On the heat transfer coefficient for transfer of internally dissipated (mechanical) energy to englacial and subglacial conduit walls

Aleah Sommers
Harihar Rajaram

Department of Civil, Environmental and Architectural Engineering
University of Colorado, Boulder

MOTIVATION

Most general formulation of governing equations for englacial and subglacial hydrologic systems – Coupled PDEs for mass, momentum and energy conservation - e.g. Spring-Hutter (1981) model, Clarke (2003) enhancement of the Spring-Hutter model

Approximations to the full equations are often used for computational efficiency and simplicity (e.g. Nye model of englacial and subglacial conduits, most subglacial hydrology models – 2D)

Conversion of dissipated mechanical energy (“energy loss”, “head loss”, “frictional loss”) to thermal energy and its transfer to (ice) walls of englacial and subglacial hydrologic systems – very important mechanism in their dynamics. [analogous to strain heating term in thermo-mechanical ice flow models, except dominated by turbulent dissipation in hydrologic systems – water if the fluid….]

Heat resulting from dissipated mechanical energy is essentially negligible in most other (notably engineering) contexts and applications - transfer of this heat to pipe/duct walls has not been studied rigorously in the extensive body of work on heat transfer

Our goal was to rigorously investigate and quantify this mechanism based on contemporary understanding of turbulent velocity, eddy viscosity/diffusivity (Reynolds analogy) and turbulent dissipation profiles in duct flows (circular conduit or “sheet”)

CROSS-SECTIONALLY AVERAGED THERMAL ENERGY EQUATION FOR AN ENGLACIAL CONDUIT OR SUBGLACIAL CONDUIT/SHEET (1D FOR ILLUSTRATION) – FROM SPRING-HUTTER (1981) MODEL

$\langle T_w \rangle$=cross-sectionally averaged water temperature ("bulk" temperature)

accumulation + transport of thermal energy in water

mechanical energy dissipation ("loss") = source term for thermal energy

thermal energy transfer to (ice) conduit walls, produces melt

minor terms

$$\rho_w C_w A\left(\frac{\partial\langle T_w\rangle}{\partial t}+V\frac{\partial\langle T_w\rangle}{\partial x}\right) = A\langle\Phi\rangle - Ph(\langle T_w\rangle\text{-}\mathrm{T}_i) - \dot{m}\left(C_w(\langle T_w\rangle - T_i)-\frac{V^2}{2}\right)$$

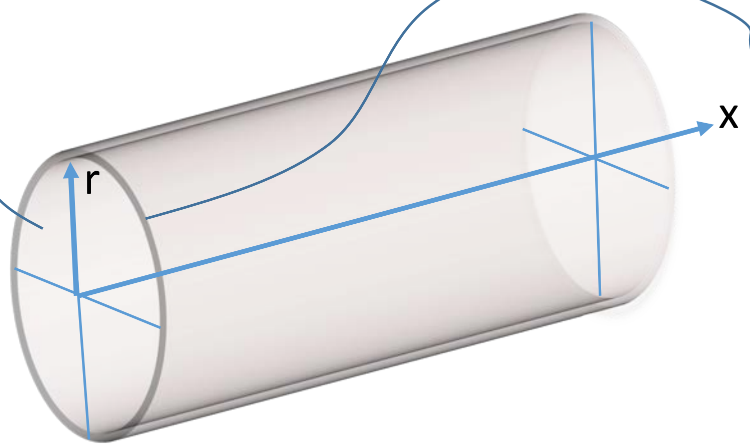
cross-section area

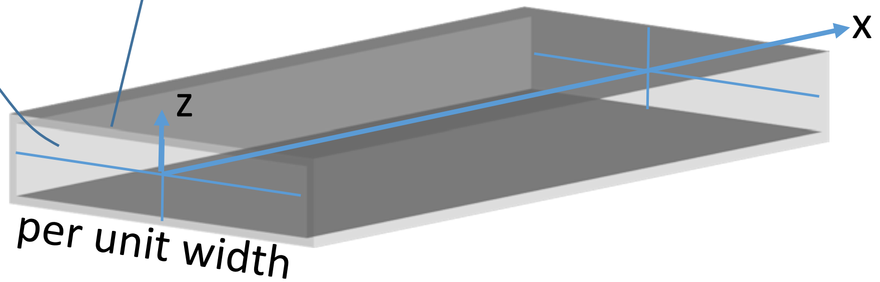
average velocity

perimeter

heat transfer coefficient

melt production rate





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melt production rate $\dot{m} = \dfrac{Ph(\langle T_w\rangle - T_i)}{L}$

Latent heat of fusion

Nye conduit model for jokulhlaups, subglacial hydrology models: neglect accumulation + transport, minor terms

$$\dot{m} = \frac{A\langle\Phi\rangle}{L}$$

All mechanical energy dissipated locally used to produce melt

Spring-Hutter model (1981), Clarke (2003) – consider all terms: not all locally dissipated mechanical energy used to produce melt locally

Clarke (2003) – suggested that unrealistically high calibrated roughness values obtained with the Nye model can be "reduced" if the Spring-Hutter model is used – specifically highlighting the limitations of $\dot{m} = A\langle\Phi\rangle / L$

Clarke (2003) – wondered if the correct heat transfer coefficient *h* was being used in his and Spring-Hutter models

Engineering Heat Transfer Correlations:
(derived for quasi-steady heat transfer)

$$h = \frac{\overset{\text{Nusselt number}}{Nu}\ \underset{\text{thermal conductivity}}{k_w}}{\underset{\text{hydraulic diameter = 4*A/P}}{d_h}}$$

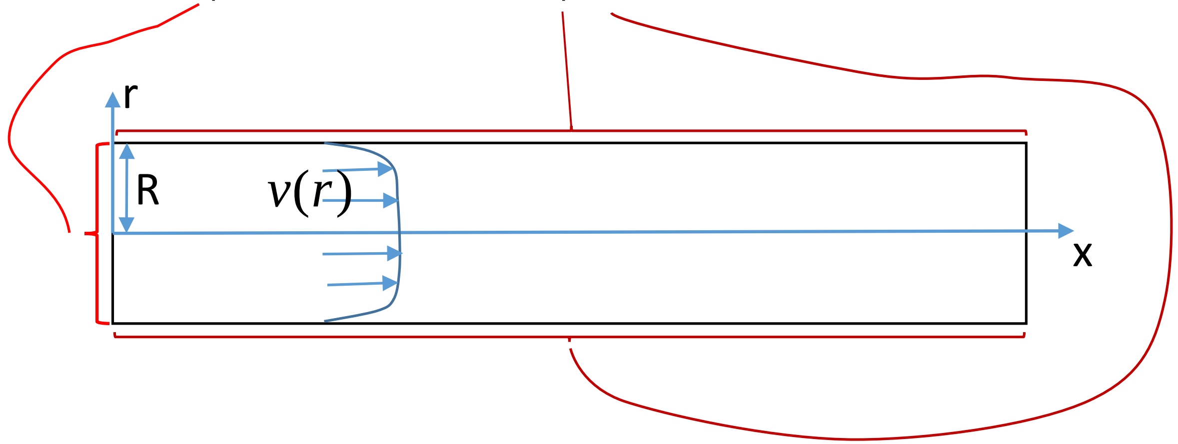
Clarke (2003) used the classical Dittus-Boelter correlation – based on heat transfer from heated walls (perimeter) of a duct to the bulk fluid, also applicable for warmer fluid and cooler walls – neglect thermal energy resulting from mechanical energy dissipation

$$\rho_w C_w A V \frac{\partial \langle T_w \rangle}{\partial x} = Ph(T_{walls} - \langle \mathrm{T}_{w(ater)} \rangle)$$ Effective cross-sectionally averaged heat transport equation

Nusselt number – can be determined experimentally, or theoretically derived by numerically solving multi-dimensional heat transport boundary value problem: (e.g. for pipe flow)

$$\underset{\substack{\text{velocity}\\\text{profile}}}{v(r)}\frac{\partial T_w}{\partial x} - \frac{1}{r}\frac{\partial}{\partial r}\left(r\left(\underset{\substack{\text{thermal}\\\text{diffusivity}}}{\kappa} + \underset{\substack{\text{eddy}\\\text{thermal diffusivity}}}{\kappa_T(r)} \right)\frac{\partial T_w}{\partial r} \right) = 0; \quad \underset{\text{entrance temperature}}{T_w(x=0,r) = T_{w0}}; \quad \underset{\text{wall temperature}}{T_w(x,r=R) = T_{wall}}$$

At “large” x, T_w (x,r) ⟶ T_{wall} across full cross-section,
< T_w >(x) cross-section average temperature ⟶ T_{wall}
The rate of approach is measured by the heat transfer coefficient (h) or Nusselt number (Nu), which depend on (turbulent) velocity and eddy diffusivity profiles, hence on Reynolds Number (Re)



Proper formulation of multi-dimensional heat transport boundary value problem for transfer of internally dissipated mechanical energy (converted to thermal energy) from fluid to conduit walls

$$v(r)\frac{\partial T_w}{\partial x} - \frac{1}{r}\frac{\partial}{\partial r}\left(r\left(\kappa + \kappa_T(r)\right)\frac{\partial T_w}{\partial r}\right) = \Phi(r); \quad T_w(x=0,r) = T_0; \quad T_w(x, r=R) = T_0$$

not zero   equal entrance fluid and wall temperature

The corresponding effective cross-section averaged transport equation is:

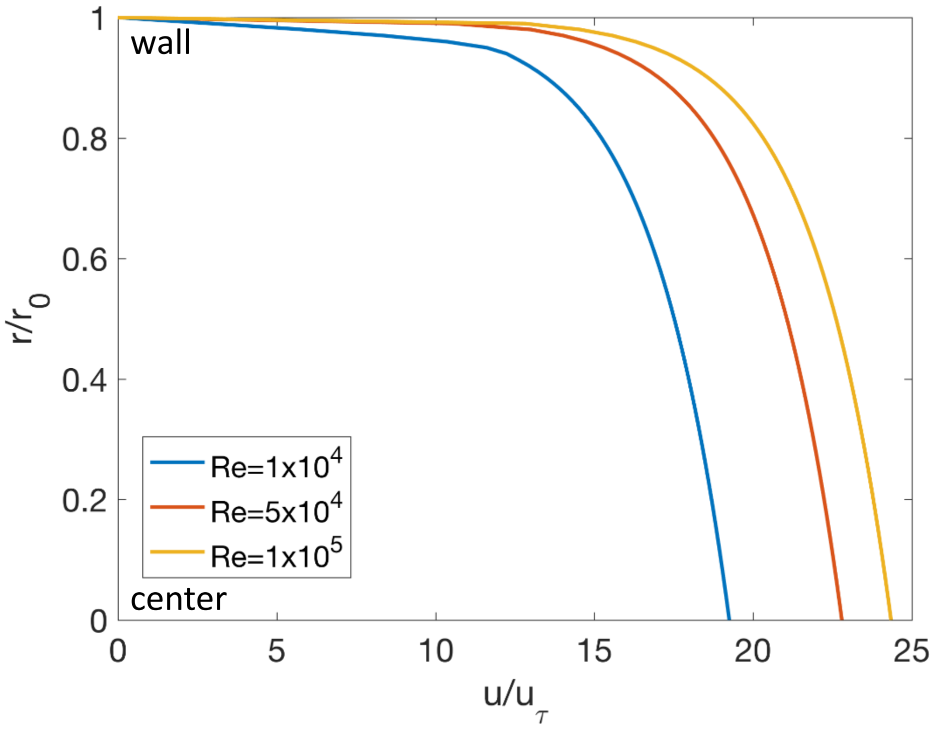
$$\rho_w C_w A V \frac{\partial \langle T_w \rangle}{\partial x} = A\langle \Phi \rangle - Ph(\langle T_{w(ater)} \rangle \text{-} \mathrm{T}_{walls})$$

(DIFFERENT) heat transfer coefficient

At “large” x, cross-section average temperature $\langle T_w \rangle \rightarrow$ constant value > T_{wall}

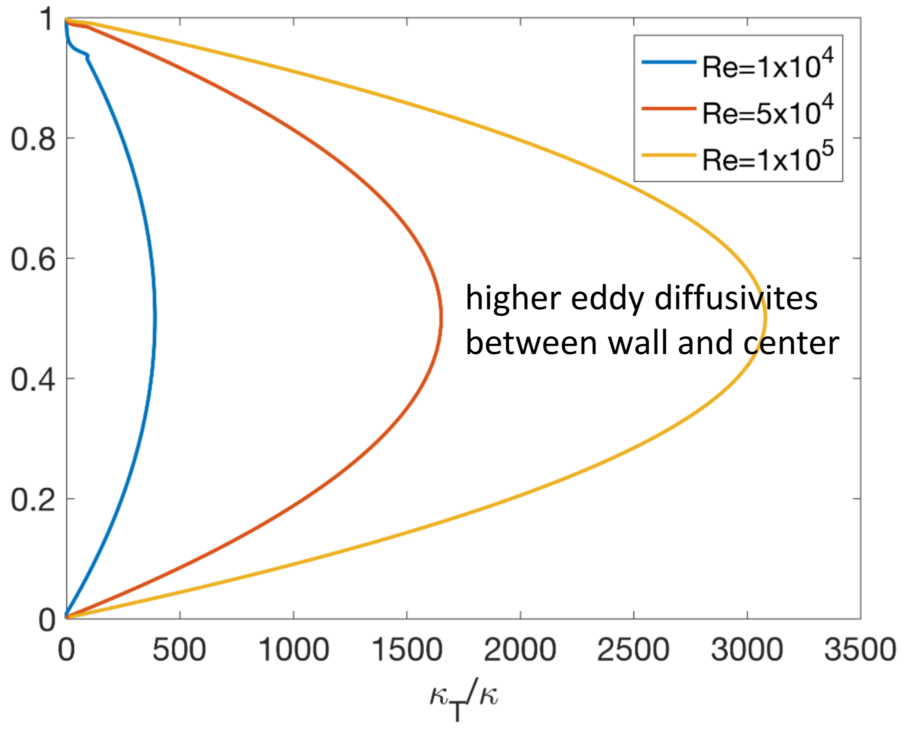
The rate of approach and the constant value attained are related to the heat transfer coefficient (h) or Nusselt number (Nu), which depend on (turbulent) velocity and eddy diffusivity profiles, hence on Reynolds Number (Re)

VELOCITY PROFILE



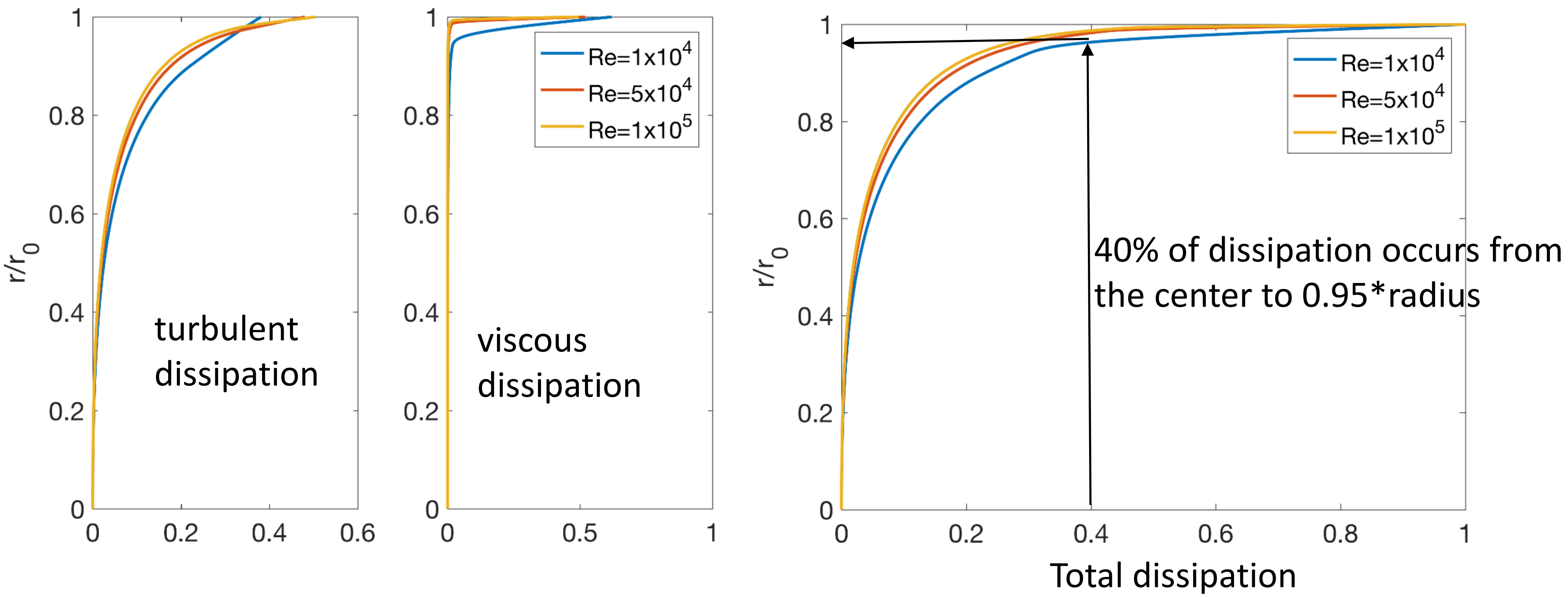
(non-dimensionalized by shear velocity)

EDDY DIFFUSIVITY PROFILE



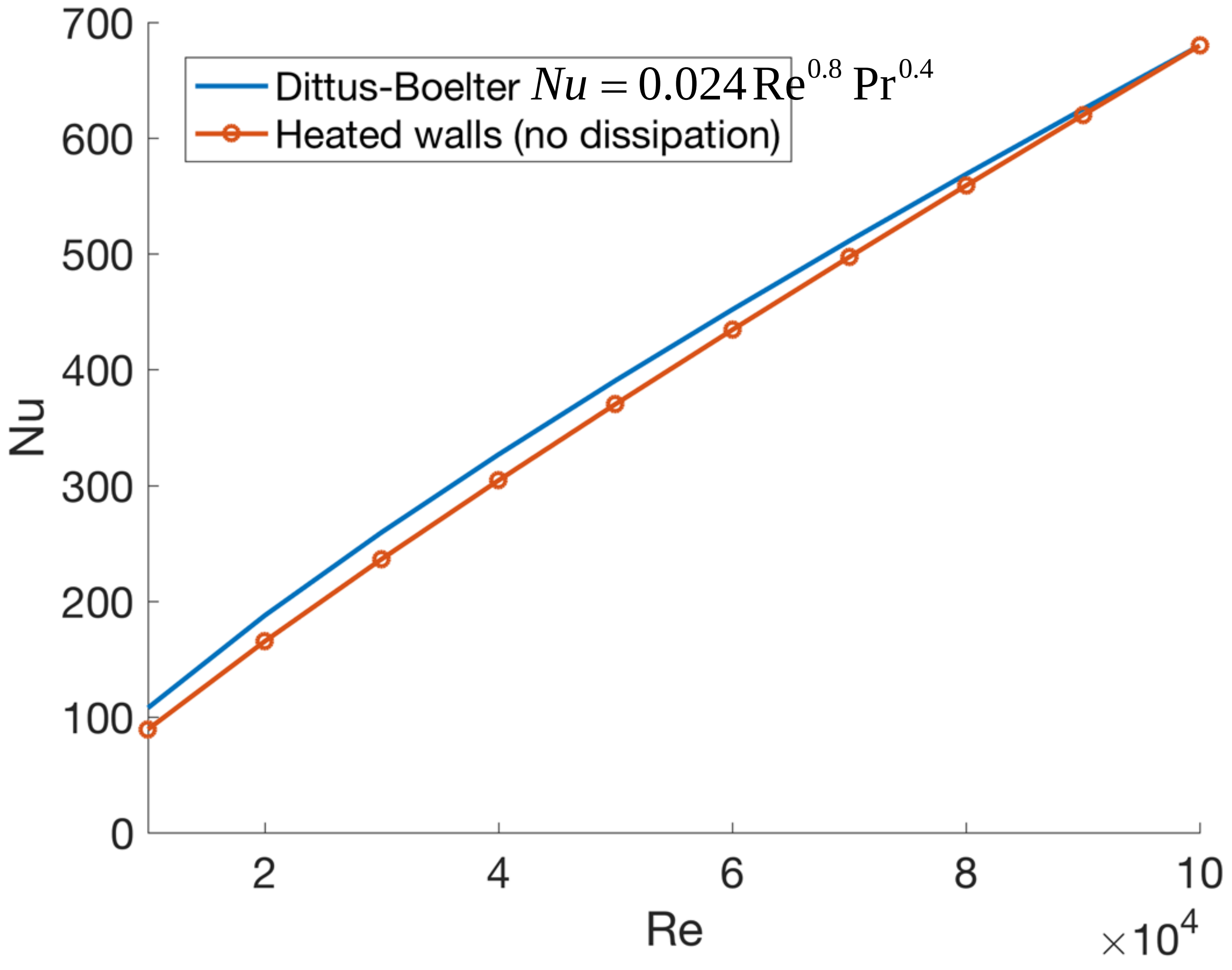
(non-dimensionalized by molecular thermal diffusivity)

Cumulative energy dissipation from center to radius r as a fraction of total dissipation

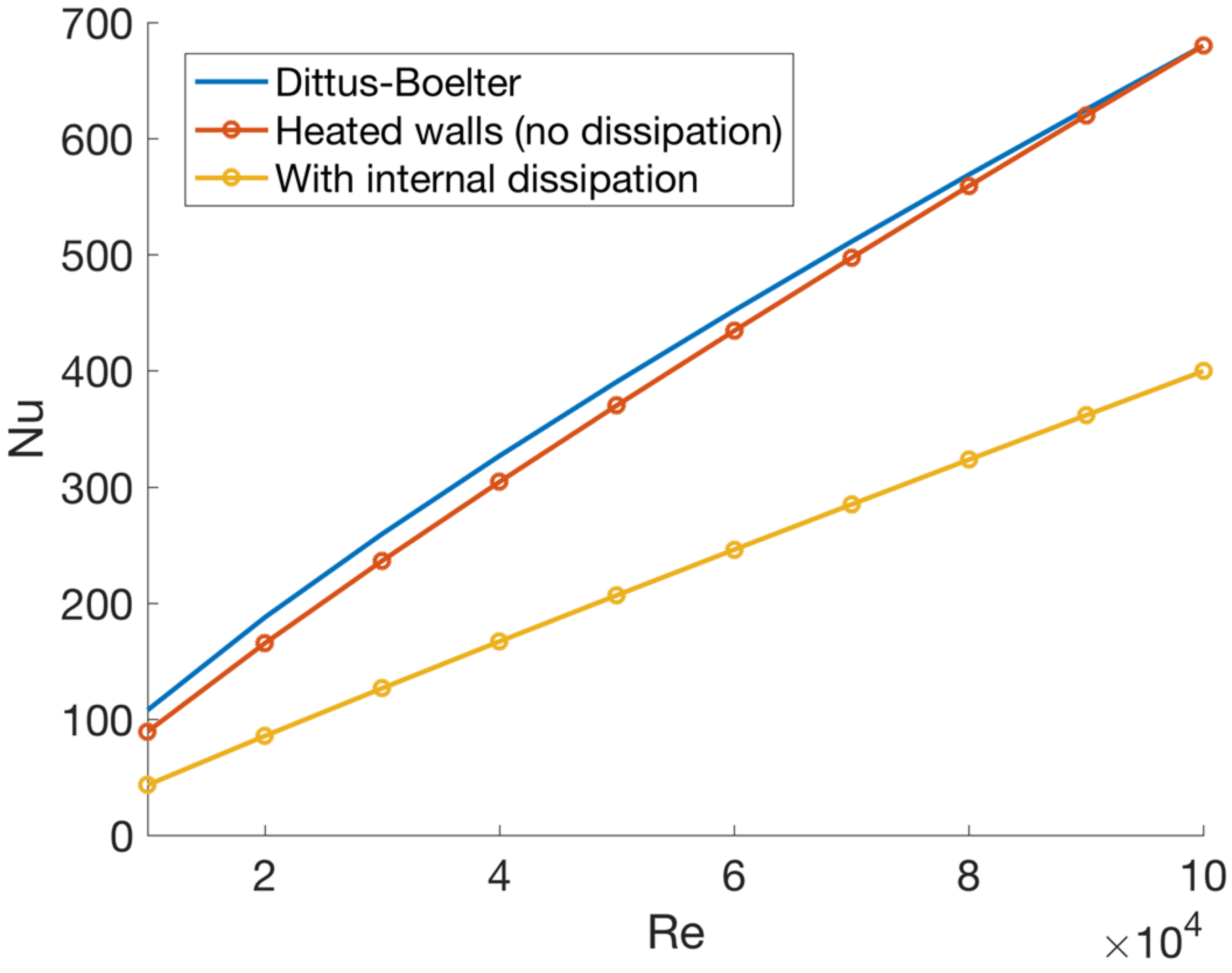


Based on Direct Numerical Simulation database of Kim, Moin and Moser (JFM 1987) and Lee and Moser (JFM 2015), analyzed by Abe and Antonia (JFM 2016)

For heat transfer from a heated wall to bulk fluid (CIRCULAR PIPE), our theoretical calculations match the Dittus-Boelter heat transfer correlation very well

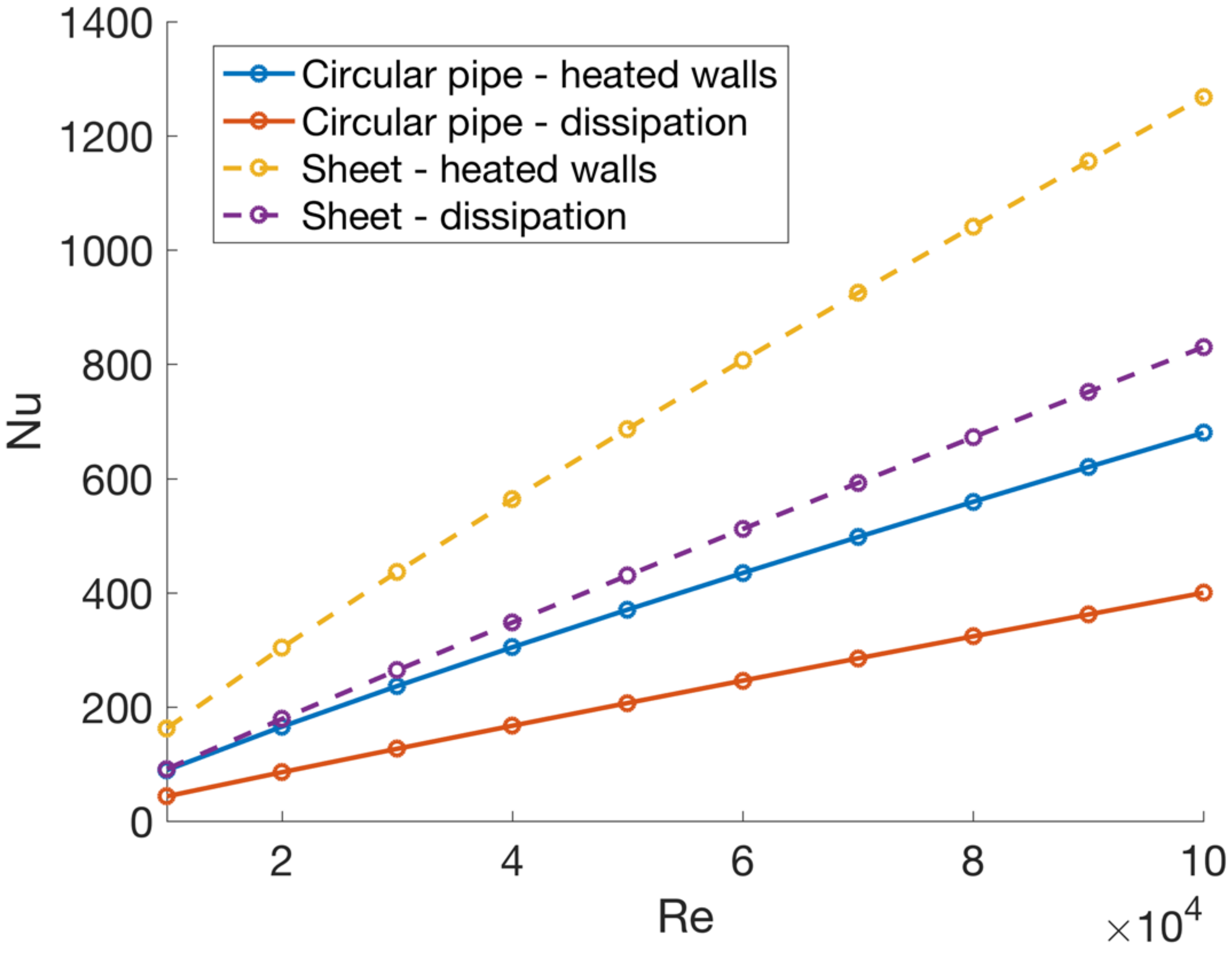


Nu COMPARISON
TRANSFER FROM HEATED WALLS
VERSUS
TRANSFER OF DISSIPATED ENERGY



Much lower Nusselt number!
Suggested correlation:
$$Nu = 0.0032\,\mathrm{Re}^{0.93}\,\mathrm{Pr}^{0.4}$$

COMPARE CIRCULAR PIPE AND (VERY WIDE, i.e. no side-wall effects) SHEET



Nu for dissipated energy transfer is smaller than for transfer from heated walls even in sheet geometry

Nu values for sheet geometry are higher than corresponding values in circular conduit geometry

IN CONCLUSION……….

We theoretically derived the Nusselt number appropriate for transfer of dissipated mechanical energy to the walls of an ice conduit/sheet – the derivation accounts for the cross-sectional variation of velocity, eddy diffusivity and dissipation rate

Our theoretical approach consistently reproduced the Dittus-Bolter correlation for the wall-heat transfer case

We show that the Nusselt number for transfer of dissipated energy to walls is much lower that predicted by the classical Dittus-Bolter correlation

In situations where the approximation of immediate local transfer of locally dissipated energy to conduit/sheet walls is inaccurate (e.g. very high water flow rates, jokulhlaups), the appropriate Nusselt numbers used in Spring-Hutter models should be revised

For hydrologic systems in cold ice, locally dissipated energy does not all go towards producing melt (dissipated energy → heat → partitioned between conduction into cold ice and melting of wall; dissipated energy needs to be large enough to counteract refreezing)