



Concurrent Ice & Embedded Ice Coupling: A Solution to Address the Numerical Stability of Ice/Ocean Coupling

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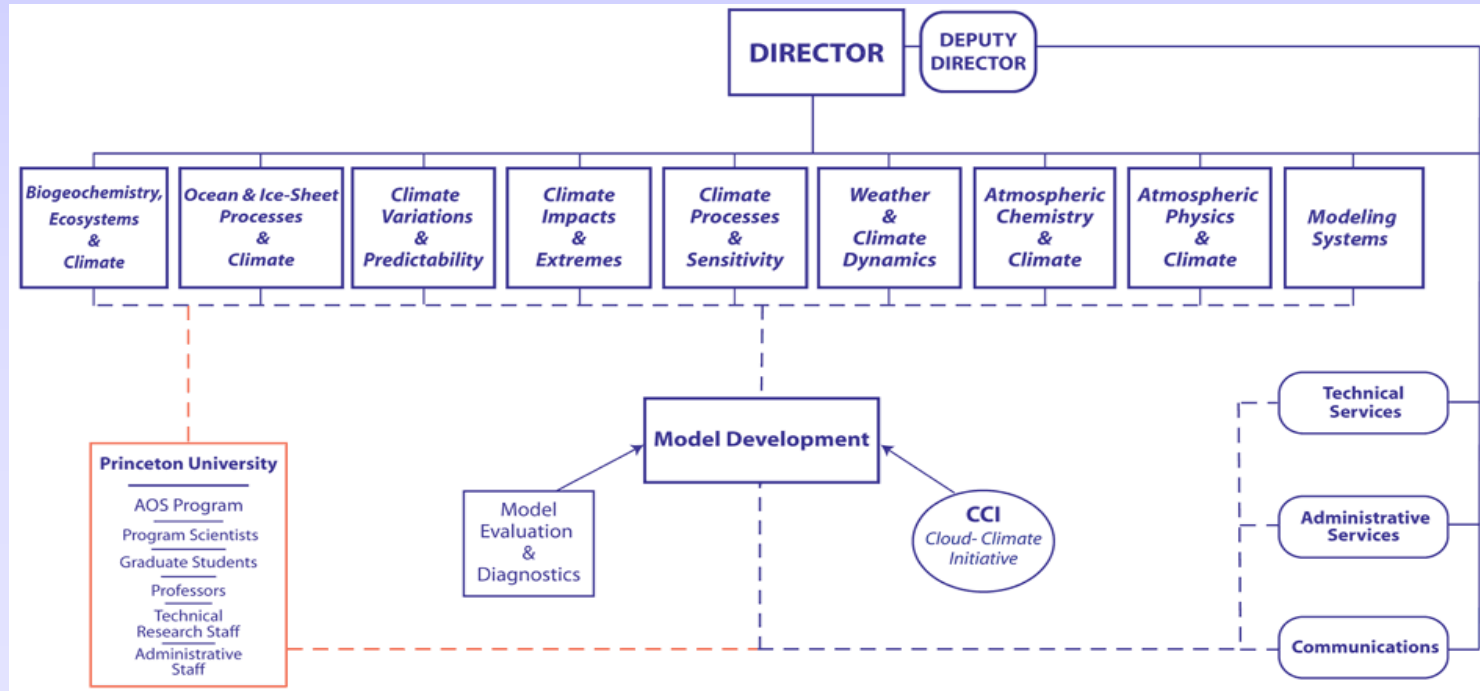
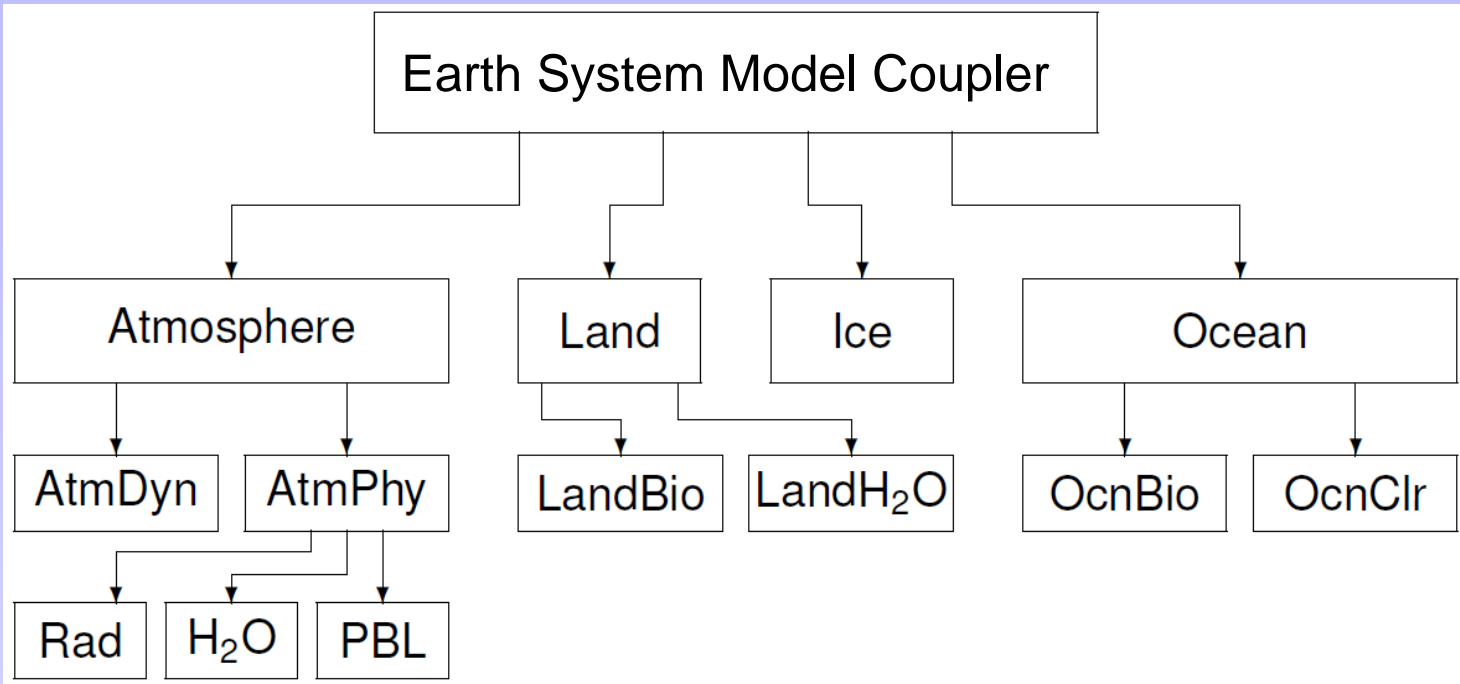
Objectives of Climate Model Couplers

- **Manage Complexity**
 - Separate the climate system into disciplinary components
 - Interchange different component models with minimal changes to other components



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- **Achieve Social Harmony**
 - “Good fences make good neighbors”



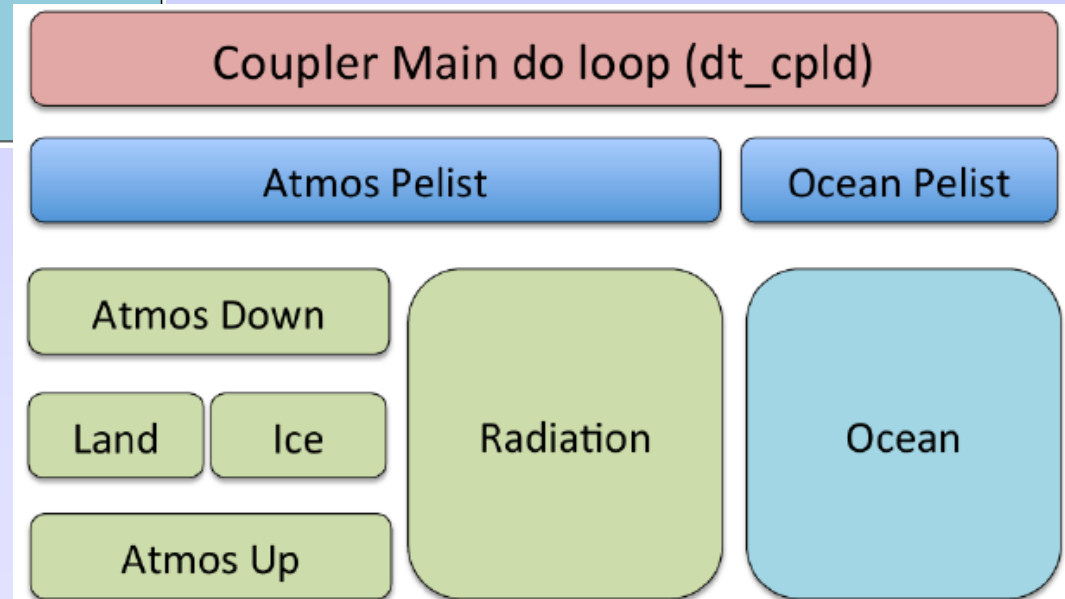
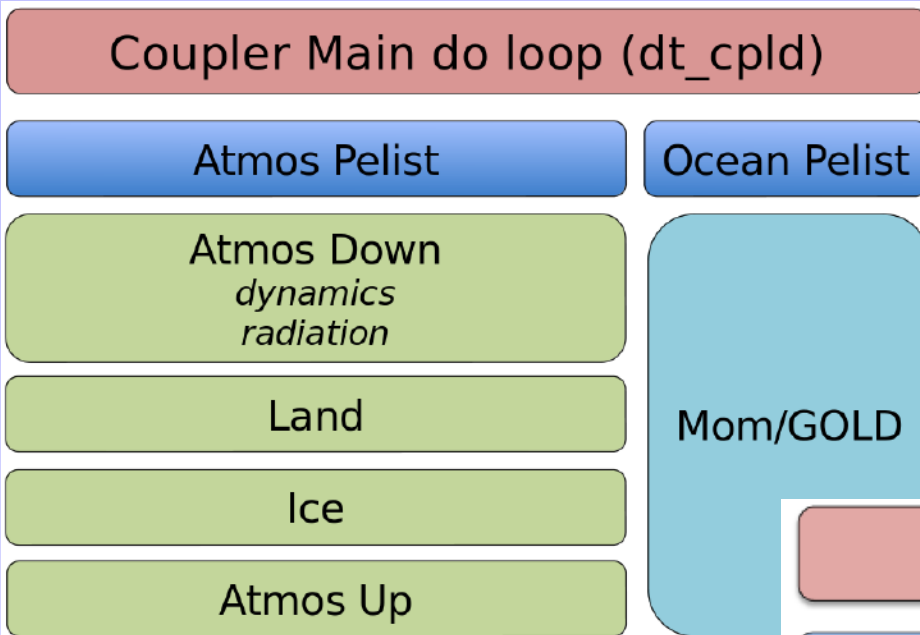


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 - “Good fences make good neighbors”
- **Computational Efficiency**
 - Find concurrency - “Many hands make light work”



Seeking Greater Concurrency





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- **Computational Efficiency**
 - Find concurrency - “Many hands make light work”
- **Achieve physically correct behavior of the coupled system dynamics**
 - Avoid coupled instabilities

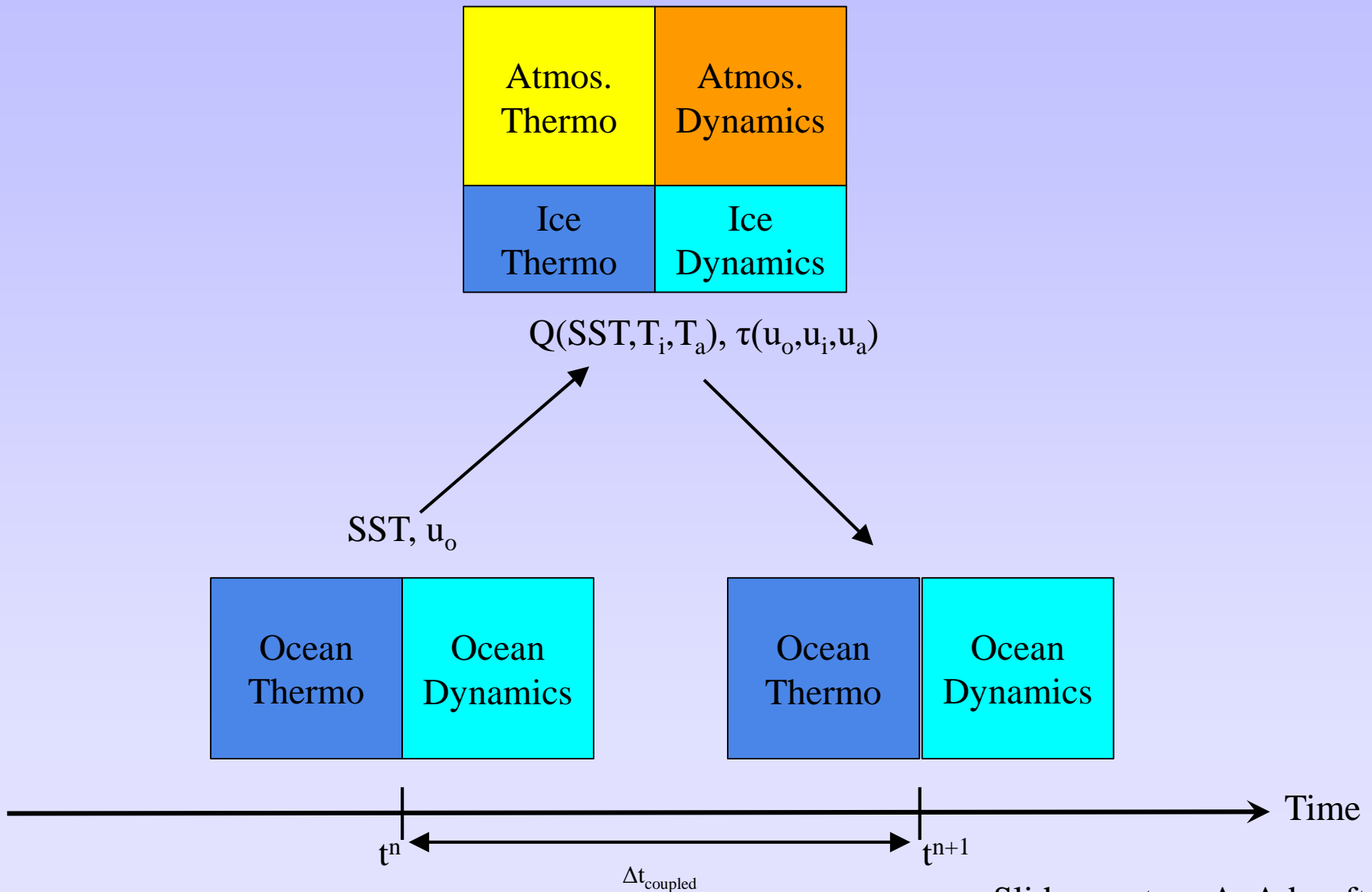


Key Coupler Considerations:

COUPLING TIME-STEPPING STRATEGIES

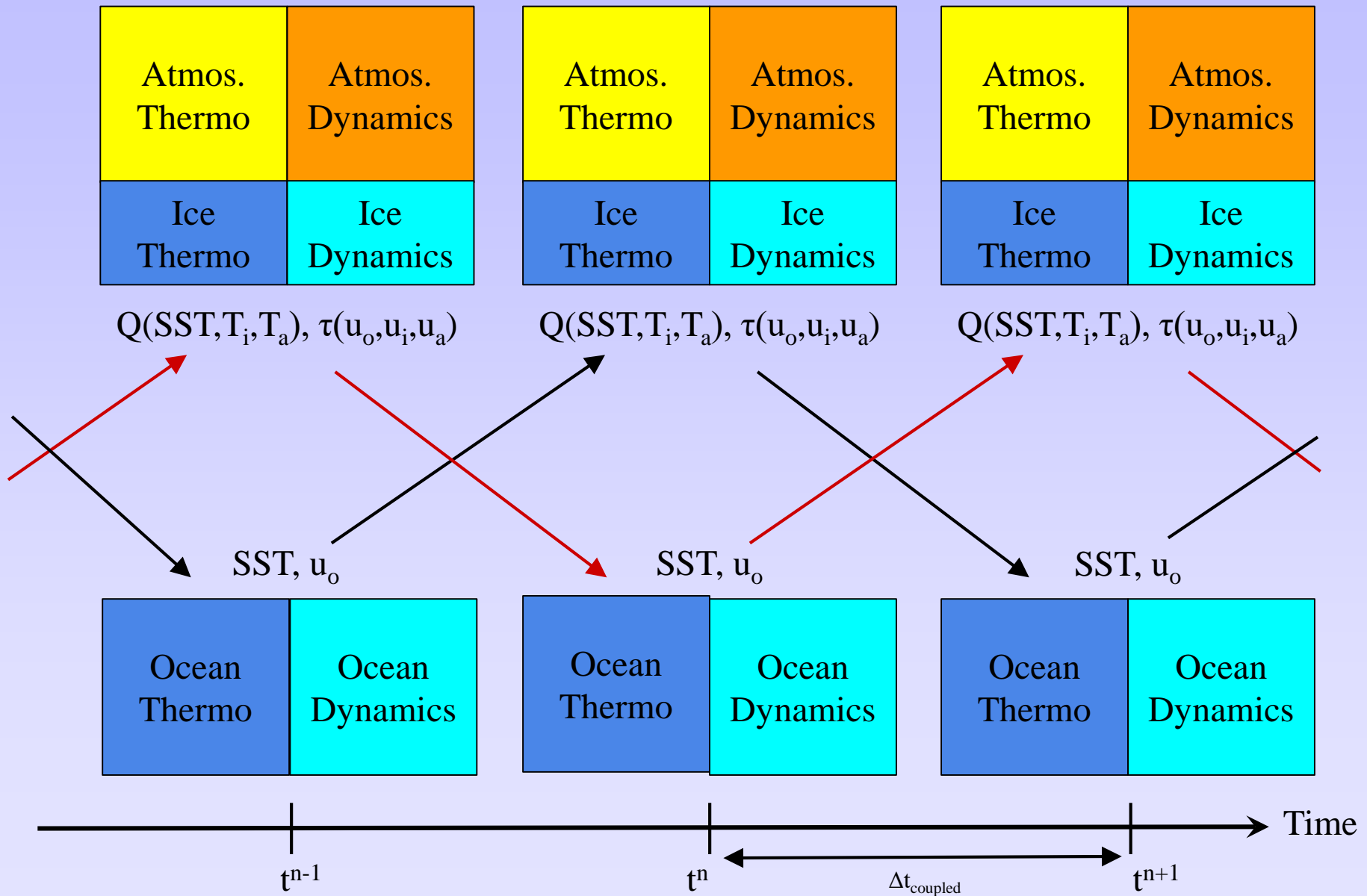


Sequential Coupling



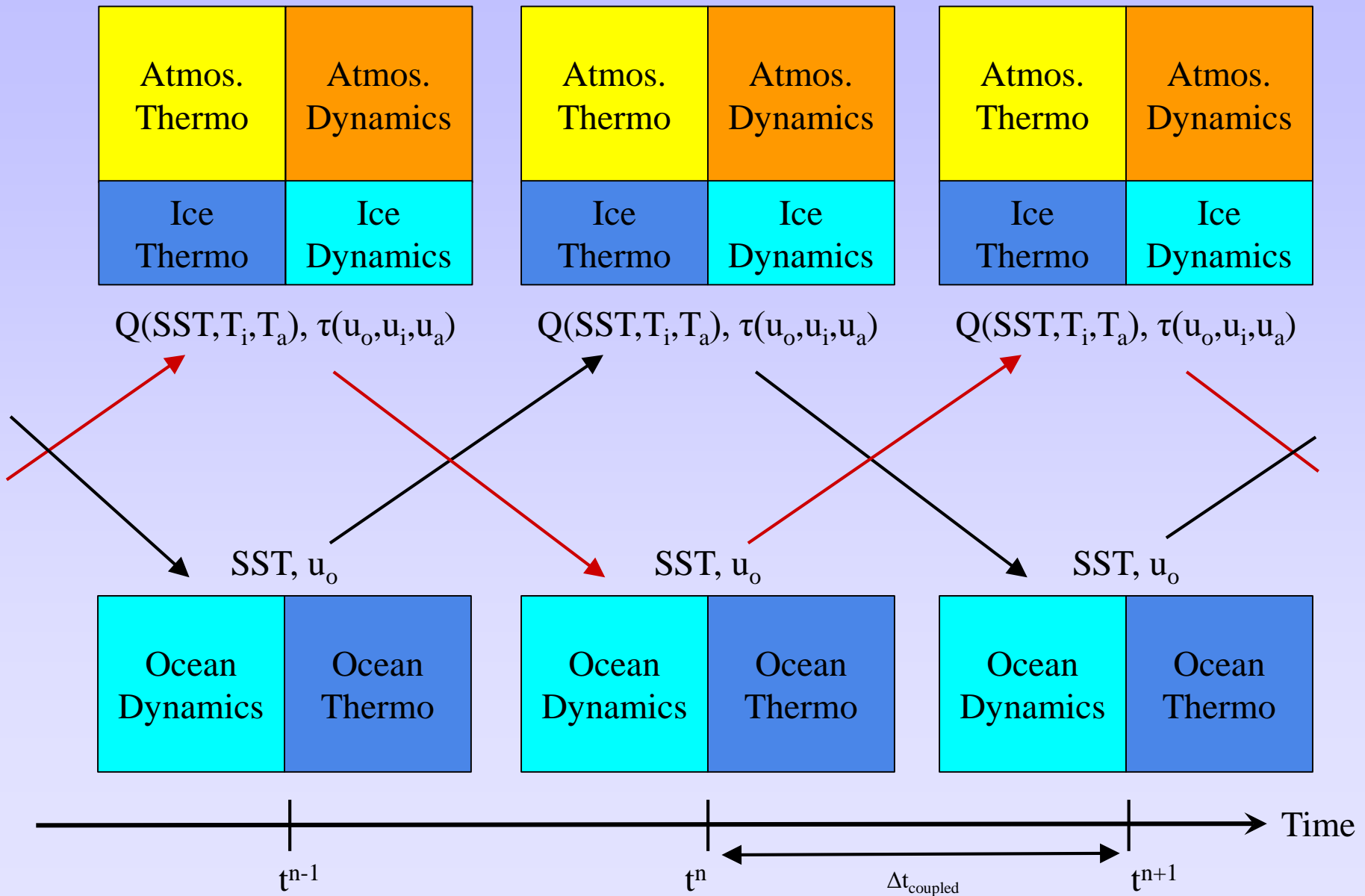


Concurrent Coupling



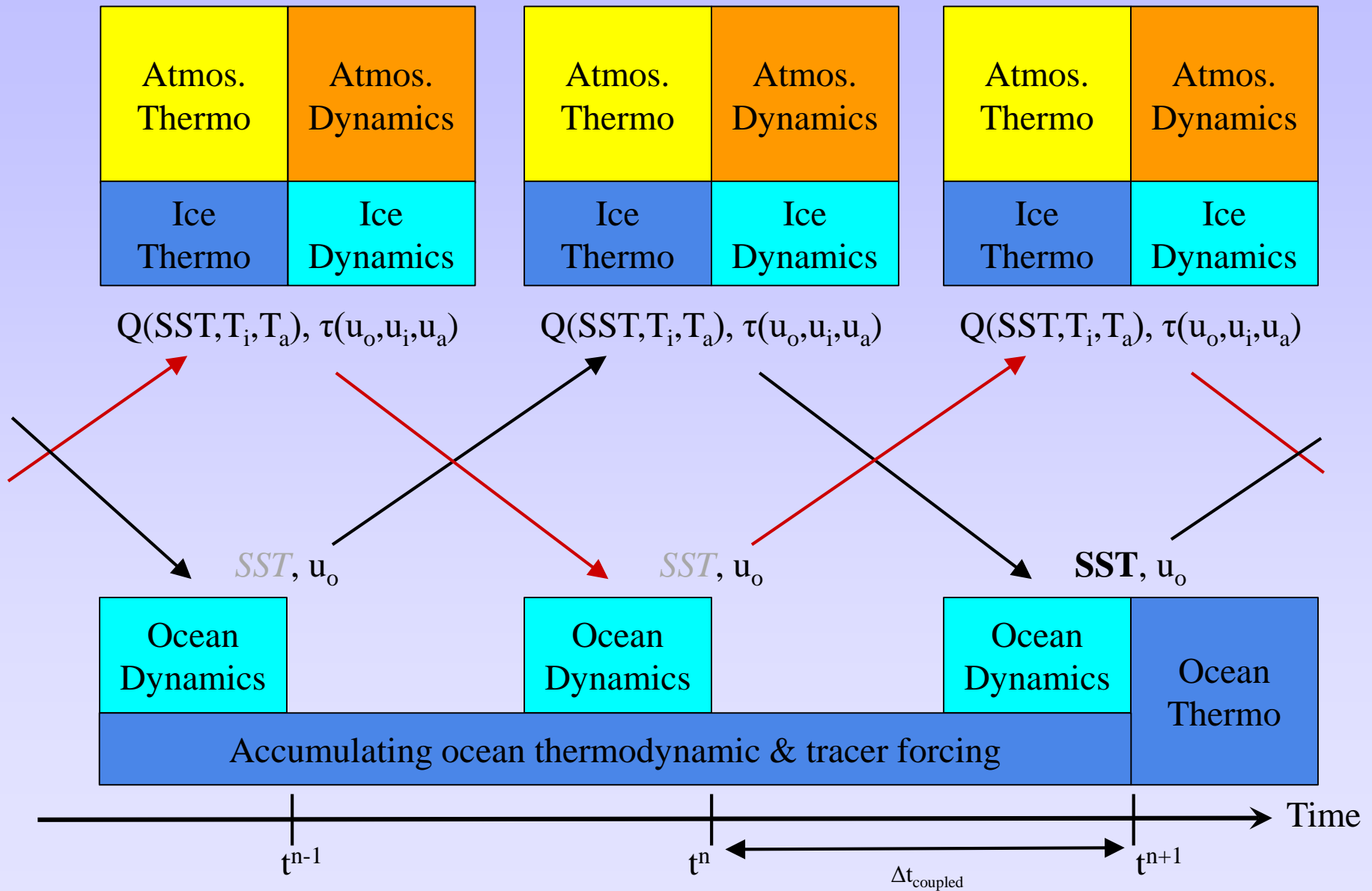


Concurrent Coupling





Concurrent Coupling with MOM6





A simplified history of sea-ice ocean coupling

- Rigid lid ocean models could not handle divergent flows or mass loss or gain at the surface (1970s).

Problem – sea-ice grows by taking fresh water from the ocean

Solution – use a virtual salt flux to get the equivalent brine rejection

$$F_{Salt} = -SF_{Water}$$

Advantages – Massless sea ice does not exert pressure on the ocean or participate in dynamics; Sea ice can be treated as a completely independent component.

Liabilities – Freezing & melting at different S give inconsistent forcing

- Free surface ocean models allowed climate models to return to the “natural boundary condition” (~2000).

– Z-coordinate models still require limits on ice pressure: $P_{Ice} < O(0.5)g\rho_{Oce}\Delta z_{Sfc}$

– Artificial Stommel-Goldsborough circulation results where the pressure-limited ice melts; sea-ice grounding is not permitted.

- Z*-coordinates & other ocean model developments allow for increasingly realistic sea-ice models... (Today)



Traditional (GFDL) Approach to Ocean/Ice Coupling

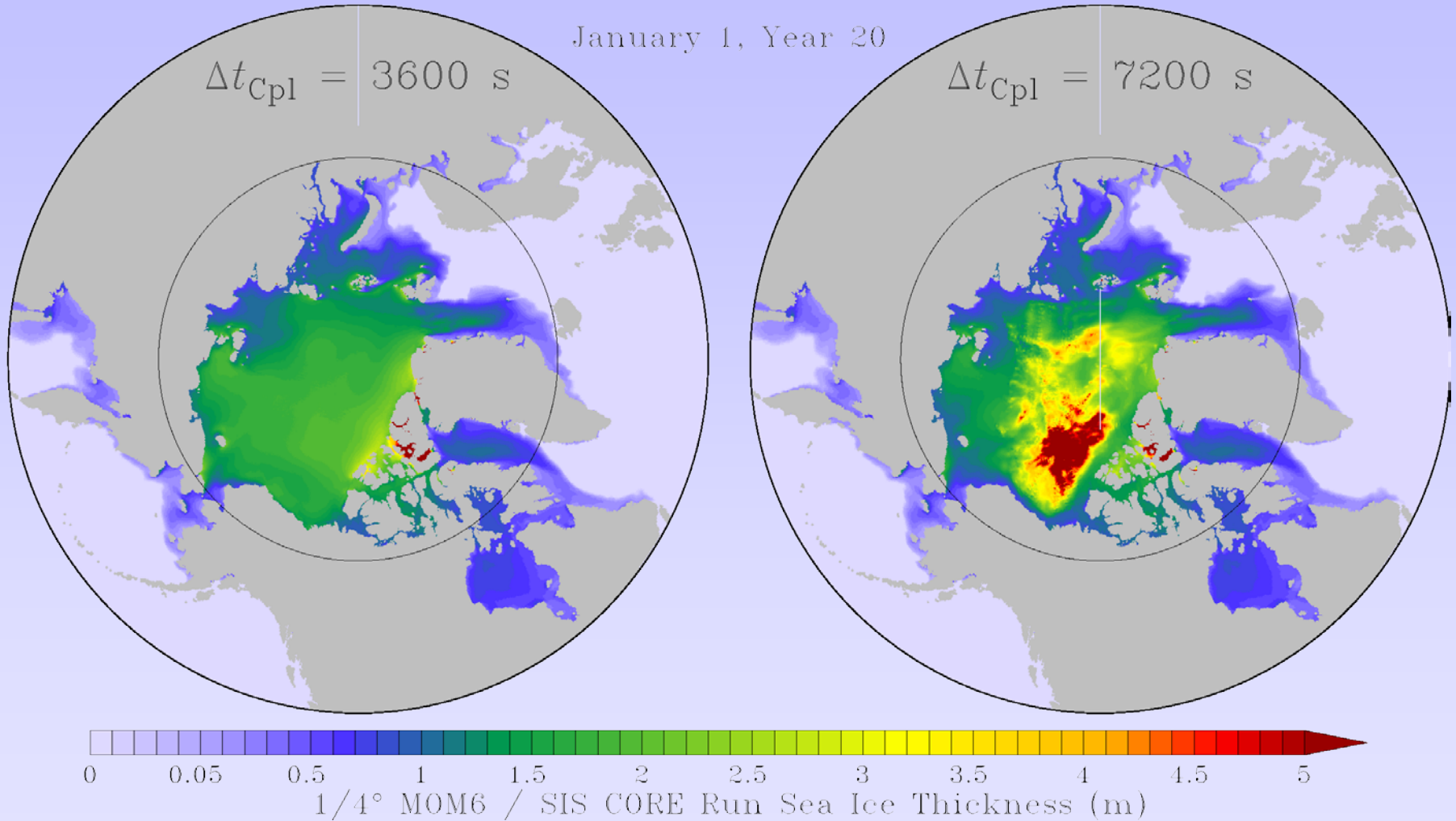
- Sea-ice (SIS or SIS2) is advanced implicitly with the atmosphere, for skin temperatures consistent with atmosphere.
- Ocean (MOM4, MOM5, GOLD or MOM6) is forced by prescribed fluxes from the sea-ice.
- Air-sea fluxes are based on ocean properties from 1 (sequential) or 2 (concurrent) time-steps before they are applied to the ocean.
- Ice displacement is similarly lagged.
- Icebergs are point masses embedded in the sea-ice.
- Ice can displace a limited thickness of ocean; more than ~2-5 m of ice “levitates” to avoid numerical problems.

Ocean model (MOM6), sea-ice (SIS2), icebergs, and GFDL coupler are all being restructured to allow this approach to be revised.

These revisions may provide a template for consideration in CESM.



Evidence of Lagged Stress-Inertial Coupling Instability in Sea-Ice Thickness



Sequentially coupled data-driven ice-ocean model

Hallberg (2014, *Clivar Exchanges*)



Symptoms of problems with GFDL's traditional coupling approach

- Numerical instability of high resolution coupled models, especially in Spring when thick sea-ice becomes unlocked from the pack of thin ice.
- Avoiding “surfing” icebergs and marginal sea-ice requires “levitation” of the ice
- “Levitation” in turn introduces undesirable consequences
 - Icebergs and sea-ice can not ground
 - Unlimited growth of sea-ice (to 1000s of m) in certain embayments
 - No dynamic ice-sheet coupling, or else tabular icebergs must be treated differently from ice-shelves
- Short coupling time-step required at higher resolutions
 - E.g., 1200 s for GFDL's $\frac{1}{4}^\circ$ CM4 with concurrent coupling



Numerical Ice-Ocean Coupling Instabilities

1. Lagged stress / inertial oscillation instability

$$\frac{\partial u}{\partial t} + ifu = \frac{c_d U}{H} (u_{Atm} - u^n)$$

$$u' = u - u_{Steady}$$

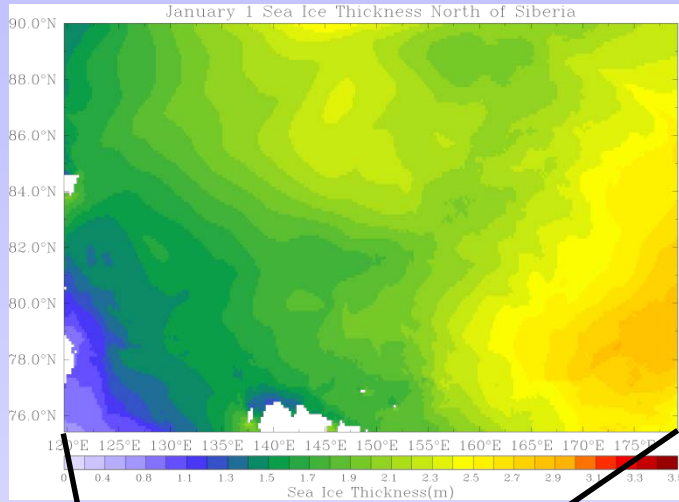
$$u'(t^{n+1}) = \left[e^{-if\Delta t} + i \frac{c_d U}{Hf} (1 - e^{-if\Delta t}) \right] u'(t^n) = Au'(t^n)$$

$$\|A\|^2 = 1 - 2 \frac{c_d U}{Hf} \sin(f\Delta t) + 2 \left(\frac{c_d U}{Hf} \right)^2 (1 - \cos(f\Delta t))$$

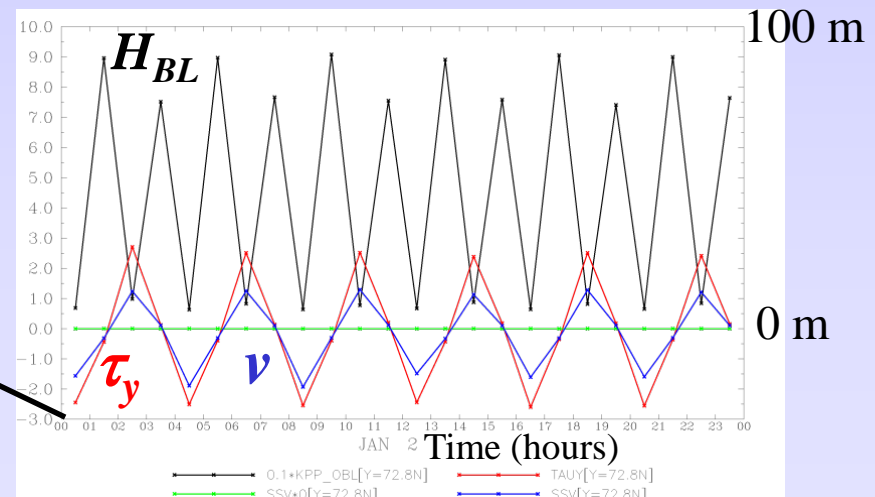
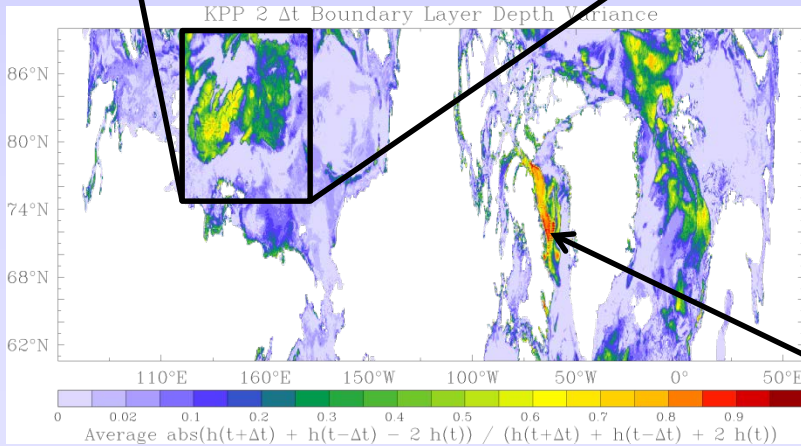
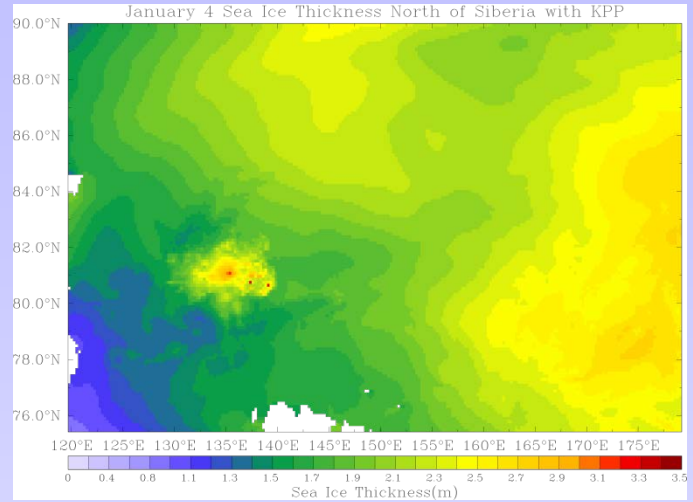


Explosive Sea-Ice Growth as a Manifestation of a Sea Ice-Ocean Coupling Instability

Jan. 1 Sea Ice Thickness of Siberia



Jan. 4 Sea Ice Thickness of Siberia



$$\left| \frac{2H_t - H_{t-\Delta t} - H_{t+\Delta t}}{2H_t + H_{t-\Delta t} + H_{t+\Delta t}} \right|$$

H = Ocean boundary layer depth from KPP; determined from initial bulk Ri consideration.



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$$\|A\|^2 = 1 - 2 \frac{c_d U}{Hf} \sin(f\Delta t) + 2 \left(\frac{c_d U}{Hf} \right)^2 (1 - \cos(f\Delta t))$$

2. Thermal forcing instability

$$\frac{\partial \theta_1}{\partial t} = -\frac{\lambda}{H_1} (\theta_1 - \theta_2) \quad \frac{\theta_1^{n+1} - \theta_1^n}{\Delta t} = -\frac{\lambda}{H_1} (\theta_1^{n+1} - \theta_2^n)$$

$$\frac{\partial \theta_2}{\partial t} = +\frac{\lambda}{H_2} (\theta_1 - \theta_2) \quad \frac{\theta_2^{n+1} - \theta_2^n}{\Delta t} = +\frac{\lambda}{H_2} (\theta_1^{n+1} - \theta_2^n)$$

Eigenvalues:

$$A_1 = \frac{1}{1 + \lambda\Delta t / H_1}$$

$$A_2 = 1 - \lambda\Delta t / H_2$$

3. Gravity wave instability

- Sea-ice and icebergs participate in barotropic gravity waves
- Stability analysis analogous to split-explicit ocean time stepping (e.g., Hallberg, 1997)
- Instability growth rate proportional to the sea-ice external gravity wave CFL ratio based on the *coupling time step*. $\frac{\sqrt{gH_{Ice}\Delta T}}{\Delta x} < O(1)$



Ice in a Greenland Fjord (Rink Isbrae)



(Photo Credit: R. Hallberg 2015 pretending to be an observationalist.)



A coupled gravity-wave toy model

2-layer (sea-ice & ocean) linear nonrotating flat-bottom channel flow with no viscosity.

$$\frac{\partial u_1}{\partial t} = -g \frac{\partial \eta_{1/2}}{\partial x}$$

$$= -g \frac{\partial}{\partial x} (h_1 + h_2)$$

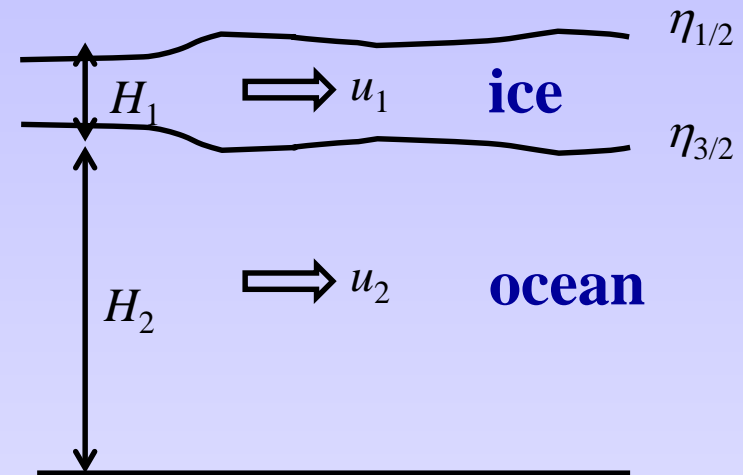
$$\frac{\partial u_2}{\partial t} = -g \frac{\rho_I}{\rho_o} \frac{\partial \eta_{1/2}}{\partial x} - g \frac{\rho_o - \rho_I}{\rho_o} \frac{\partial \eta_{3/2}}{\partial x}$$

$$= -(g - g') \frac{\partial}{\partial x} (h_1 + h_2) - g' \frac{\partial h_2}{\partial x}$$

$$= -(g - g') \frac{\partial h_1}{\partial x} - g \frac{\partial h_2}{\partial x}$$

$$\frac{\partial h_1}{\partial t} = -H_1 \frac{\partial u_1}{\partial x}$$

$$\frac{\partial h_2}{\partial t} = -H_2 \frac{\partial u_2}{\partial x}$$





A coupled gravity-wave toy model

Sequential coupling of gravity waves only:

$$\begin{aligned} \frac{\partial h_1}{\partial t} &= -H_1 \frac{\partial u_1}{\partial x} & \frac{\partial u_1}{\partial t} &= -g \frac{\partial h_1}{\partial x} - g \frac{\partial h_2^n}{\partial x} \\ \frac{\partial h_2}{\partial t} &= -H_2 \frac{\partial u_2}{\partial x} & \frac{\partial u_2}{\partial t} &= -g \frac{\partial h_2}{\partial x} - (g - g') \frac{\partial h_1^{n+1}}{\partial x} \end{aligned}$$

Sequential coupling:

Marginally stable if waves are treated analytically in each component.

$$\omega_1 \equiv \sqrt{gH_1}k \quad ; \quad \omega_2 \equiv \sqrt{gH_2}k$$

$$0 \leq \omega_2 \Delta T < \sim 100$$

Concurrent (forward) coupling:

$$\begin{aligned} \frac{\partial h_1}{\partial t} &= -H_1 \frac{\partial u_1}{\partial x} & \frac{\partial u_1}{\partial t} &= -g \frac{\partial h_1}{\partial x} - g \frac{\partial h_2^n}{\partial x} \\ \frac{\partial h_2}{\partial t} &= -H_2 \frac{\partial u_2}{\partial x} & \frac{\partial u_2}{\partial t} &= -g \frac{\partial h_2}{\partial x} - (g - g') \frac{\partial h_1^n}{\partial x} \end{aligned}$$

Concurrent forward coupling:

$$\begin{aligned} &\text{Unconditionally unstable, growth rate:} \\ &\approx \frac{(g - g')}{g\Delta T} [1 - \cos(\omega_1 \Delta T)] [1 - \cos(\omega_2 \Delta T)] \end{aligned}$$

Sequential (filtered) coupling:

$$\begin{aligned} \frac{\partial h_2}{\partial t} &= -H_2 \frac{\partial u_2}{\partial x} & \frac{\partial u_2}{\partial t} &= -g \frac{\partial h_2}{\partial x} - (g - g') \frac{\partial h_1^n}{\partial x} \\ \frac{\partial h_1}{\partial t} &= -H_1 \frac{\partial u_1}{\partial x} & \frac{\partial u_1}{\partial t} &= -g \frac{\partial h_1}{\partial x} - g \frac{\partial}{\partial x} \left(\frac{1}{\Delta T} \int_0^{\Delta T} h_2 dt \right) \end{aligned}$$

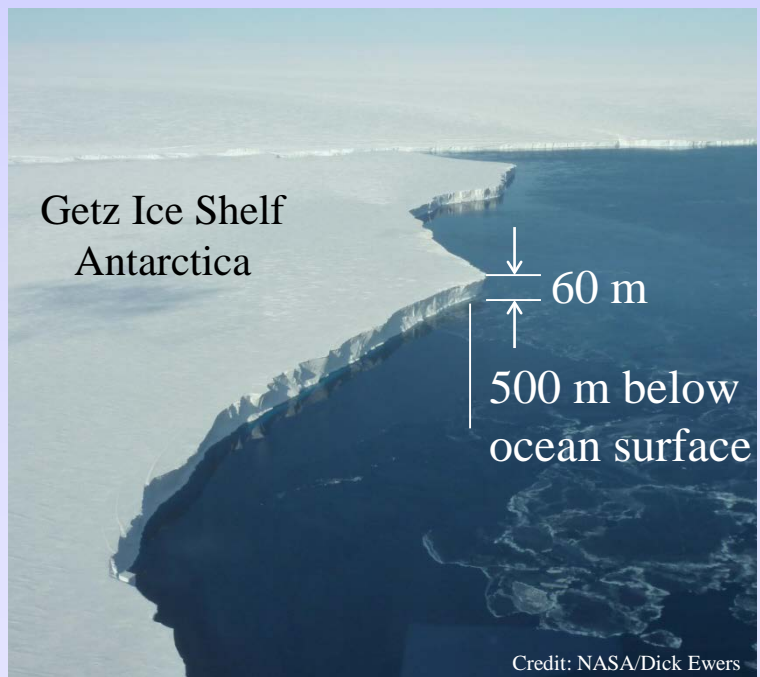
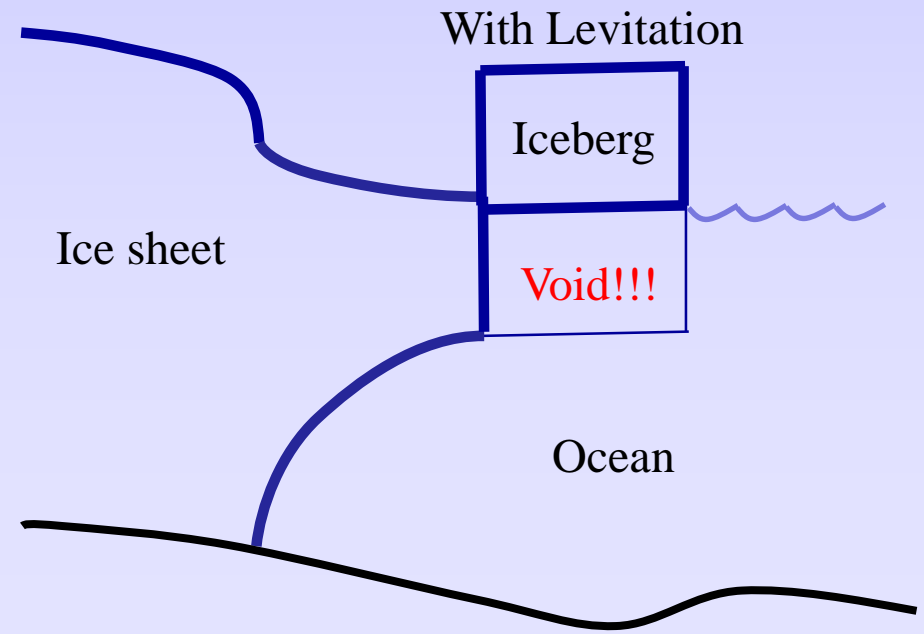
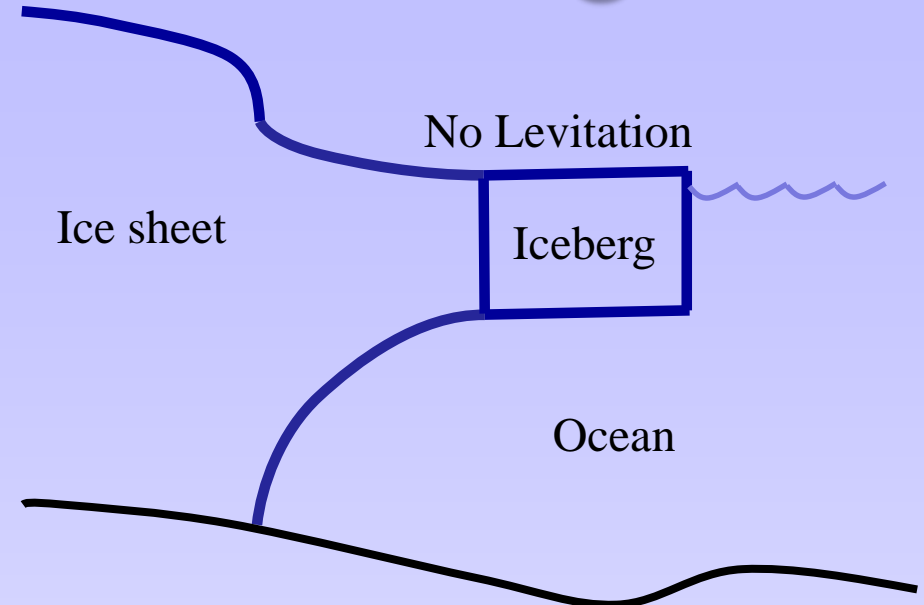
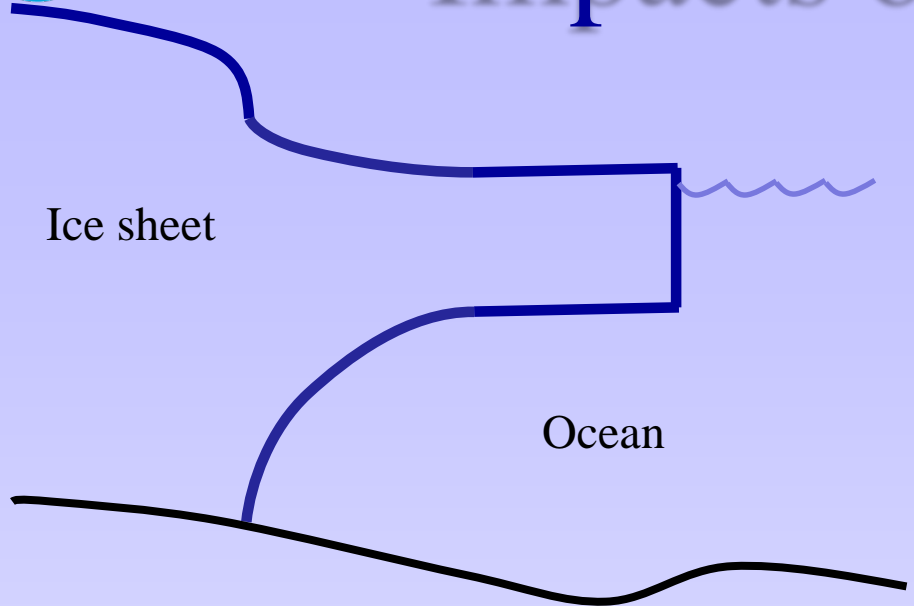
Sequential filtered coupling:

$$\begin{aligned} &\text{Unconditionally unstable, growth rate:} \\ &\approx \frac{1}{2} \text{Concurrent growth rate for small } \omega_2 \Delta T \\ &\propto \frac{1}{\omega_2 \Delta T}, \text{ for large } \omega_2 \Delta T \end{aligned}$$

Damping from an ice-pack can locally stabilize the instability.



Impacts of “Levitating” Ice

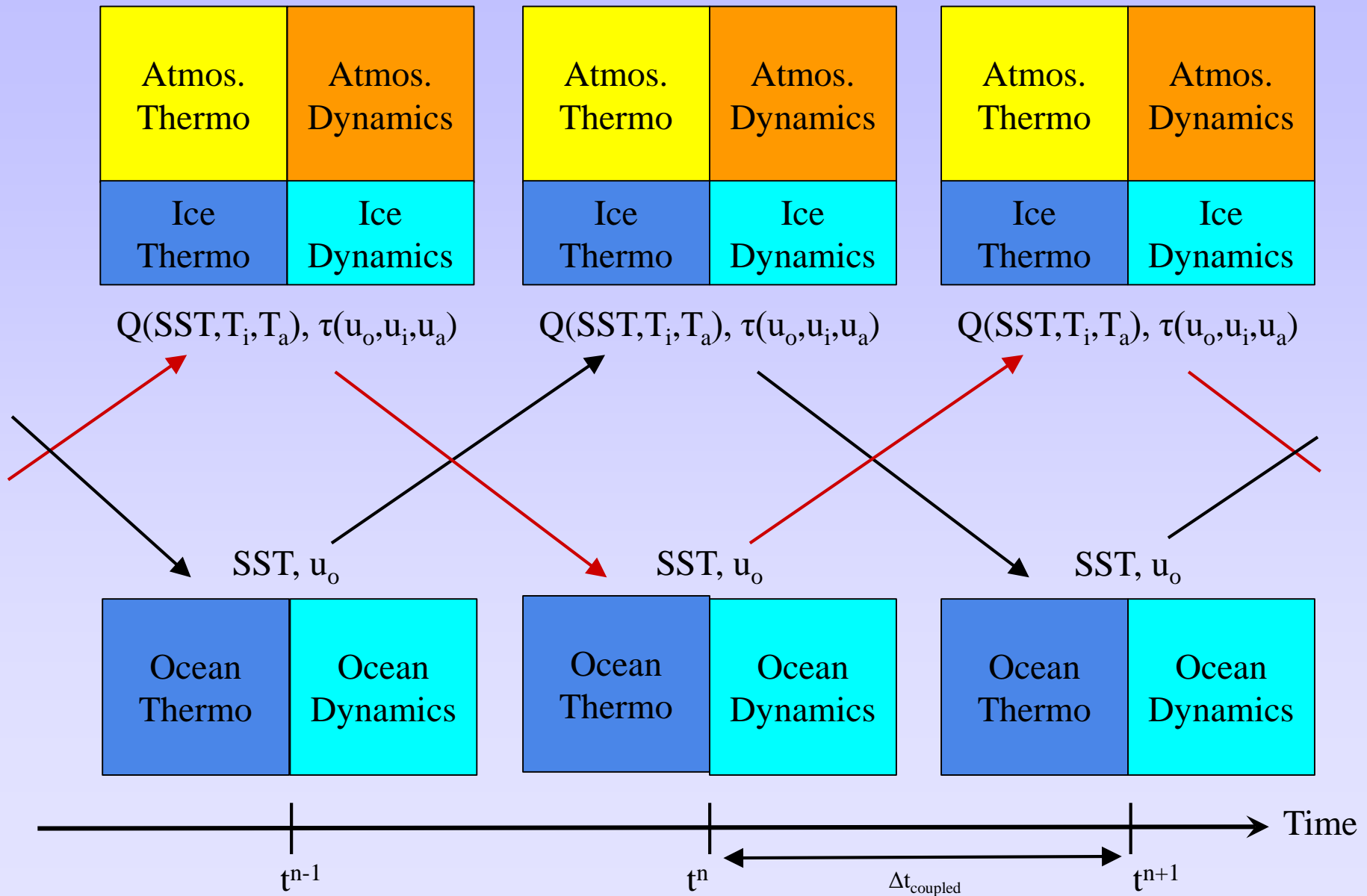




A NEW ICE / OCEAN COUPLING STRATEGY



Concurrent Coupling



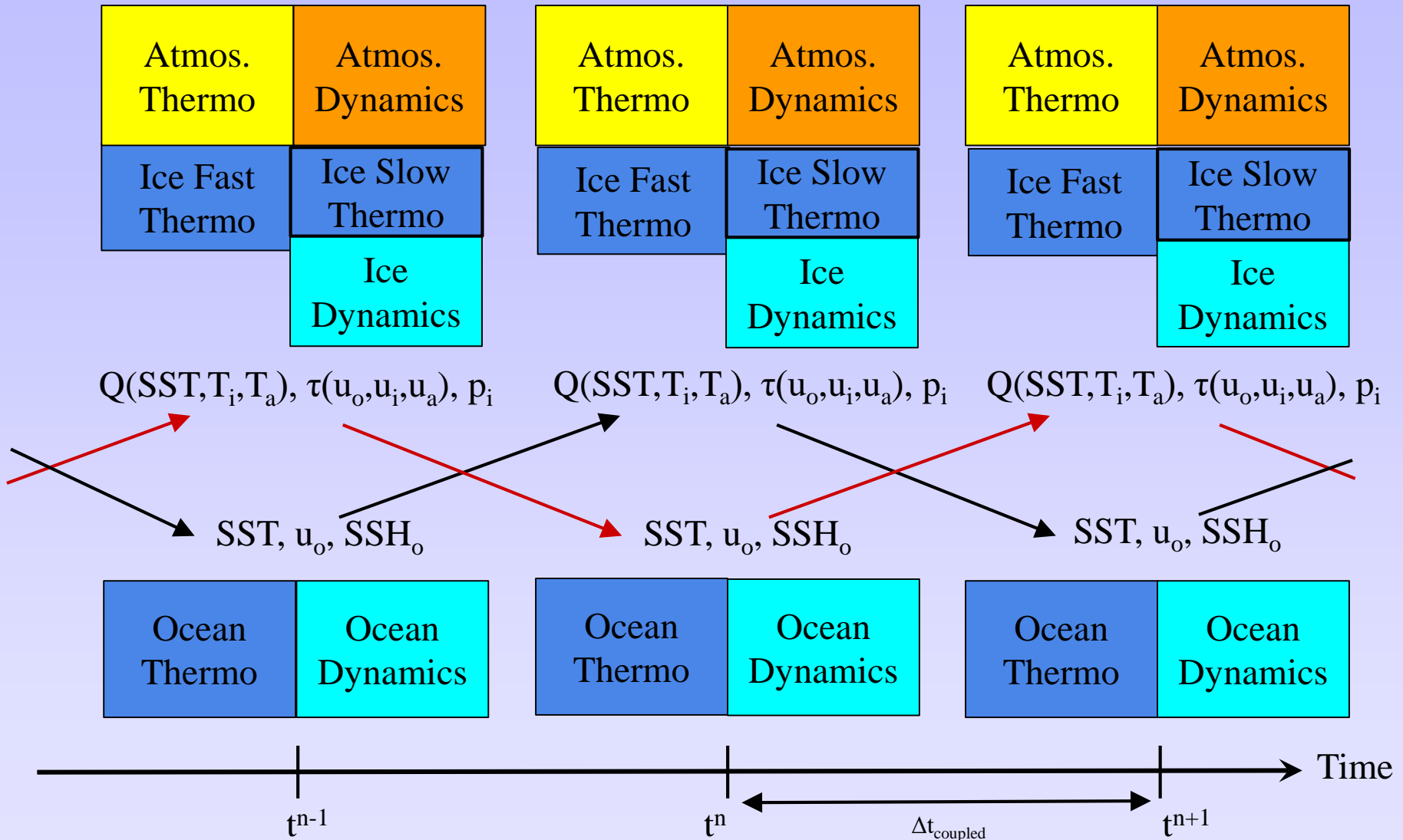


A Subcomponent Decomposition of Sea-ice Processes

- Fast thermal processes (almost immediate)
 - Surface skin temperature calculation
 - Determines atmospheric boundary layer stability
- Slow thermodynamic processes (hours to years)
 - Melting, Freezing
 - Ice salinity changes
- Dynamics and Rheology (minutes to days)
 - Ice-pack stress fields and momentum budget
- Transport and ridging (hours to days)



Concurrent Coupling in more detail





A solution to the ice-ocean coupling issues?

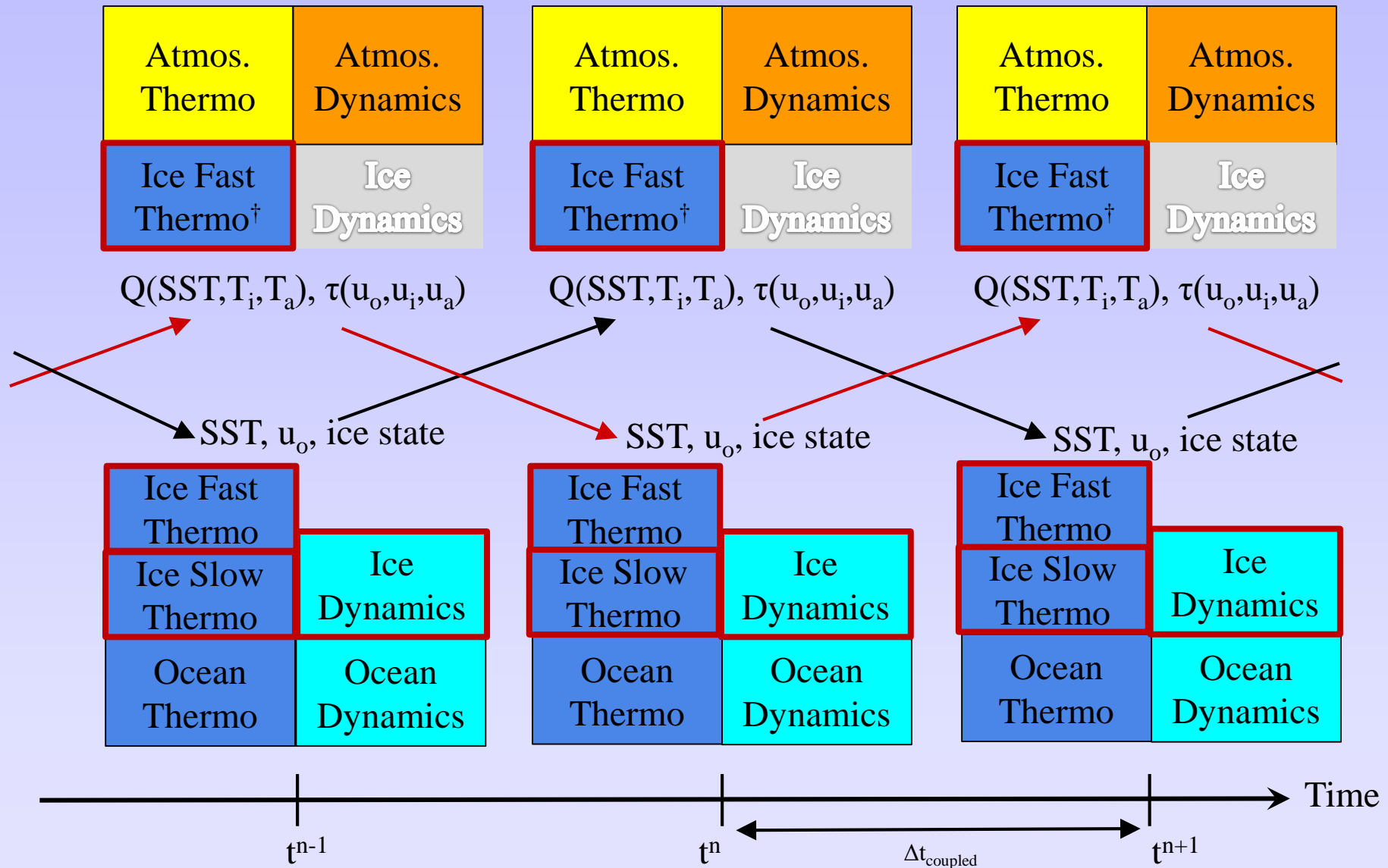
The (SIS2) sea-ice is being embedded in MOM6, while the atmosphere interacts with its own estimate of the sea ice state.

AMIP runs are effectively unchanged!

- Atmosphere calculates air-sea and air-ice fluxes implicitly (as before), but based on an ice-surface state provided by the slow-ice / ocean PEs
- Fast fluxes are conservatively recalculated to update the slow ice state.
 - Fluxes to ice categories are based on ice state and atmospheric boundary layer
 - Fluxes to the ocean are corrected to match the total fluxes found by the atmosphere
- Slow ice thermodynamics are tightly coupled with ocean thermodynamics
- Tight coupling (cycling or embedding) of ice and ocean dynamics
- Sea ice and icebergs dynamically participate in the ocean's barotropic solver with embedding – no gravity wave instability
- Ice-ocean dynamic and thermodynamic coupling can be implicit on both sides, allowing grounding of icebergs and sea ice – NO LEVITATION!
- Ice shelf and tabular iceberg thermodynamics treated equivalently
- Icebergs can interact with the ocean over their full depth range
- Add ~1 m “mud-layer” to avoid thermal instabilities during wetting & drying



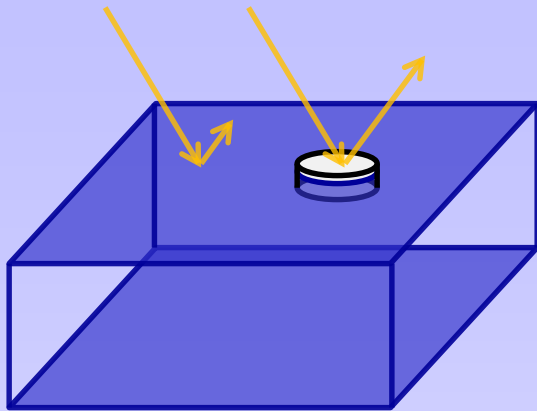
Concurrent/Embedded Ice Coupling



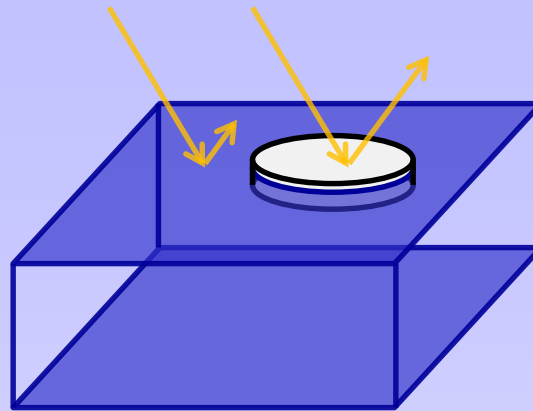


Conservatively Recalculating Solar Heating

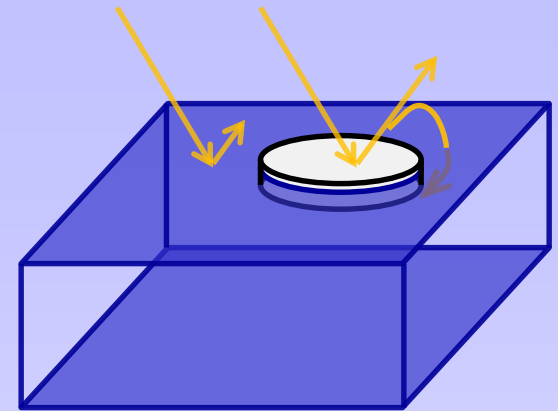
Increasing sea-ice area or albedo → Apply excess reflected shortwave to ocean



Previous ice state

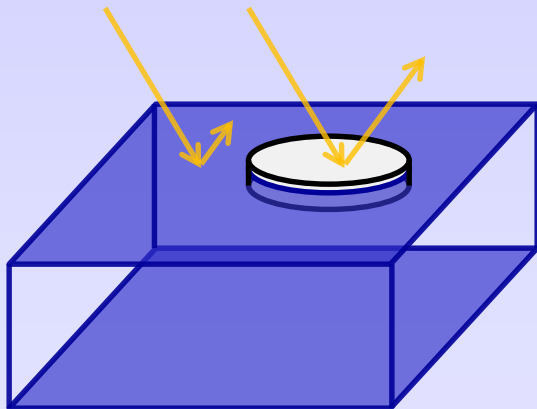


Current ice state

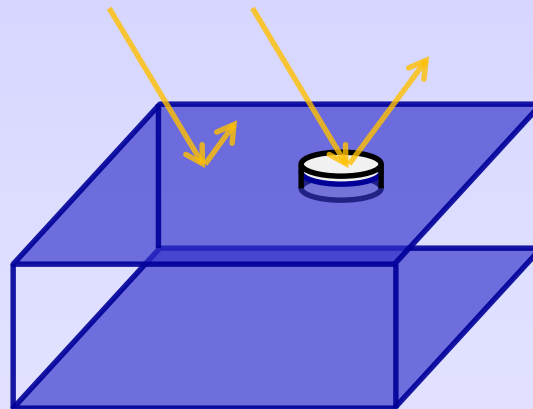


Shortwave applied to current ice state

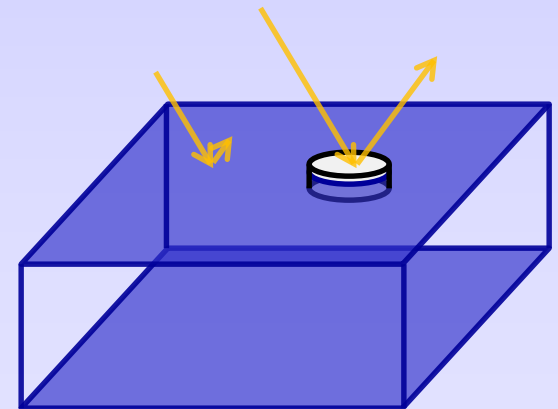
Decreasing ice area or albedo → Reduce incident shortwave to ocean



Previous ice state



Current ice state



Shortwave applied to current ice state



Stable and Quasi-Conservative Thermal Coupling:

$$\frac{\theta_1^{n+1} - \theta_1^n}{\Delta t} = -\frac{\lambda}{H_1} (\theta_1^{n+1} - \tilde{\theta}_2^{n+1}) + \frac{\lambda}{H_1} (\theta_2^n - \tilde{\theta}_2^n)$$

$$\frac{\theta_2^{n+1} - \theta_2^n}{\Delta t} = +\frac{\lambda}{H_2} (\tilde{\theta}_1^{n+1} - \theta_2^{n+1}) - \frac{\lambda}{H_2} (\theta_1^n - \tilde{\theta}_1^n)$$

$\frac{\lambda}{H_1} (\theta_2^n - \tilde{\theta}_2^n)$ and $\frac{\lambda}{H_2} (\theta_1^n - \tilde{\theta}_1^n)$ Correct for last step's flux mismatch.

$\tilde{\theta}_1^n = \theta_1^{n-1} \Rightarrow$ *Quartic Eigenvalue Equation* Conditionally stable.

$\tilde{\theta}_1^n$ Implicit Estimate \Rightarrow No (linear) correction terms; Linearly stable.

- **With only a single component, this is simply implicit flux calculation.**
- **Essentially a linearized variant of the “fast-physics” implicit coupling between the land/ice and atmosphere.**
- **Atmosphere and ice/ocean could each calculate air-ocean/ice fluxes**
- **Conservation is lagged, analogous to concurrent coupling**



Considerations in Revising Coupling

- To correct coupling problems, seek verisimilitude before palliative approximations
- Base coupling algorithms on understanding the dynamics of the coupled system
- Defy disciplinary component boundaries as necessary
- Respect tradition and social harmony, but not to the point of compromising the dynamics
- Algorithm changes primarily for computational efficiency need to be carefully analyzed, especially in extreme situations



Consequences of Embedded / Concurrent Ice Coupling

- Dramatic revisions to sea-ice code structure
 - Separate sea ice model into 4 distinct pieces, while also permitting the sea ice to be used as a single component (Done for SIS2, not Icepack?)
 - Revise of sea-ice code for consistency with ocean code to permit embedding ice dynamics in ocean (Done for SIS2 and MOM6)
- Reformulate coupler for new call sequence options
 - Partially complete/underway for GFDL coupler
- Separation of dynamic and thermodynamic interfaces to ocean
 - Also retaining extant interfaces and solutions
 - Partially complete/underway for MOM6
- To embed: incorporate ice dynamics solver into ocean model
 - Dramatic changes to ocean & ice dynamic cores, while preserving the option to generate existing solutions and behavior
 - Open questions about how to actually handle transport interactions
 - Not started yet for MOM6/SIS2/icebergs

