Mesoscale Eddy Vertical Structure & GM Parameterization

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The thing to be parameterized is the SGS tracer flux:

$$\boldsymbol{F}_C = \overline{\boldsymbol{u}C} - \overline{\boldsymbol{u}}\overline{C}.$$

Turbulent diffusion parameterization:

$$\boldsymbol{F}_C \approx -\mathbf{J}\nabla\overline{C}.$$

The symmetric part of **J** corresponds to diffusive/irreversible.
The antisymmetric part is advective/reversible:

$$\frac{\mathbf{J} - \mathbf{J}^T}{2} = \mathbf{A} = \left[\begin{array}{c|c} \mathbf{0} & -\boldsymbol{\Upsilon} \\ \hline \boldsymbol{\Upsilon}^T & 0 \end{array}\right].$$

$$\text{Advective part} = \boldsymbol{u}^* \cdot \nabla\overline{C}, \quad \boldsymbol{u}^* = (\partial_z \Upsilon^x, \partial_z \Upsilon^y, -\nabla_\perp \boldsymbol{\Upsilon}).$$

The GM/Redi parameterization consists of specific choices for the symmetric and antisymmetric parts of **J**:

The Redi parameterization constructs the symmetric part of **J** to have zero diffusion across isopycnal/neutral surfaces.

The GM parameterization wants the antisymmetric part to remove resolved potential energy, and it does that by setting

$$\boldsymbol{\Upsilon} = -\mathbf{K}\boldsymbol{s}, \quad \boldsymbol{s} = -\frac{\nabla_{\perp}\bar{\rho}}{\partial_z \bar{\rho}}.$$

If **K** is positive definite then the GM parameterization extracts resolved potential energy, mimicking baroclinic instability.

The focus of this investigation is the antisymmetric/ advective/ GM component, in particular the vertical structure of **K**.

If we focus on SGS flux of density (or buoyancy) then *there is no diffusive component* and we can write

$$\boldsymbol{F}_\rho \approx \mathbf{A}\nabla\bar{\rho}$$

or using just the horizontal component

$$\overline{\boldsymbol{u}_\perp \rho} - \bar{\boldsymbol{u}}_\perp \bar{\rho} \approx -\mathbf{K}\nabla_\perp \bar{\rho}.$$

Even if we diagnose $\boldsymbol{F}_\rho$ and $\nabla_\perp \bar{\rho}$, we still can't unambiguously solve for **K**. Solution might not be unique; also might not exist!

Attempts to diagnose **K** have to

- Get more data. E.g. use multiple tracers, or use many times, etc.
- Get a 'best fit' to the data, since it typically won't be possible to get an exact match between GM model and the data.

The approach taken here is to constrain the vertical structure of **K** and then do a least-squares fit across depth.

E.g. let **K** be depth-independent, then find the **K** that minimizes

$$\int_0^H |\boldsymbol{F}_\rho + \mathbf{K}\nabla_\perp \bar{\rho}|^2 \mathrm{d}z.$$

Standard baroclinic modes are denoted ψ_0 (barotropic) and ψ_1 (first baroclinic).

'Surface' modes (LaCasce GRL 17) (aka equivalent barotropic modes) are solutions of

$$\partial_z \left(\frac{f^2}{N^2} \partial_z \phi \right) = -k^2 \phi, \ \phi(0) = 0, \partial_z \phi(h) = 0$$

The first surface mode is denoted ϕ_1.

Our data is from a 0.1° ocean-ice POP model simulation (Johnson et al. JPO 2016)

- 15 year spinup using CORE NYF, then 33 years CORE IAF
- 5-day averages
- We use last 5 years
- Every 25 days starting January 25 of every year (14 time points per year)
- Exclude regions with less than 2 km depth and regions closer than 200 grid points to shore
- Only use data from 6.2° S to 64°S

Spatial filter is a Gaussian kernel moving average

$$\exp\left\{-\frac{1}{2}\frac{r^2}{L^2}\right\}$$

where L is 1°, $1.\overline{3}^\circ$, or 2°.

Grid points on land **not set to zero**; instead they are ignored, i.e. not included in the average.

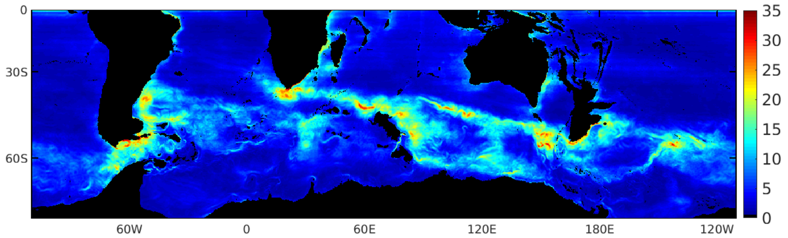
Results at all L are similar, so we only show results for $L = 1.\overline{3}^\circ$.

We start by looking at vertical structure of $\boldsymbol{u}'$ and ρ', since this will inform the vertical structure of $\boldsymbol{F}_\rho$ and of $\mathbf{K}$.

We first do a least-squares fit across depth, then look at the relative error.

For velocity we restrict statistics to places with at least 2 cm/s of eddy velocity averaged across depth.

For density we restrict statistics to places with at least 5×10^{-5} g/cm^3 of RMS eddy density anomaly averaged across depth.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35
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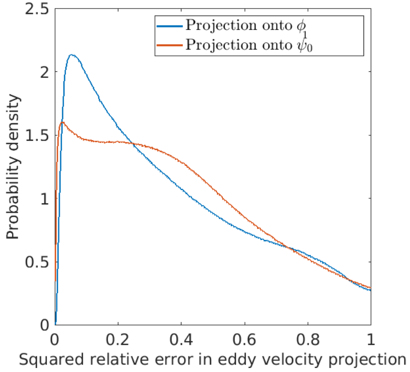
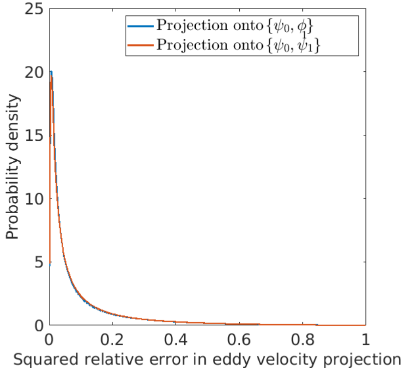
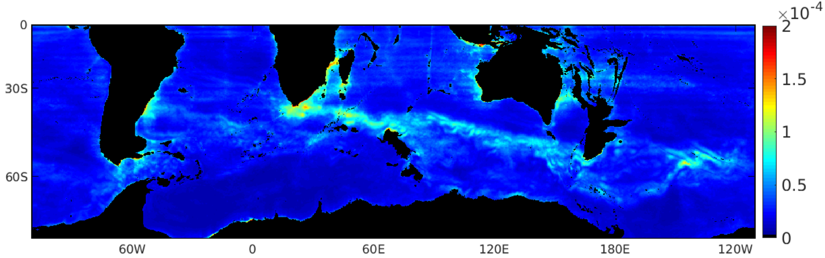

Projection onto ϕ_1
Projection onto ψ_0
Probability density
Squared relative error in eddy velocity projection


Projection onto $\{\psi_0, \phi_1\}$
Projection onto $\{\psi_0, \psi_1\}$
Probability density
Squared relative error in eddy velocity projection



0
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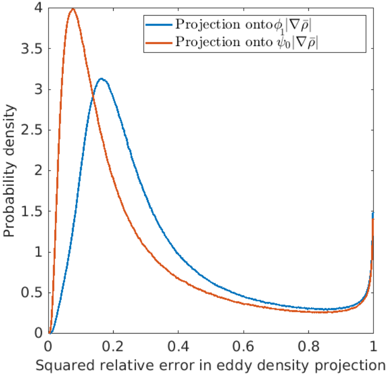
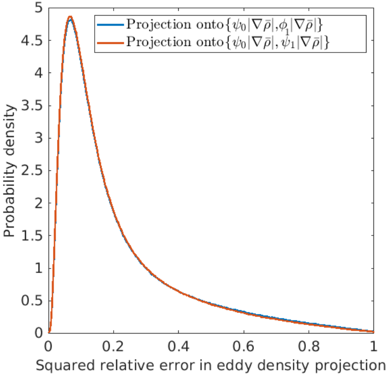
×10^-4
2
1.5
1
0.5
0



Projection onto $\phi_1|\nabla\bar{\rho}|$
Projection onto $\psi_0|\nabla\bar{\rho}|$
Probability density
Squared relative error in eddy density projection



Projection onto $\{\psi_0|\nabla\bar{\rho}|, \phi_1|\nabla\bar{\rho}|\}$
Projection onto $\{\psi_0|\nabla\bar{\rho}|, \psi_1|\nabla\bar{\rho}|\}$
Probability density
Squared relative error in eddy density projection

Eddy density seems to have vertical structure $\rho' \sim |\nabla_\perp \bar{\rho}|$, and eddy velocity requires two modes.

This suggests the eddy density flux should have vertical structure

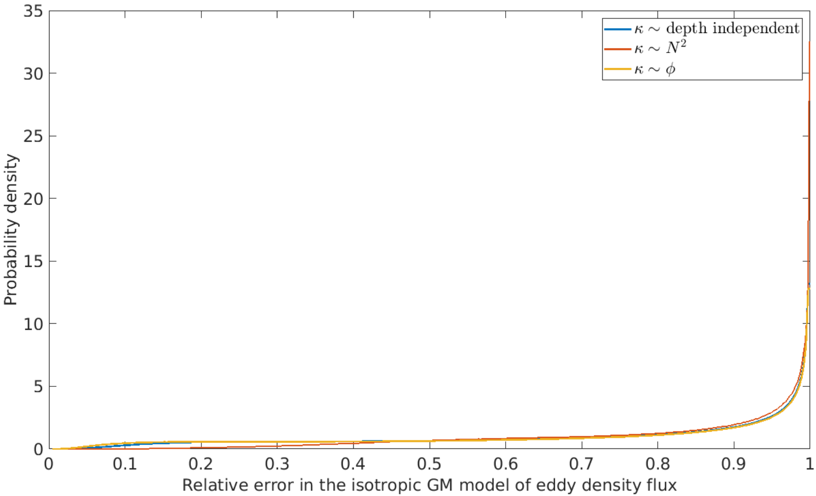
$$\boldsymbol{F}_\rho \sim \text{ combination of } |\nabla_\perp \bar{\rho}| \text{ and } \phi_1 |\nabla_\perp \bar{\rho}|.$$

Since $\boldsymbol{F}_\rho \approx -\mathbf{K}\nabla_\perp \bar{\rho}$ this suggests for the vertical structure of $\mathbf{K}$

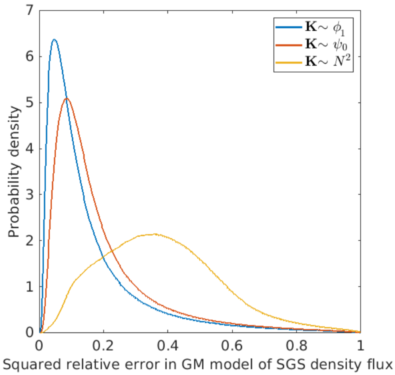
$$\mathbf{K} \sim \text{ combination of } \psi_0 \text{ and } \phi_1$$

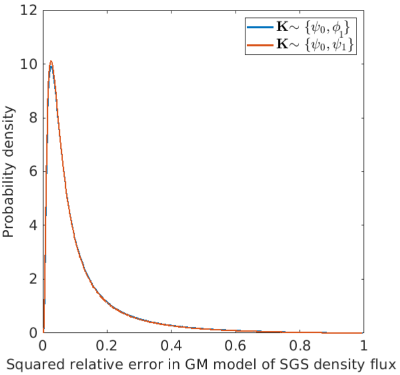
i.e. the same vertical structure as eddy velocity.

It would be really nice if **K** had only *one* vertical structure, so we'll try that first.



Isotropic GM is a bad fit to data regardless of vertical structure.





$\mathbf{K}\sim\phi_1$ is good, depth-independent is surprisingly good, $\mathbf{K}\sim N^2$ is bad.

Having 2 modes improves the fit, but makes it harder to parameterize. Fit with one mode is already pretty good.

Summary

- GM **K** with vertical structure ϕ_1 fits data quite well.
- GM **K** depth-independent fits data much better than vertical structure $\sim N^2$.
- Isotropic GM does not fit data, unless perhaps after lots of averaging. (Iso/aniso GM is **not the same** as iso/aniso Redi!)

The diagnosed **K** typically has one positive and one negative eigenvalue, about the same size. Not sure what this means, and not sure if directions are related to mean flow.

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