

# BIOMASS PARTITIONING USING AN OPTIMIZATION APPROACH FROM ECONOMIC THEORY

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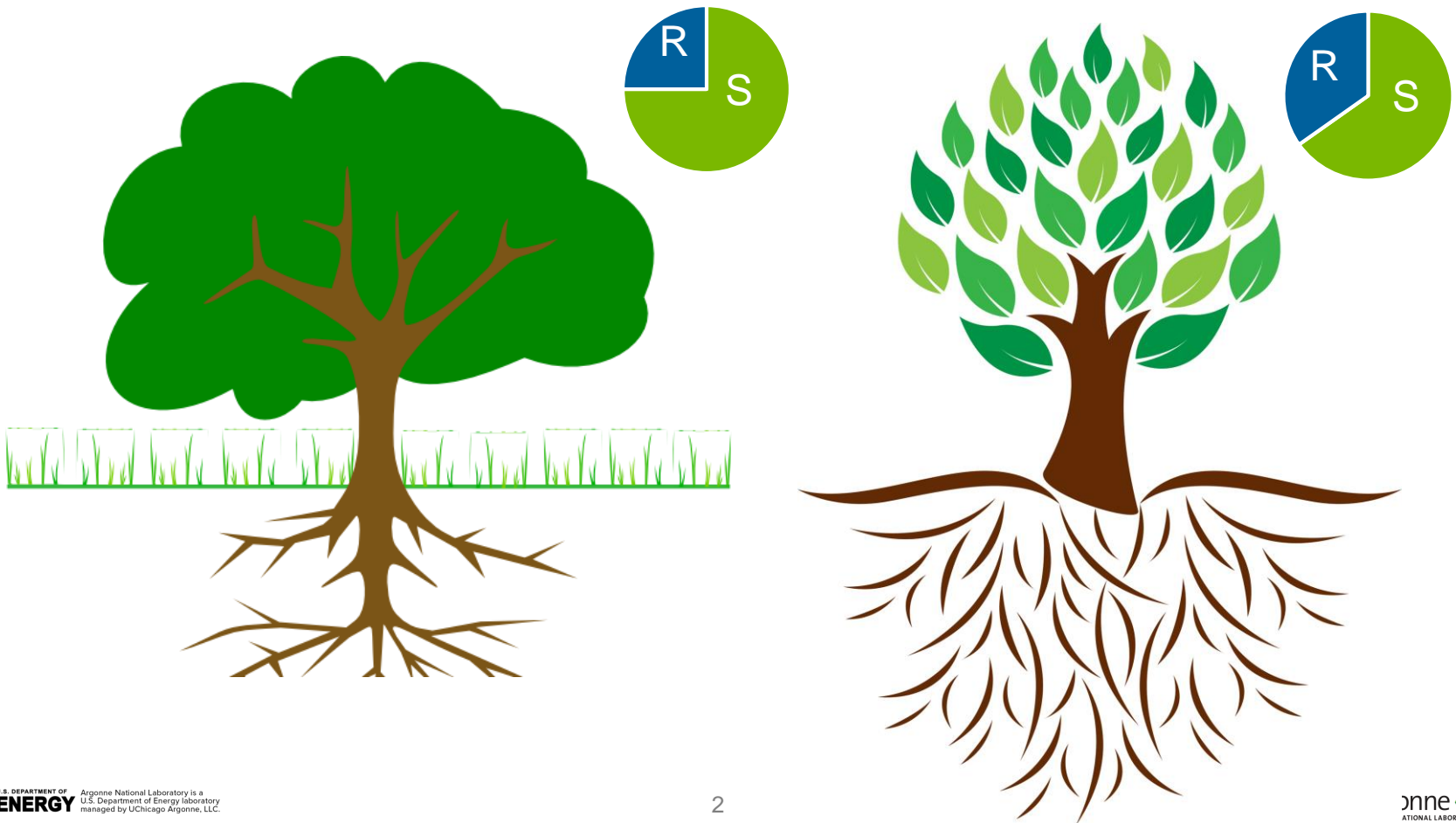
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# MOTIVATION AND BACKGROUND

- Optimal partitioning theory: plants allocate biomass to most limiting resource
- Most LSMs used fixed ratios for biomass partitioning

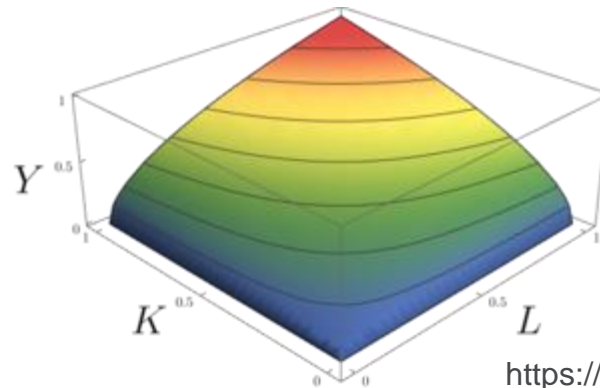


# PARALLELS BETWEEN ECOLOGY AND ECONOMICS

- Bloom et al., 1985:
  - Plants acquire resources when they are cheap and store them for later use
  - Plants produce roots and leaves until they cannot benefit from further growth of that component
  - Plants adjust allocation such that growth limitation is equal for all resources
  - Plants adjust phenology to changes in resources

# COBB-DOUGLAS PRODUCTION FUNCTION

$$Y = K^{\alpha}L^{\beta}$$



[https://en.wikipedia.org/wiki/Cobb%E2%80%93Douglas\\_production\\_function](https://en.wikipedia.org/wiki/Cobb%E2%80%93Douglas_production_function)

- Currency for plants can be carbon, nitrogen, water, etc.
- The Caveat:
  - This is a first order highly simplified approach – a proof-of-concept
  - Two resources: carbon and nitrogen
  - Two plant components: leaves and fine roots
  - Solve for fine root:leaf ratio

# CARBON PARTITIONING WITH COBB-DOUGLAS EQUATIONS

Lynch, 2015

$$P(u_s, u_r) = \pi_C^\alpha \pi_N^\beta \dots \pi_x^\lambda$$

$\alpha$  and  $\beta$  are fixed based on CN ratios in the model  
 $\alpha + \beta = 1$

Goal: Optimize NPP.

Inputs: carbon and nitrogen.

$$\pi_C = H_C(u_s) - c_C(u_s) - c_C(u_r)$$

$\pi$  is the harvest of carbon or nitrogen

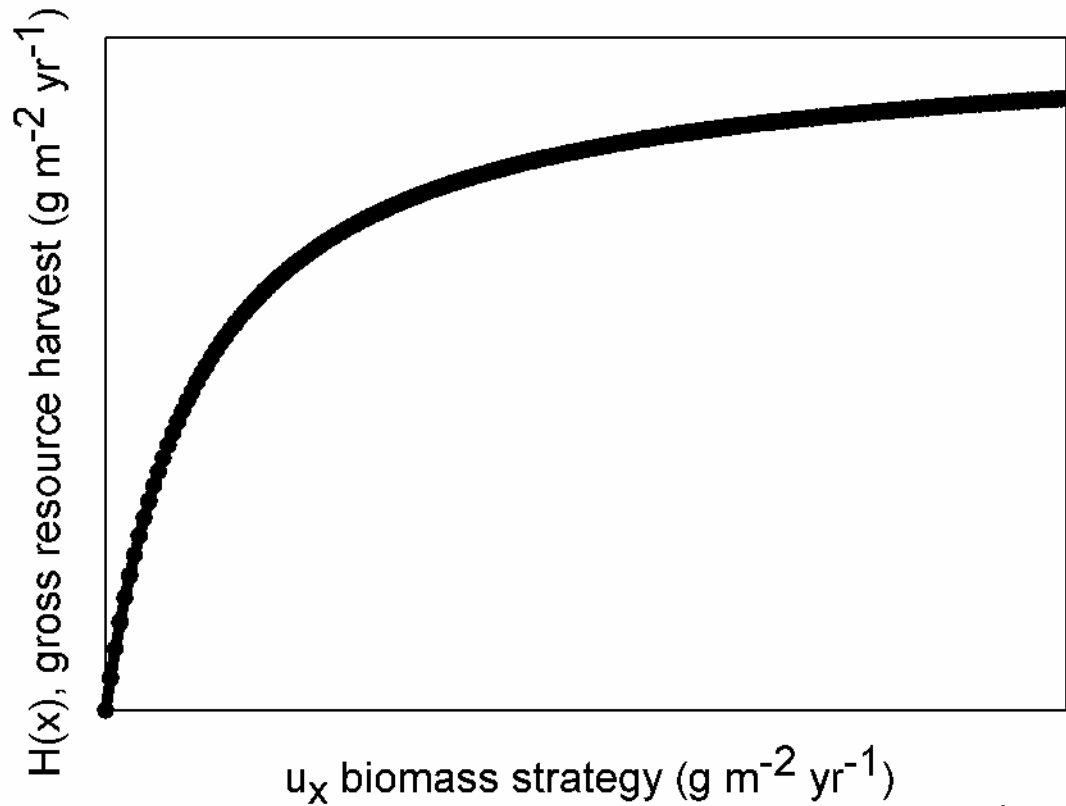
$$\pi_N = \sum_{j=1}^m H_{jN}(u_r) - c_N(u_s) - c_N(u_r)$$

Must follow the law of diminishing returns

$$\frac{\partial P(u_l, u_r)}{\partial u_l} = \frac{\partial P(u_l, u_r)}{\partial u_r} = 0$$

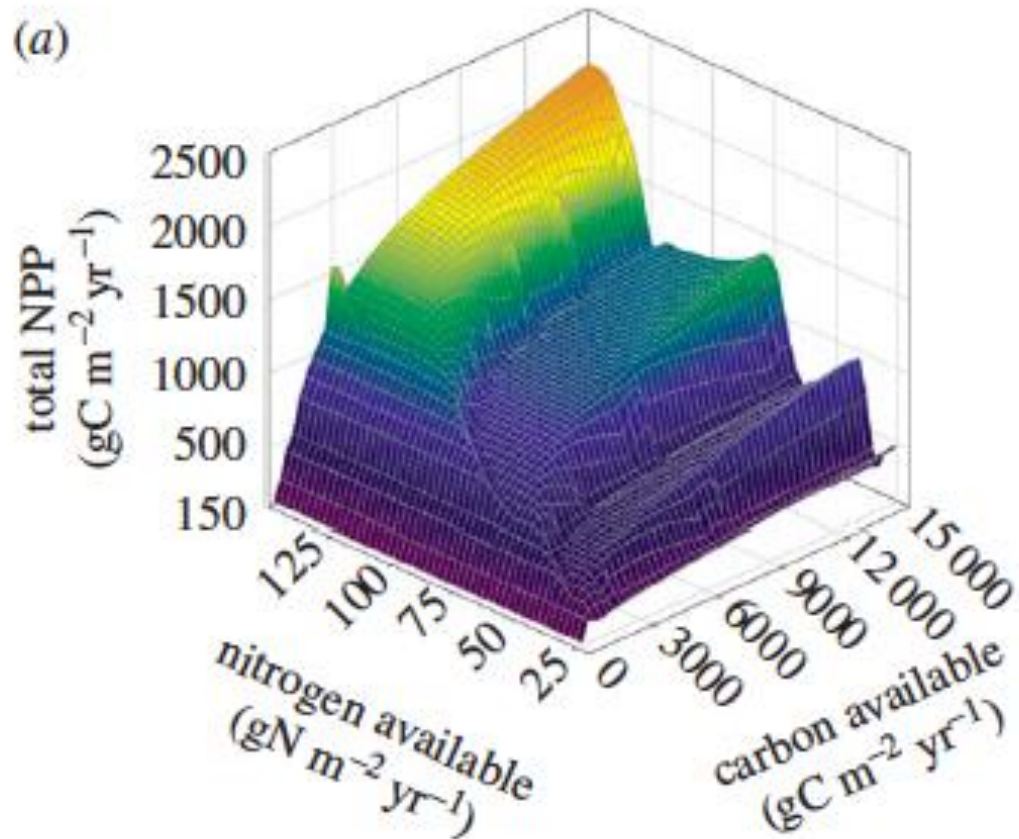
Solver uses Newton-Raphson with a finite difference approximation for the derivative

# EXAMPLE OF SINGLE RESOURCE



Lynch, 2015

# EXAMPLE OF MULTIPLE RESOURCES



McNickel et al., 2016

# DYNAMIC ALLOCATION, UPDATED ANNUALLY (GRASS EXAMPLE)

- $H_c(l) = GPPp_{ot} * (1 - e^{-l})$
- $cc(l) = l * (mr + gr)$
- $cc(fr) = fr * (mr + gr)$
- $H_n(fr) = Nallocp_{ot} * (1 - e^{-fr})$
- $cn(l) = (mr + gr) * (l / leafcn)$
- $cn(fr) = (mr + gr) * (fr / frootcn)$

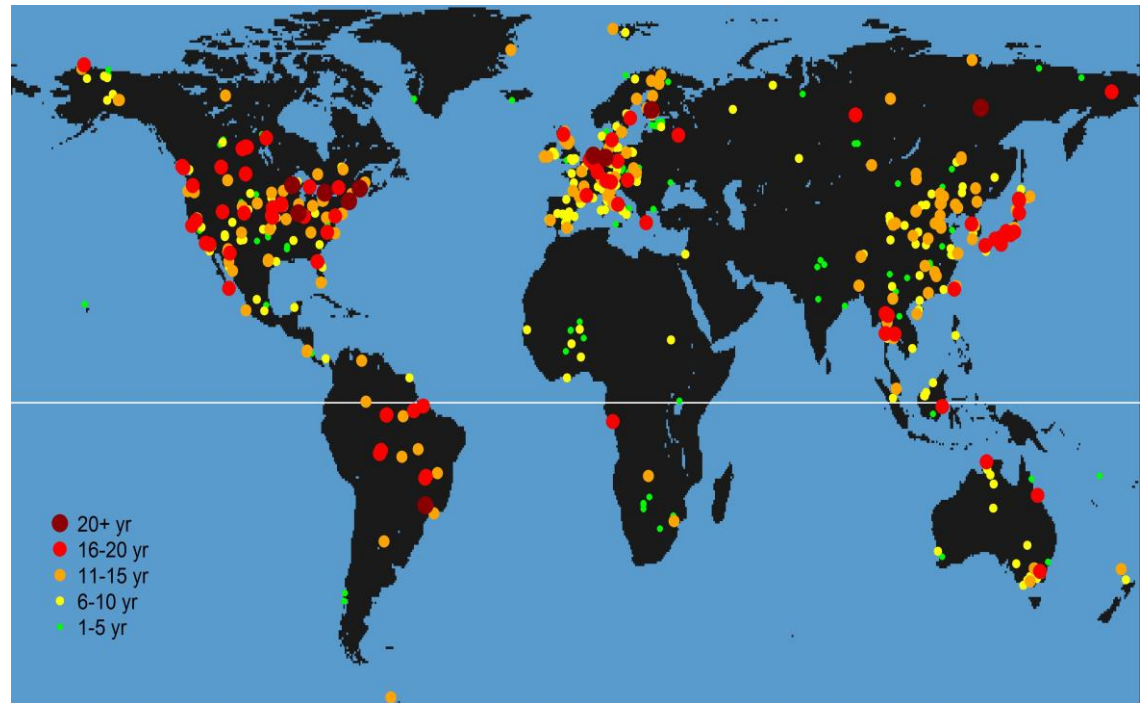


# RAN THE SIMULATION IN POINT MODE AT 30 FLUXNET2015 SITES IN ELM

30 sites  
14 countries  
11 PFTs (6 mixes sites)

DCA: dynamic carbon allocation model

CONTROL: default fixed allocation model

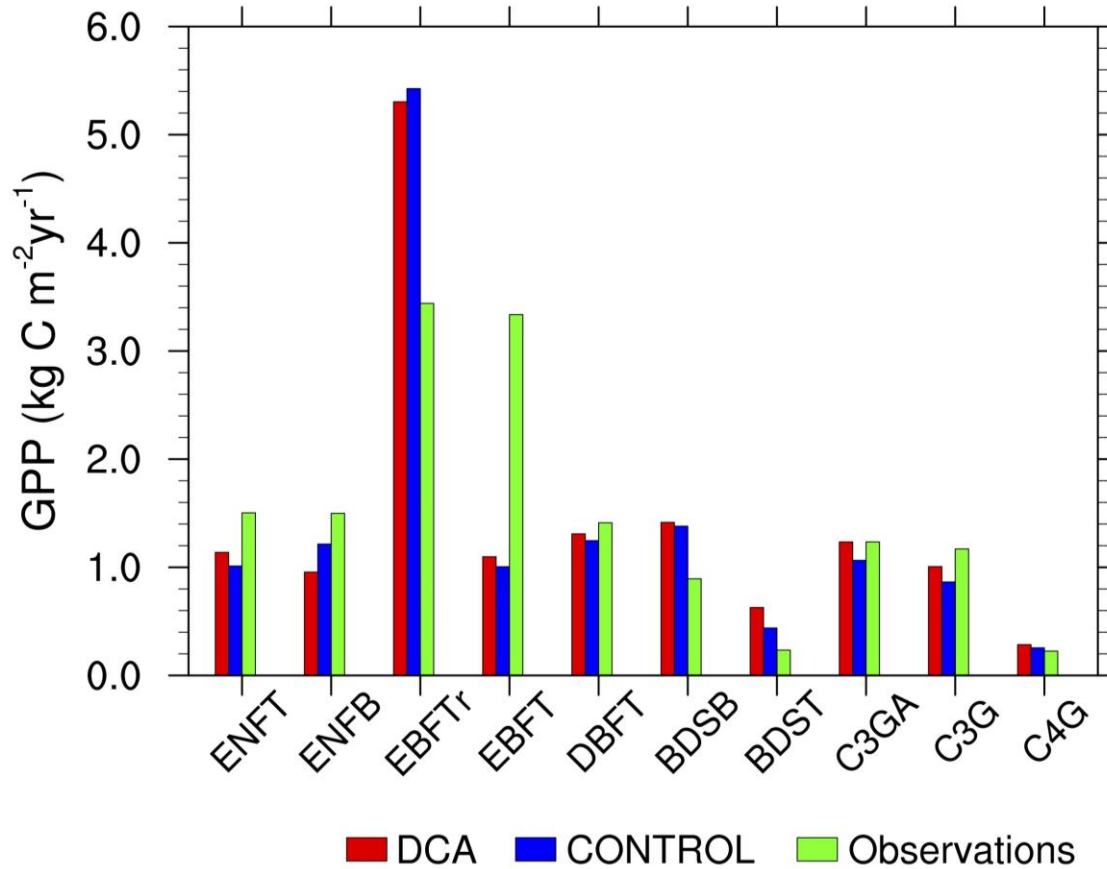


<http://fluxnet.fluxdata.org/data/fluxnet2015-dataset/>

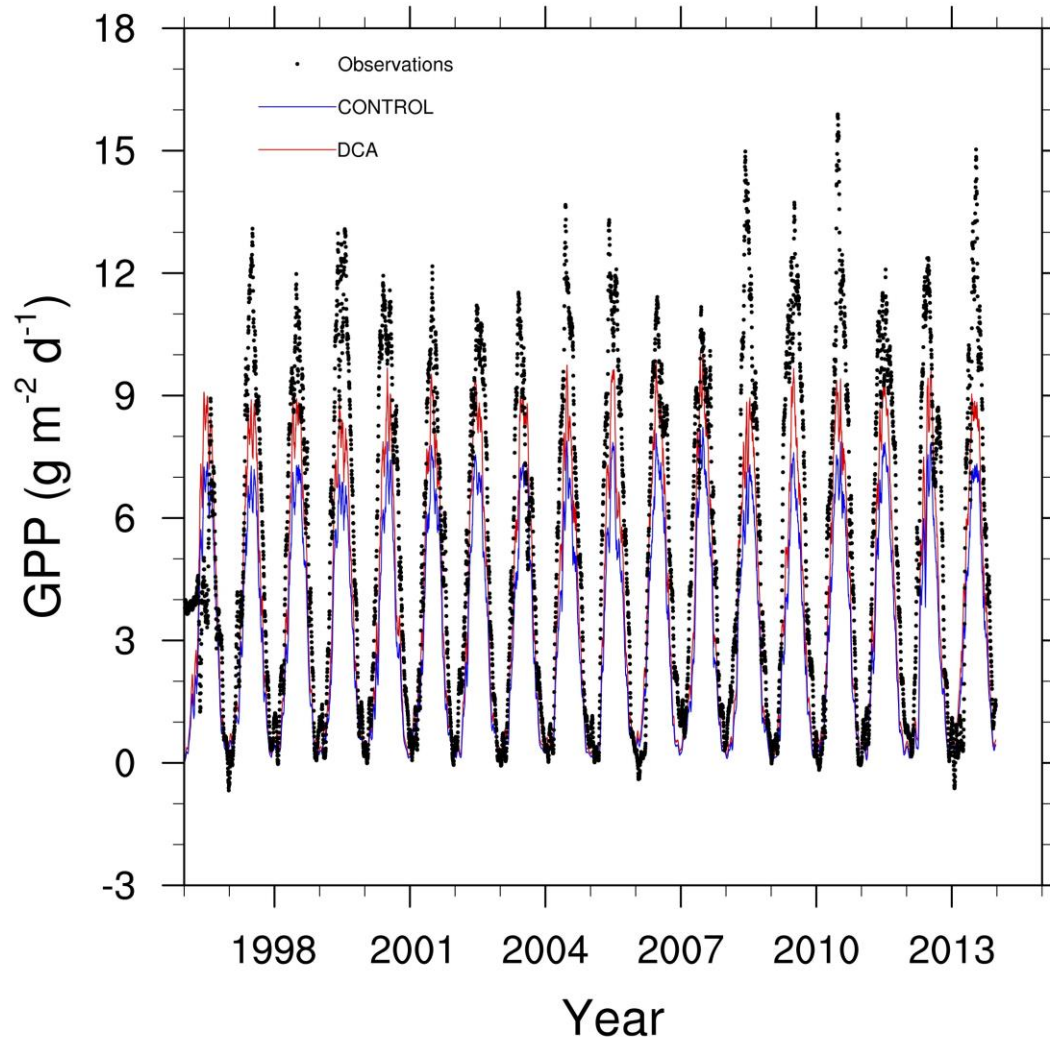
# FINE ROOT:LEAF RATIO ARE DEPENDENT ON PHENOLOGY TYPE

Phenology	Fine Root:Leaf Ratio Average	Fine Root:Leaf Range
Evergreen	0.96	0.51 – 2.18
Seasonal Deciduous	0.37	0.16 – 0.55
Stress Deciduous	0.41	0.34 - 0.51

# GENERAL IMPACTS ON GPP



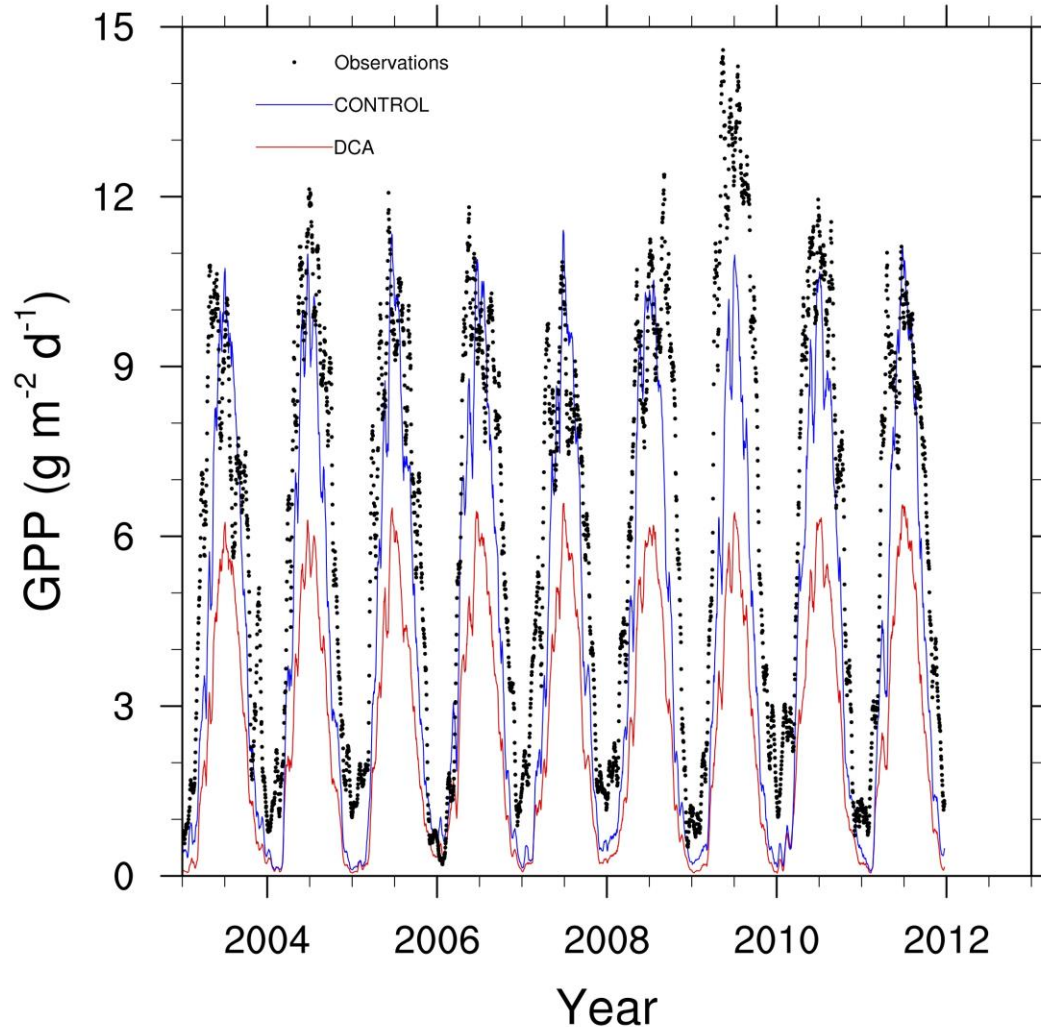
# EXAMPLE 1: DCA MODELED GPP INCREASE



DE-Tha Site:  
Evergreen Needleleaf  
Temperate

Fine Root:Leaf 0.51

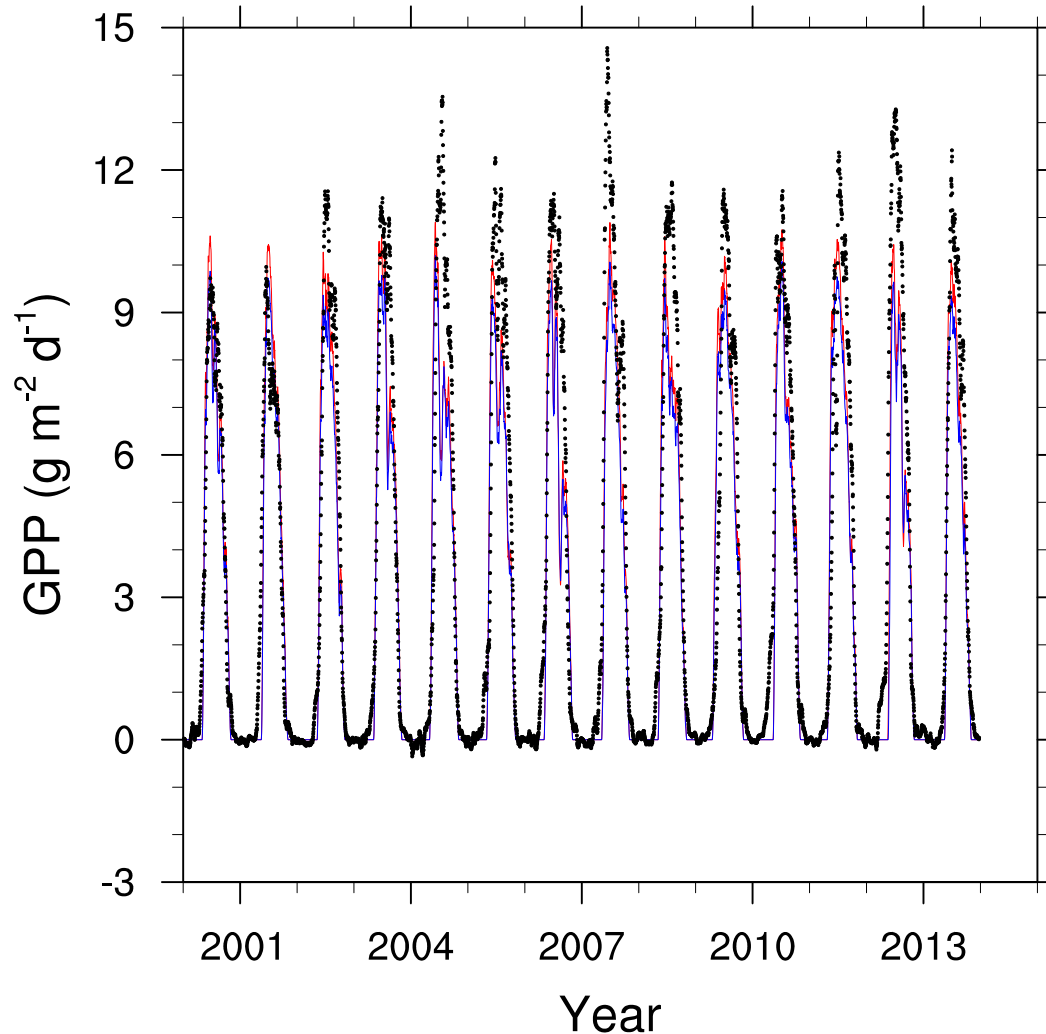
# EXAMPLE 2: DCA MODELED GPP DECREASE



IT-Lav Site:  
Evergreen Needleleaf  
Boreal

Fine Root:Leaf 2.18

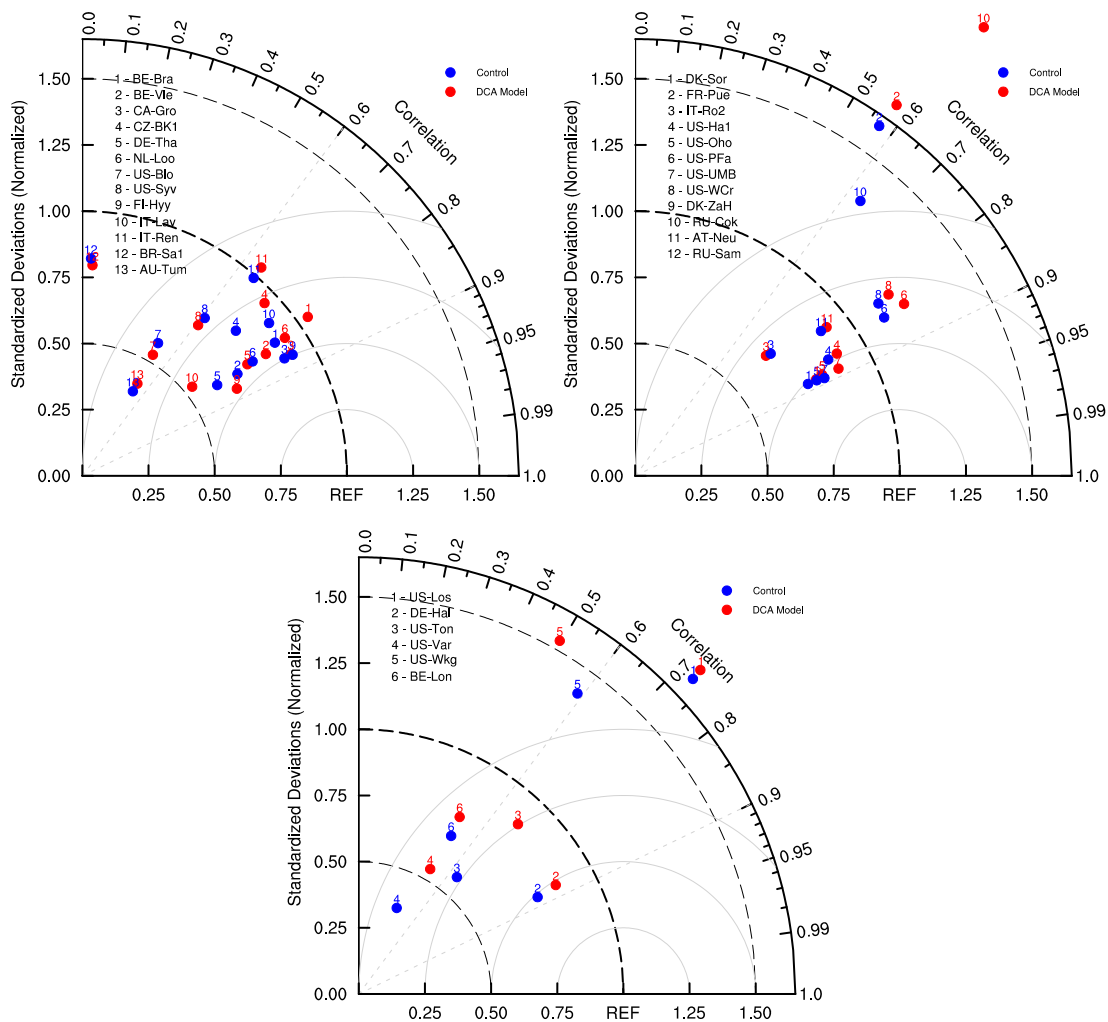
# EXAMPLE 3: DCA MODELED GPP INCREASE



US-UMB Site:  
Seasonal Deciduous  
Temperate

Fine Root:Leaf 0.16

# TAYLOR DIAGRAM OF GPP SHOWS IMPROVEMENT IN SD BUT NOT CORRELATION



# SUMMARY

Fine root:leaf ratios vary with PFT:

Evergreen phenology has highest fine root:leaf ratios (i.e., N limited).

Deciduous phenology has lowest fine root:leaf ratios (i.e., C limited).

DCA simulated *increases* in GPP at all sites with fine root:leaf ratio  $< 1$

DCA simulated *decreases* in GPP at all sites with fine root:leaf ratio  $> 1$

Changes in fine root:leaf ratio have stronger impact on evergreen than deciduous PFTs.

Standard deviation of the DCA model is closer to observations than CONTROL for most ecosystems

DCA model correlation with observations is unchanged compared with the CONTROL



# CONCLUSIONS

The Cobb-Douglas dynamic carbon allocation model shows promise for including a dynamic approach to carbon partitioning in ESMs.

## In the future

- The Cobb-Douglas algorithm should consider woody tissue (stem and coarse roots)
- Water, phosphorus, and distinguishing the nitrogen species of nitrate and ammonium should be included the Cobb-Douglas equation
- Test alternate solvers
- Test alternate equations of harvest and cost
- Bring in competition – game theory

THANK YOU

QUESTIONS?



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