Concurrent Ice & Embedded Ice Coupling: A Solution to Address the Numerical Stability of Ice/Ocean Coupling

Robert Hallberg

NOAA/GFDL and Princeton AOS/CICS

Objectives of Climate Model Couplers

- Manage Complexity
  - Separate the climate system into disciplinary components
  - Interchange different component models with minimal changes to other components

Objectives of Climate Model Couplers

- Manage Complexity
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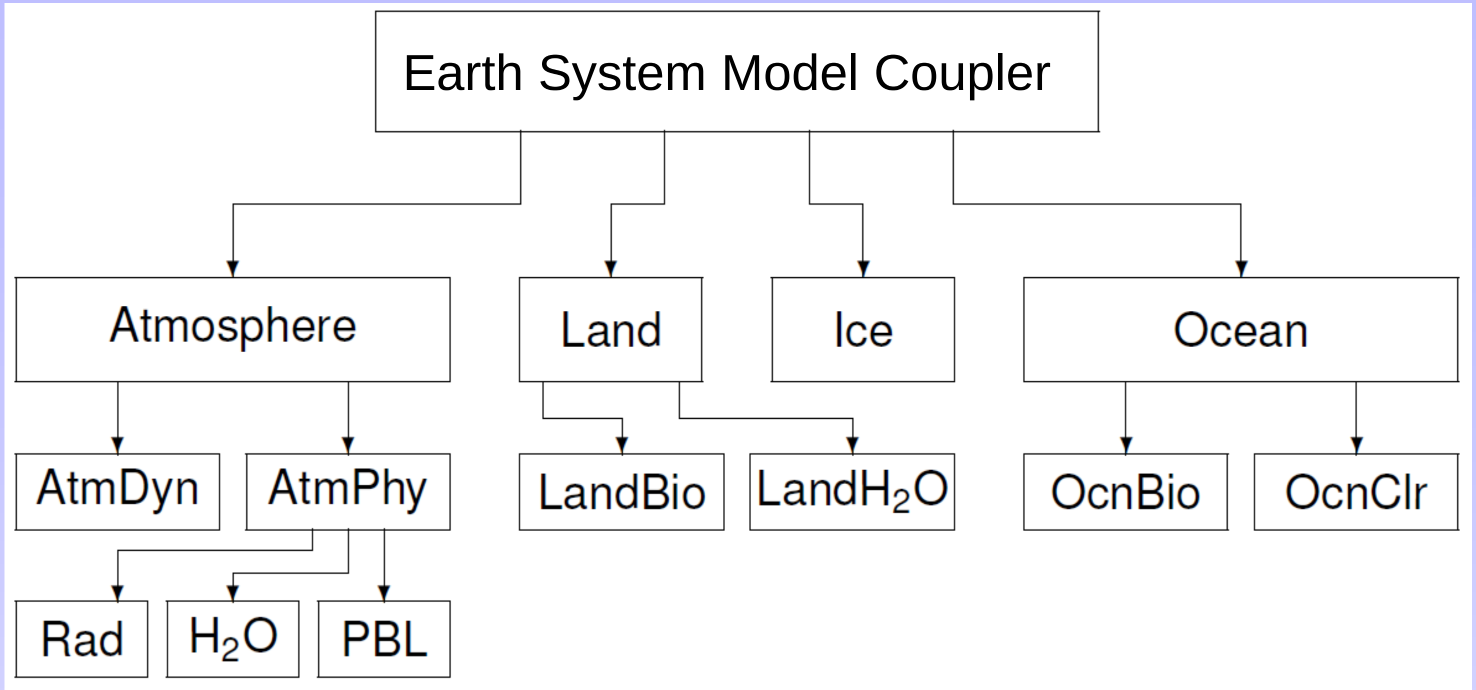
- Achieve Social Harmony
  - "Good fences make good neighbors"



Earth System Model Coupler

Atmosphere

Land

Ice

Ocean

AtmDyn

AtmPhy

LandBio

LandH_2O

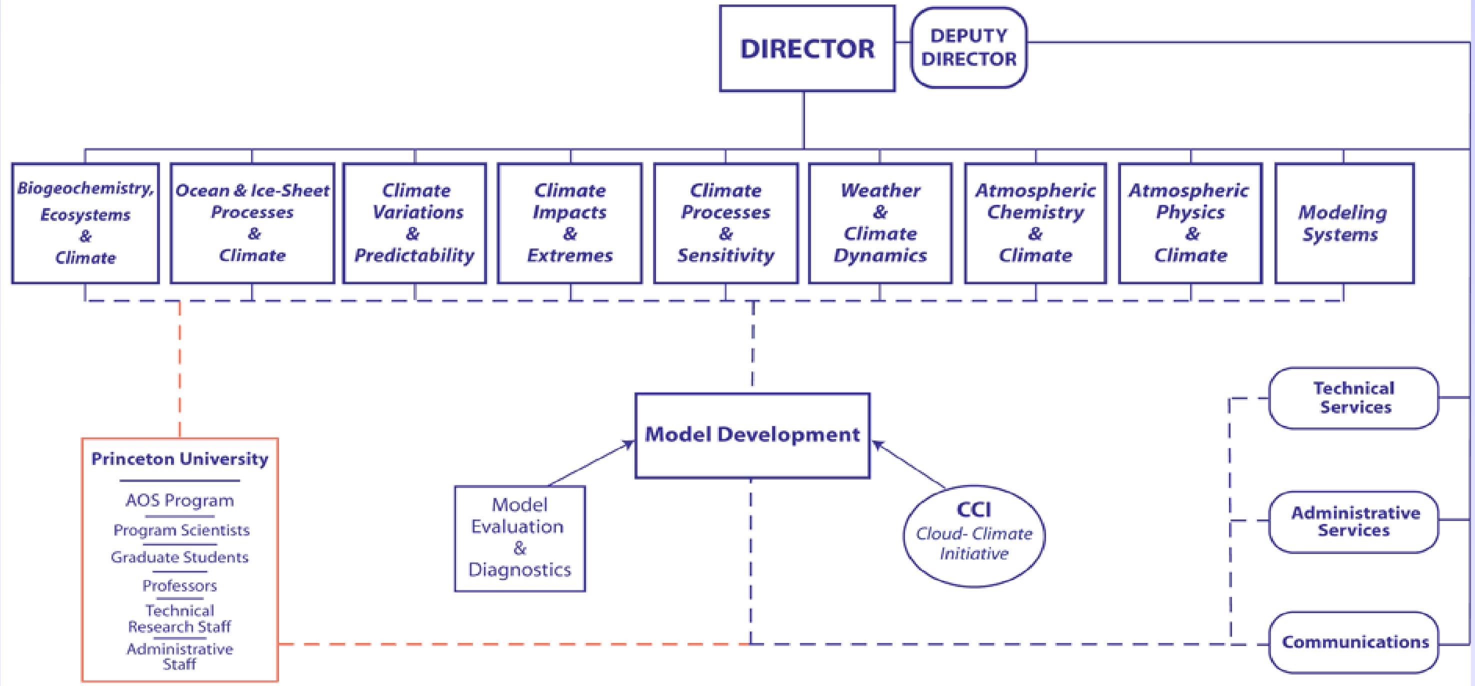
OcnBio

OcnClr

Rad

H_2O

PBL



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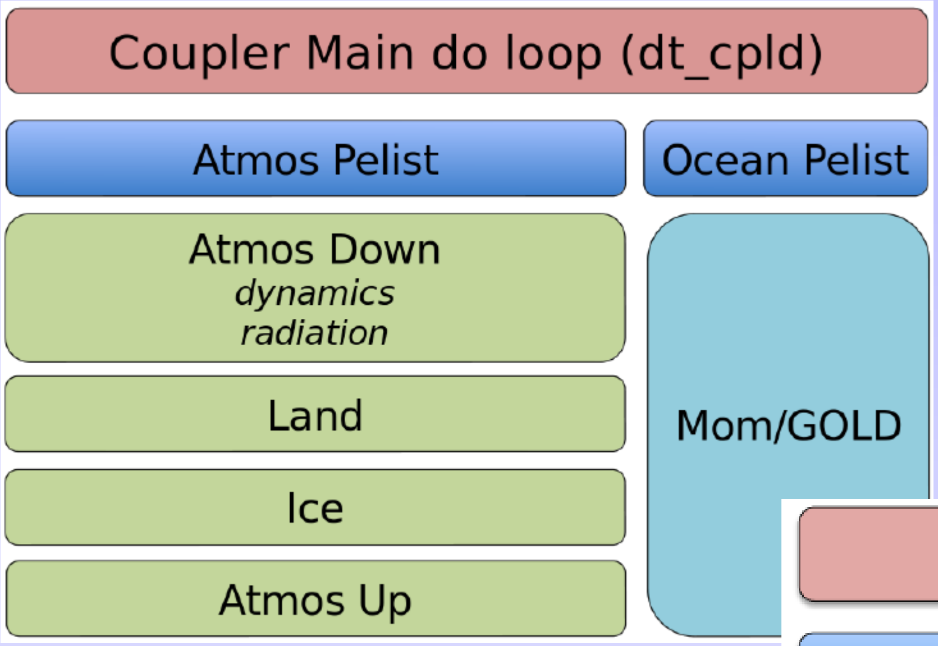
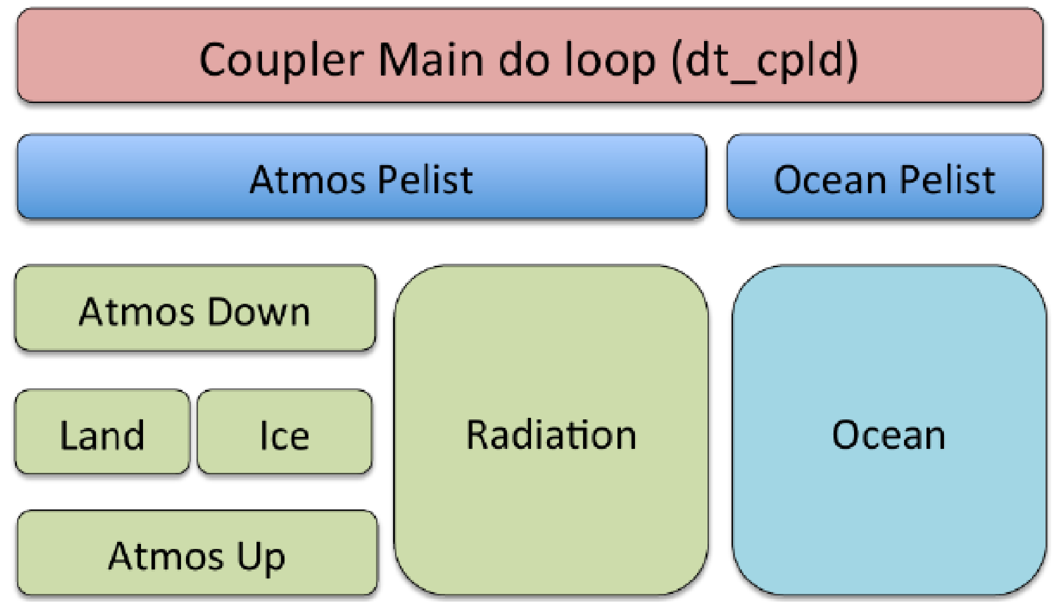
  - “Good fences make good neighbors”
- Computational Efficiency
  - Find concurrency - “Many hands make light work”

Seeking Greater Concurrency





Images courtesy R. Benson & V. Balaji

Objectives of Climate Model Couplers

- Manage Complexity
  - Separate the climate system into disciplinary components
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- Achieve Social Harmony
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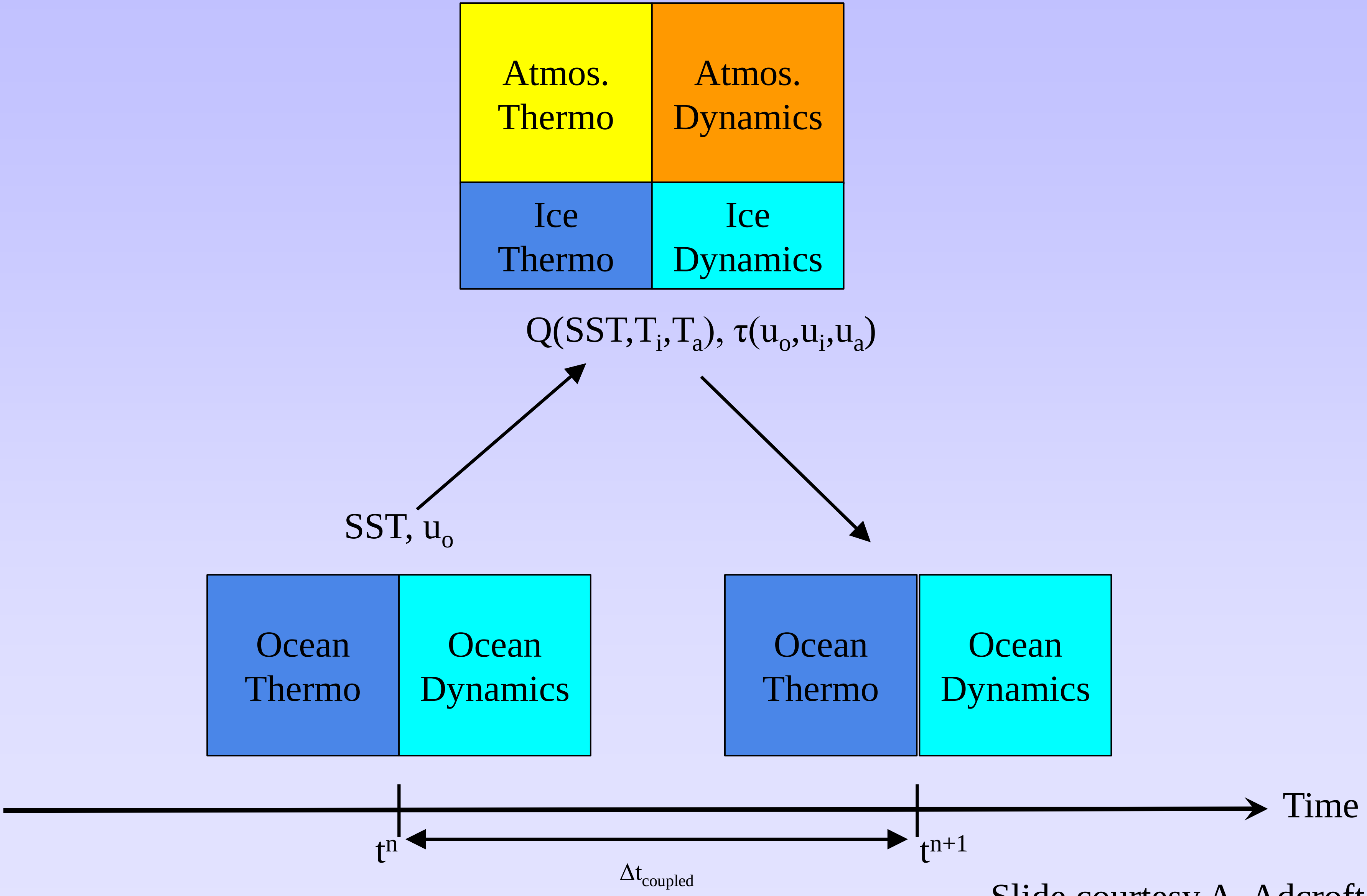
- Computational Efficiency
  - Find concurrency - “Many hands make light work”
- Achieve physically correct behavior of the coupled system dynamics
  - Avoid coupled instabilities

Key Coupler Considerations:

COUPLING TIME-STEPPING STRATEGIES

Sequential Coupling



Atmos. Thermo

Atmos. Dynamics

Ice Thermo

Ice Dynamics

$Q(SST,T_i,T_a)$, $\tau(u_o,u_i,u_a)$

SST, u_o

Ocean Thermo

Ocean Dynamics

Ocean Thermo

Ocean Dynamics

Time

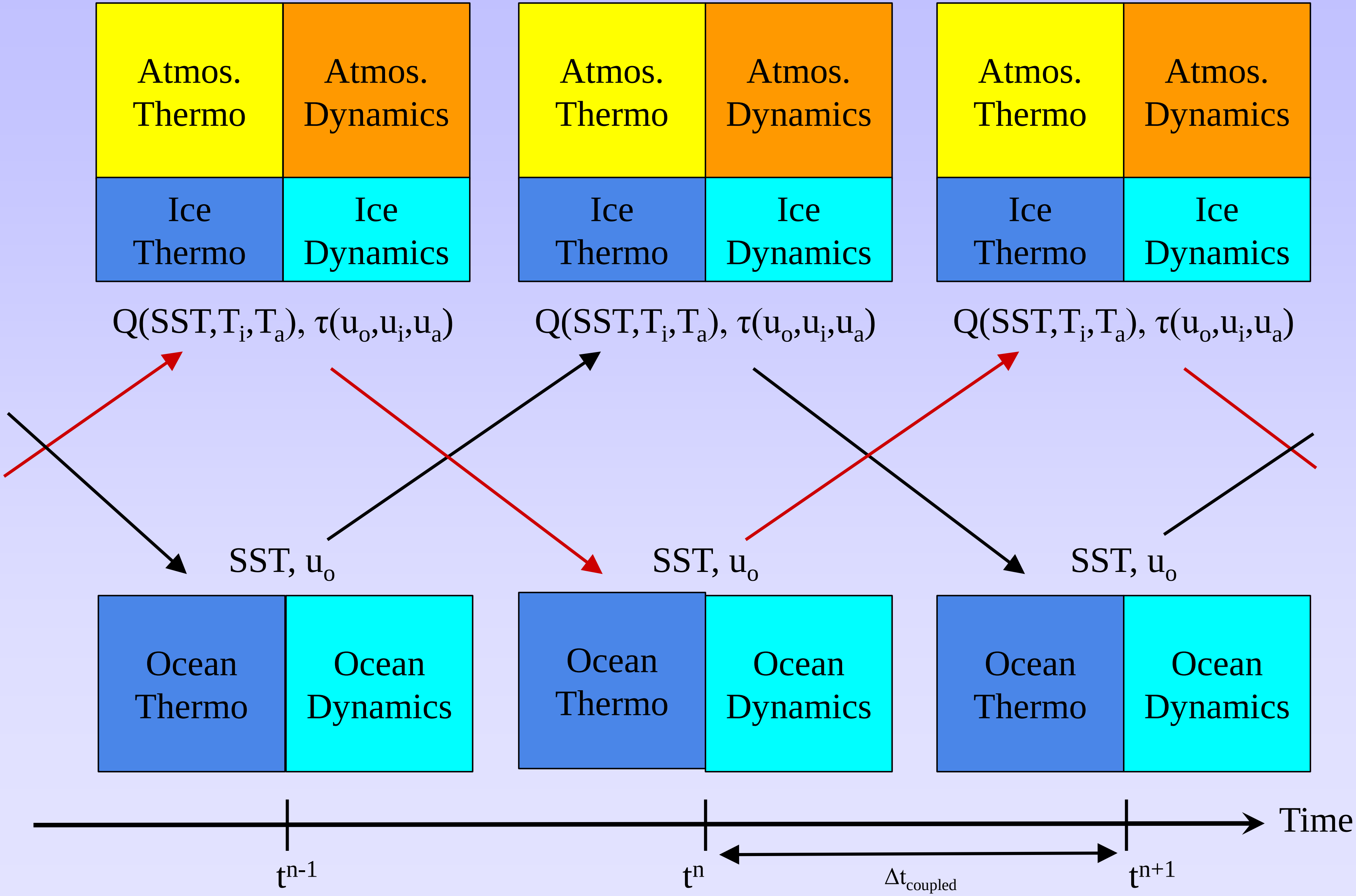
t^n

t^{n+1}

$\Delta t_{coupled}$

Slide courtesy A. Adcroft

Concurrent Coupling



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$Q(SST,T_i,T_a)$, $\tau(u_o,u_i,u_a)$

SST, u_o

SST, u_o

SST, u_o

Ocean Thermo

Ocean Dynamics

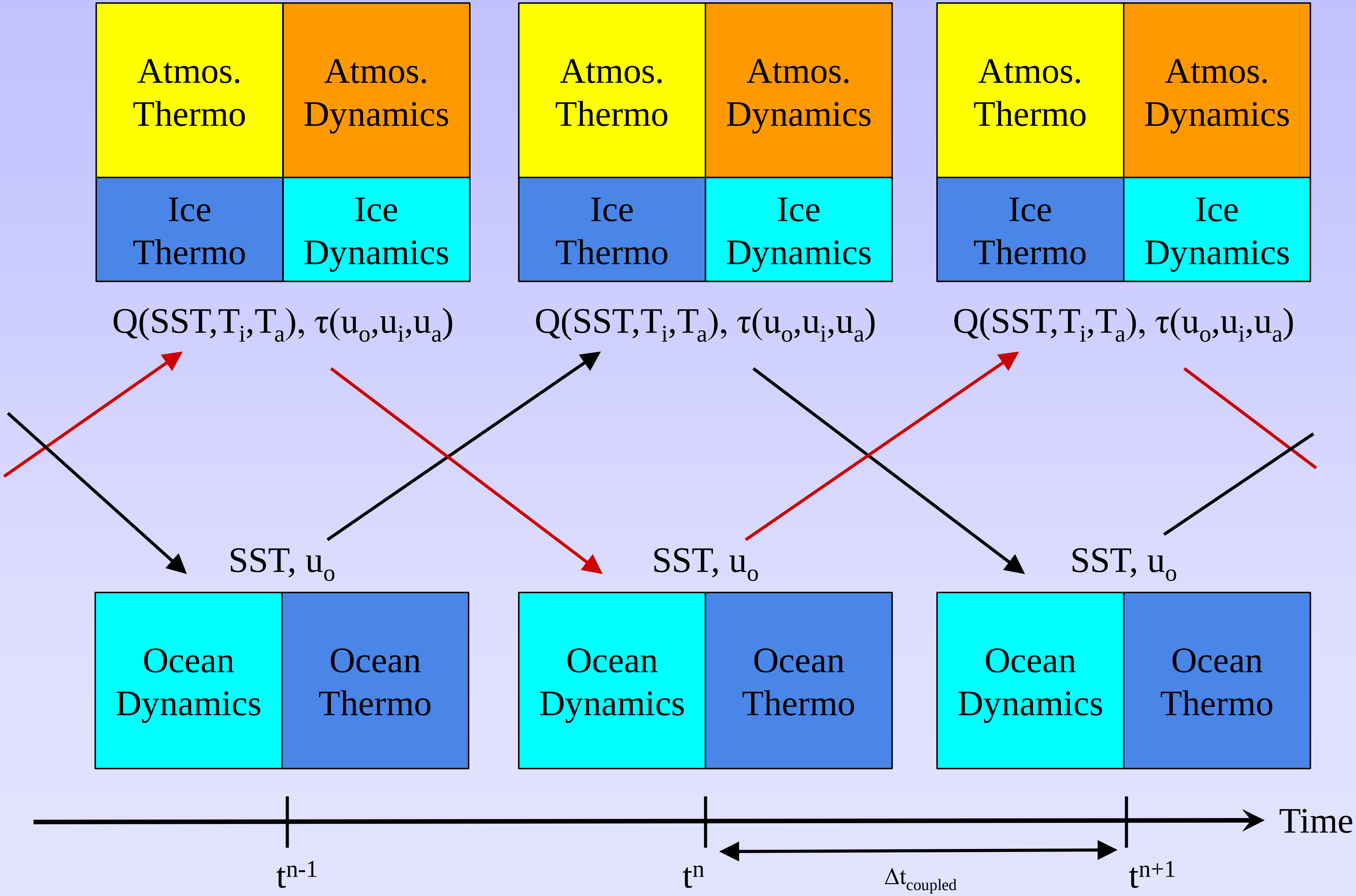
Ocean Thermo

Ocean Dynamics

Ocean Thermo

Ocean Dynamics

Concurrent Coupling



Atmos. Thermo
Atmos. Dynamics
Ice Thermo
Ice Dynamics

Atmos. Thermo
Atmos. Dynamics
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$Q(SST,T_i,T_a), \tau(u_o,u_i,u_a)$ $Q(SST,T_i,T_a), \tau(u_o,u_i,u_a)$ $Q(SST,T_i,T_a), \tau(u_o,u_i,u_a)$

SST, u_o SST, u_o SST, u_o

Ocean Dynamics
Ocean Thermo

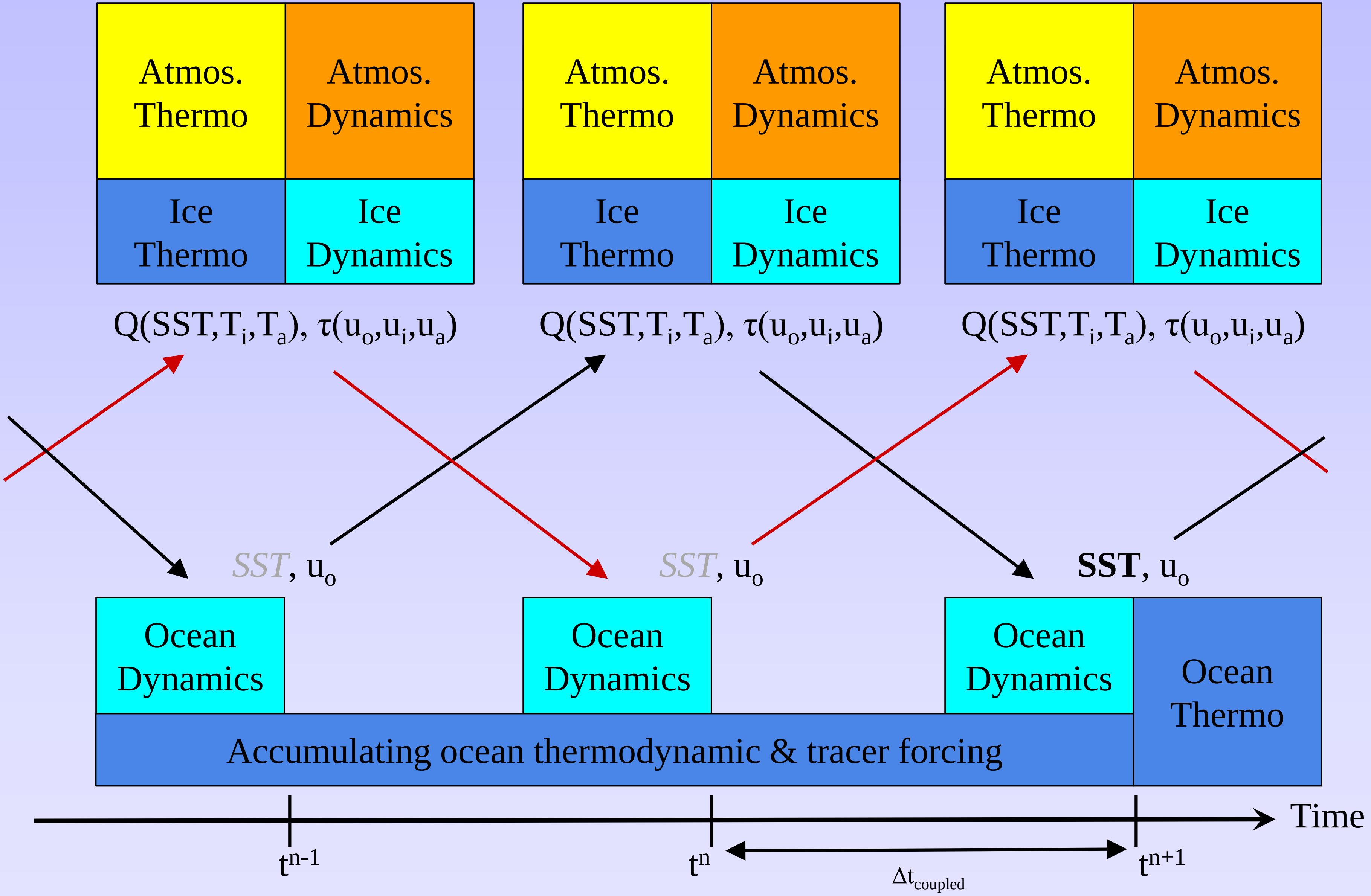
Ocean Dynamics
Ocean Thermo

Ocean Dynamics
Ocean Thermo

Time

t^{n-1} t^{n} $\Delta t_{coupled}$ t^{n+1}

Concurrent Coupling with MOM6



Atmos. Thermo | Atmos. Dynamics | Atmos. Thermo | Atmos. Dynamics | Atmos. Thermo | Atmos. Dynamics

Ice Thermo | Ice Dynamics | Ice Thermo | Ice Dynamics | Ice Thermo | Ice Dynamics

$Q(SST,T_i,T_a), \tau(u_o,u_i,u_a)$

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$Q(SST,T_i,T_a), \tau(u_o,u_i,u_a)$

SST, u_o

SST, u_o

SST, u_o

Ocean Dynamics | Ocean Dynamics | Ocean Dynamics | Ocean Thermo

Accumulating ocean thermodynamic & tracer forcing

Time

t^{n-1}

t^n

$\Delta t_{coupled}$

t^{n+1}

A simplified history of sea-ice ocean coupling

- Rigid lid ocean models could not handle divergent flows or mass loss or gain at the surface (1970s).
  Problem – sea-ice grows by taking fresh water from the ocean

  Solution – use a virtual salt flux to get the equivalent brine rejection

  $$F_{Salt} = -SF_{Water}$$

  Advantages – Massless sea ice does not exert pressure on the ocean or participate in dynamics; Sea ice can be treated as a completely independent component.

  Liabilities – Freezing & melting at different S give inconsistent forcing
- Free surface ocean models allowed climate models to return to the “natural boundary condition” (~2000).
  - Z-coordinate models still require limits on ice pressure: $P_{Ice} < O(0.5) g \rho_{Oce} \Delta z_{Sfc}$
  - Artificial Stommel-Goldsborough circulation results where the pressure-limited ice melts; sea-ice grounding is not permitted.
- Z*-coordinates & other ocean model developments allow for increasingly realistic sea-ice models… (Today)

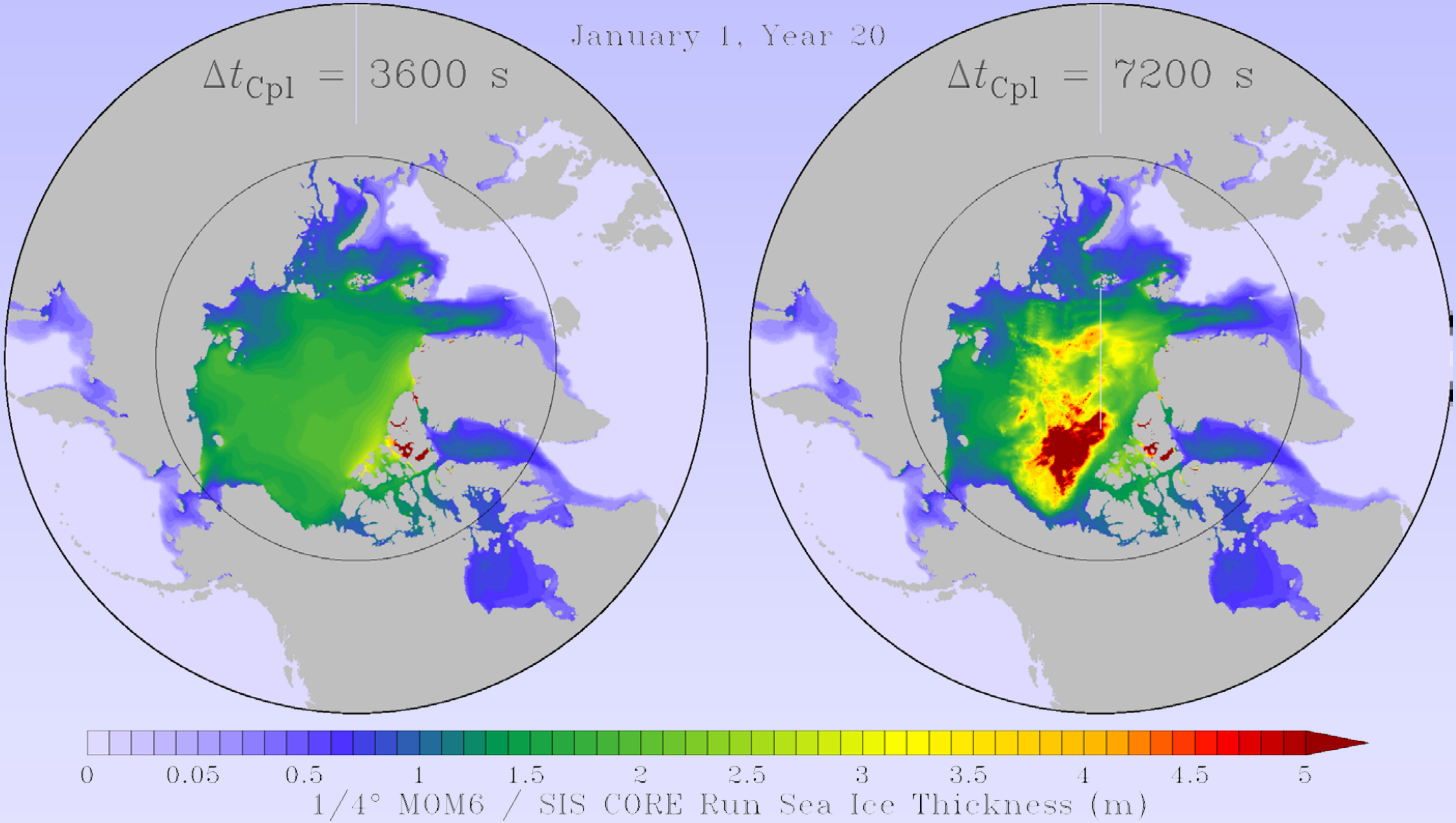
Traditional (GFDL) Approach to Ocean/Ice Coupling

- Sea-ice (SIS or SIS2) is advanced implicitly with the atmosphere, for skin temperatures consistent with atmosphere.
- Ocean (MOM4, MOM5, GOLD or MOM6) is forced by prescribed fluxes from the sea-ice.
- Air-sea fluxes are based on ocean properties from 1 (sequential) or 2 (concurrent) time-steps before they are applied to the ocean.
- Ice displacement is similarly lagged.
- Icebergs are point masses embedded in the sea-ice.
- Ice can displace a limited thickness of ocean; more than ~2-5 m of ice “levitates” to avoid numerical problems.

Ocean model (MOM6), sea-ice (SIS2), icebergs, and GFDL coupler are all being restructured to allow this approach to be revised.

These revisions may provide a template for consideration in CESM.

Evidence of Lagged Stress-Inertial Coupling Instability in Sea-Ice Thickness



Sequentially coupled data-driven ice-ocean model

Hallberg (2014, *Clivar Exchanges*)

Symptoms of problems with GFDL's traditional coupling approach

- Numerical instability of high resolution coupled models, especially in Spring when thick sea-ice becomes unlocked from the pack of thin ice.
- Avoiding "surfing" icebergs and marginal sea-ice requires "levitation" of the ice
- "Levitation" in turn introduces undesirable consequences
  - Icebergs and sea-ice can not ground
  - Unlimited growth of sea-ice (to 1000s of m) in certain embayments
  - No dynamic ice-sheet coupling, or else tabular icebergs must be treated differently from ice-shelves
- Short coupling time-step required at higher resolutions
  - E.g., 1200 s for GFDL's ¼° CM4 with concurrent coupling

Numerical Ice-Ocean Coupling Instabilities

1. Lagged stress / inertial oscillation instability

$$u' = u - u_{Steady}$$

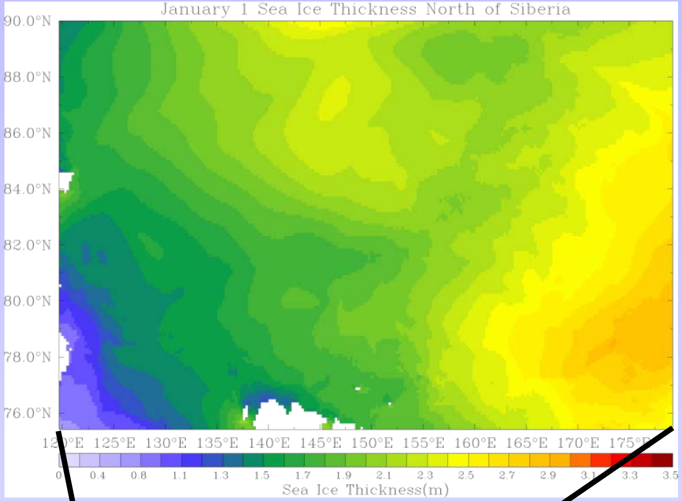
$$\frac{\partial u}{\partial t} + ifu = \frac{c_d U}{H}\left(u_{Atm} - u^n\right)$$

$$u'\left(t^{n+1}\right) = \left[e^{-if\Delta t} + i\frac{c_d U}{Hf}\left(1 - e^{-if\Delta t}\right)\right]u'\left(t^n\right) = Au'\left(t^n\right)$$

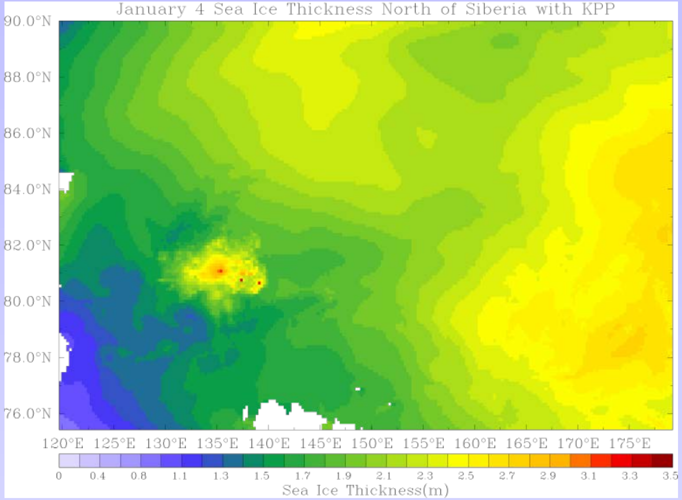
$$\|A\|^2 = 1 - 2\frac{c_d U}{Hf}\sin\left(f\Delta t\right) + 2\left(\frac{c_d U}{Hf}\right)^2\left(1 - \cos\left(f\Delta t\right)\right)$$

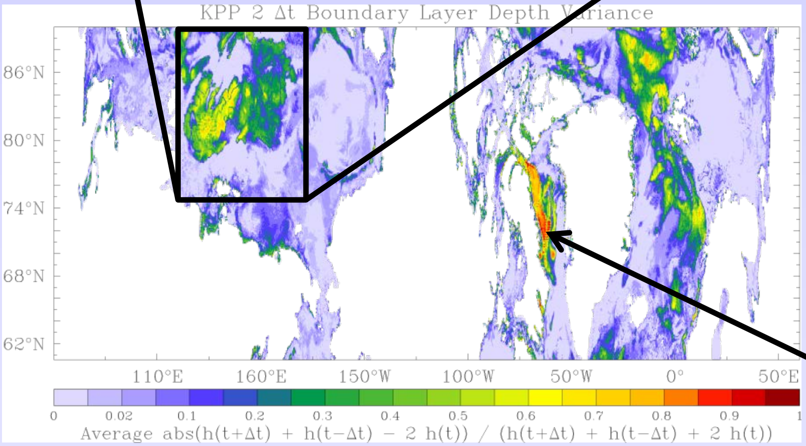
Explosive Sea-Ice Growth as a Manifestation of a Sea Ice-Ocean Coupling Instability

Jan. 1 Sea Ice Thickness of Siberia

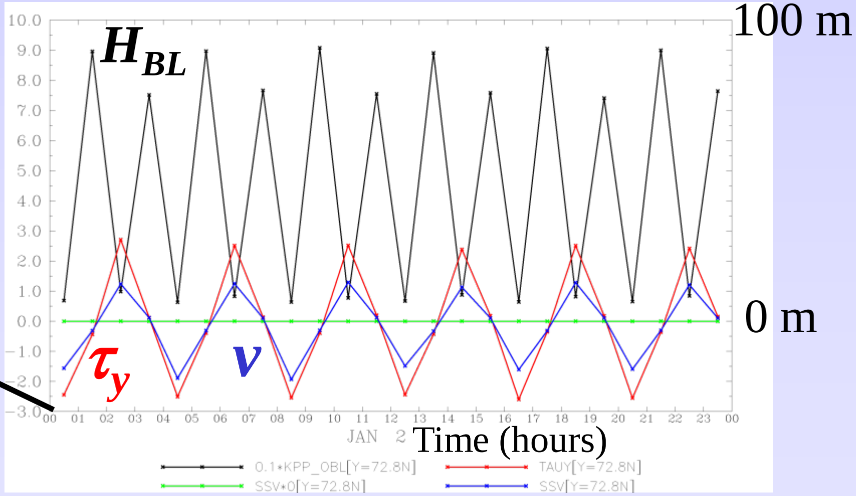


Jan. 4 Sea Ice Thickness of Siberia





$$\overline{\left|2H_t - H_{t-\Delta t} - H_{t+\Delta t}\right| / \left(2H_t + H_{t-\Delta t} + H_{t+\Delta t}\right)}$$



H = Ocean boundary layer depth from KPP; determined from initial bulk Ri consideration.

Numerical Ice-Ocean Coupling Instabilities

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$$\left\|A\right\|^2 = 1 - 2\frac{c_d U}{Hf}\sin\left(f\Delta t\right) + 2\left(\frac{c_d U}{Hf}\right)^2\left(1 - \cos\left(f\Delta t\right)\right)$$

2. Thermal forcing instability

$$\frac{\partial \theta_1}{\partial t} = -\frac{\lambda}{H_1}\left(\theta_1 - \theta_2\right) \qquad \frac{\theta_1^{n+1} - \theta_1^n}{\Delta t} = -\frac{\lambda}{H_1}\left(\theta_1^{n+1} - \theta_2^n\right)$$

$$\frac{\partial \theta_2}{\partial t} = +\frac{\lambda}{H_2}\left(\theta_1 - \theta_2\right) \qquad \frac{\theta_2^{n+1} - \theta_2^n}{\Delta t} = +\frac{\lambda}{H_2}\left(\theta_1^{n+1} - \theta_2^n\right)$$

$Eigenvalues:$

$$A_1 = \frac{1}{1 + \lambda\Delta t / H_1}$$

$$\mathrm{A}_2 = 1 - \lambda\Delta t / H_2$$

3. Gravity wave instability
   - Sea-ice and icebergs participate in barotropic gravity waves
   - Stability analysis analogous to split-explicit ocean time stepping (e.g., Hallberg, 1997)
   - Instability growth rate proportional to the sea-ice external gravity wave CFL ratio based on the ***coupling time step***. $\frac{\sqrt{gH_{Ice}}\Delta T}{\Delta x} < O(1)$

Ice in a Greenland Fjord (Rink Isbrae)

(Photo Credit: R. Hallberg 2015 pretending to be an observationalist.)

A coupled gravity-wave toy model

2-layer (sea-ice & ocean) linear nonrotating flat-bottom channel flow with no viscosity.

$$\frac{\partial u_1}{\partial t} = -g\frac{\partial \eta_{1/2}}{\partial x}$$

$$= -g\frac{\partial}{\partial x}\left(h_1 + h_2\right)$$

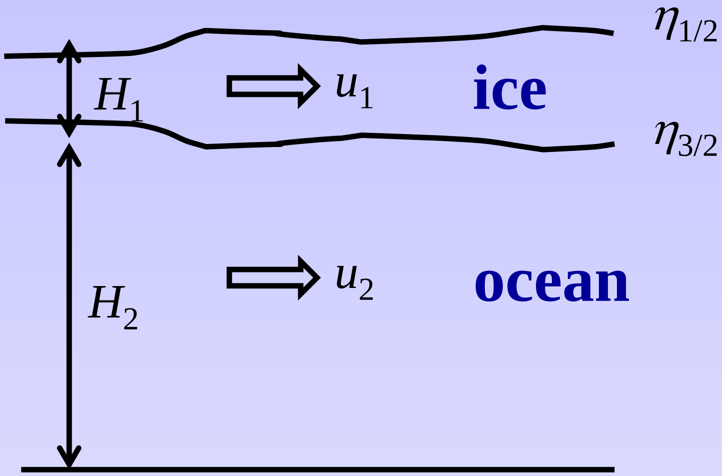
$$\frac{\partial u_2}{\partial t} = -g\frac{\rho_I}{\rho_o}\frac{\partial \eta_{1/2}}{\partial x} - g\frac{\rho_o - \rho_I}{\rho_o}\frac{\partial \eta_{3/2}}{\partial x}$$

$$= -(g - g')\frac{\partial}{\partial x}\left(h_1 + h_2\right) - g'\frac{\partial h_2}{\partial x}$$

$$= -(g - g')\frac{\partial h_1}{\partial x} - g\frac{\partial h_2}{\partial x}$$

$$\frac{\partial h_1}{\partial t} = -H_1\frac{\partial u_1}{\partial x}$$

$$\frac{\partial h_2}{\partial t} = -H_2\frac{\partial u_2}{\partial x}$$



A coupled gravity-wave toy model

Sequential coupling of gravity waves only:

$$\frac{\partial h_1}{\partial t} = -H_1 \frac{\partial u_1}{\partial x} \qquad \frac{\partial u_1}{\partial t} = -g\frac{\partial h_1}{\partial x} - g\frac{\partial h_2^n}{\partial x}$$

$$\frac{\partial h_2}{\partial t} = -H_2 \frac{\partial u_2}{\partial x} \qquad \frac{\partial u_2}{\partial t} = -g\frac{\partial h_2}{\partial x} - (g-g')\frac{\partial h_1^{n+1}}{\partial x}$$

Sequential coupling:

Marginally stable if waves are treated analytically in each component.

$$\omega_1 \equiv \sqrt{gH_1}k \quad ; \quad \omega_2 \equiv \sqrt{gH_2}k$$

$$0 \le \omega_2 \Delta T <\sim 100$$

Concurrent (forward) coupling:

$$\frac{\partial h_1}{\partial t} = -H_1 \frac{\partial u_1}{\partial x} \qquad \frac{\partial u_1}{\partial t} = -g\frac{\partial h_1}{\partial x} - g\frac{\partial h_2^n}{\partial x}$$

$$\frac{\partial h_2}{\partial t} = -H_2 \frac{\partial u_2}{\partial x} \qquad \frac{\partial u_2}{\partial t} = -g\frac{\partial h_2}{\partial x} - (g-g')\frac{\partial h_1^{n}}{\partial x}$$

Concurrent forward coupling:

Unconditionally unstable, growth rate:

$$\approx \frac{(g-g')}{g\Delta T}\left[1-\cos(\omega_1 \Delta T)\right]\left[1-\cos(\omega_2 \Delta T)\right]$$

Sequential (filtered) coupling:

$$\frac{\partial h_2}{\partial t} = -H_2 \frac{\partial u_2}{\partial x} \qquad \frac{\partial u_2}{\partial t} = -g\frac{\partial h_2}{\partial x} - (g-g')\frac{\partial h_1^{n}}{\partial x}$$

$$\frac{\partial h_1}{\partial t} = -H_1 \frac{\partial u_1}{\partial x} \qquad \frac{\partial u_1}{\partial t} = -g\frac{\partial h_1}{\partial x} - g\frac{\partial}{\partial x}\left(\frac{1}{\Delta T}\int_0^{\Delta T} h_2 dt\right)$$

Sequential filtered coupling:

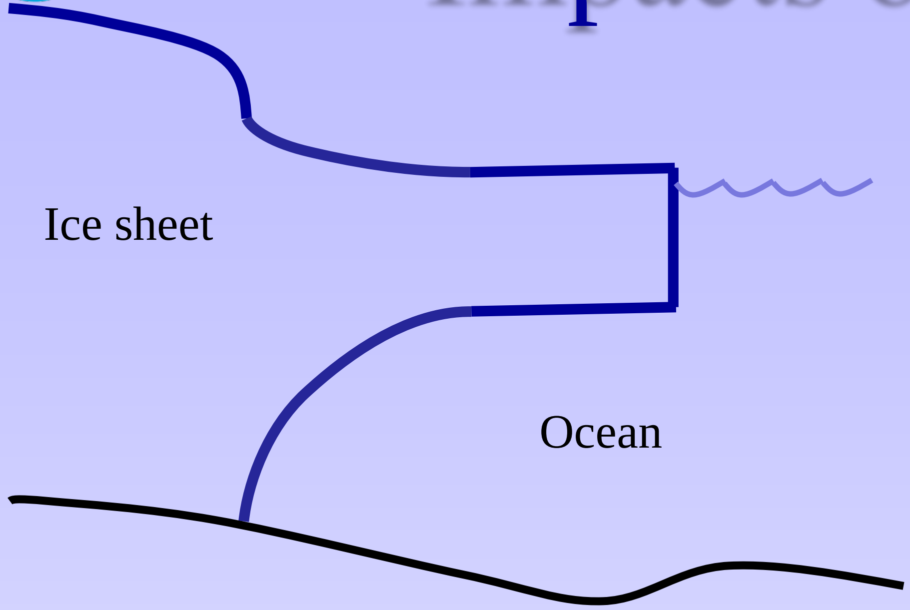
Unconditionally unstable, growth rate:

$\approx \frac{1}{2}$ Concurrent growth rate for small $\omega_2 \Delta T$

$\propto \dfrac{1}{\omega_2 \Delta T}$, for large $\omega_2 \Delta T$

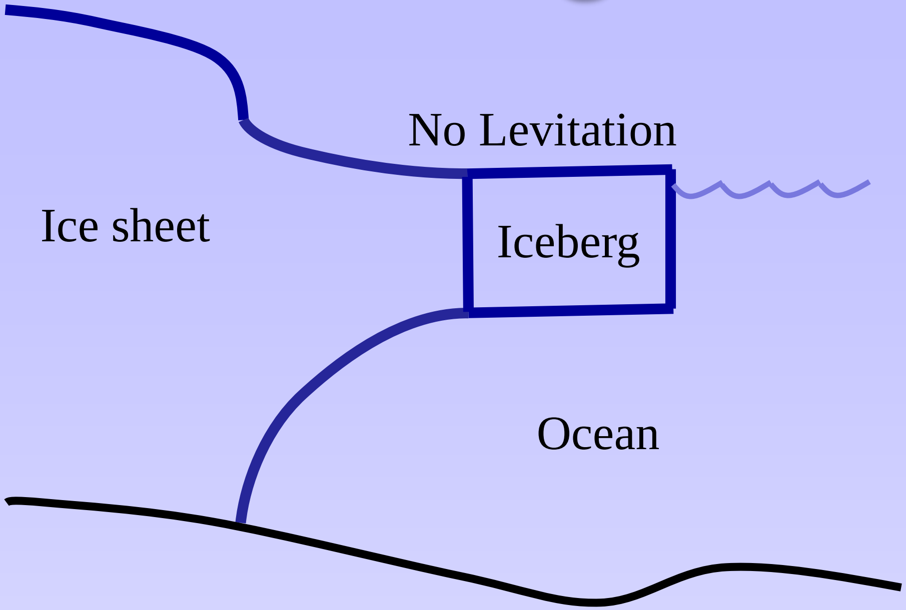
Damping from an ice-pack can locally stabilize the instability.

Impacts of "Levitating" Ice



Ice sheet

Ocean

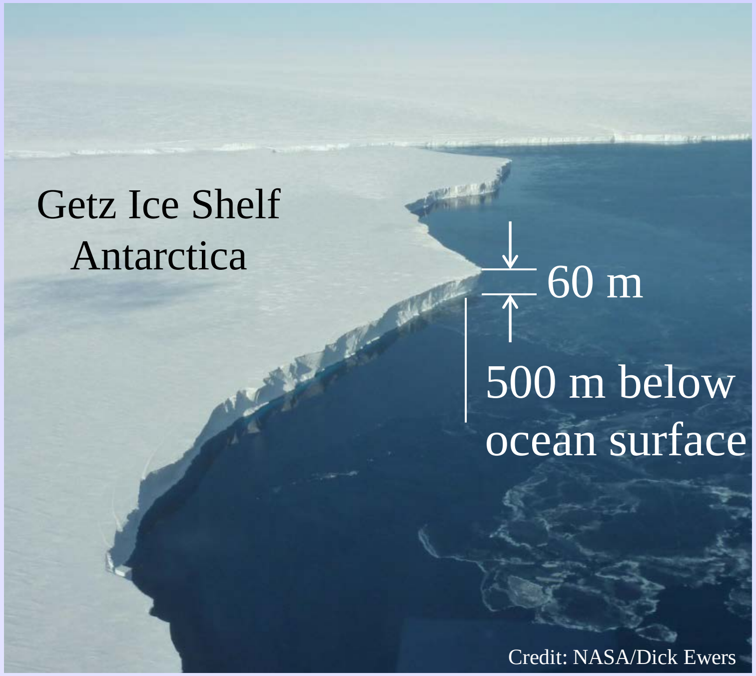


No Levitation

Ice sheet

Iceberg

Ocean

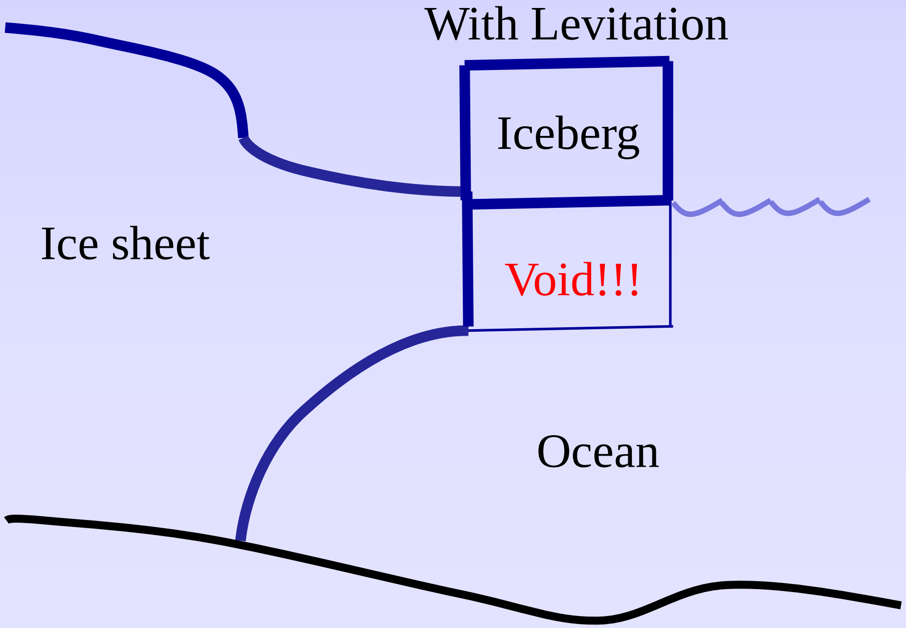


Getz Ice Shelf
Antarctica

60 m

500 m below
ocean surface

Credit: NASA/Dick Ewers



With Levitation

Iceberg

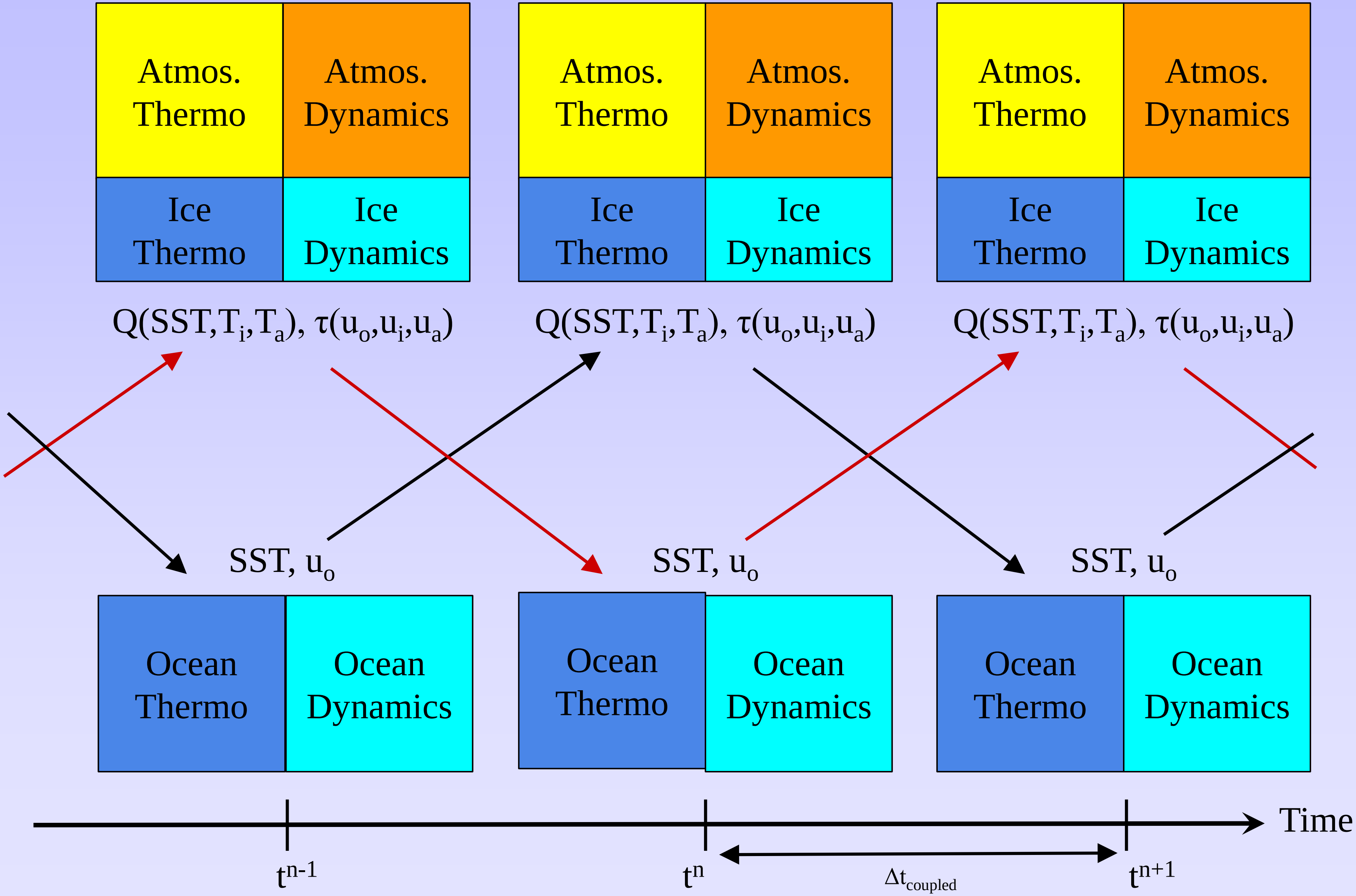
Ice sheet

Void!!!

Ocean

A NEW ICE / OCEAN COUPING STRATEGY

Concurrent Coupling



Atmos. Thermo | Atmos. Dynamics | Atmos. Thermo | Atmos. Dynamics | Atmos. Thermo | Atmos. Dynamics

Ice Thermo | Ice Dynamics | Ice Thermo | Ice Dynamics | Ice Thermo | Ice Dynamics

$Q(SST, T_i, T_a), \tau(u_o, u_i, u_a)$ $Q(SST, T_i, T_a), \tau(u_o, u_i, u_a)$ $Q(SST, T_i, T_a), \tau(u_o, u_i, u_a)$

SST, u_o SST, u_o SST, u_o

Ocean Thermo | Ocean Dynamics | Ocean Thermo | Ocean Dynamics | Ocean Thermo | Ocean Dynamics

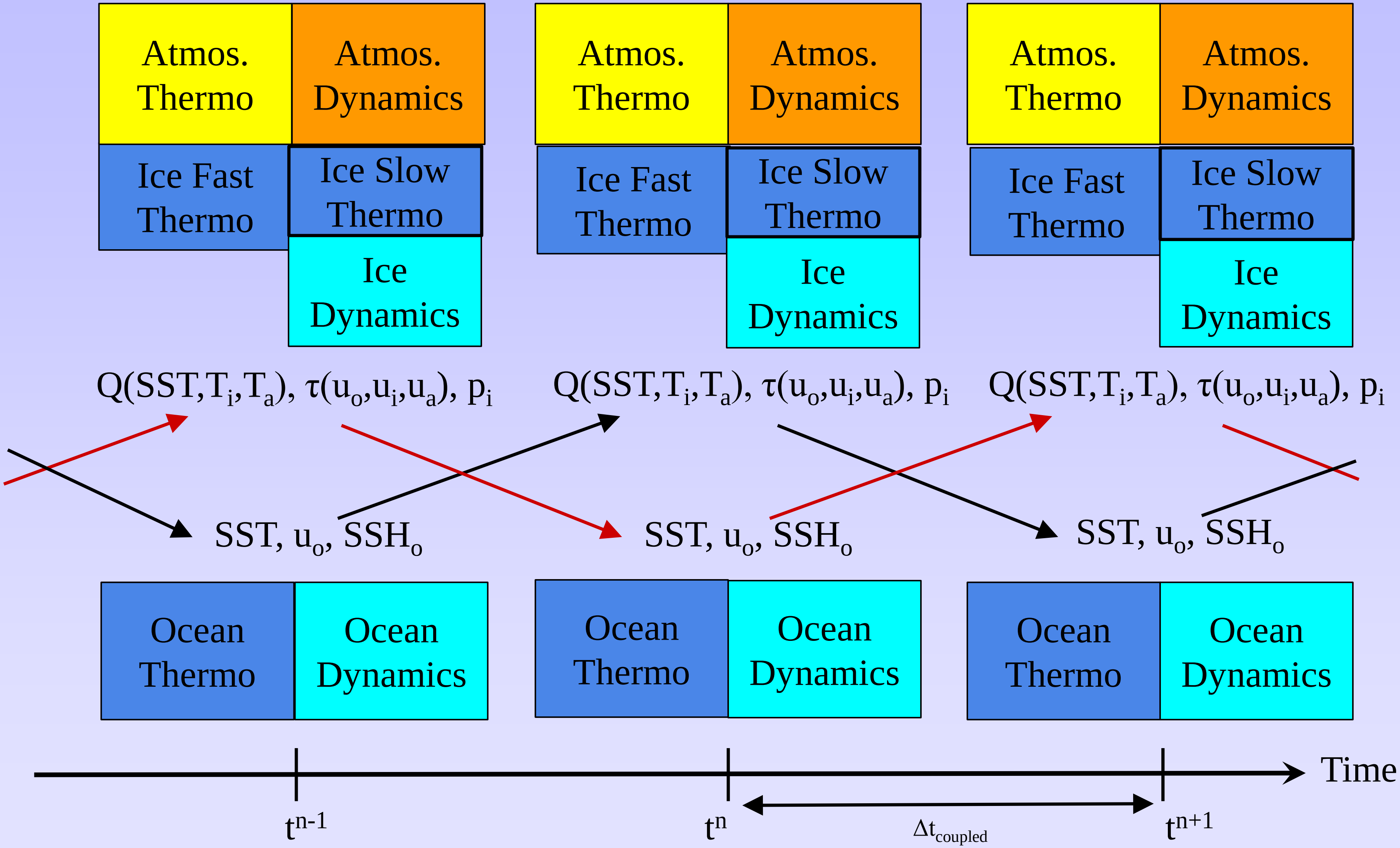
Time

t^{n-1} t^{n} $\Delta t_{coupled}$ t^{n+1}

A Subcomponent Decomposition of Sea-ice Processes

- Fast thermal processes (almost immediate)
  - Surface skin temperature calculation
  - Determines atmospheric boundary layer stability
- Slow thermodynamic processes (hours to years)
  - Melting, Freezing
  - Ice salinity changes
- Dynamics and Rheology (minutes to days)
  - Ice-pack stress fields and momentum budget
- Transport and ridging (hours to days)

Concurrent Coupling in more detail



Atmos. Thermo

Atmos. Dynamics

Ice Fast Thermo

Ice Slow Thermo

Ice Dynamics

Atmos. Thermo

Atmos. Dynamics

Ice Fast Thermo

Ice Slow Thermo

Ice Dynamics

Atmos. Thermo

Atmos. Dynamics

Ice Fast Thermo

Ice Slow Thermo

Ice Dynamics

$Q(SST,T_i,T_a)$, $\tau(u_o,u_i,u_a)$, p_i $Q(SST,T_i,T_a)$, $\tau(u_o,u_i,u_a)$, p_i $Q(SST,T_i,T_a)$, $\tau(u_o,u_i,u_a)$, p_i

SST, u_o, SSH_o SST, u_o, SSH_o SST, u_o, SSH_o

Ocean Thermo

Ocean Dynamics

Ocean Thermo

Ocean Dynamics

Ocean Thermo

Ocean Dynamics

Time

t^{n-1} t^n $\Delta t_{coupled}$ t^{n+1}

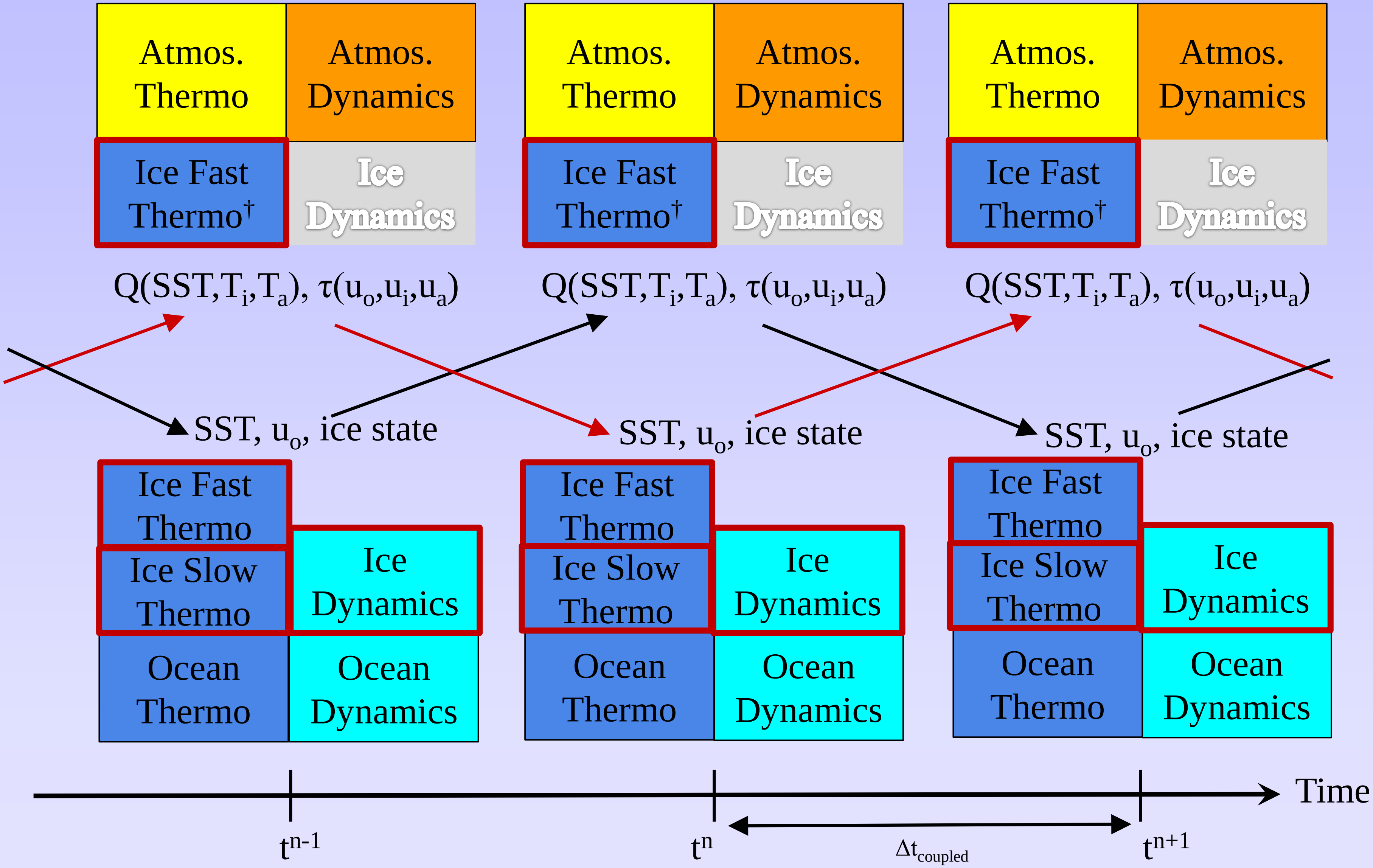
A solution to the ice-ocean coupling issues?

The (SIS2) sea-ice is being embedded in MOM6, while the atmosphere interacts with its own estimate of the sea ice state.

AMIP runs are effectively unchanged!

- Atmosphere calculates air-sea and air-ice fluxes implicitly (as before), but based on an ice-surface state provided by the slow-ice / ocean PEs
- Fast fluxes are conservatively recalculated to update the slow ice state.
  - Fluxes to ice categories are based on ice state and atmospheric boundary layer
  - Fluxes to the ocean are corrected to match the total fluxes found by the atmosphere
- Slow ice thermodynamics are tightly coupled with ocean thermodynamics
- Tight coupling (cycling or embedding) of ice and ocean dynamics
- Sea ice and icebergs dynamically participate in the ocean’s barotropic solver with embedding – no gravity wave instability
- Ice-ocean dynamic and thermodynamic coupling can be implicit on both sides, allowing grounding of icebergs and sea ice – NO LEVITATION!
- Ice shelf and tabular iceberg thermodynamics treated equivalently
- Icebergs can interact with the ocean over their full depth range
- Add ~1 m “mud-layer” to avoid thermal instabilities during wetting & drying

Concurrent/Embedded Ice Coupling



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$Q(SST,T_i,T_a)$, $\tau(u_o,u_i,u_a)$

$Q(SST,T_i,T_a)$, $\tau(u_o,u_i,u_a)$

$Q(SST,T_i,T_a)$, $\tau(u_o,u_i,u_a)$

SST, u_o, ice state

SST, u_o, ice state

SST, u_o, ice state

Ice Fast Thermo
Ice Slow Thermo | Ice Dynamics
Ocean Thermo | Ocean Dynamics

Ice Fast Thermo
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Ocean Thermo | Ocean Dynamics

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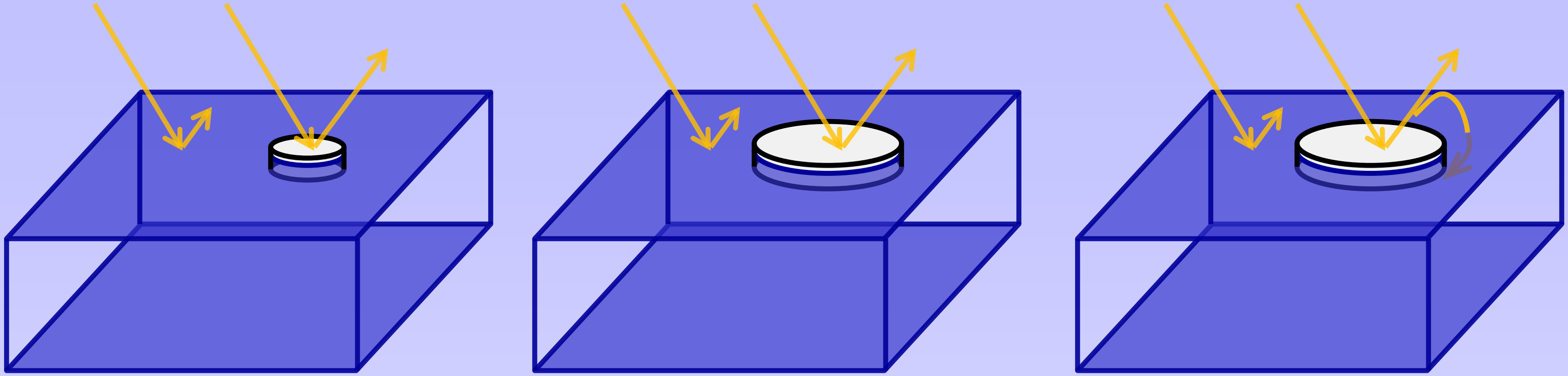
Time

t^{n-1} t^{n} $\Delta t_{coupled}$ t^{n+1}

Conservatively Recalculating Solar Heating

Increasing sea-ice area or albedo ➔ Apply excess reflected shortwave to ocean

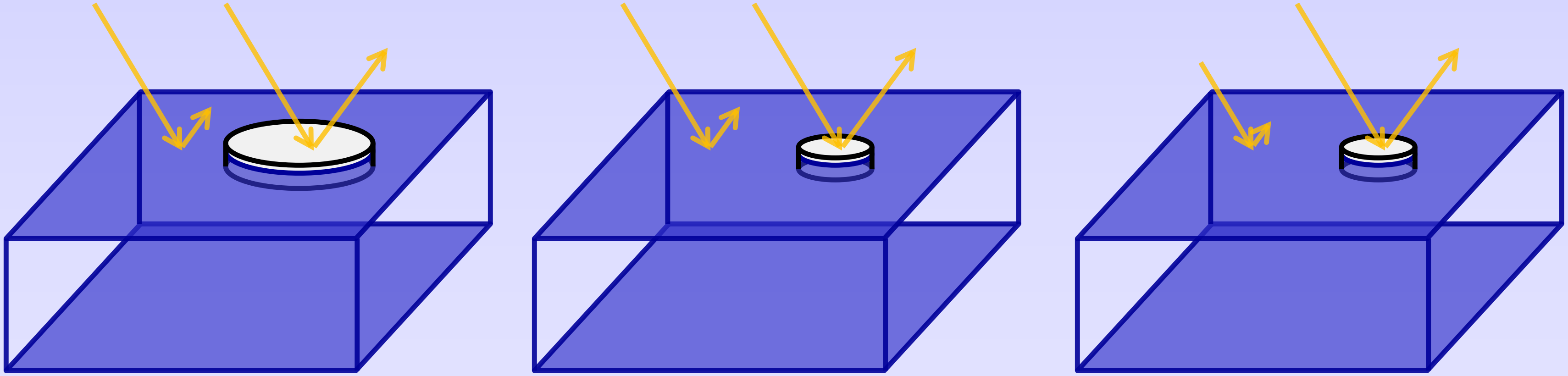


Previous ice state

Current ice state

Shortwave applied to
current ice state

Decreasing ice area or albedo ➔ Reduce incident shortwave to ocean



Previous ice state

Current ice state

Shortwave applied to
current ice state

Stable and Quasi-Conservative Thermal Coupling:

$$\frac{\theta_1^{n+1}-\theta_1^{n}}{\Delta t} = -\frac{\lambda}{H_1}\left(\theta_1^{n+1}-\tilde{\theta}_2^{n+1}\right)+\frac{\lambda}{H_1}\left(\theta_2^{n}-\tilde{\theta}_2^{n}\right)$$

$$\frac{\theta_2^{n+1}-\theta_2^{n}}{\Delta t} = +\frac{\lambda}{H_2}\left(\tilde{\theta}_1^{n+1}-\theta_2^{n+1}\right)-\frac{\lambda}{H_2}\left(\theta_1^{n}-\tilde{\theta}_1^{n}\right)$$

$\frac{\lambda}{H_1}\left(\theta_2^{n}-\tilde{\theta}_2^{n}\right)$ *and* $\frac{\lambda}{H_2}\left(\theta_1^{n}-\tilde{\theta}_1^{n}\right)$ Correct for last step's flux mismatch.

$\tilde{\theta}_1^{n} = \theta_1^{n-1} \Rightarrow \mathit{Quartic\ Eigenvalue\ Equation}$ Conditionally stable.

$\tilde{\theta}_1^{n}$ Implicit Estimate $\Rightarrow$ No (linear) correction terms; Linearly stable.

- **With only a single component, this is simply implicit flux calculation.**
- **Essentially a linearized variant of the "fast-physics" implicit coupling between the land/ice and atmosphere.**
- **Atmosphere and ice/ocean could each calculate air-ocean/ice fluxes**
- **Conservation is lagged, analogous to concurrent coupling**

Considerations in Revising Coupling

- To correct coupling problems, seek verisimilitude before palliative approximations
- Base coupling algorithms on understanding the dynamics of the coupled system
- Defy disciplinary component boundaries as necessary
- Respect tradition and social harmony, but not to the point of compromising the dynamics
- Algorithm changes primarily for computational efficiency need to be carefully analyzed, especially in extreme situations

Consequences of Embedded / Concurrent Ice Coupling

- Dramatic revisions to sea-ice code structure
  - Separate sea ice model into 4 distinct pieces, while also permitting the sea ice to used as a single component (Done for SIS2, not Icepack?)
  - Revise of sea-ice code for consistency with ocean code to permit embedding ice dynamics in ocean (Done for SIS2 and MOM6)
- Reformulate coupler for new call sequence options
  - Partially complete/underway for GFDL coupler
- Separation of dynamic and thermodynamic interfaces to ocean
  - Also retaining extant interfaces and solutions
  - Partially complete/underway for MOM6
- To embed: incorporate ice dynamics solver into ocean model
  - Dramatic changes to ocean & ice dynamic cores, while preserving the option to generate existing solutions and behavior
  - Open questions about how to actually handle transport interactions
  - Not started yet for MOM6/SIS2/icebergs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