An Unconventional Approach to Modeling Subglacial Hydrology with Flexible Geometry

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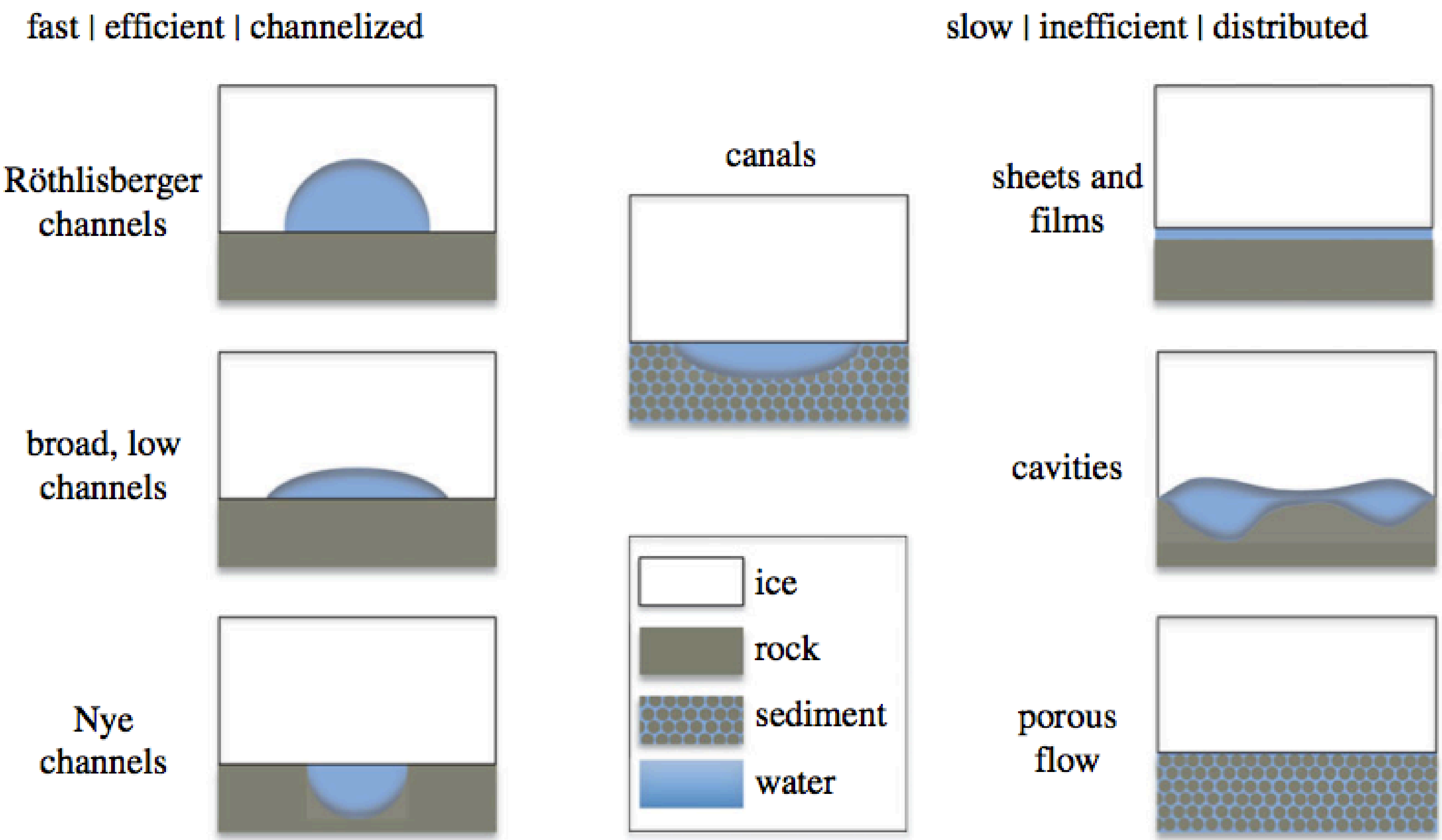
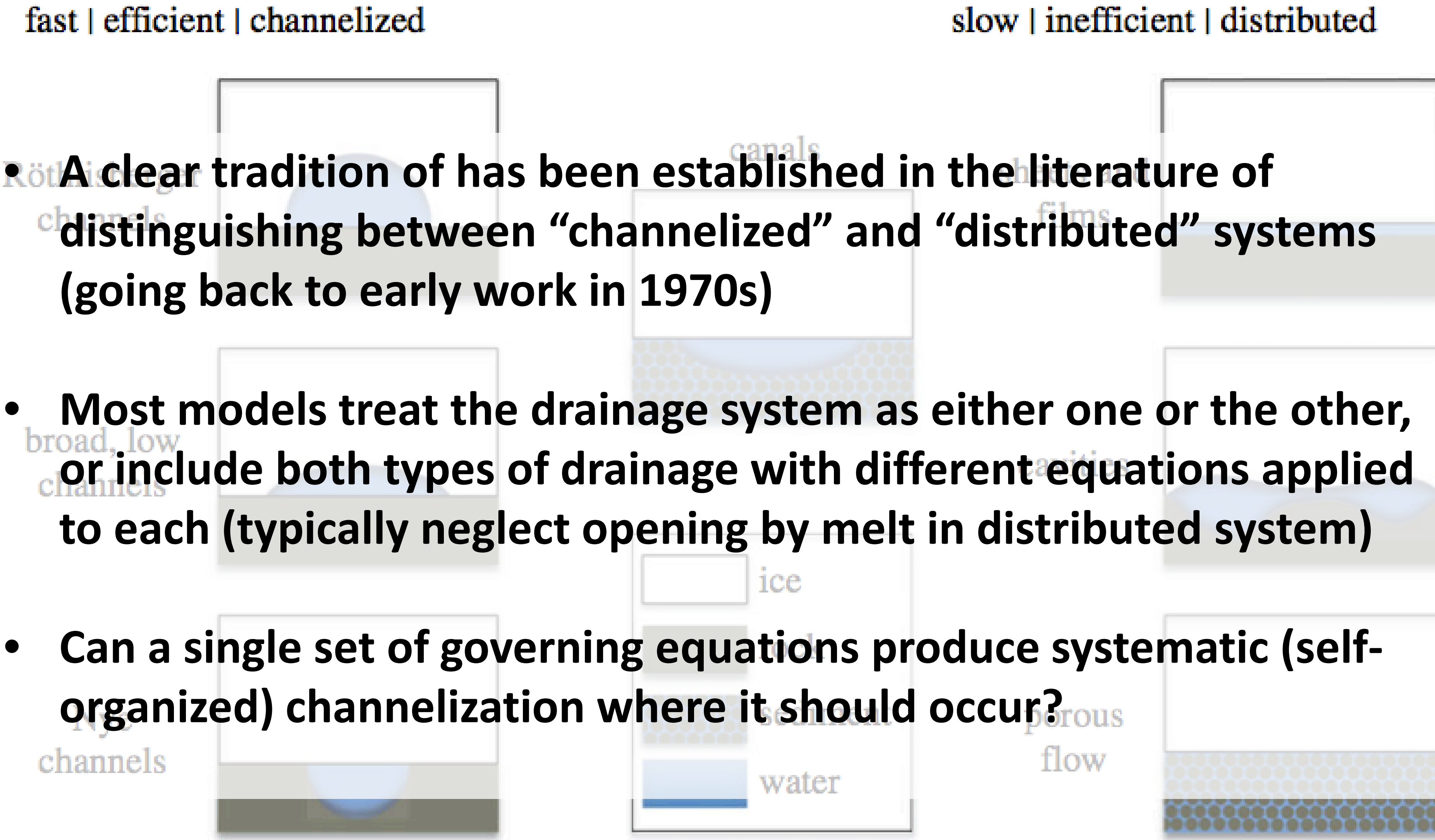


Flowers GE. (2015), Modelling water flow under glaciers and ice sheets. *Proc. R. Soc. A*



- A clear tradition of has been established in the literature of distinguishing between “channelized” and “distributed” systems (going back to early work in 1970s)
- Most models treat the drainage system as either one or the other, or include both types of drainage with different equations applied to each (typically neglect opening by melt in distributed system)
- Can a single set of governing equations produce systematic (self-organized) channelization where it should occur?

Flowers GE. (2015), Modelling water flow under glaciers and ice sheets. *Proc. R. Soc. A*

Our model formulation:

- **Produces stable drainage configurations and pressure fields for steady and transient inputs, while including melt opening term everywhere in the domain**

- **Allows for natural evolution of subglacial geometry between distributed drainage, channels, isolated regions of the bed, or any other configuration**

- **Handles transient meltwater inputs (distributed input, localized moulin inputs, or incoming basal flux from boundaries)**

- **Handles realistic bed topography and ice geometry**

We hope this "unified" formulation will be a useful contribution that can provide additional insight into seasonal evolution of the subglacial system and its influence on ice dynamics

A Brief Summary of Model Equations…

Water mass balance (**continuity equation**):

$$\frac{\partial b}{\partial t} + \frac{\partial b_e}{\partial t} + \nabla \cdot \boldsymbol{q} = \frac{\dot{m}}{\rho_w} + i_{e \to b}$$

b	subglacial gap height	[m]
b_e	englacial storage	[m]
q	basal water flux (discharge)	[m^2 s^{-1}]
m	internal melt rate	[m s^{-1}]
$i_{e \to b}$	meltwater input rate from englacial to basal system	[m s^{-1}]

Basal gap dynamics:

$$\frac{\partial b}{\partial t} = \frac{\dot{m}}{\rho_i} - A|p_i - p_w|^{n-1}(p_i - p_w)b + \beta u_b$$

b	subglacial gap height	[m]
m	internal melt rate	[kg m^{-2} s^{-1}]
ρ_i	bulk density of ice	[kg m^{-3}]
A	flow law parameter	[Pa^{-3} s^{-1}]
p_i	ice overburden pressure, $p_i=\rho_w gH$	[Pa]
p_w	subglacial water pressure, $p_w=\rho_w g(h-z_b)$	[Pa]
u_b	sliding velocity	[m s^{-1}]
n	flow law exponent	[ ]
β	parameter to control opening by sliding, $\beta=(b-b_r)/l_r$	[ ]

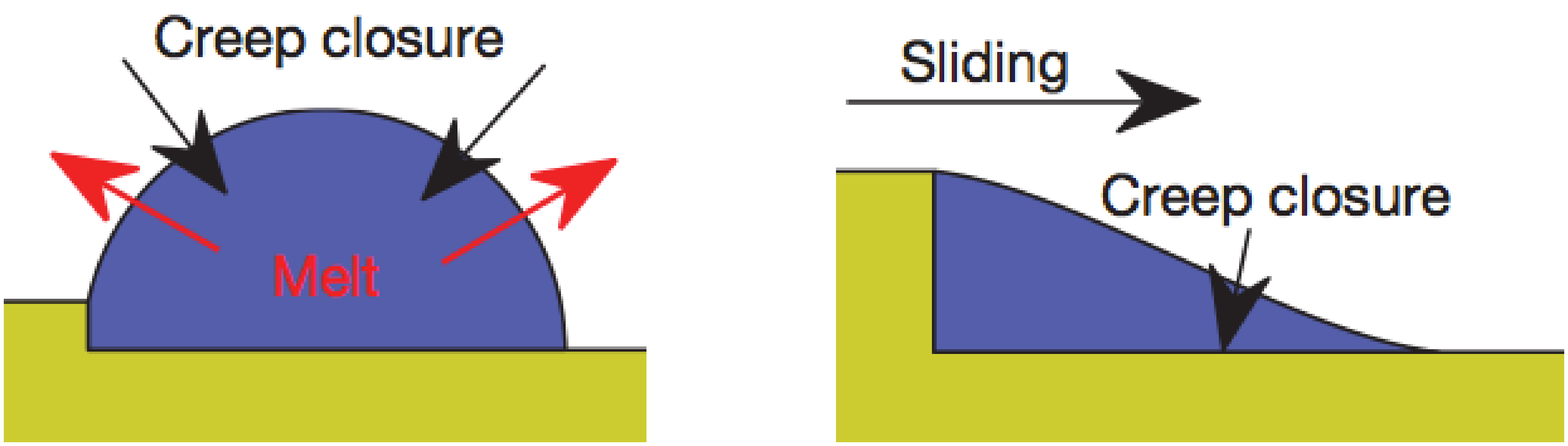
Basal gap dynamics:

$$\frac{\partial b}{\partial t} = \frac{\dot{m}}{\rho_i} - A|p_i - p_w|^{n-1}(p_i - p_w)b + \beta u_b$$

Opening by melt

Closing by creep

Opening by sliding over bedrock bumps



Schoof, C. (2010), Ice-sheet acceleration driven by melt supply variability, *Nature*

Basal water flux (approximate **momentum equation**):

$$\vec{q} = \frac{-b^3 g}{12\nu(1 + \omega Re)} \nabla h$$

q	basal water flux	[$m^2\ s^{-1}$]
b	subglacial gap height	[m]
g	gravitational acceleration	[$m\ s^{-2}$]
υ	kinematic viscosity of water	[$m^2\ s^{-1}$]
h	hydraulic head	[m]

Basal water flux (approximate **momentum equation**):

$$\vec{q} = \frac{-b^3 g}{12\nu(1 + \omega Re)} \nabla h$$

ω dimensionless parameter controlling transition between laminar and turbulent flow

Re Reynolds' number, $\mathrm{Re} = \frac{|\vec{q}\,|}{\upsilon}$

- Allows for transition between laminar and turbulent flow (turbulent flow depends on the square root of the head gradient)

Internal melt generation (**energy equation** based on energy balance at the bed):

$$\dot{m} = \frac{1}{L}(G + \boldsymbol{u_b} \cdot \boldsymbol{\tau_b} - \rho_w g \boldsymbol{q} \cdot \nabla h - c_t c_w \rho_w \boldsymbol{q} \cdot \nabla p_w)$$

m	internal melt rate	[kg m^{-2} s^{-1}]
L	latent heat of fusion of water	[J kg^{-1}]
G	geothermal flux	[W m^{-2}]
u_b	sliding velocity	[m s^{-1}]
τ_b	basal shear stress, $\tau_b = \alpha^2(p_i - p_w)u_b$	[Pa]
q	basal water flux (discharge)	[m^2 s^{-1}]
h	hydraulic head	[m]
c_t	Clapeyron slope	[K Pa^{-1}]
c_w	Specific heat capacity of water	[J kg^{-1} K^{-1}] ρ_i
	bulk density of water	[kg m^{-3}]
p_w	subglacial water pressure, $p_w = \rho_w g(h - z_b)$	[Pa]

Internal melt generation (**energy equation** based on energy balance at the bed):

$$\dot{m} = \frac{1}{L}(G + \boldsymbol{u_b} \cdot \boldsymbol{\tau_b} - \rho_w g \boldsymbol{q} \cdot \nabla h - c_t c_w \rho_w \boldsymbol{q} \cdot \nabla p_w)$$

+	+	+	-
Geothermal flux	Frictional heat from sliding	Heat generated by water flux (internal dissipation)	Heat consumed by changes in pressure melting point (about 1/3 of heat produced)

- Assumes all heat generated is converted locally to melt
- Assumes ice and water are isothermal (at pressure melting point)

Combine to form nonlinear PDE in terms of head:

$$\nabla \cdot \left[\frac{-b^3 g}{12\nu(1+\omega Re)} \cdot \nabla h \right] + \frac{\partial e_v (h - z_b)}{\partial t} = \dot{m} \left[\frac{1}{\rho_w} - \frac{1}{\rho_i} \right] + A|p_i - p_w|^{n-1}(p_i - p_w)b - \beta u_b + i_{e \to b}$$

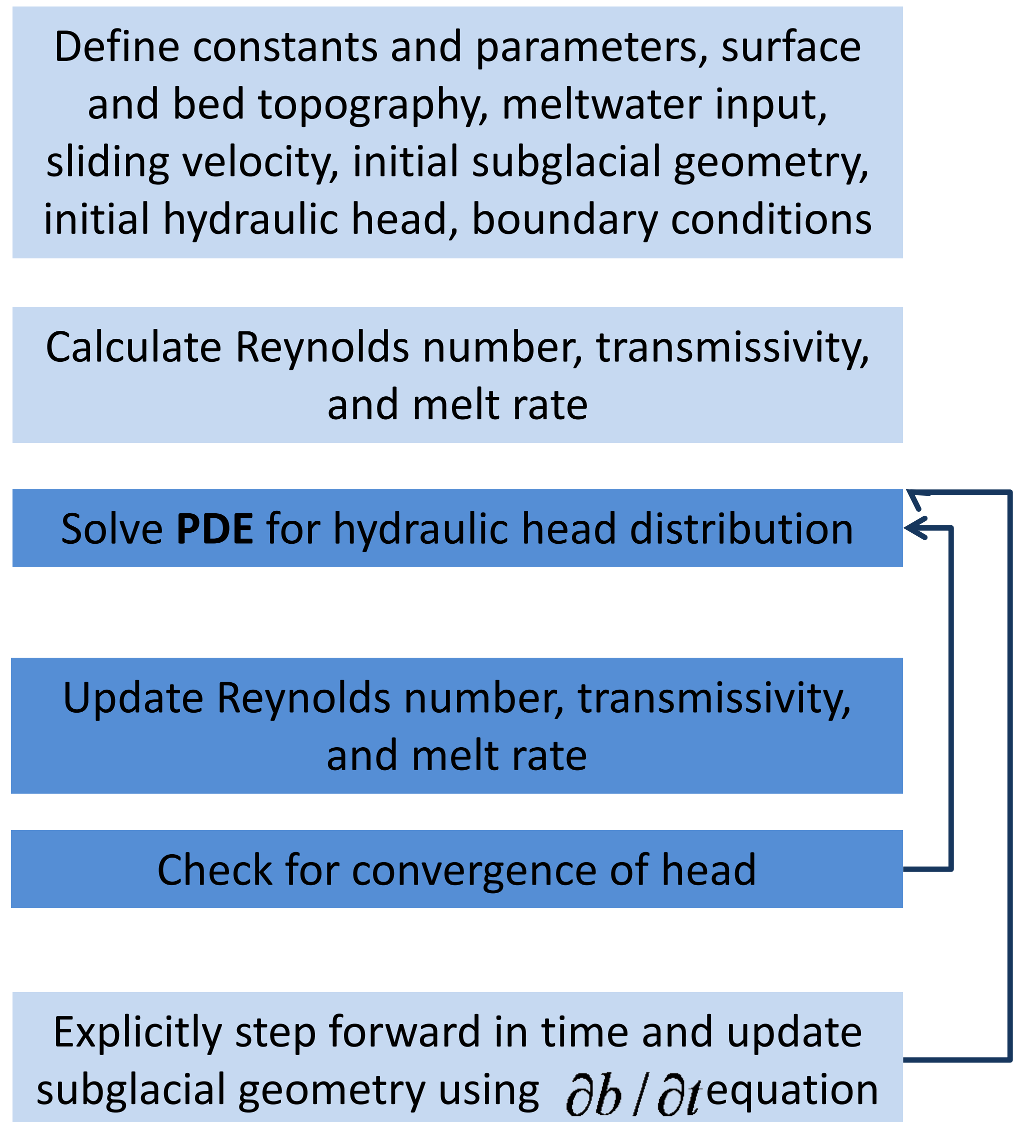
$$\nabla \cdot \left[\frac{-b^3 g}{12\nu(1+\omega Re)} \cdot \nabla h \right] + \frac{\partial e_v (h - z_b)}{\partial t} = \dot{m} \left[\frac{1}{\rho_w} - \frac{1}{\rho_i} \right] + A|p_i - p_w|^{n-1}(p_i - p_w)b - \beta u_b + i_{e \to b}$$

$$\boldsymbol{K} = \frac{-b^3 g}{12\nu(1+\omega Re)}$$

- "Transmissivity" (*K*) depends on *h*
- The forcing (*RHS*) is a function of the relevant dependent variables

↓

$$\nabla \cdot (\boldsymbol{K} \cdot \nabla h) + \frac{\partial e_v (h - z_b)}{\partial t} = \dot{m}\left(\frac{1}{\rho_w} - \frac{1}{\rho_i}\right) + A|p_i - p_w|^{n-1}(p_i - p_w)b - \beta u_b + i_{e \to b}$$


Define constants and parameters, surface and bed topography, meltwater input, sliding velocity, initial subglacial geometry, initial hydraulic head, boundary conditions
Calculate Reynolds number, transmissivity, and melt rate
Solve PDE for hydraulic head distribution
Update Reynolds number, transmissivity, and melt rate
Check for convergence of head
Explicitly step forward in time and update subglacial geometry using $\partial b / \partial t$ equation

Model details:

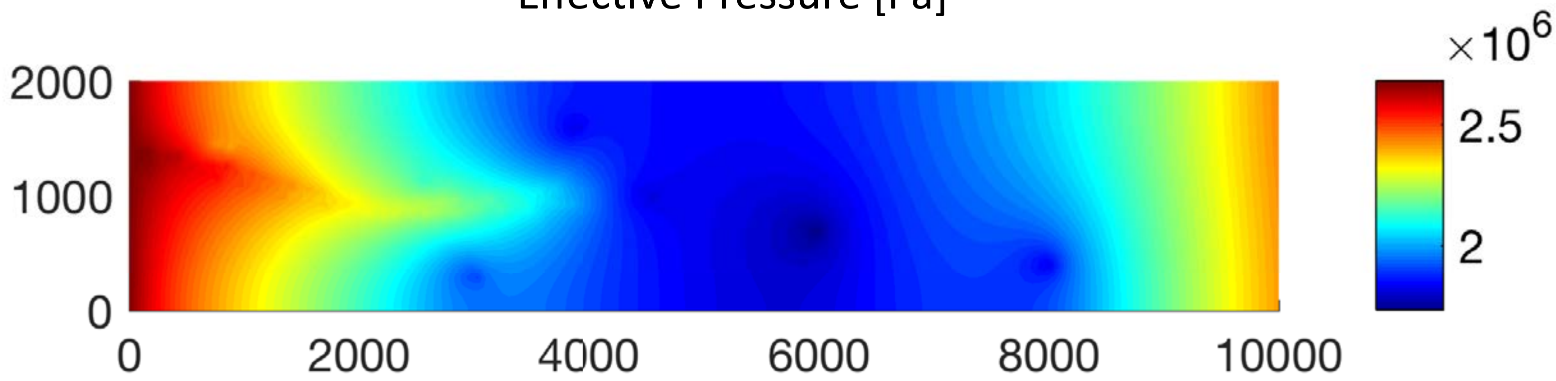
- **SHaKTI:** Subglacial Hydrology as Kinetic Transient Interplay

- Implemented as a solution in the Ice Sheet System Model (ISSM)
    - Parallel architecture using linear finite elements (i.e. P1 triangular Lagrange finite elements)
    - Unstructured mesh
    - Source code is written in C++
    - Data structures and solvers provided by PETSc
    - MATLAB user interface
    - Nonlinear iteration performed to solve for hydraulic head using the direct linear solver MUMPS in PETSc

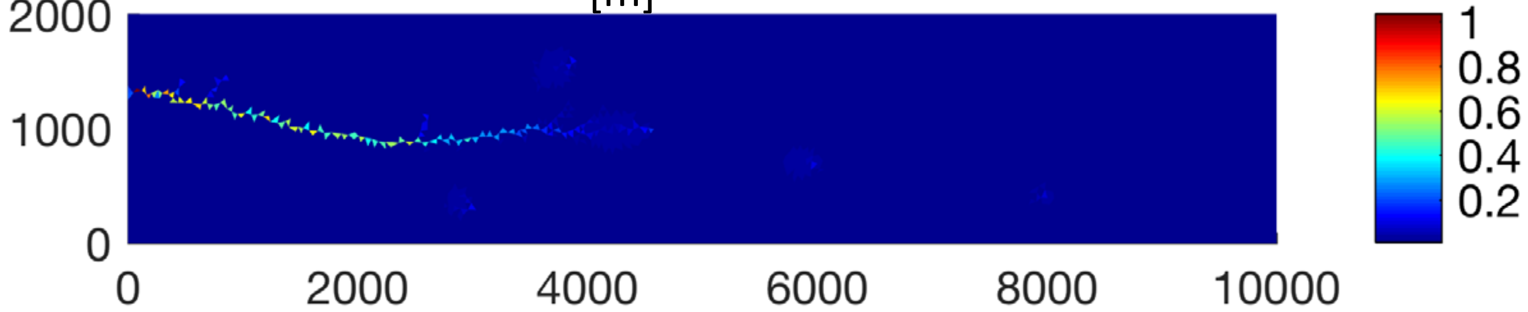
A Few Sample Simulations…

Steady input into 10 moulins

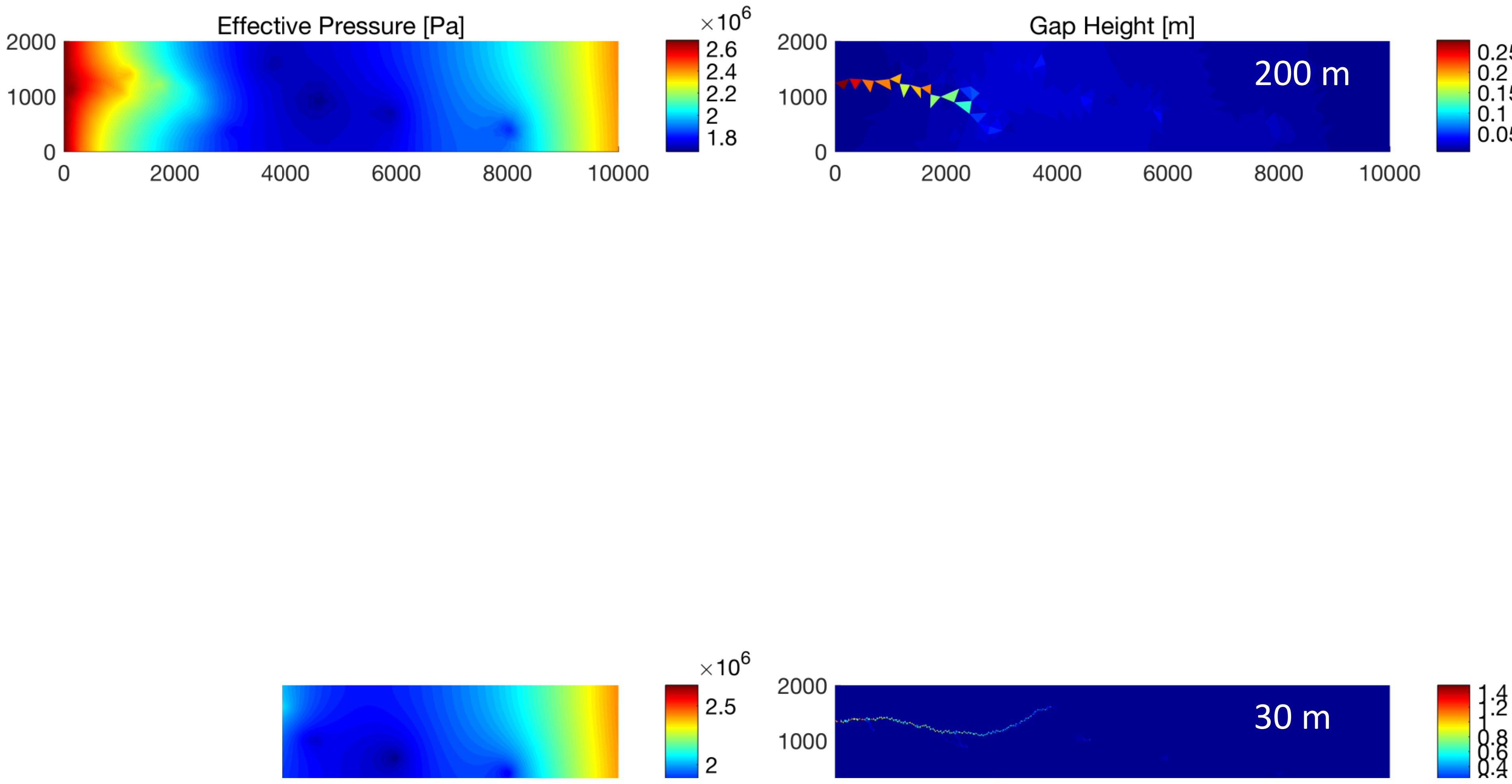
Effective Pressure [Pa]



Gap Height [m]



Steady input into 10 moulins – a qualitative look at mesh refinement



Seasonal cycles of distributed melt input

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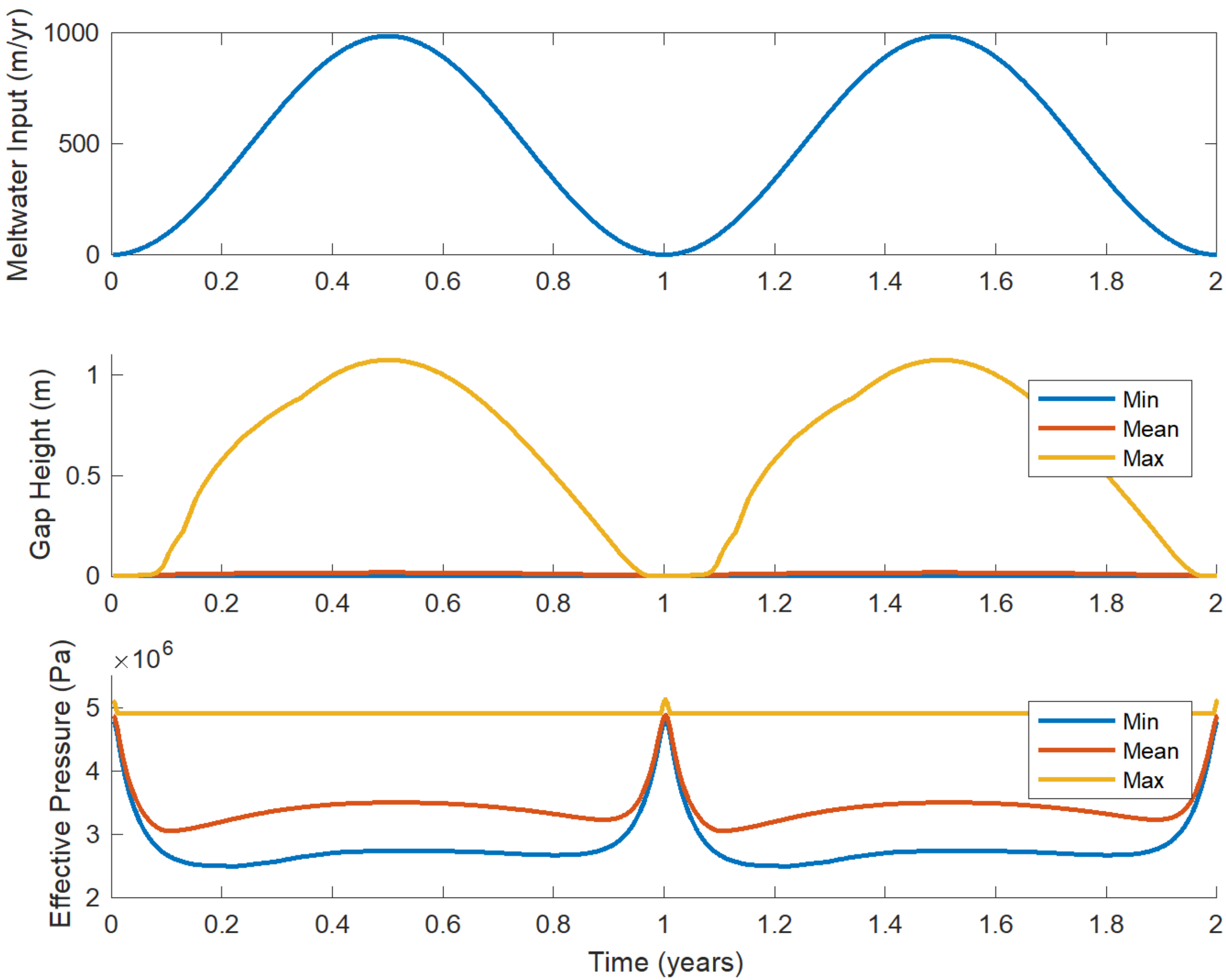

Upcoming work:

- Publication of model formulation (manuscript in preparation)
- Publication of results in the Subglacial Hydrology Model Intercomparison Project (SHMIP, https://shmip.bitbucket.io)
- Application to marine-terminating Store Glacier in west Greenland
- Two-way coupling to ice dynamics in ISSM


Thank you.
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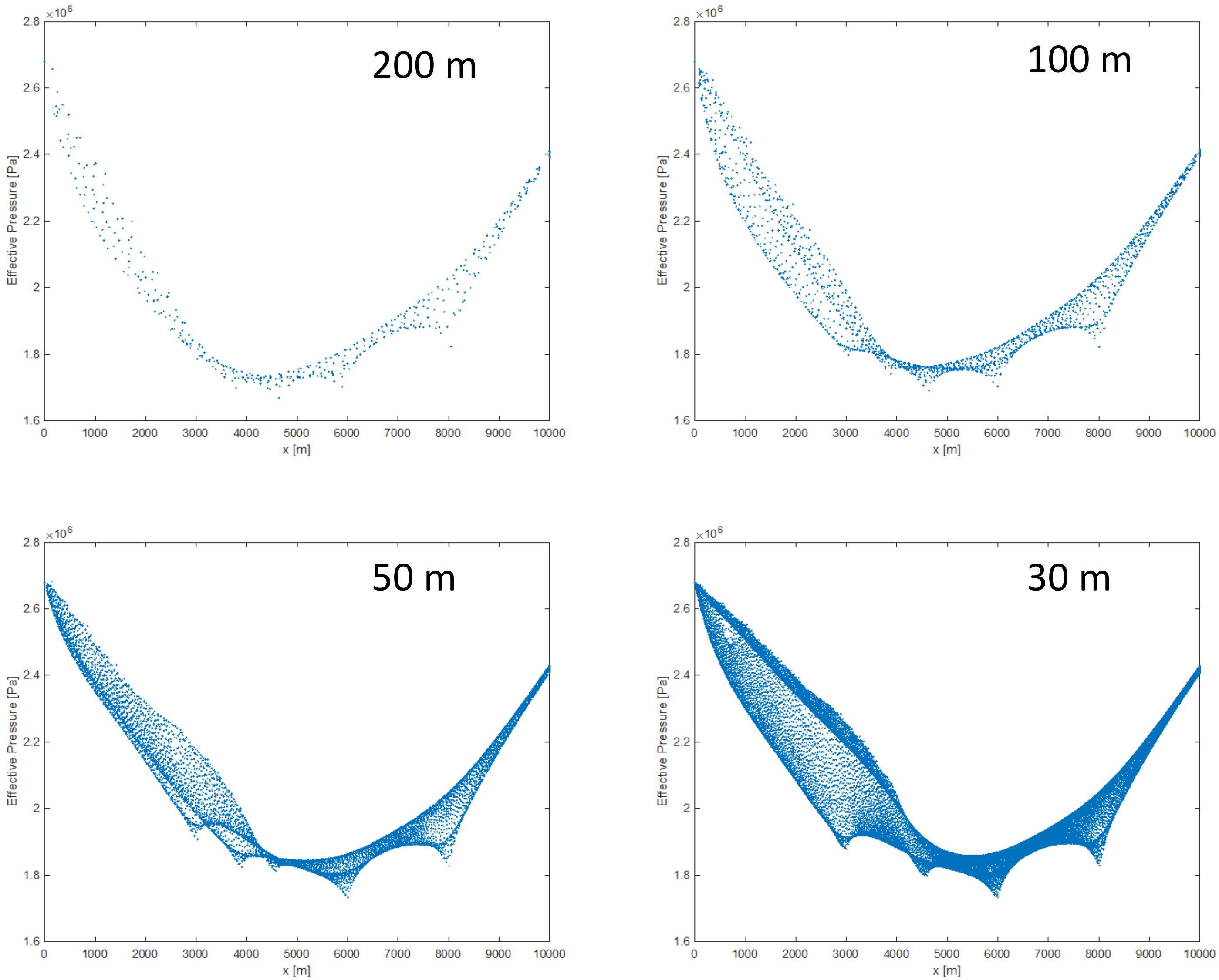
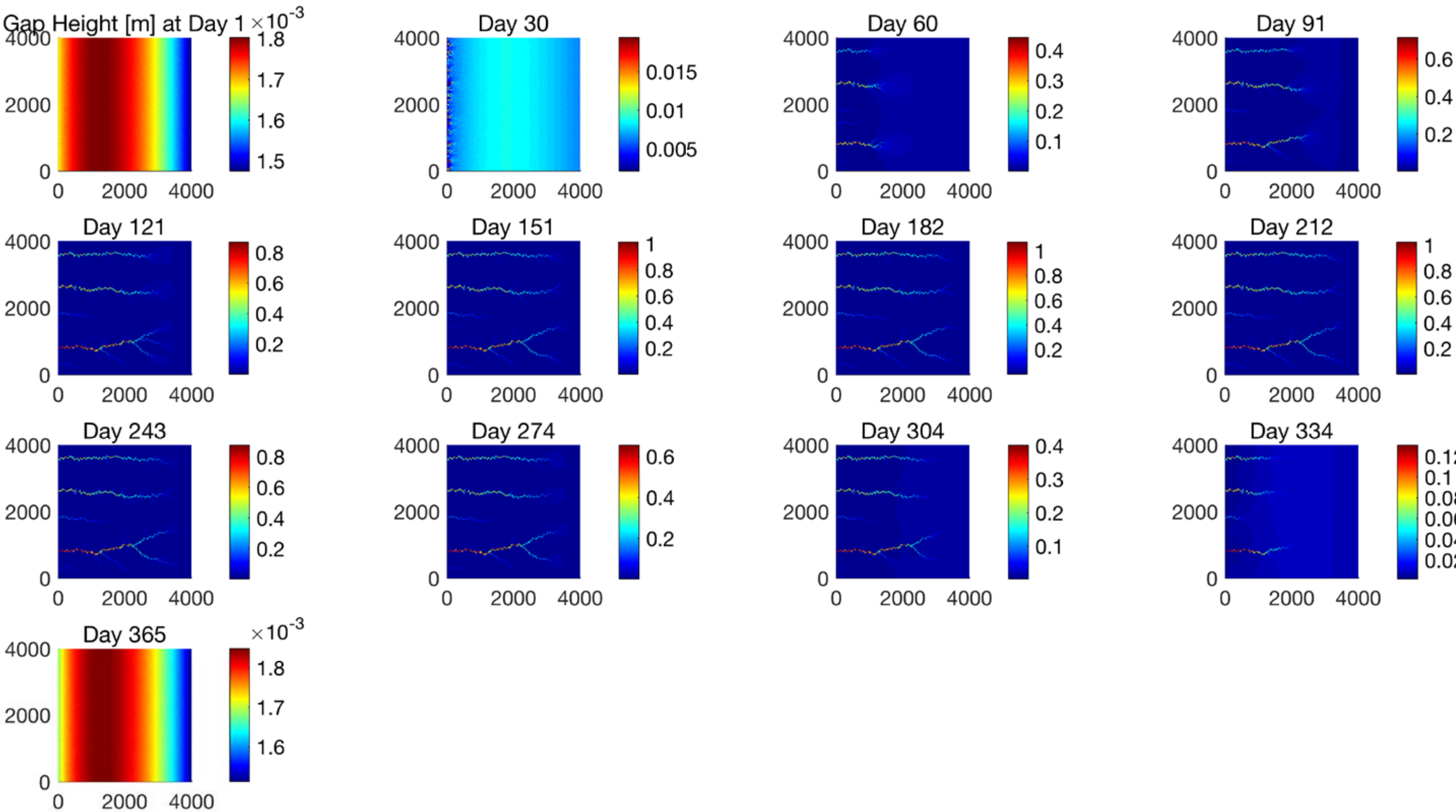
Steady input into 10 moulins – a look at mesh refinement



Seasonal cycles of distributed melt input



Seasonal cycles of distributed melt input