 2 | Projected future changes in multi-model averaged significant wave height.** **a**, Averaged multi-model annual significant wave height ( $H_S$ , m) for the time-slice representing present climate (~1979-2009). **b-d**, Averaged multi-model projected changes in annual (**b**), JFM (**c**) and JAS (**d**) mean  $H_S$  for the future time-slice (~2070-2100) relative to the present climate time-slice (~1979-2009) (% change). Stippling denotes areas where the magnitude of the multi-model ensemble mean exceeds the inter-model standard deviation. Results for individual models are included in the Supplementary Information.



# Hydrostatic Pressure is the weight of the air & water above you.

Short & dense stack

Weighs same as

Tall & low-density stack



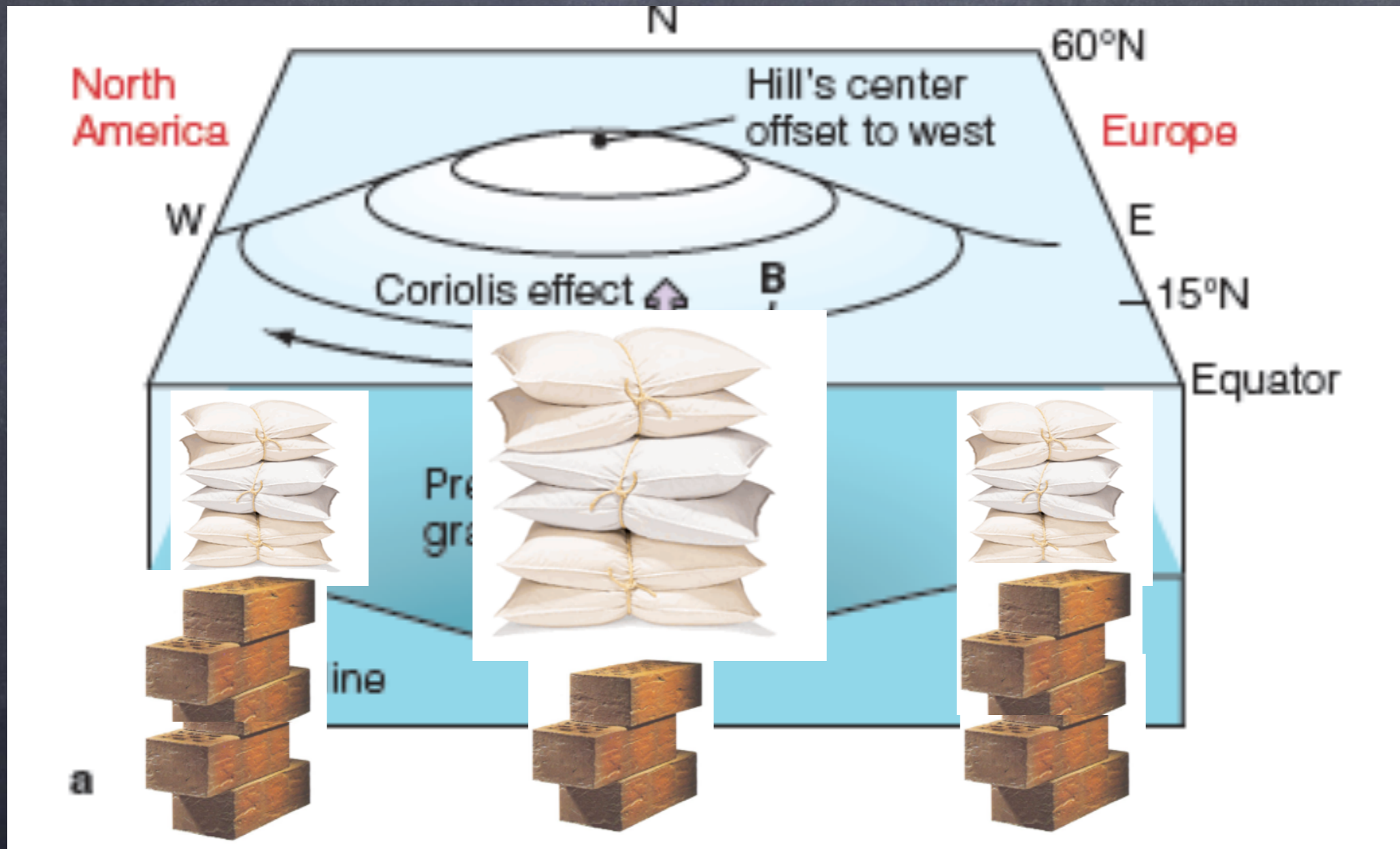
Med. Pressure



Med. Pressure



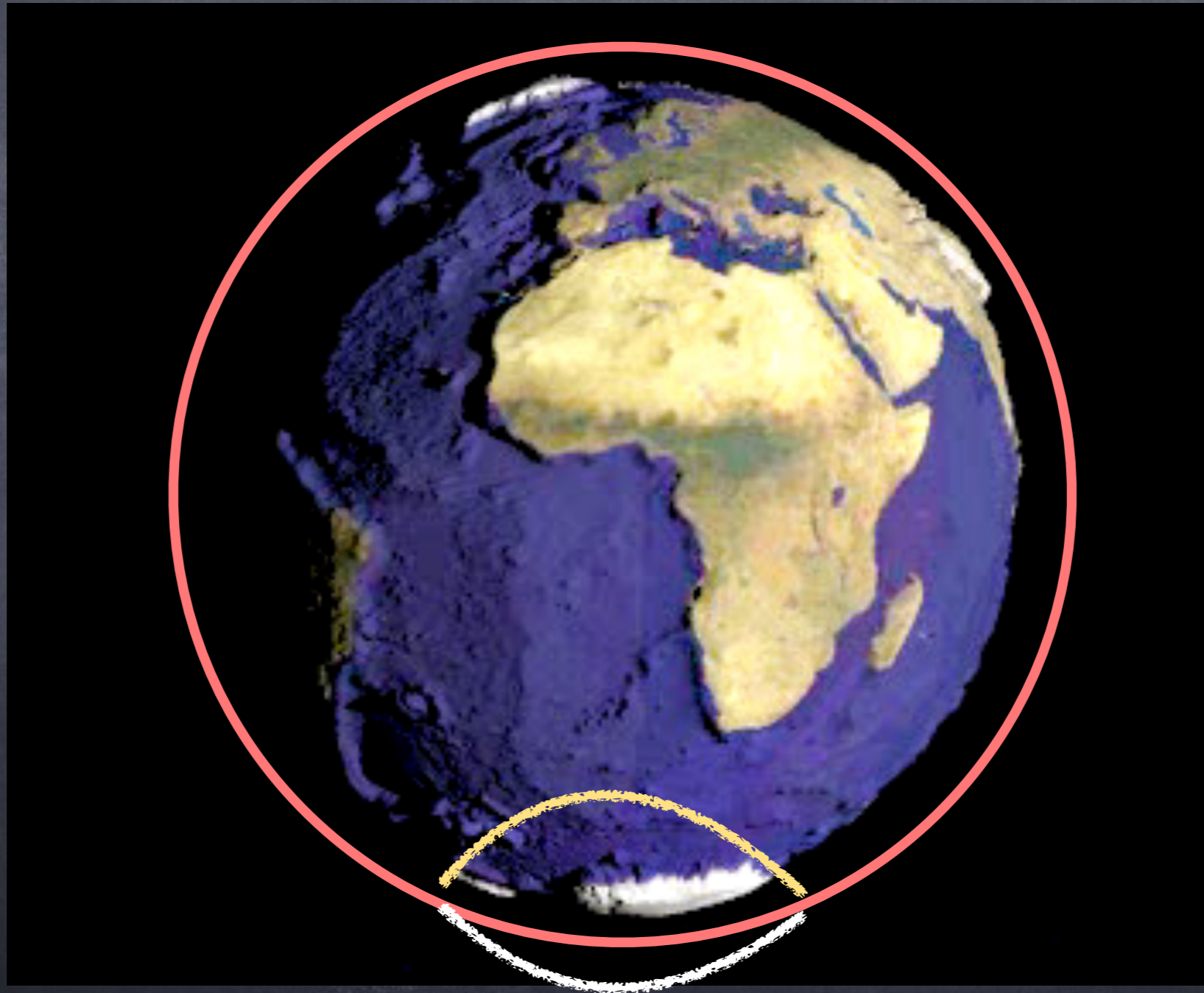
The Thermal Wind/Margules relation is at play here as well.



Here, the elevated surface height \*raises\* pressure while the depressed Pycnocline indicates light water concentrated below. At the bottom of this 'sea', there will be little pressure differences at a given depth.



If there are no bottom pressure anomalies, i.e., no  
integrated mass anomalies—  
in a thin ocean approximation, the Geoid is unaffected

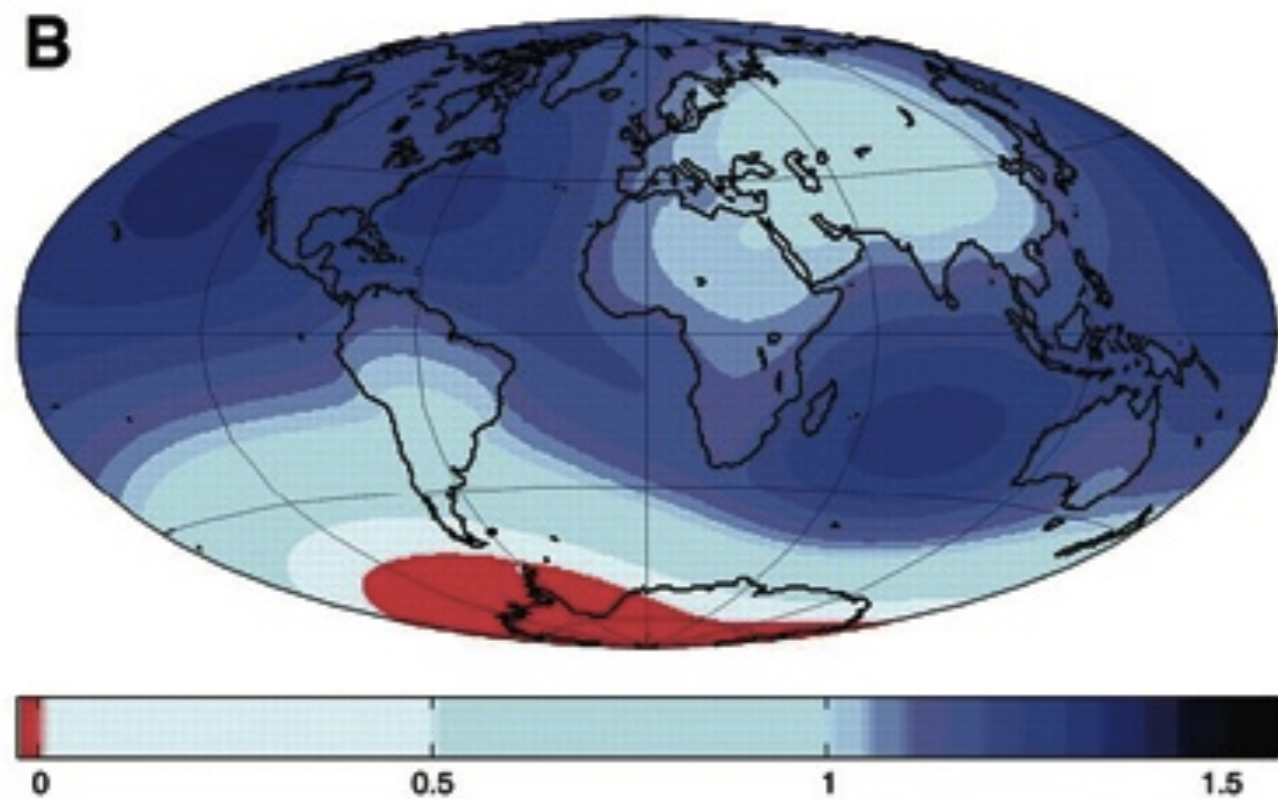


Then just consider glacial effects on geoid & solid earth: **LIWG**



# "The Sea-Level Fingerprint of West Antarctic Collapse" Mitrovica et al. 2009

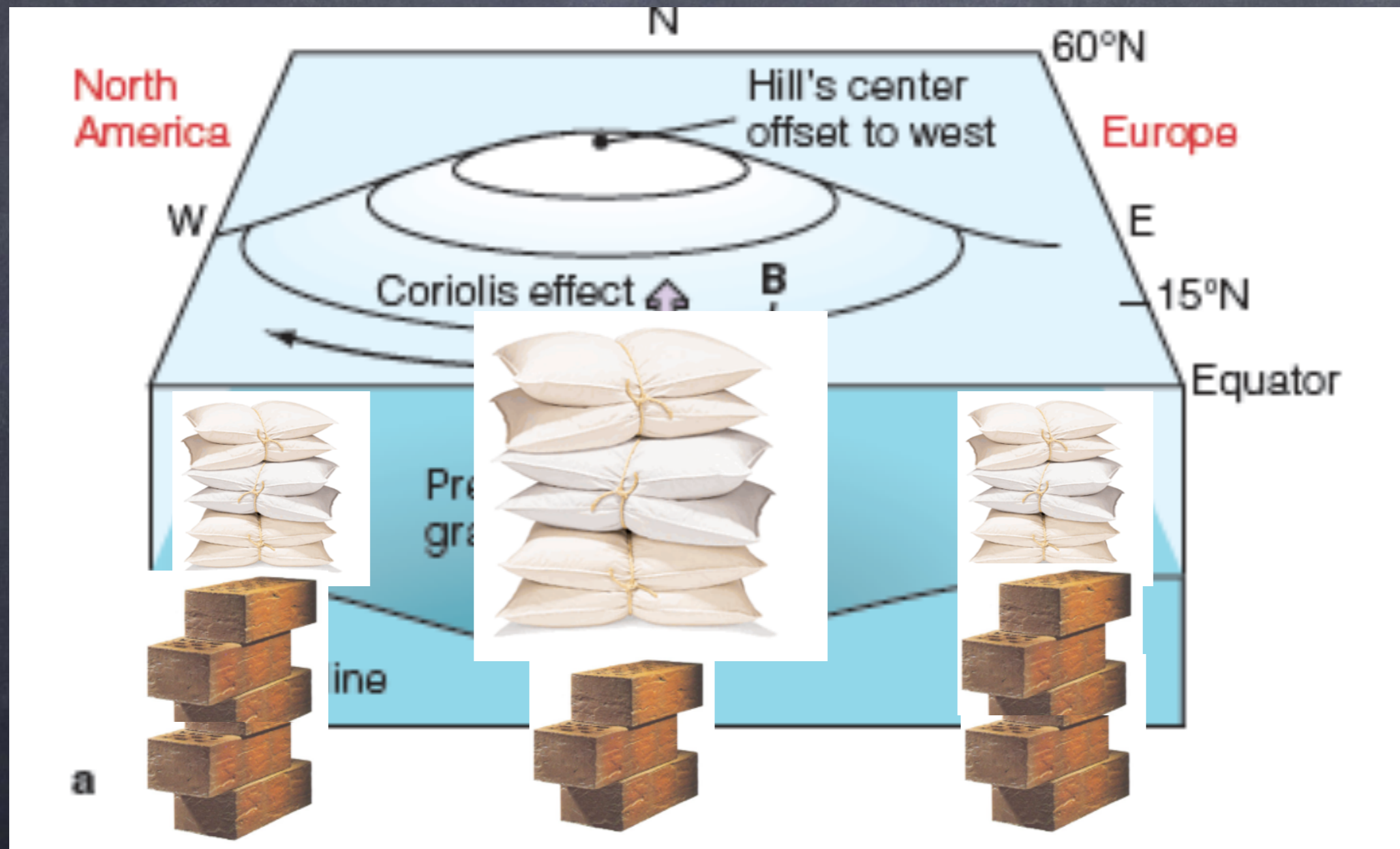
Sea Level response to melting W. Antarctica has this bullseye on US East Coast...



This map certainly got the attention of US news reports!



This "Self-Attraction and Loading" also occurs with ocean dynamics, tides, etc.



At the bottom of this 'sea',  
if there are residual pressure differences on the bottom, it means there  
are mass anomalies in the horizontal—which affect the geoid & depress solid earth.



# Greatbatch (94) & Griffies & Greatbatch (12)

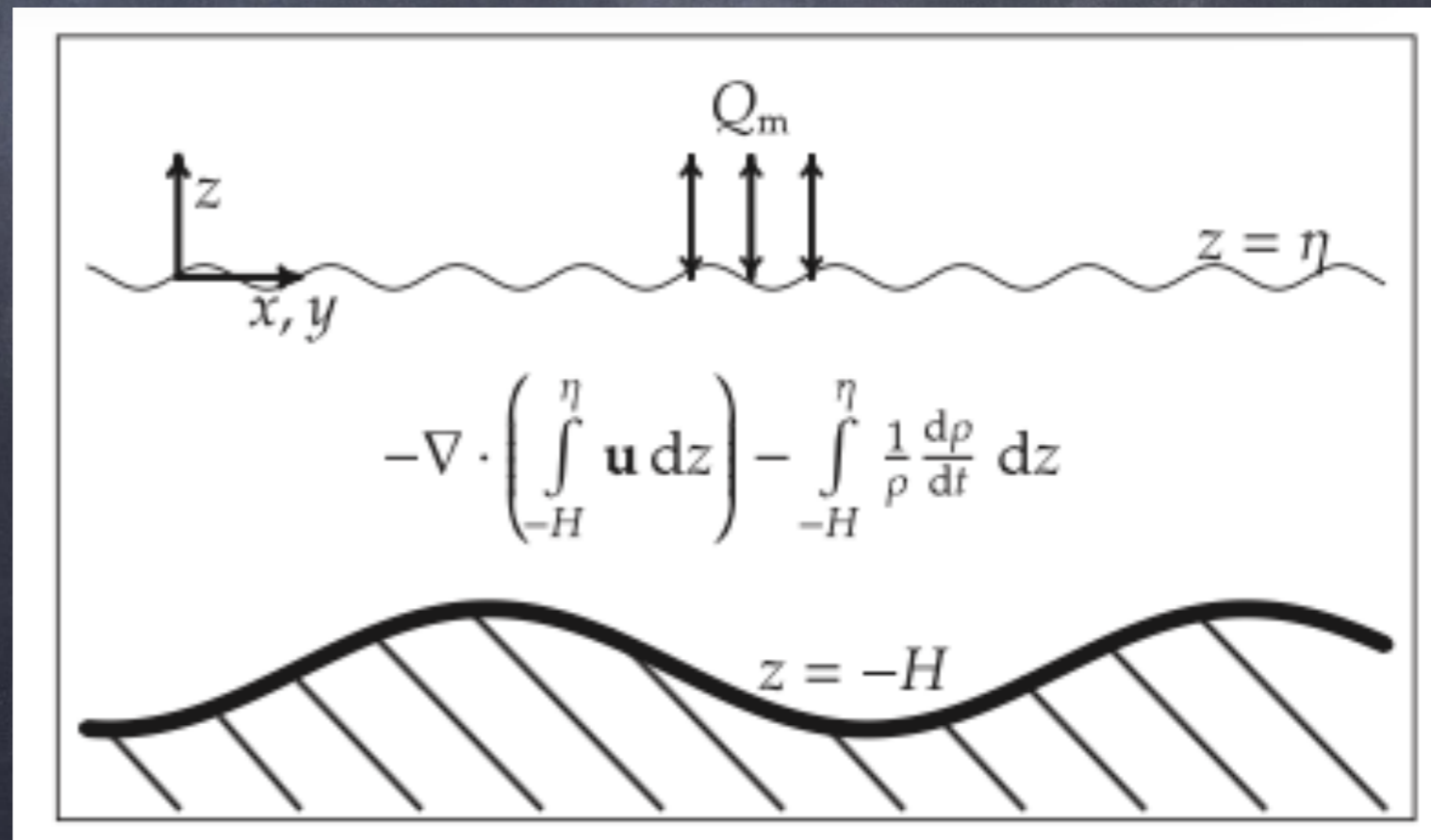
These authors address the challenge of volume-conserving (Boussinesq) models—  
by finding an effective  
“kinematic thickness equation”

Volume-conserving  
like POP

$$\frac{\partial \eta^B}{\partial t} = Q_m / \rho_0 - \nabla \cdot \mathbf{U}.$$

with

$$\mathbf{U} = \int_{-H}^{\eta} \mathbf{u} dz,$$





# Greatbatch (94) & Griffies & Greatbatch (12)

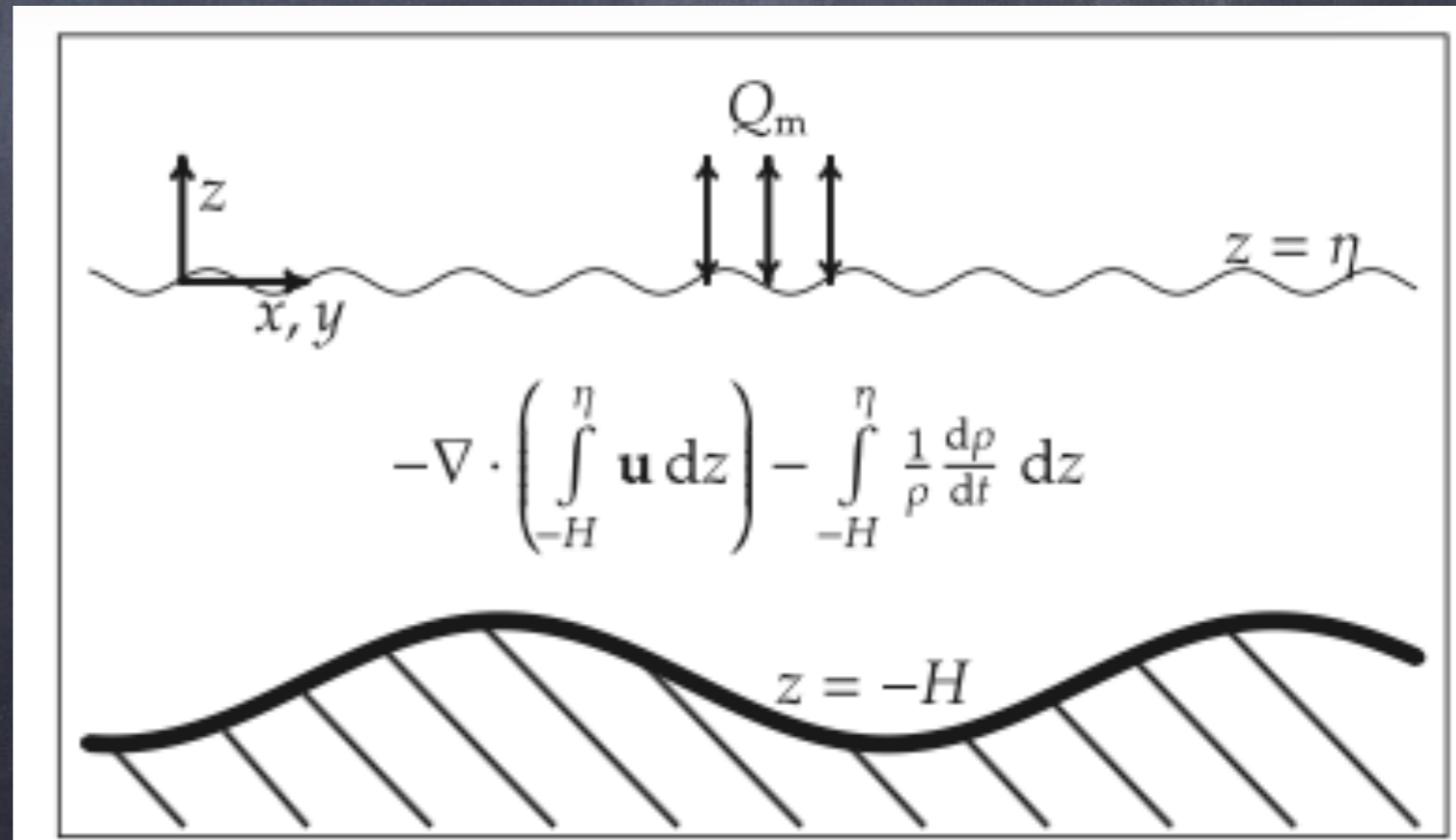
These authors address the challenge of volume-conserving (Boussinesq) models—  
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“Kinematic” Correction  
Greatbatch (94)

$$\frac{\partial \eta}{\partial t} = \frac{Q_m}{\rho(\eta)} - \nabla \cdot \mathbf{U} - \int_{-H}^{\eta} \frac{1}{\rho} \frac{d\rho}{dt} dz$$

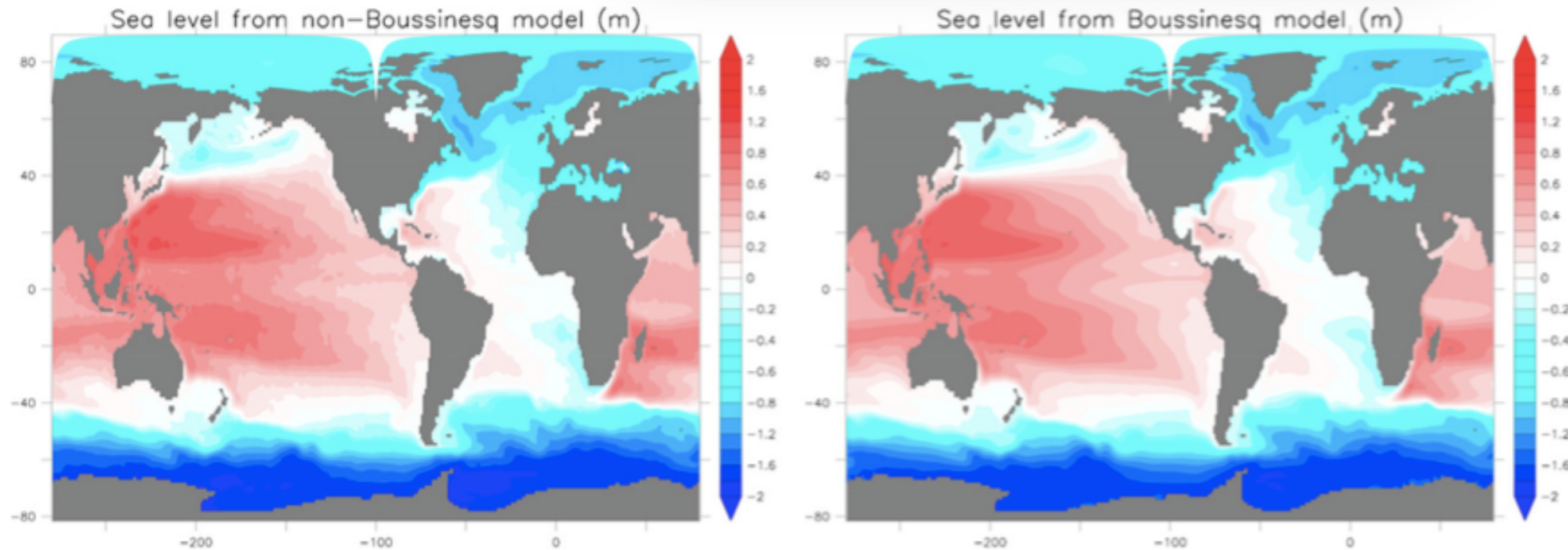
with

$$\mathbf{U} = \int_{-H}^{\eta} \mathbf{u} dz,$$





# Greatbatch (94) & Griffies & Greatbatch (12)



Good diagnostic agreement in Sea Level between  
Boussinesq and non-Boussinesq models

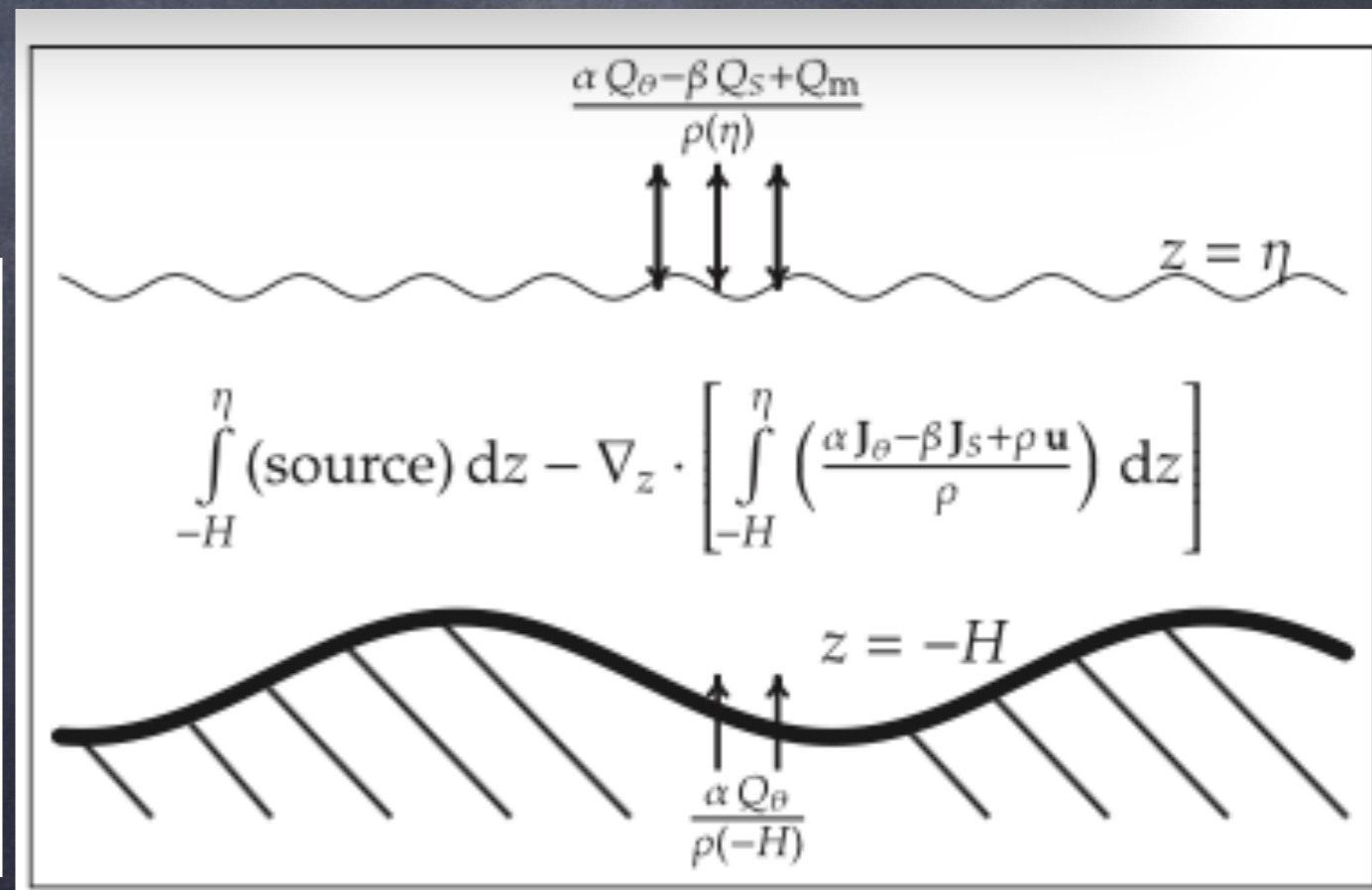


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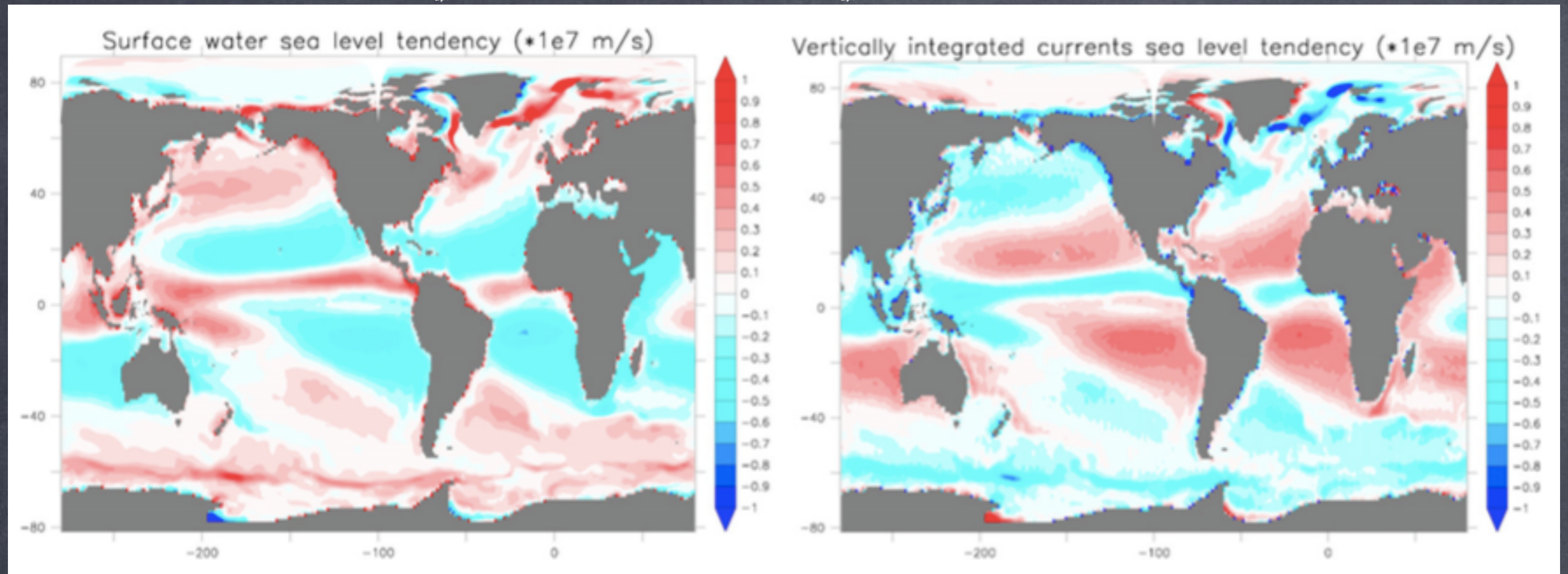
“Diagnostic” formulation  
Griffies & Greatbatch

$$\begin{aligned} \frac{\partial \eta}{\partial t} = & -\nabla_z \cdot \left( \int_{-H}^{\eta} \left( \frac{(\alpha \mathbf{J}^{(\theta)} - \beta \mathbf{J}^{(S)} + \rho \mathbf{u})}{\rho} \right) dz \right) \\ & + \int_{-H}^{\eta} \left( \mathbf{J}^{(\theta)} \cdot \nabla(\alpha/\rho) - \mathbf{J}^{(S)} \cdot \nabla(\beta/\rho) \right. \\ & \left. - \frac{1}{\rho c_{\text{sound}}^2} \frac{dp}{dt} + \alpha S^{(\theta)} - \beta S^{(S)} \right) dz \\ & + \left( \frac{\alpha Q^{(\theta)} - \beta Q^{(S)} + Q_m}{\rho} \right)_{z=\eta} + \left( \frac{\alpha Q^{(\theta)} - \beta Q^{(S)} + Q_m}{\rho} \right)_{z=-H}, \end{aligned}$$

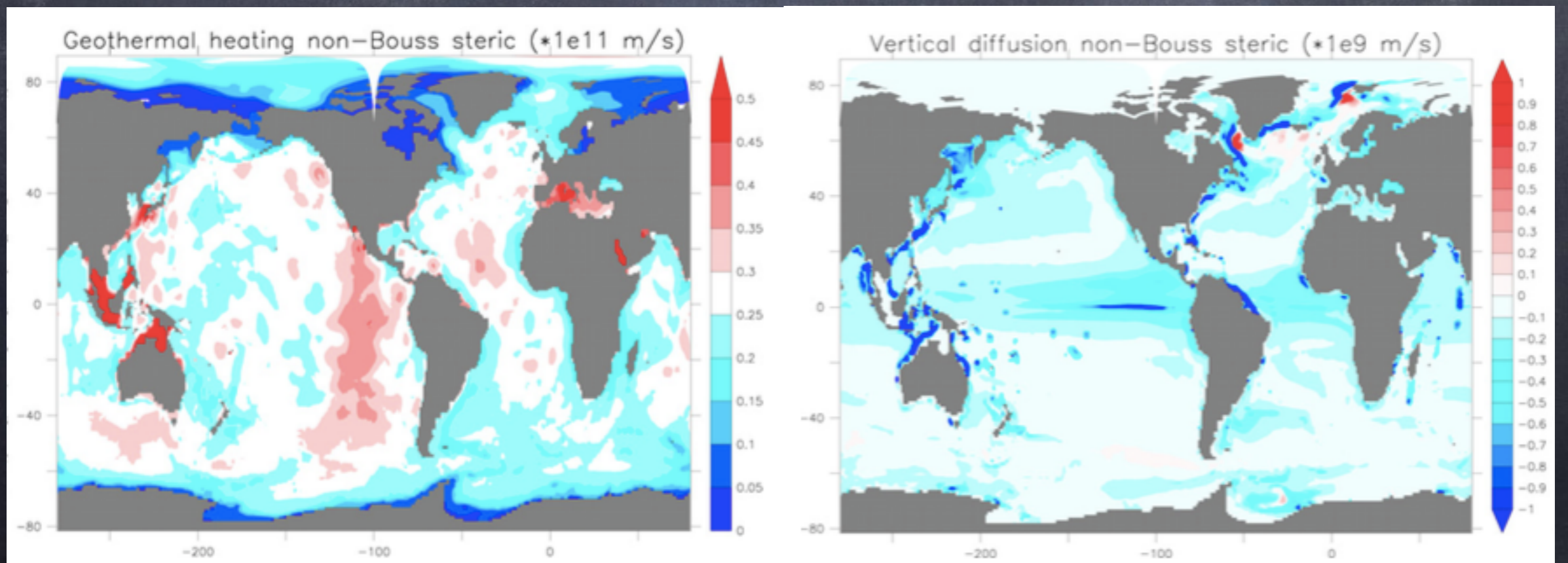




Directly Simulated by POP/CESM2



Would be inferred from POP/CESM2





# But, what about mass anomalies?

Greatbatch (94)

Griffies & Greatbatch (12) also diagnose  
the relevant bottom pressure

$$\frac{\partial(p_b - p_a)_{\text{diag}}^B}{\partial t} = \frac{\partial(p_b - p_a)^B}{\partial t} - \frac{gV\partial_t\langle\rho\rangle}{A},$$

Which is just what's needed for  
the self-attraction & loading

Combine with LI Model?



# Laplace Tidal Eqns.

- Ray (1998) reviews tidal modeling.  
The critical equations are:

$$\frac{\partial u}{\partial t} + f \times u = -g \nabla \left( \zeta_g - \frac{\Gamma}{g} \right) - \mathcal{F}$$

"geocentric"—not consistent with  
"thickness" equation

$$\frac{\partial u}{\partial t} + f \times u = -g \nabla (\zeta - \zeta_{EQ} - \zeta_{SAL}) - \mathcal{F}$$

Relative—consistent with "thickness"



$$\frac{\partial u}{\partial t} + f \times u = -g \nabla (\zeta - \zeta_{\text{EQ}} - \zeta_{\text{SAL}}) - \mathcal{F}$$

How to get the new forcings?

$$\zeta_{\text{EQ}} = \frac{\gamma_2 \Phi}{g}$$

"equilibrium tide"—not needed for low freq. SLR

$$\zeta_{\text{SAL}} = \sum_n \gamma'_n \alpha_n \zeta_n$$

"Self-Attraction & Loading"

Requires Global convolution, or Spher. Harmonic projection of bottom pressure.



# Equilibrium Vs. Tidal Eqns.

Stepanov & Hughes (04) show that for low frequencies, just using the equilibrium balance is good approximation:

$$\left( \zeta_g - \frac{\Gamma}{g} \right) = (\zeta - \zeta_{\text{SAL}}) \text{ diagnostic SLR}$$

Model potential



# Footprinting vs. Dynamics and Regional Variability

- This part is most speculative, as properly the SAL effects should be part of the dynamics, not just diagnosed after the fact (Vinogradov et al., 2015)
- E.G. the Stammer "Slow set-up of baroclinic footprints" controversy.
- This is a failure of the SAL community to calculate response in baroclinic models.
- True Global Mean SLR vs. ENSO (Leben & Hamlington)  
—another dynamics vs. forcing question
- To Be Continued...



# Conclusions

- Thanks to Greatbatch & Griffies, there is a diagnostic technique for evaluating sea level rise in Boussinesq models such as CESM2/POP
- Interestingly, they have it broken down by process, allowing per-process assessments of subgrid effects.
- In order to correctly account for climate perturbations, geoid & solid earth perturbations need to be accounted for. These effects are  $O(30\%)$  of the signal, and can be importantly unintuitive.
- On low frequencies (longer than the crossing time of all relevant waves), a combined LI & OM diagnosis is OK, but baroclinic waves are slow, so doing it online is more realistic (although harder & costly). Decision?