Unified Formula for Land Biogeochemical Models

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Matrix equation of CLM4.5

$$\frac{\boldsymbol{dX(t)}}{\boldsymbol{dt}} = \boldsymbol{B(t)I(t)} - \boldsymbol{A\xi(t)KX(t)} - \boldsymbol{V(t)X(t)}$$

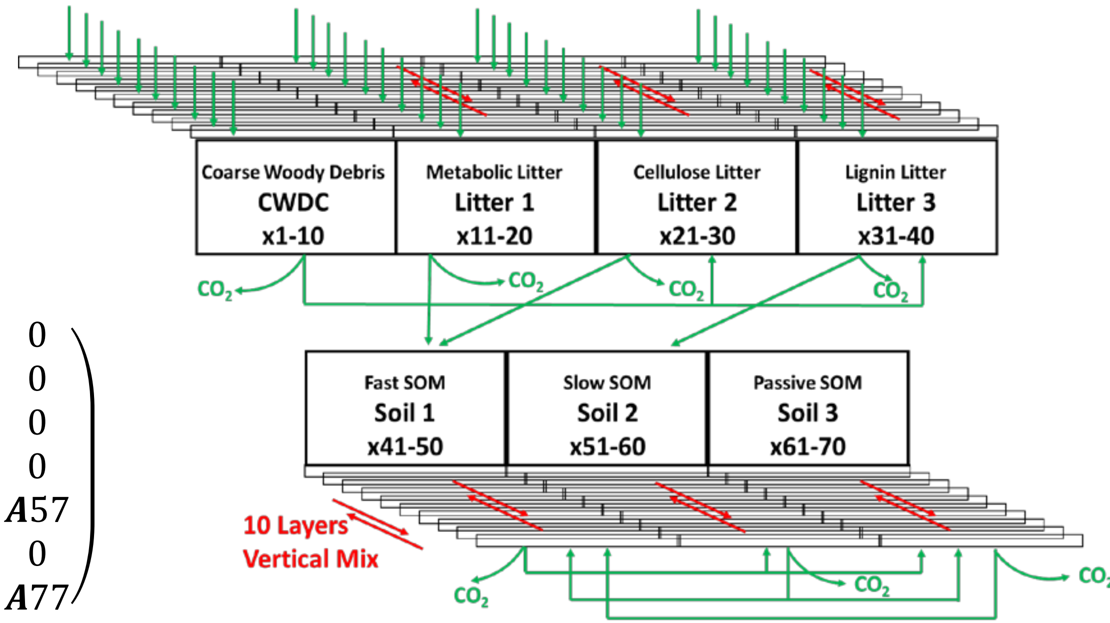
$$X(t) = (X_1(t), X_2(t), X_3(t), \dots, X_{70}(t))^T$$

$$\mathbf{A} = \begin{pmatrix} \boldsymbol{A}11 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \boldsymbol{A}22 & 0 & 0 & 0 & 0 & 0 \\ \boldsymbol{A}31 & 0 & \boldsymbol{A}33 & 0 & 0 & 0 & 0 \\ \boldsymbol{A}41 & 0 & 0 & \boldsymbol{A}44 & 0 & 0 & 0 \\ 0 & \boldsymbol{A}52 & \boldsymbol{A}53 & 0 & \boldsymbol{A}55 & \boldsymbol{A}56 & \boldsymbol{A}57 \\ 0 & 0 & 0 & \boldsymbol{A}64 & \boldsymbol{A}65 & \boldsymbol{A}66 & 0 \\ 0 & 0 & 0 & 0 & \boldsymbol{A}75 & \boldsymbol{A}76 & \boldsymbol{A}77 \end{pmatrix}$$

$$\mathbf{A}_{31} = diag(-f_{31}, -f_{31}, -f_{31}, -f_{31}, -f_{31}, -f_{31}, -f_{31}, -f_{31}, -f_{31}, -f_{31})$$

$$\mathbf{V(t)} = \begin{pmatrix} \boldsymbol{V}11 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \boldsymbol{V}22(\boldsymbol{t}) & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \boldsymbol{V}33(\boldsymbol{t}) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \boldsymbol{V}44(\boldsymbol{t}) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \boldsymbol{V}55(\boldsymbol{t}) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \boldsymbol{V}66(\boldsymbol{t}) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \boldsymbol{V}77(\boldsymbol{t}) \end{pmatrix}$$

$$\boldsymbol{V}22 = \boldsymbol{diag}(\boldsymbol{z}_1, \boldsymbol{z}_2, \dots, \boldsymbol{z}_{10})^{-1} \begin{pmatrix} \boldsymbol{g}_1 & -\boldsymbol{g}_1 & 0 & 0 & \cdots & 0 & 0 & 0 \\ -\boldsymbol{h}_2 & \boldsymbol{h}_2 + \boldsymbol{g}_2 & -\boldsymbol{g}_2 & 0 & \cdots & 0 & 0 & 0 \\ 0 & -\boldsymbol{h}_3 & \boldsymbol{h}_3 + \boldsymbol{g}_3 & -\boldsymbol{g}_3 & \cdots & 0 & 0 & 0 \\ 0 & 0 & -\boldsymbol{h}_4 & \boldsymbol{h}_4 + \boldsymbol{g}_4 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & \boldsymbol{h}_8 + \boldsymbol{g}_8 & -\boldsymbol{g}_8 & 0 \\ 0 & 0 & 0 & 0 & \cdots & -\boldsymbol{h}_9 & \boldsymbol{h}_9 + \boldsymbol{g}_9 & -\boldsymbol{g}_9 \\ 0 & 0 & 0 & 0 & \cdots & 0 & -\boldsymbol{h}_{10} & \boldsymbol{h}_{10} \end{pmatrix}$$



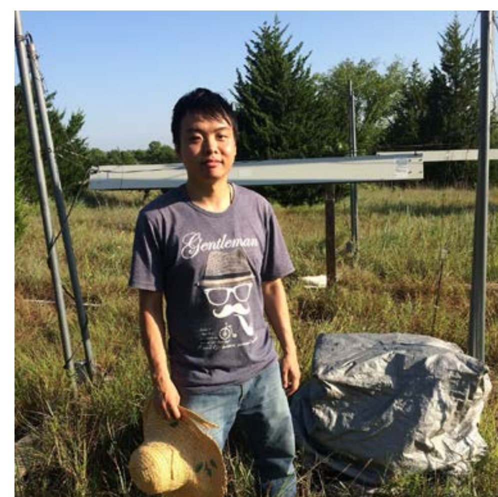
Huang et al. 2018
Global Change Biology

What have we done in the past year?

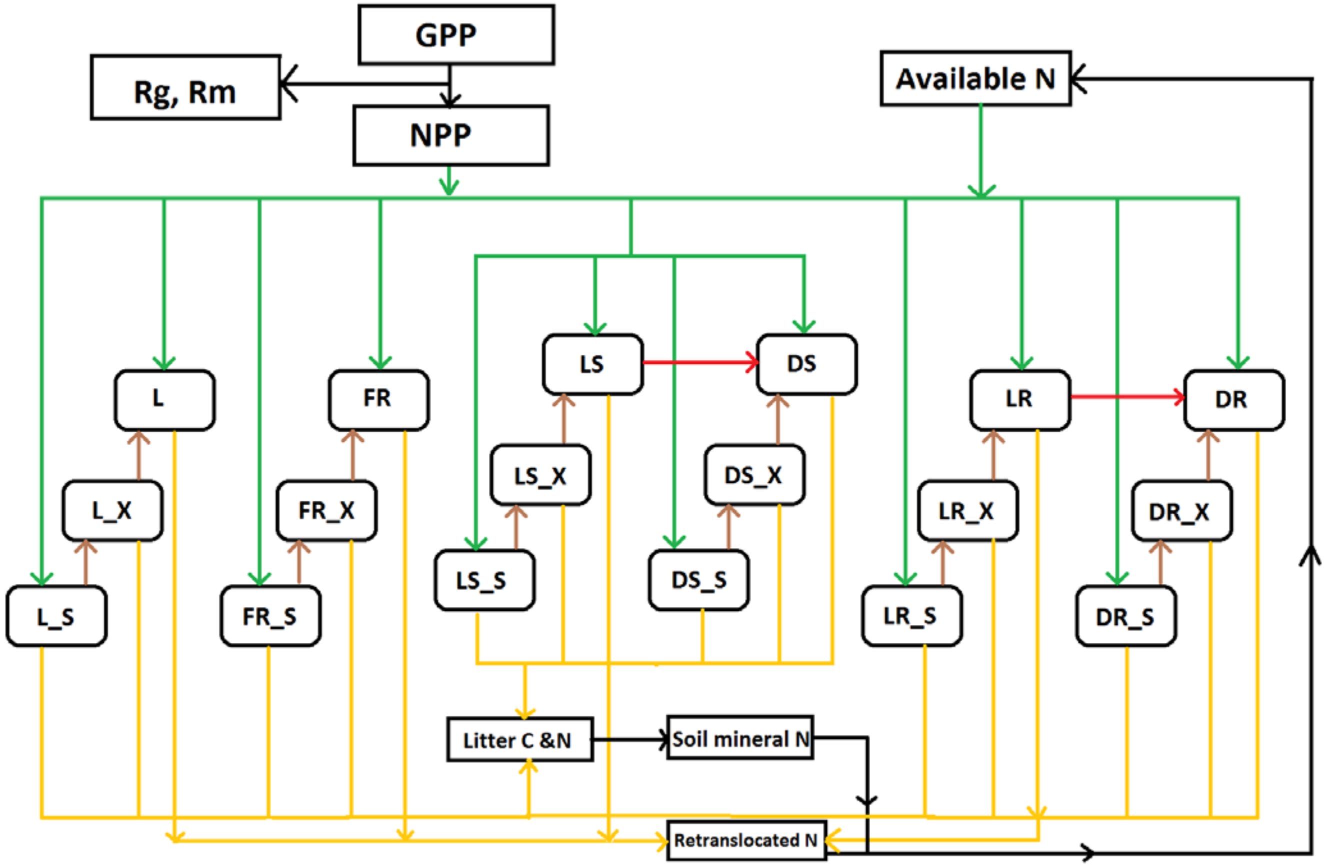
General equation for C and N model

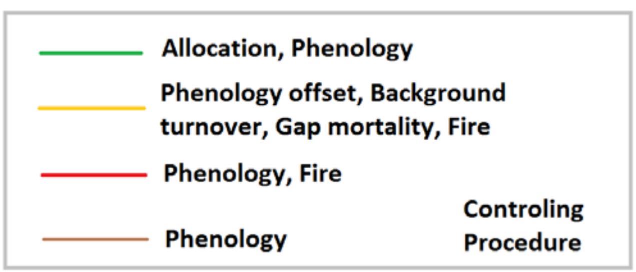
$$\begin{cases} \frac{d}{dt}X(t) = A_C\xi(t)K_C X(t) + u(N,t)B \\ \frac{d}{dt}N(t) = A_N\xi(t)K_N N(t) + k_u F\prod \end{cases}$$

$$X(t=0)=X_0$$
$$N(t=0)=N_0$$



CLM vegetation C&N: phenology, fire etc.





L: leaf; L_X: leaf transfer; L_S: leaf storage
FR: fine root; FR_X: fine root transfer; FR_S: fine root storage
LS: live stem; LS_X: live stem transfer; LS_S: live stem storage
DS: dead stem; DS_X: dead stem transfer; DS_S: dead stem storage
LR: live coarse root; LR_X: live coarse root transfer; LR_S: live coarse root storage
DR: dead coarse root; DR_X: dead coarse root transfer; DR_X: dead coarse root storage

Matrix equation of vegetation C&N dynamics

$$\frac{d}{dt}\boldsymbol{X}(t) = (\boldsymbol{A}_{ph}(t)\boldsymbol{K}_{ph}(t) + \boldsymbol{A}_{gm}(t)\boldsymbol{K}_{gm}(t) + \boldsymbol{A}_{fi}(t)\boldsymbol{K}_{fi}(t))\boldsymbol{X}(t) + \boldsymbol{B}(\mathrm{t})\boldsymbol{F}(t)$$

C transfer of phenology

C transfer of gap mortality

C transfer of fire

pool state

input

C turnover of phenology

C turnover of gap mortality

C turnover of fire

allocation

Matrix equation of soil C&N dynamics

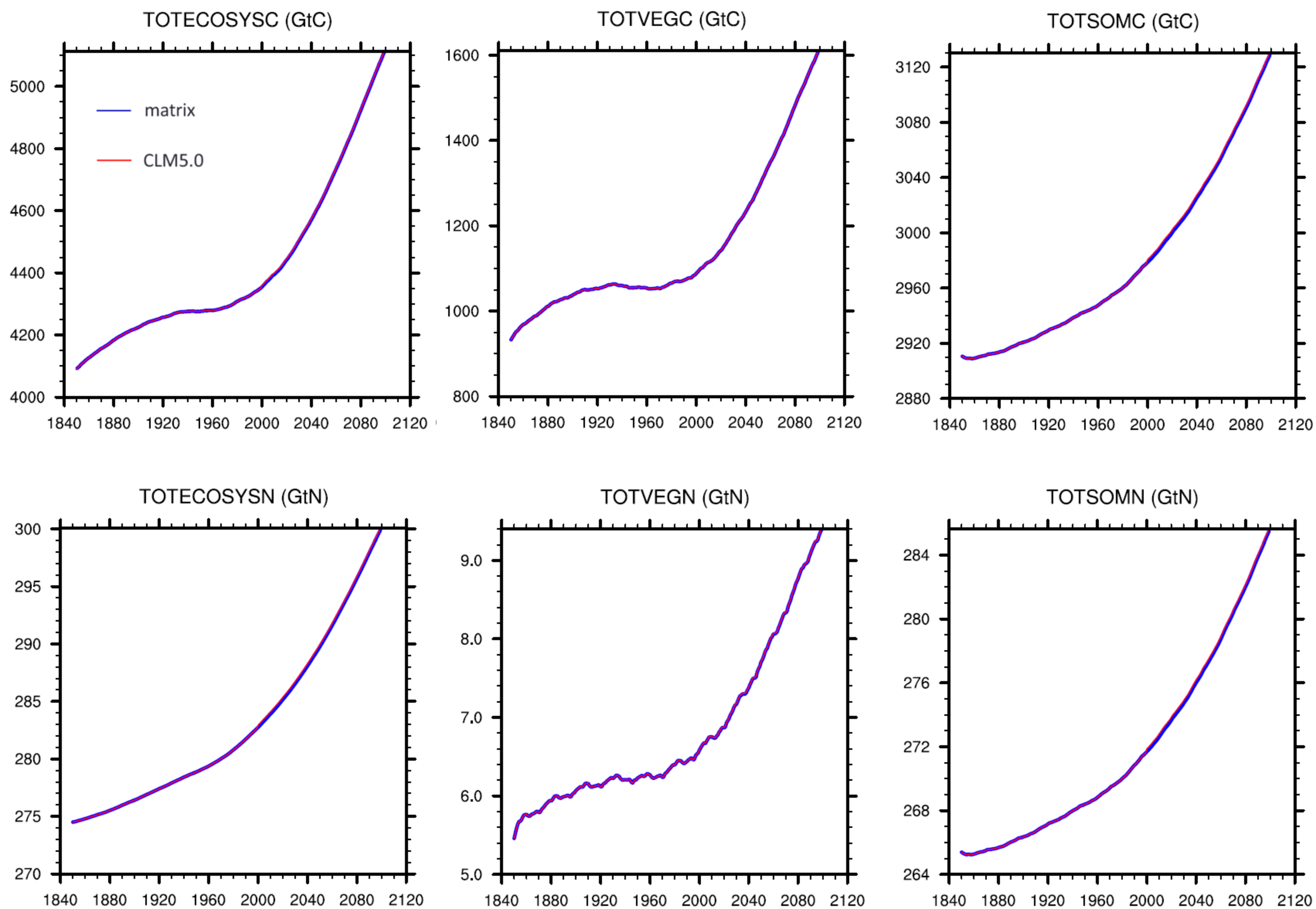
C&N pools — Transfer matrix — Scalar — Tridiagonal matrix (diffusion and advection) — allocation

$$\frac{\boldsymbol{d}}{\boldsymbol{dt}}\boldsymbol{X}(\boldsymbol{t}) = \left(\boldsymbol{A}\boldsymbol{\xi}(\boldsymbol{t})\boldsymbol{K} - \boldsymbol{V}(\boldsymbol{t}) - \boldsymbol{V}_{\boldsymbol{f}}(\boldsymbol{t})\right)\boldsymbol{X}(\boldsymbol{t}) + \boldsymbol{B}(\boldsymbol{t})\boldsymbol{I}(\boldsymbol{t})$$

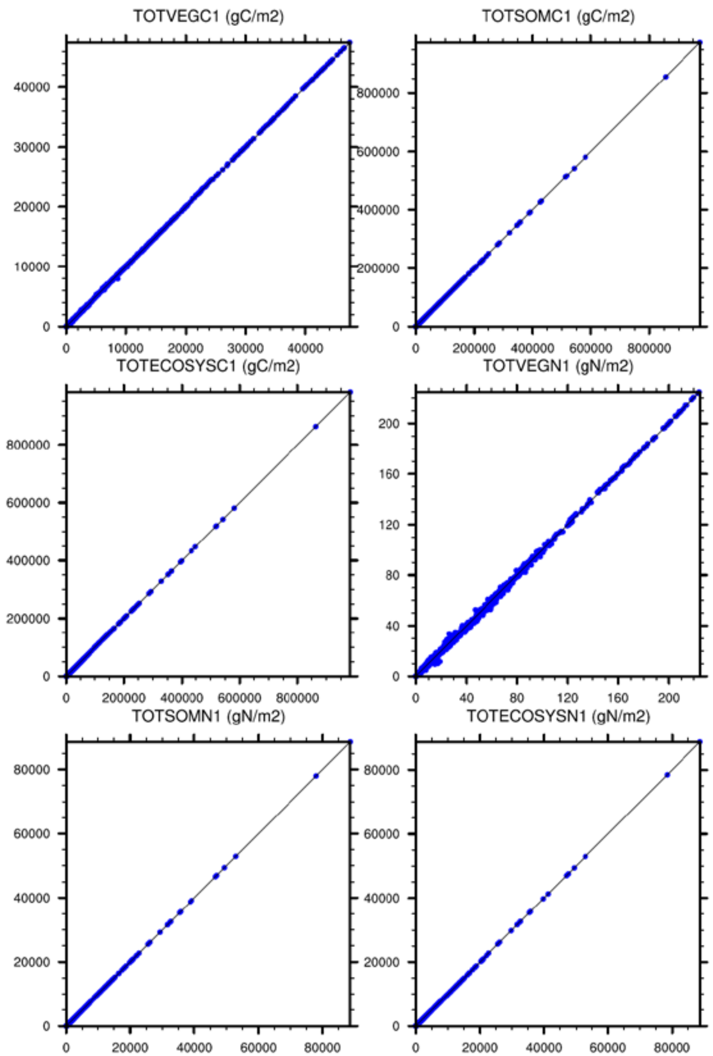
Decomposition rate — Tridiagonal matrix (fire) — input

- C&N pools: $X(t)$
- Transfer matrix: A
- Scalar: $\xi(t)$
- Decomposition rate: K
- Tridiagonal matrix (diffusion and advection): $V(t)$
- Tridiagonal matrix (fire): $V_f(t)$
- allocation: $B(t)$
- input: $I(t)$

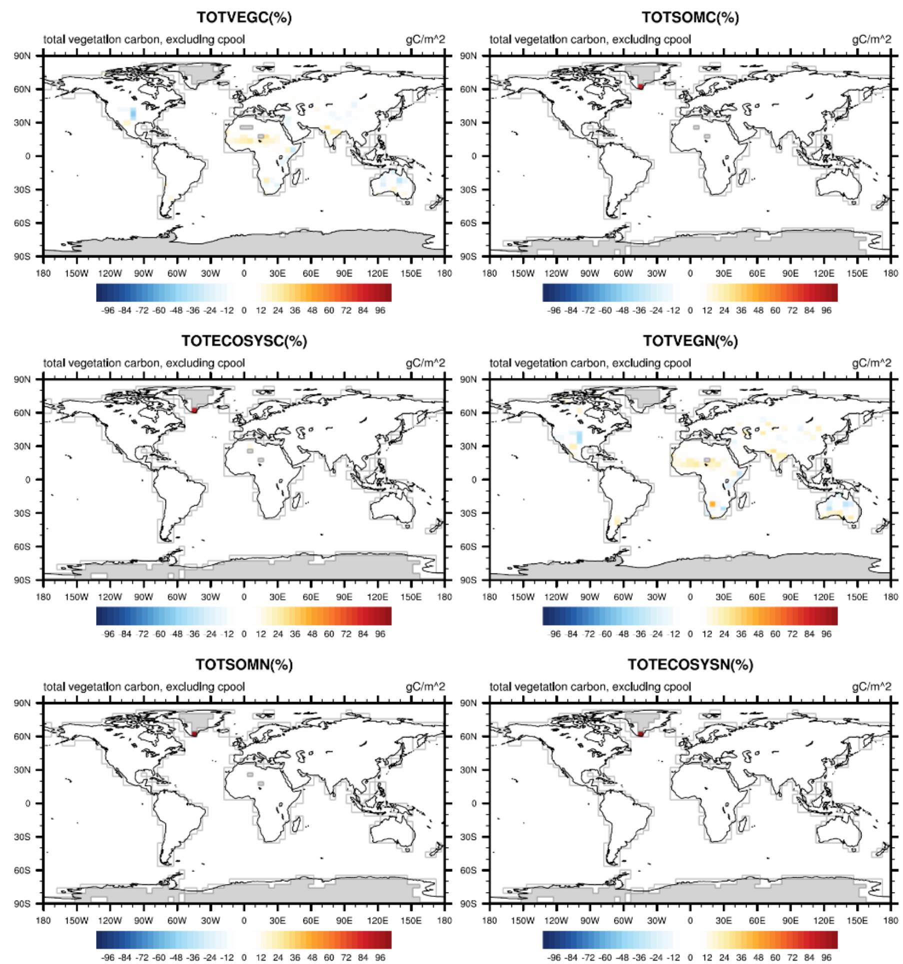
Matrix Vs default RCP 8.5 (global-level)



Matrix Vs default bias in 2099 RCP8.5 (Global pattern)



(Matrix – Default)/Default * 100%



Diagnostic variables related to C storage Capacity (X_C) and C storage potential (X_P)

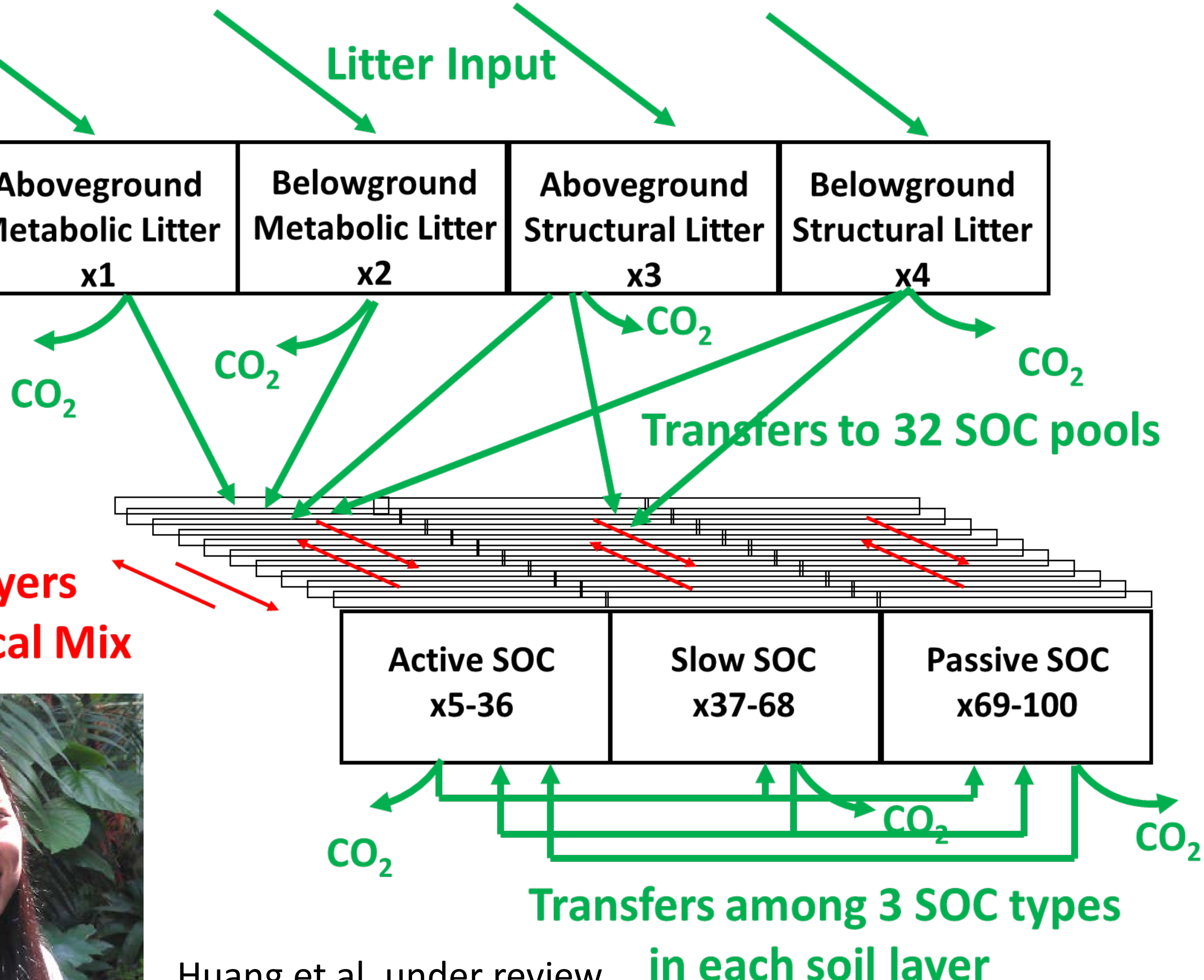
$$X_C = -(A\xi K)^{-1}BI$$
$$X_P = X_C - X$$

Luo et al. 2017

ξ: Environmental scalar
A: Carbon transfer coefficient
K: Turnover rate
B: Partitioning coefficients for influx
I: Influx
X: state variable of C storage

Add 100 variables: 36 Vegetation C output variables, 36 Vegetation N output variables (18 vegetation pools), 14 Soil C variables and 14 Soil N variables (7 soil pools) for both capacity and potential.

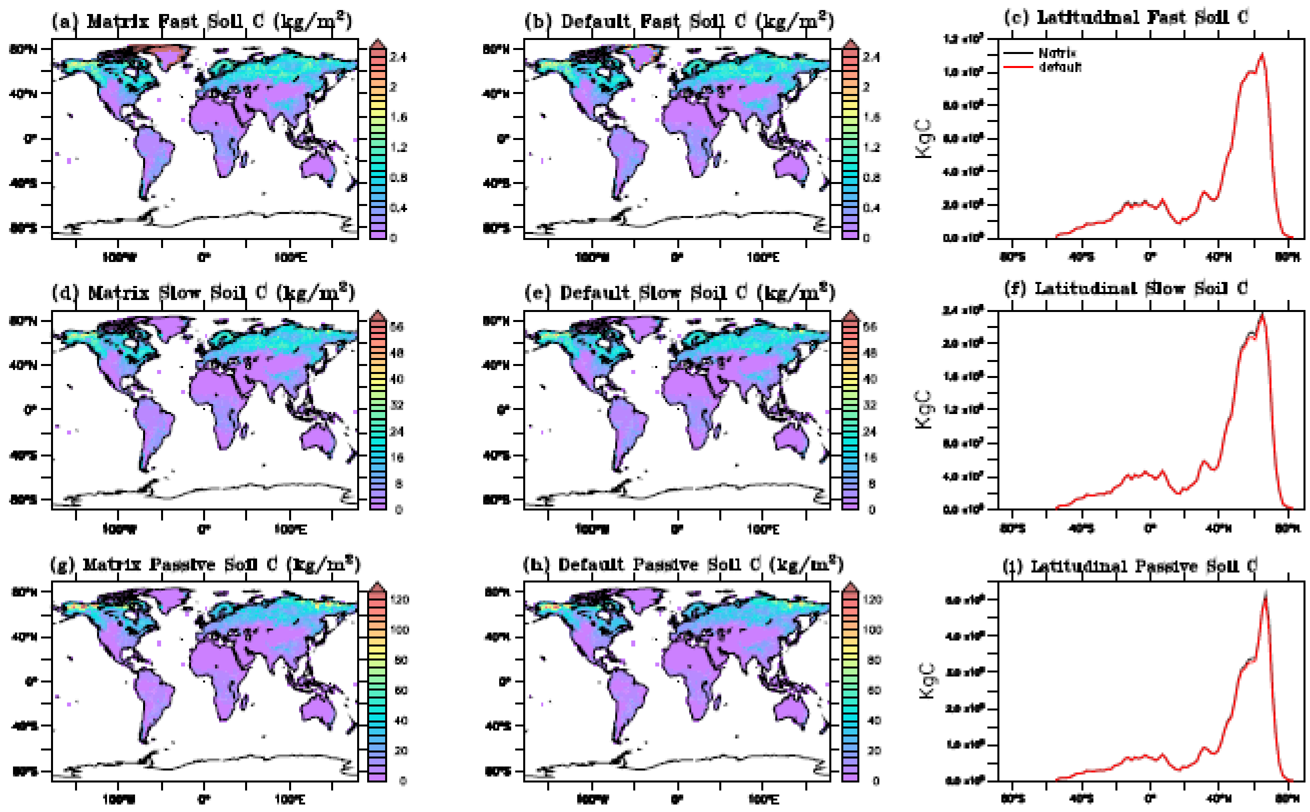
ORCHIDEE matrix model



Huang et al. under review
JAMES

What is the use of it?

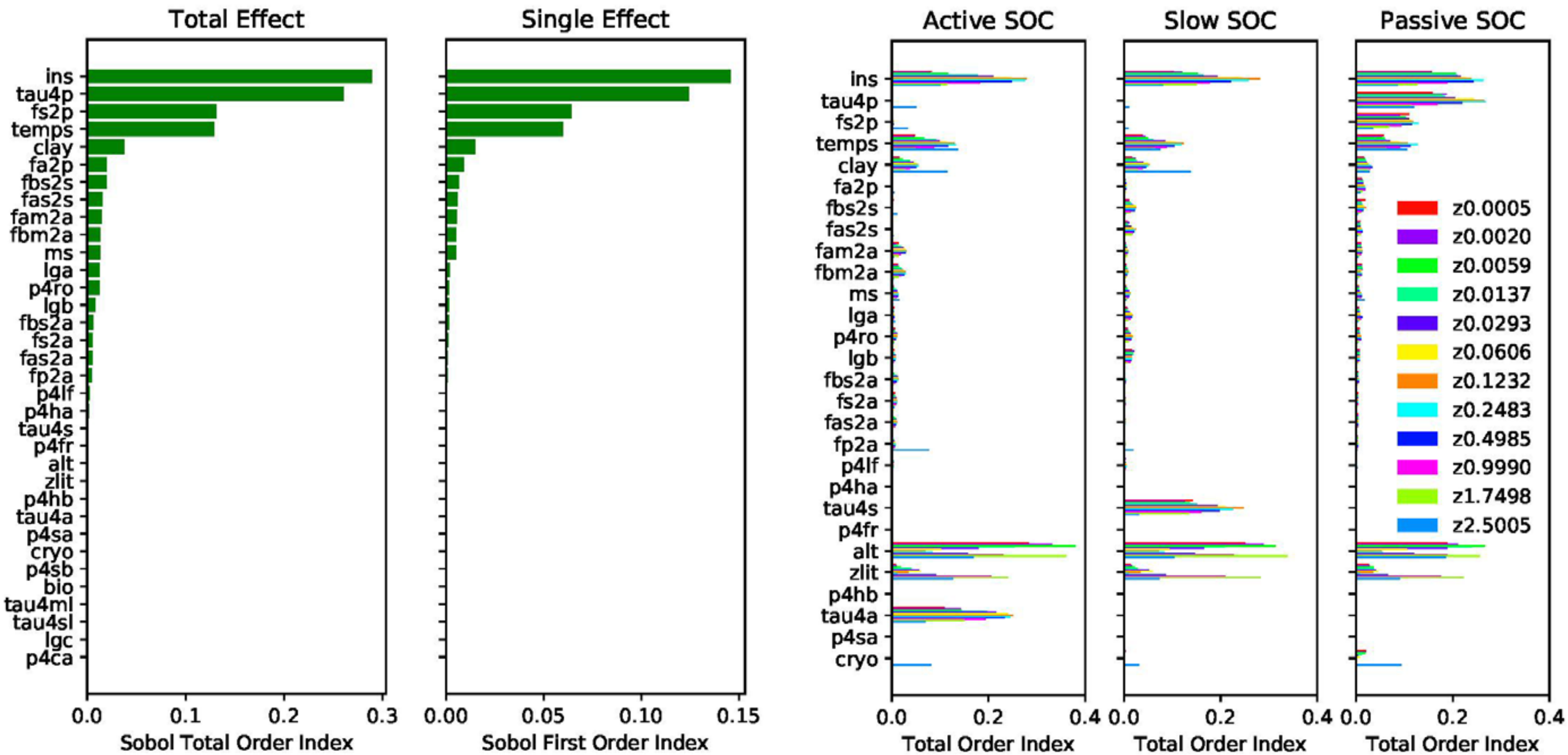
1. Semi-analytical spinup



400 years semi-analytical matrix result vs. 200,000 years ORCHIDEE-MICT

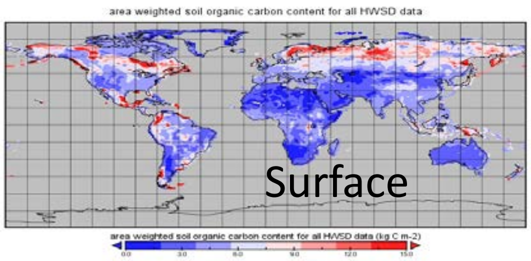
Acceleration of spin-up by 500 times

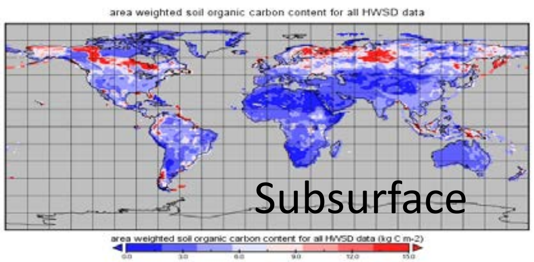
2. Comprehensive sensitivity analyses



Huang et al. under review
JAMES

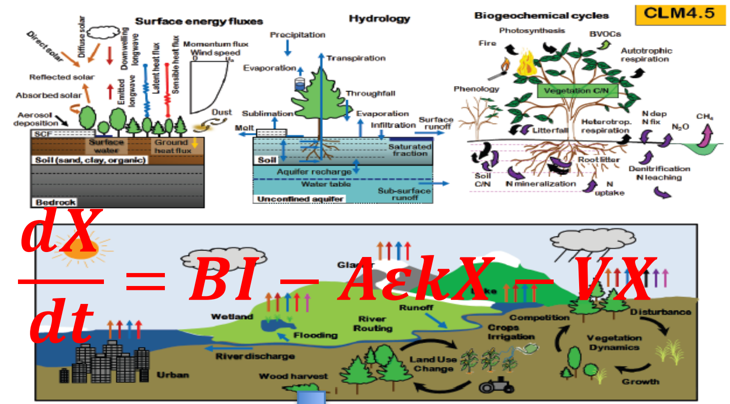
3.Data assimilation for both flux- and pool-related data to constrain global SOM



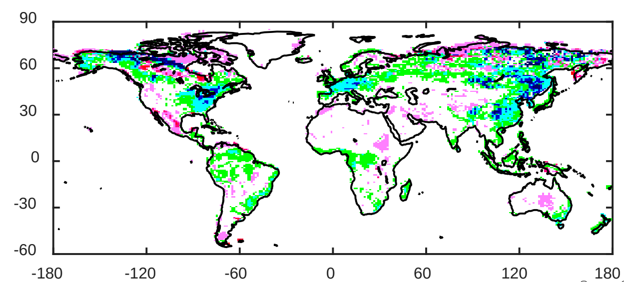


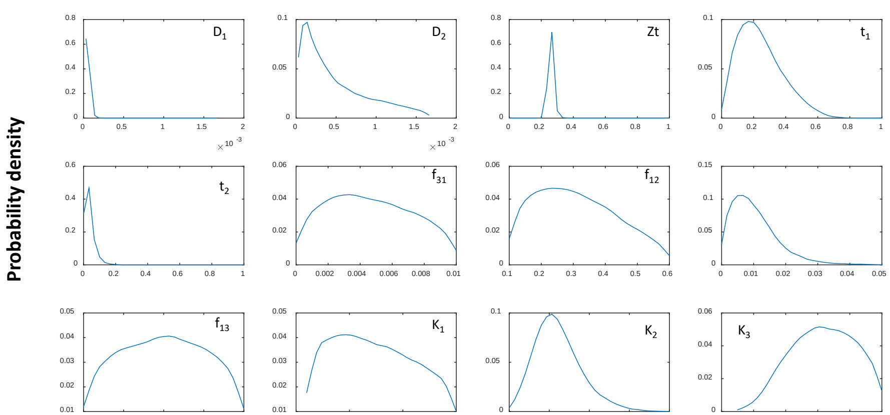
Harmonized soil C content by Wieder et al., (2015)

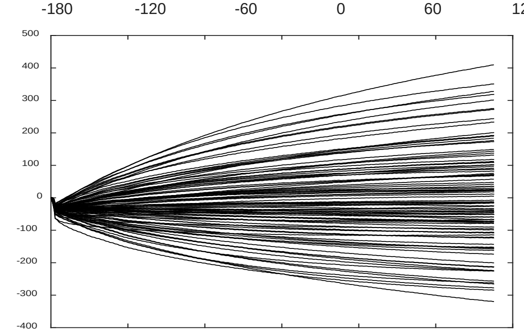
MCMC

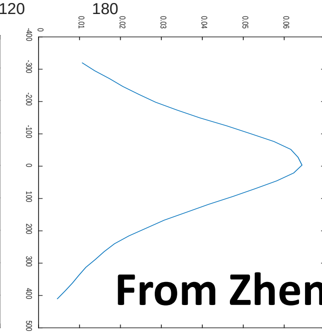


$$\frac{dX}{dt} = BI - A\varepsilon kX - VX$$



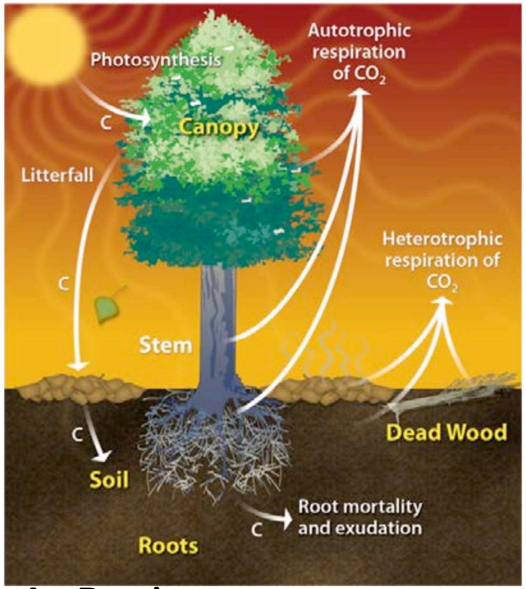




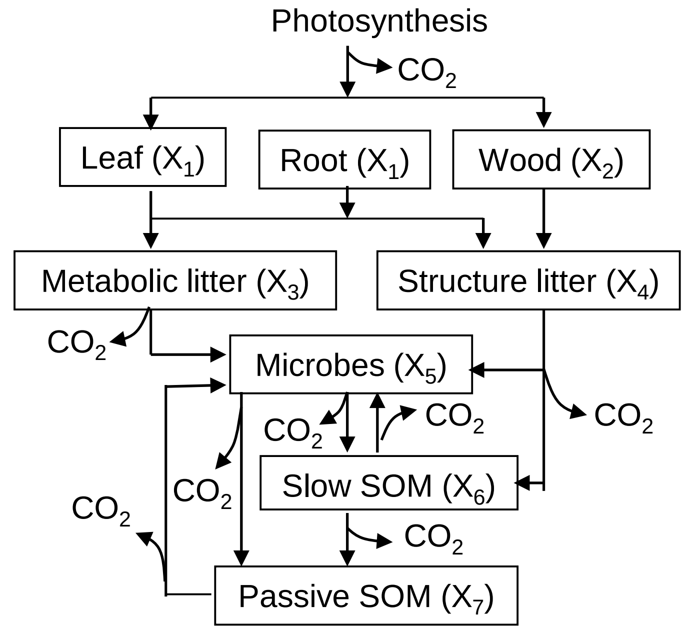


From Zheng Shi

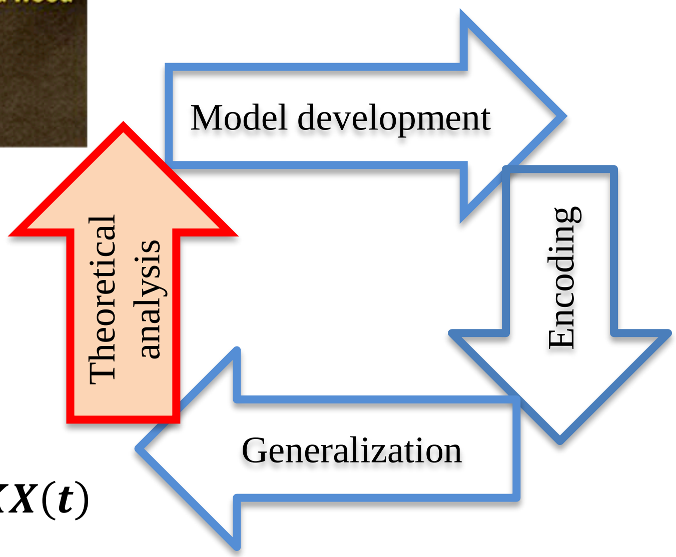
4. One formula to unify all land carbon cycle models



A: Basic processes



B: Shared model structure



D: General model

$$\frac{dX(t)}{dt} = BI(t) - A\xi(t)KX(t)$$

C: Similar algorithm

$$Plant \begin{cases} dX_1(t)/dt = b_1U(t) - \xi c_1 X_1(t) \\ \end{cases}$$

$$a_{75}x_5(t) + c_6a_{76}x_6(t) - c_7X_7(t)]$$

$$dX_8(t)/dt = \xi[c_6a_{86}x_6(t) + c_7a_{87}x_7(t) - c_8X_8(t)]$$

Luo et al. 2001
Luo et al. 2003 GBC
Luo and Weng 2011 TREE
Luo et al. 2012
Luo et al. 2015
Luo et a. 2017

5. Hierarchical models

Vertical profile

$$\frac{d\boldsymbol{X}(\boldsymbol{t})}{d\boldsymbol{t}} = (\boldsymbol{A}\boldsymbol{\xi}(\boldsymbol{t})\boldsymbol{K} + \boldsymbol{V}(\boldsymbol{t}))\boldsymbol{X}(\boldsymbol{t}) + \boldsymbol{B}(\boldsymbol{t})u(\boldsymbol{t})$$

$$\frac{dX(t)}{dt} = A\xi(t)KX(t) + Bu(t)$$

C transfer of phenology

C transfer of gap mortality

C transfer of fire

pool state

input

$$\frac{\boldsymbol{d}}{\boldsymbol{dt}}\boldsymbol{X}(\boldsymbol{t}) = (\boldsymbol{A}_{\boldsymbol{ph}}(\boldsymbol{t})\boldsymbol{K}_{\boldsymbol{ph}}(\boldsymbol{t}) + \boldsymbol{A}_{\boldsymbol{gm}}(\boldsymbol{t})\boldsymbol{K}_{\boldsymbol{gm}}(\boldsymbol{t}) + \boldsymbol{A}_{\boldsymbol{fi}}(\boldsymbol{t})\boldsymbol{K}_{\boldsymbol{fi}}(\boldsymbol{t}))\boldsymbol{X}(\boldsymbol{t}) + \boldsymbol{B}(\mathbf{t})\boldsymbol{F}(\boldsymbol{t})$$

C turnover of phenology

C turnover of gap mortality

C turnover of fire

allocation

Vegetation dynamics

Developing models at different levels of complexity under one overarching theory

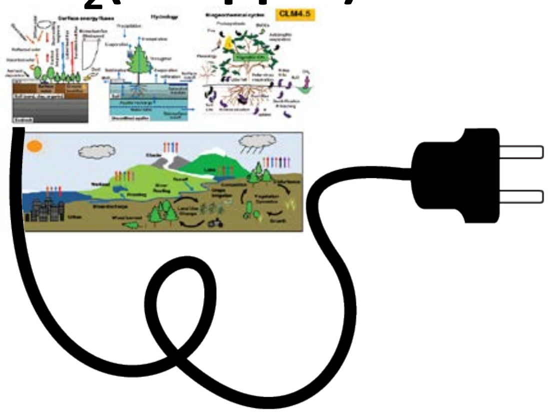
6. Unified Diagnostic System
Or 1-3-5 scheme

- One (1) formulae unifies all land C cycle models
- One 3-D space to evaluate all model outputs
- Five (5) Traceable components to pinpoint uncertainty sources

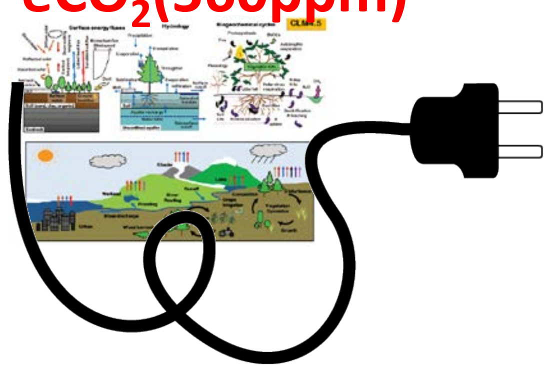
Luo et al. 2017 AGU

7. Analytic attribution of CO_2 impacts

CLM Run 1
CO_2(280ppm)



CLM Run 2
eCO_2(560ppm)

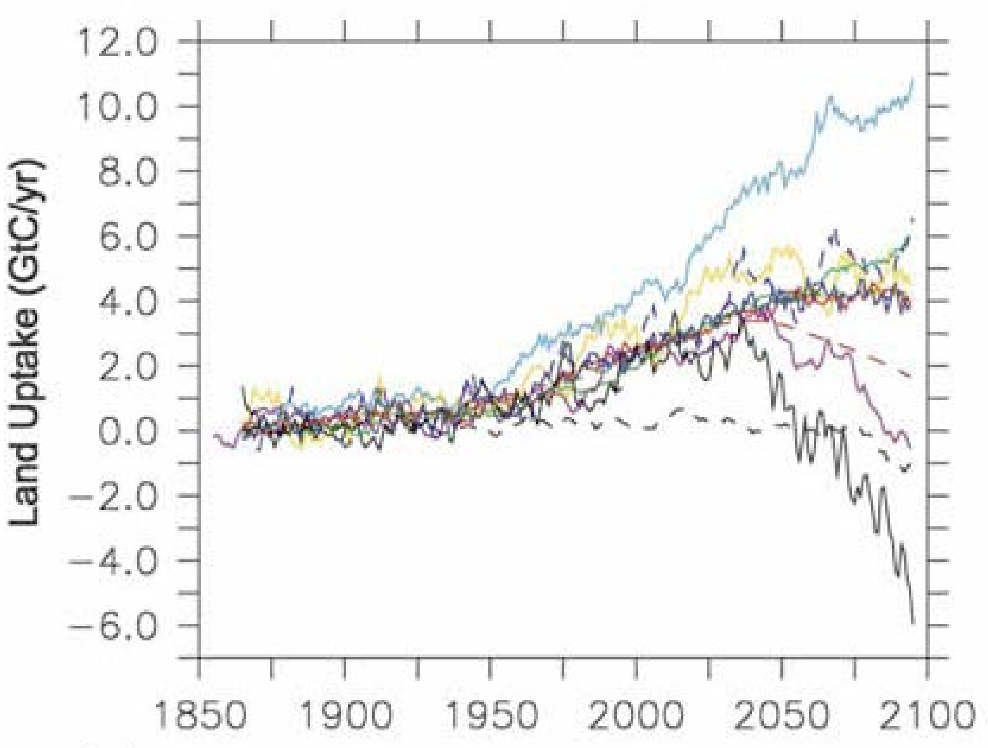


Matrix Simulations (matlab)

	S0	S1	S2	S3	S4	S5
I	I0	Ie	Ie	Ie	Ie	Ie
B	B0	B0	Be	Be	Be	Be
N	N0	N0	N0	Ne	Ne	Ne
ε	ε0	ε0	ε0	ε0	εe	εe
V	V0	V0	V0	V0	V0	Ve

Huang et al. 2018 GCB

8. Modeling conundrum



Friedlingstein et al. 2006

- Models behave so differently;
- Uncertainty has been documented in almost all model intercomparison projects (MIPs);
- Uncertainty becomes larger instead of smaller as we incorporate more processes into models
- We become more confused with uncertainty as we invest more time to address this issue.

8. Modeling conundrum

- ✓ High degree of complexity and sophistication of model implementations hinders our understanding of holistic system behavior

- ✓ Matrix approach provides a general framework for the qualitative understanding of models without compromising detail in process representation

What is the future?

CLM5.0 matrix CN model

- Done all tests
- Ready to be incorporated into trunk of CLM
- Will make the offline version of CLM5.0-CN matrix model publically available
- Welcome more people to use matrix models
- Willing to help the community for the transformation
- Summer training course at Flagstaff in May

Challenges and opportunities for you

Challenges

- Luring you out of your comfort zone for modeling
- Basic training in matrix algebra
- Other obstacles?

Benefits and opportunities

- Most likely to make your life easier
  - Simplicity in coding
  - Cleaner and more efficient code
  - Faster for spin-up
- Enabling you do analysis (e.g., Sobol) you usually can not do
- Understanding your model results much easier

General equation for biogeochemical models

Matrix models

1. CLM 3.5
2. CLM4.0
3. CLM4.5
4. CLM5.0
5. CABLE
6. LPJ-GUESS
7. ORCHIDEE
8. BEPS

In progress

1. JULES
2. LM3V-N

10 more models to participate in the summer training course

10 nonlinear Microbial models by Carlos Sierra

General dynamical equation of terrestrial carbon cycle

1. Transforming land carbon cycle modeling into more theoretically based science
2. Asking a whole set of new questions on land carbon cycle dynamics (e.g., dynamic disequilibrium, predictability)
3. Implications on empirical research are yet to be uncovered