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Land Ice Working Group Meeting
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A generalized interpolation material point (GIMP) method for shallow shelf approximation of ice flow

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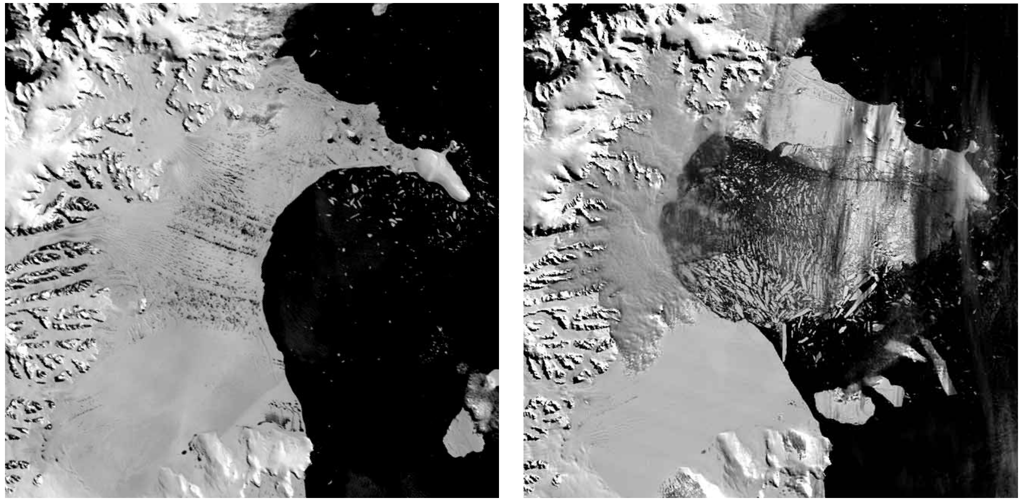
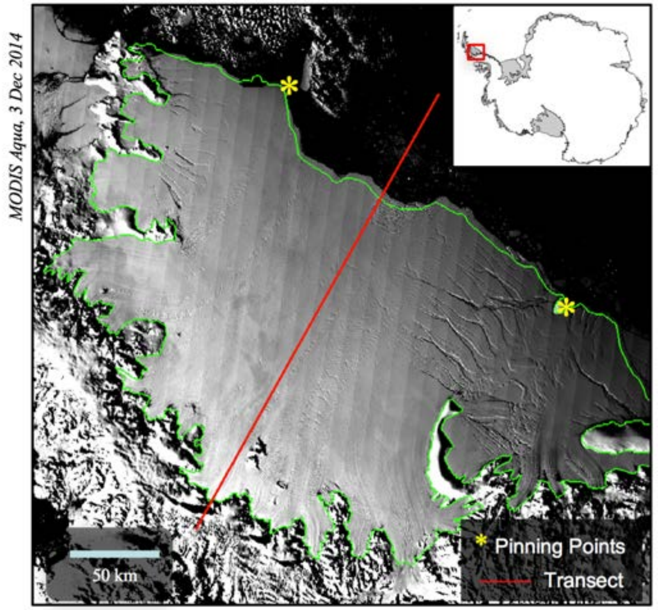
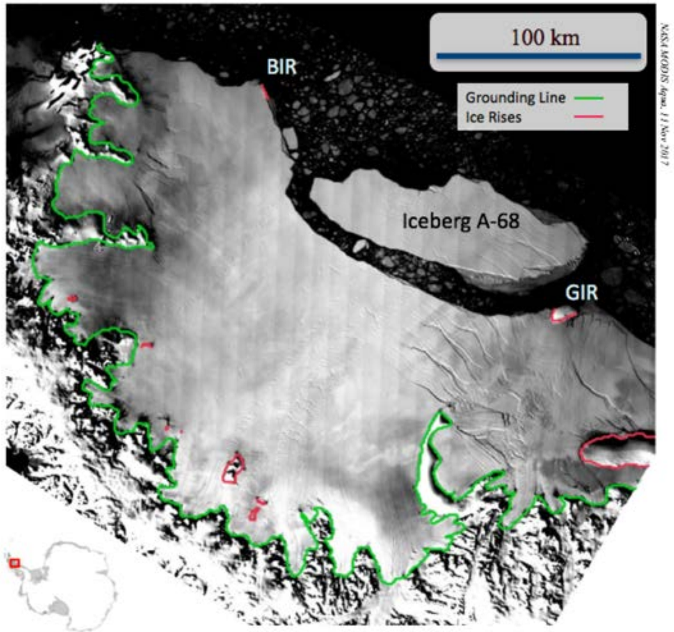
Ice Shelf Fracture and Iceberg Calving Accelerates Ice Sheet Mass Loss



Larsen B ice shelf on January 31, 2002 (left), and on March 5, 2002



Larsen C ice shelf on December 3, 2017 with transverse crevasses



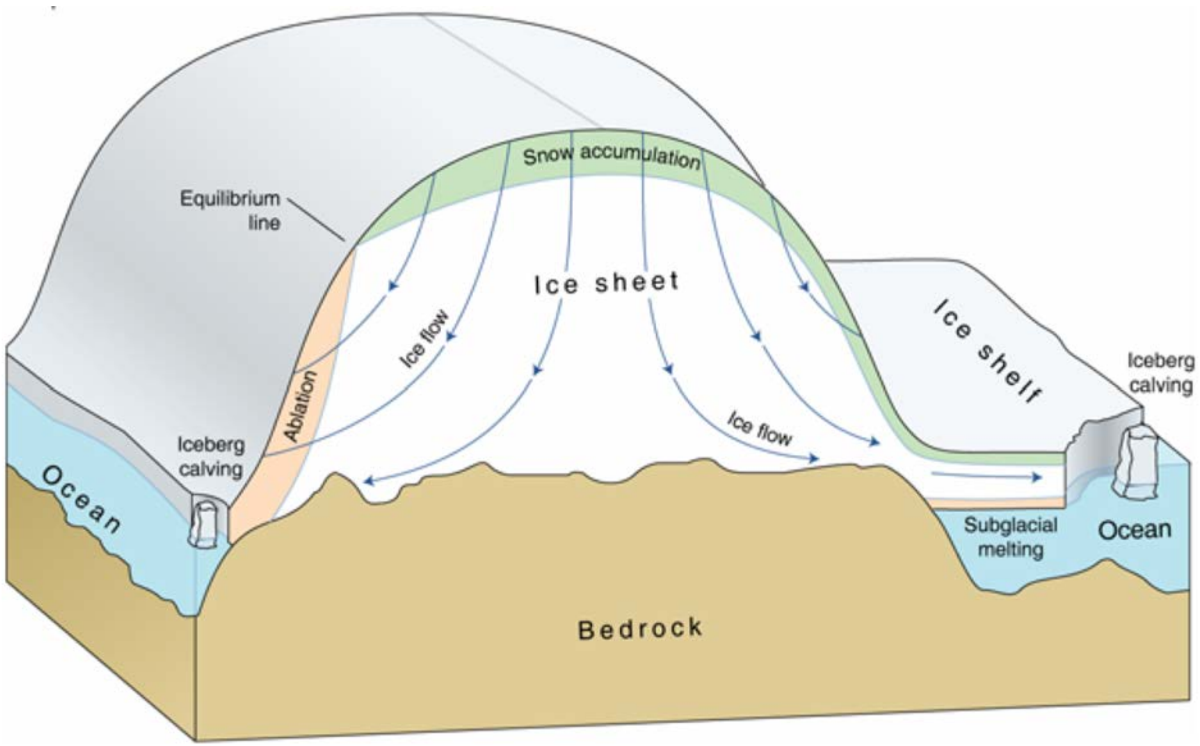
November 2017, four months after calving of iceberg A-68

- Calving from floating ice shelves increases surrounding land ice flow and contributes to sea level rise
- Thinning due to surface and basal melting enhances fracture and surface melt can cause hydrofracture
- Ice shelf fracture is poorly understood and calving is not well represented in ice sheet models

Scale Separation of Ice Sheet Processes

Time scale separation: viscous flow (longer and continuous) and fracture (shorter and sporadic)

Length scale separation: viscous flow (larger, entire ice sheet) and fracture (smaller, ice margins)



Changes to Antarctic ice shelves are controlled by boundary conditions:

- Thinning due to basal melting
- Calving due to ice shelf fracture
- Contact with bay walls or ice rises

These three conditions are also coupled

Limitations of Calving Laws in Ice Sheet Models

"No response of an ice-sheet/ice-shelf system to climate variations can be computationally forecasted, unless this (calving front) boundary condition is properly parameterized... The difficulty with parameterizations of the calving rate is its intermittent non-smooth occurrence in nature" – *Weis and others (1999)*

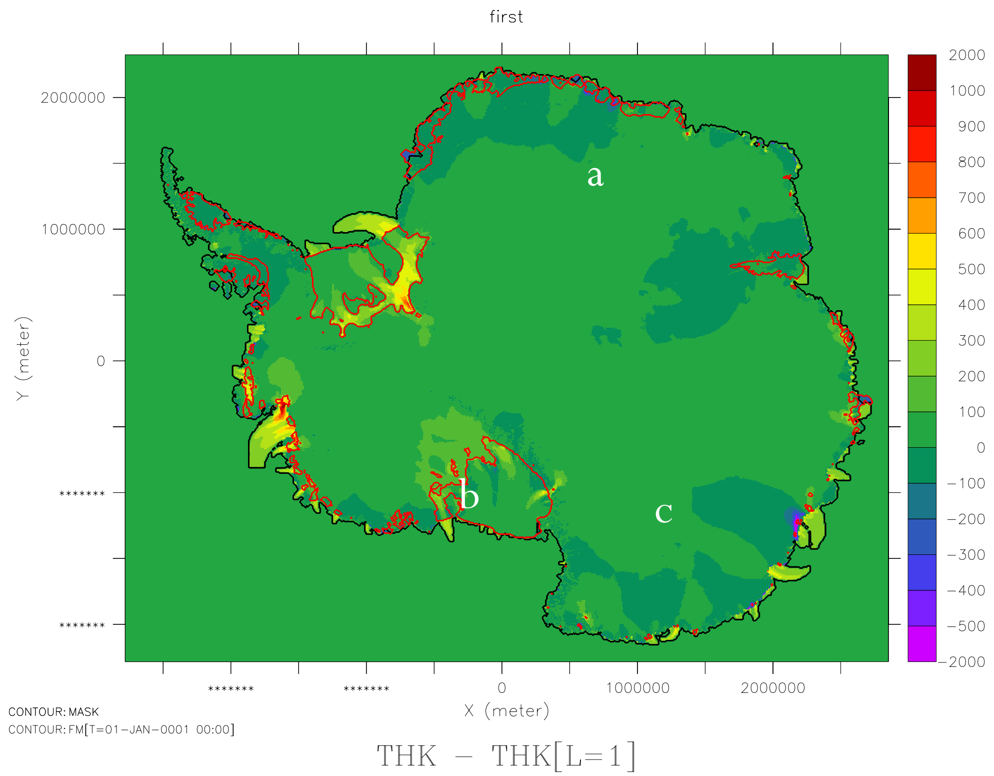
"The great unsolved problem in ice-shelf dynamics (perhaps the whole of glaciology) is the treatment of the shelf front" – *Keller and Hutter (2014)*

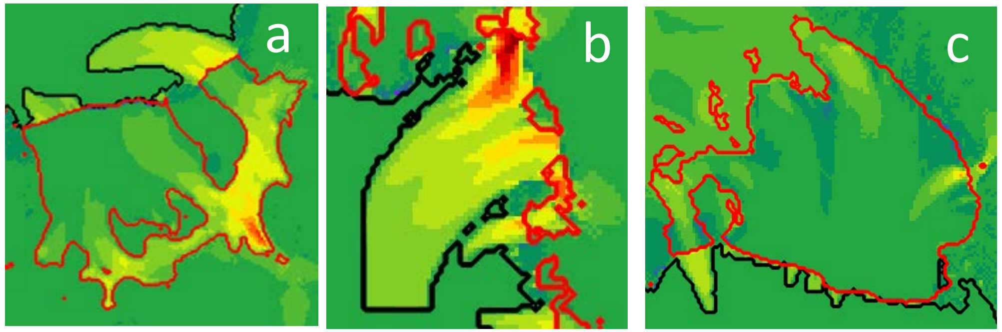
Weis M, Greve R and Hutter K (1999) Theory of shallow ice shelves. Contin. Mech. Thermodyn., **11**(1), 15–50

Keller A and Hutter K (2014) Conceptual thoughts on continuum damage mechanics of shallow ice shelves. J. Glac. 60(222), 685–693

Limitations of Calving Schemes in CISM

Community Ice Sheet Model – Community Earth System Model





Snapshots of unrealistic floating ice regions at the end of a 3000-year preliminary simulation of Antarctic Ice Sheet in CISM: (a) Filchner-Ronne; (b) Thwaites and Pine Island; (c) Ross.

- Calving schemes used: no-advance and thickness-based
- Calibration was done with the Ross ice shelf (c) terminus location
- Unrealistic floating ice tongues are predicted for other two regions: Filchner-Ronne (a) and Thwaites and Pine Island (b)

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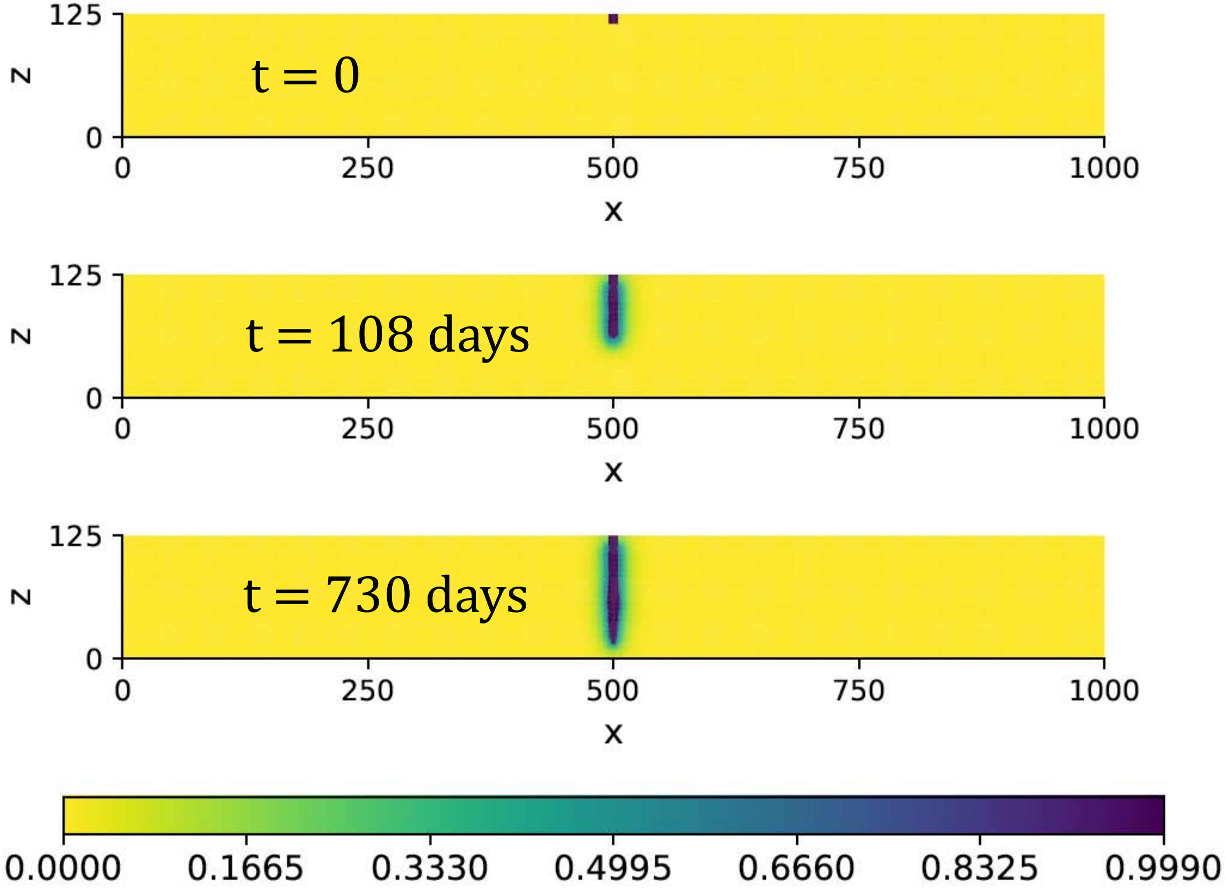
Fracture-mechanics-based Calving Models

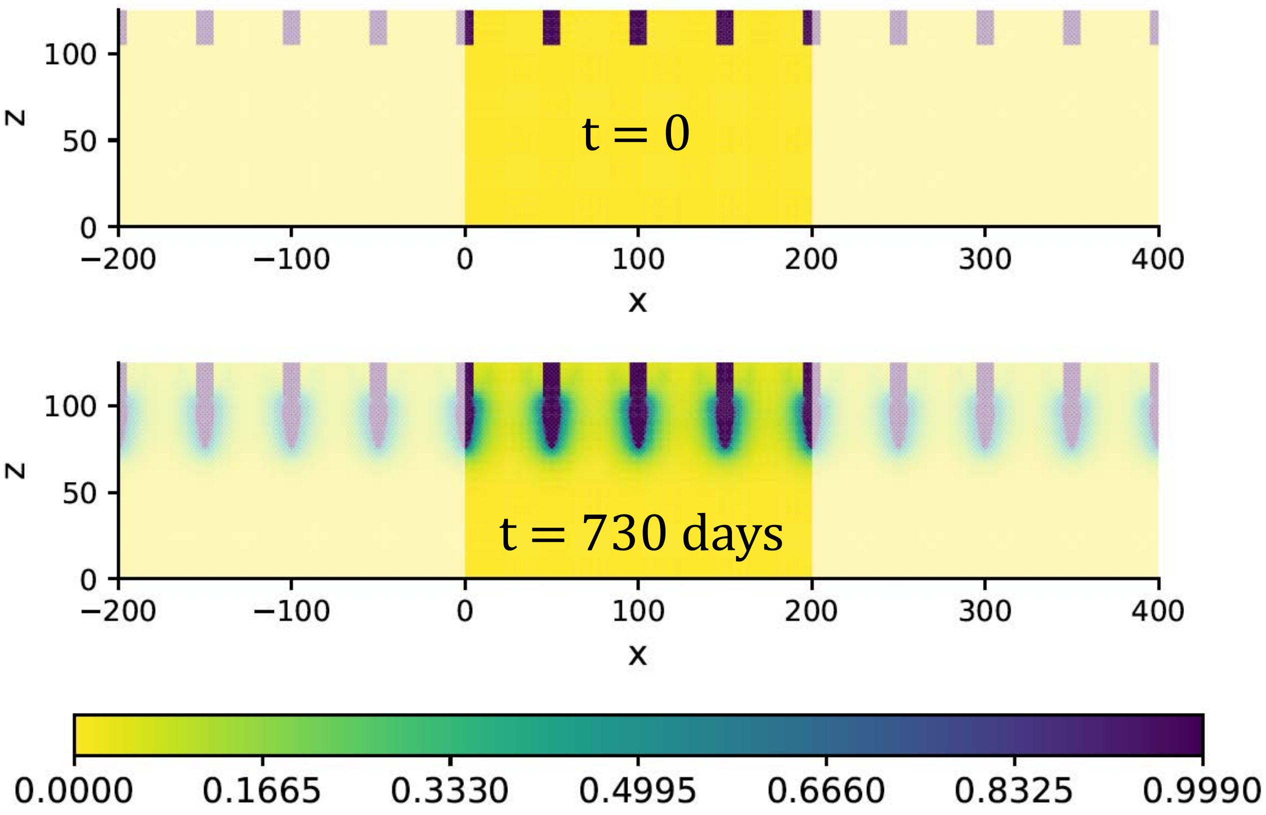
- **Zero-stress** [Nye, 1957; Nick et al., 2010; Sun et al., 2017]
  - Describes calving of glacier or ice shelf with quasi-uniform field of closely-spaced crevasses
- **Linear elastic fracture mechanics** [Weertman, 1973; van der Veen, 1998; Krug et al, 2014; Yu et al., 2017]
  - Describes calving of glacier or ice shelf with isolated or widely-spaced crevasses
- **Continuum damage mechanics** [Pralong and Funk, 2005; Duddu and Waisman, 2013; Borstad et al., 2012, Keller and Hutter, 2014, Duddu et al., 2020]
  - Describes calving of glacier or ice shelf with closely-or widely-spaced water-filled crevasses
  - Can it enable efficient parametrization of calving in ice sheet models over longer times scales?

Hydrofracture of Surface Crevasses Filled with Meltwater

Isolated crevasse



Closely-spaced crevasses



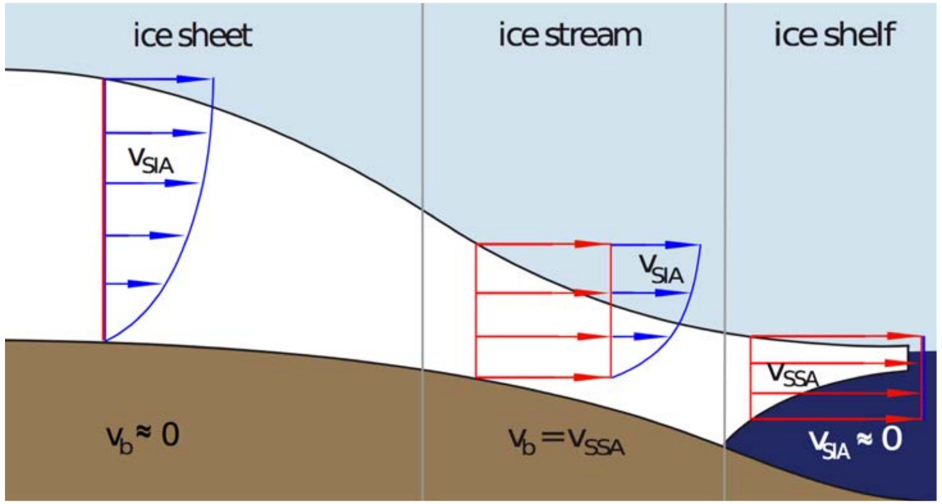
Duddu R, Jimenez S, Bassis J (2020) A nonlocal continuum poro-damage mechanics model for hydrofracturing of surface crevasses in grounded glaciers Journal of Glaciology, under review.

Ice Shelf Fracture Parameterization in an Ice Sheet Model

1. Use shallow shelf approximation of Stokes equations and fracture-based calving law
2. Define crevasse depth ratio as damage to avoid explicit description of crevasses
3. Modify depth-integrated ice viscosity with depth averaged damage to account for the feedback between flow and fracture
4. Incorporate hydrofracture parameterization based on water pressure in crevasses

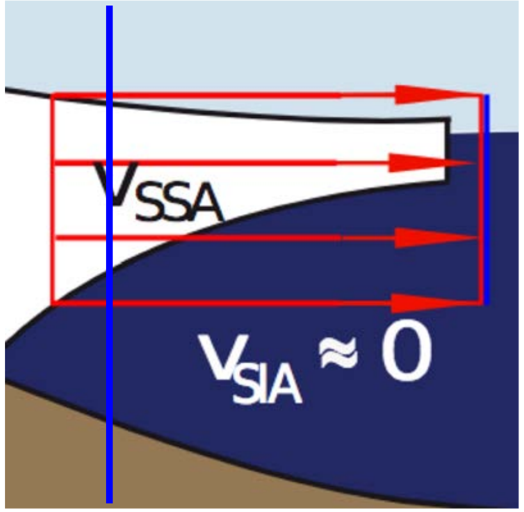
Sun, S., Cornford, S. L., Moore, J. C., Gladstone, R., and Zhao, L (2017) Ice shelf fracture parameterization in an ice sheet model The Cryosphere, 11, 2543–2554

Implementation of Damage Mechanics Model in SSA in Elmer Ice

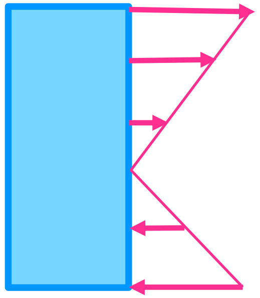


Shallow Shelf Approximation [Winkelmann et al., 2011]

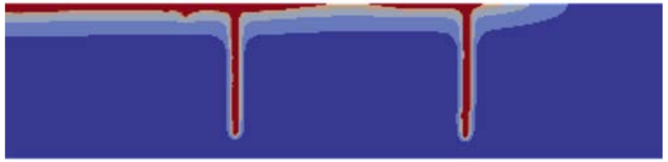
Velocity



Stress

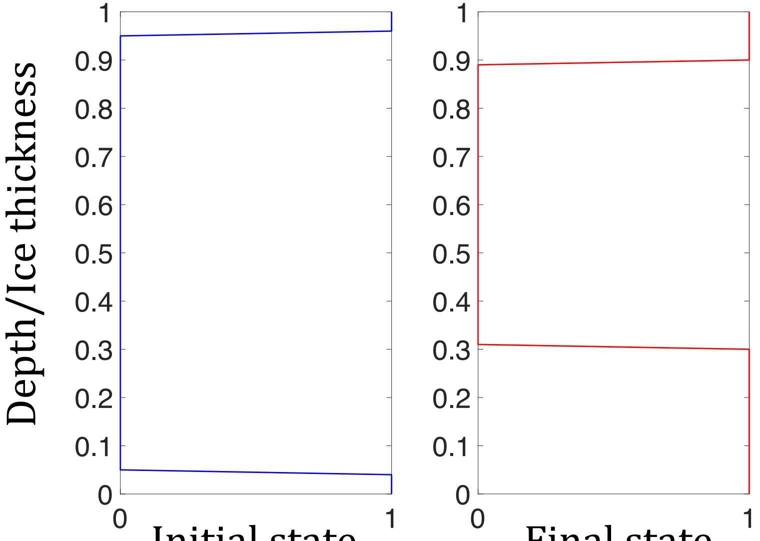


Hydraulic pressure



Creep damage evolution
[Murakami, 1983]

Crevasse depth



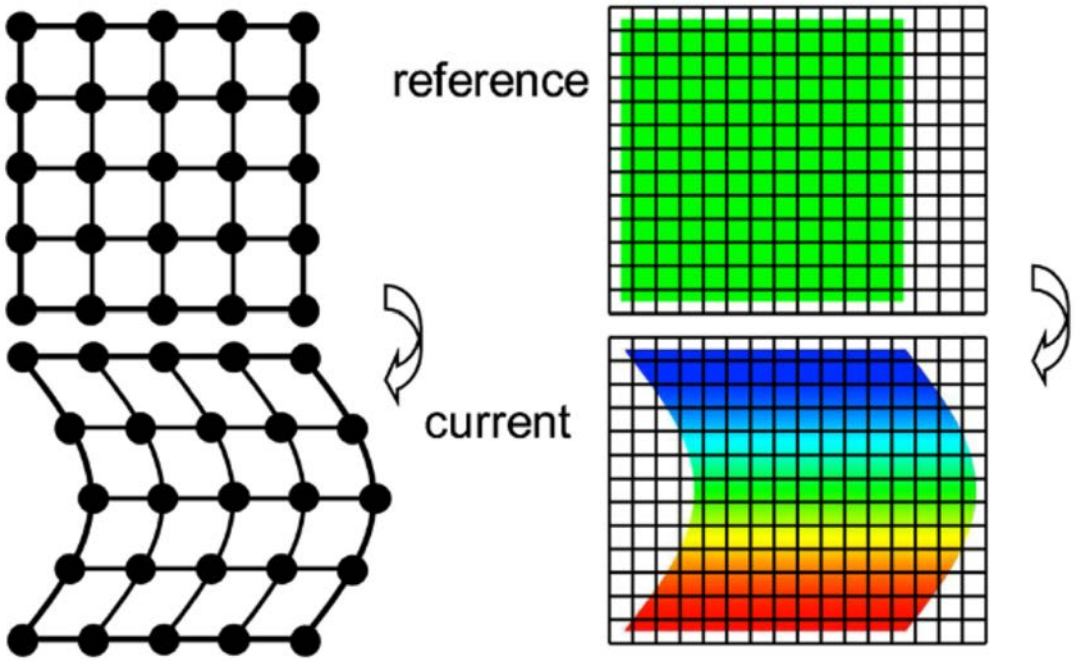
Damage

Need for a Common Continuum Framework for Flow and Fracture

Challenge: How can we model the fracture of viscous fluid?

Common framework needs to:

- handle large deformations
- maintain sharp edges while advecting damage
- track grounding line, ice front, and thickness



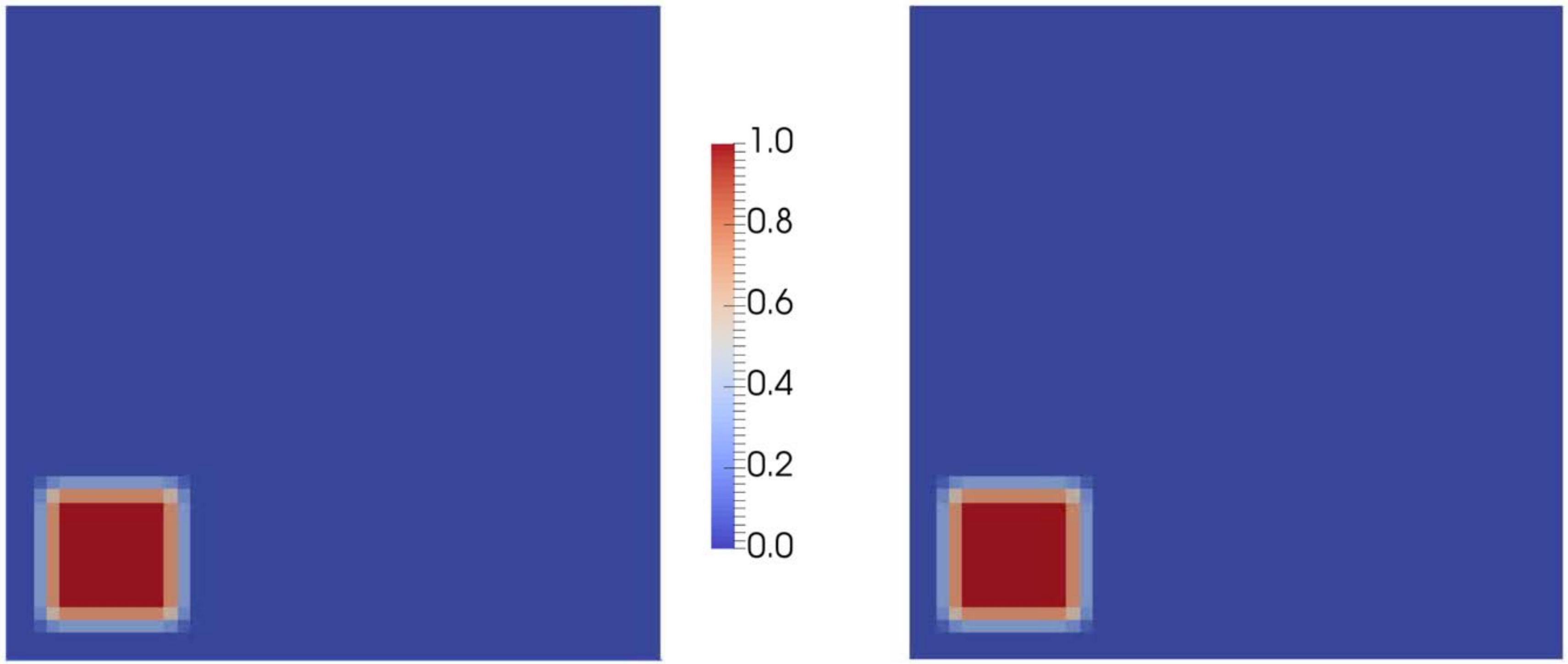
Lagrangian framework
(typically used for solid fracture)

Eulerian framework
(typically used for fluid flow)

Eulerian Versus Lagrangian Transport of Damage Variable

Lagrangian
Material Point Method

Eulerian
Discontinuous Galerkin



- 280 years
- 0.5 km grid resolution
- 4 MPs/cell

2 km

Numerical dissipation of damage is a problem even with the more accurate DG approach in Elmer Ice

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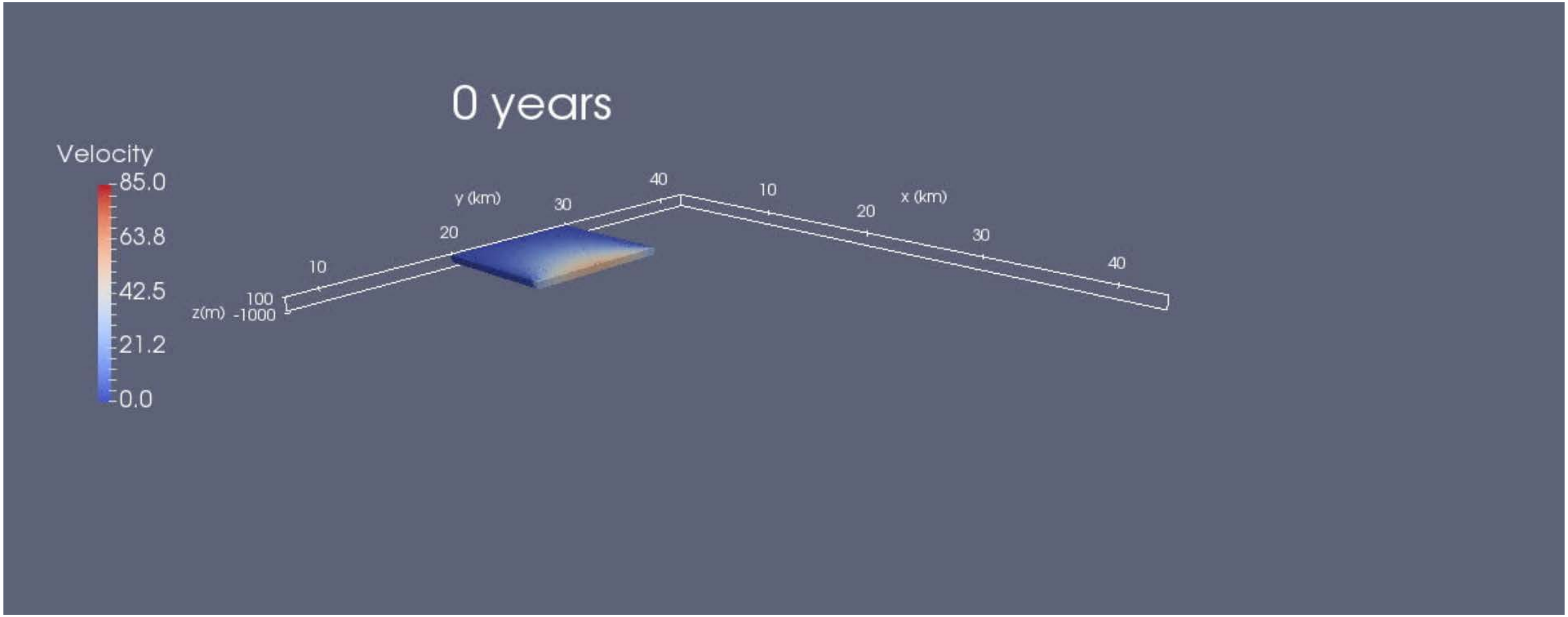
The Generalized Interpolation Material Point (GIMP) Method



The GIMP method with SSA formulation can track grounding line, ice front, and thickness

Huth, A., Smith, B. and Duddu R.(2020) A generalized interpolation material point method for shallow shelf approximation of ice flow, to be submitted.

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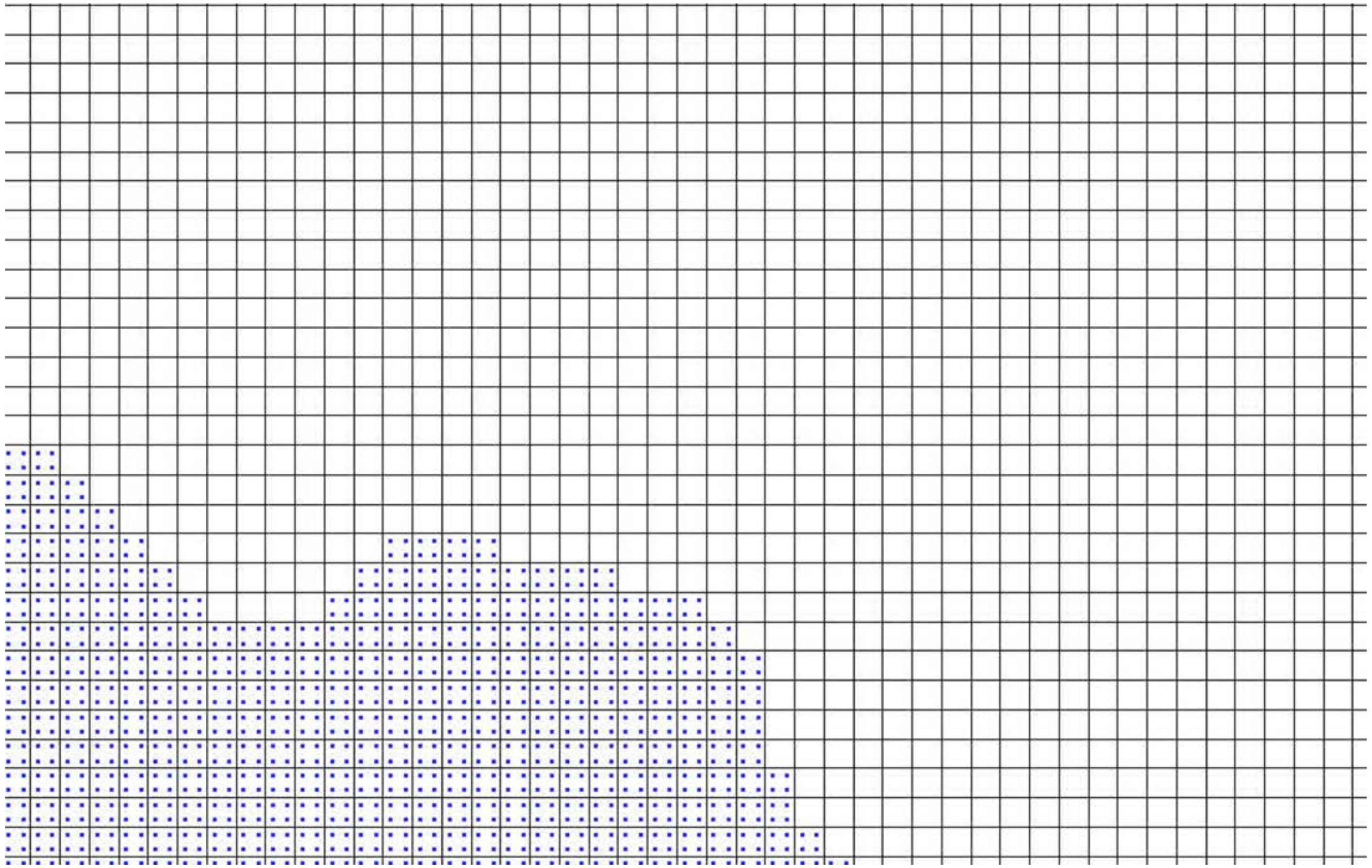
Towards a Common Framework for Simulating Flow and Fracture



The domain is discretized into a set of Lagrangian material points on a background Eulerian grid that can

- Transport material variables
- Advect and deform with the flow
- Serve as moving integration points for an Eulerian finite element formulation of SSA equations to the

The GIMP method with SSA formulation combines Lagrangian and Eulerian Frameworks

Huth, A., Smith, B. and Duddu R.(2020) A generalized interpolation material point method for shallow shelf approximation of ice flow, to be submitted.

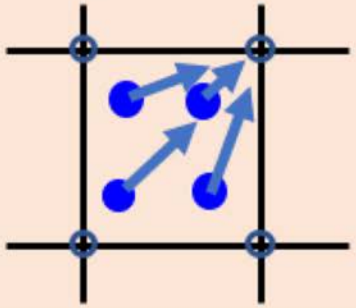
Numerical Solution Strategy with the GIMP Method

1. Update material point history variables:

e.g. thickness, damage, temperature, etc.

2. Material points to grid: H, momentum, mass

Calculate mesh surface heights and **initial velocity** for the SSA solution.

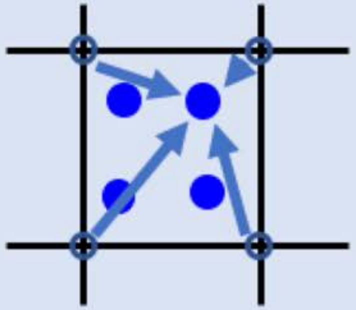


3. Grid to material points:

surface height gradients
velocity gradients

If needed: friction parameter, enhancement factor, temperature, etc.

Update grounded mask

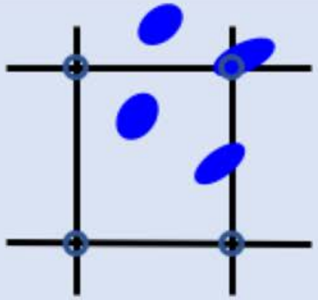


4. Solve SSA for velocity

using material points for integration

5. Material point updates:

velocity, position, deformation gradient, volume, and splitting



Grid processes (Eulerian)

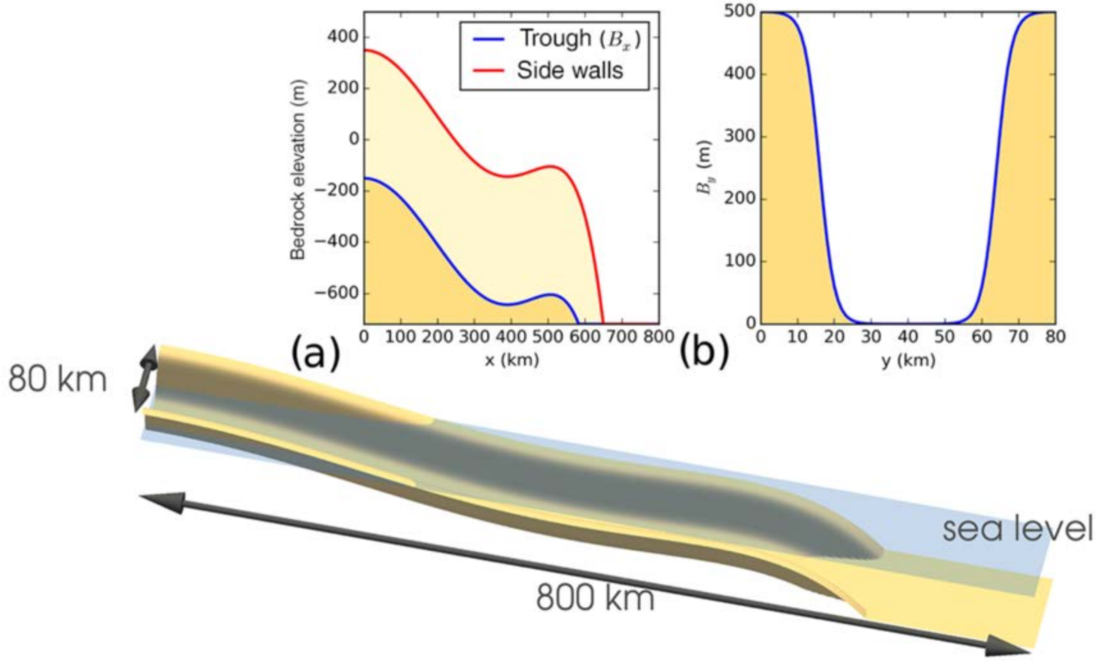
Material point processes (Lagrangian)

Thickness update:

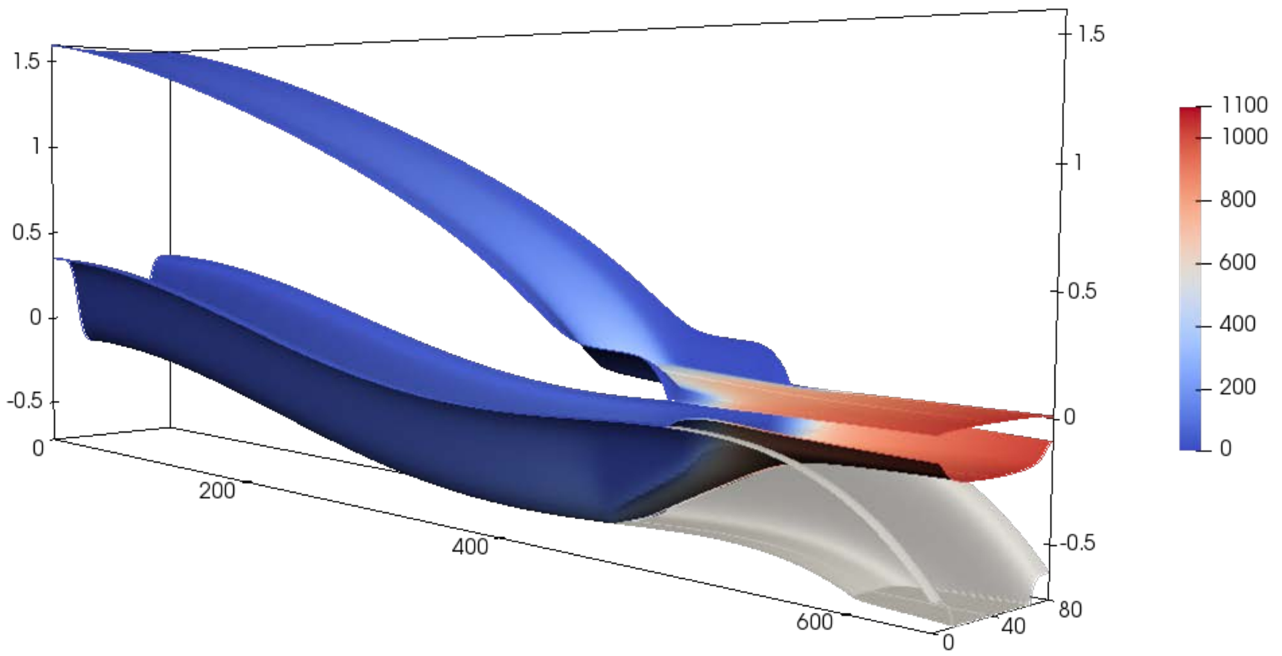
$$\frac{dH}{dt} = \dot{b} - \nabla \cdot vH$$

Huth, A. (2020) A material point method and anisotropic creep damage model for shallow ice shelves, Ph. D. Thesis, to be submitted.

Benchmark Simulation: Idealized Marine Ice Sheet MISMIP+



Bed geometry (Asay-Davis and others, 2016)



3D steady state near grounding line (km)

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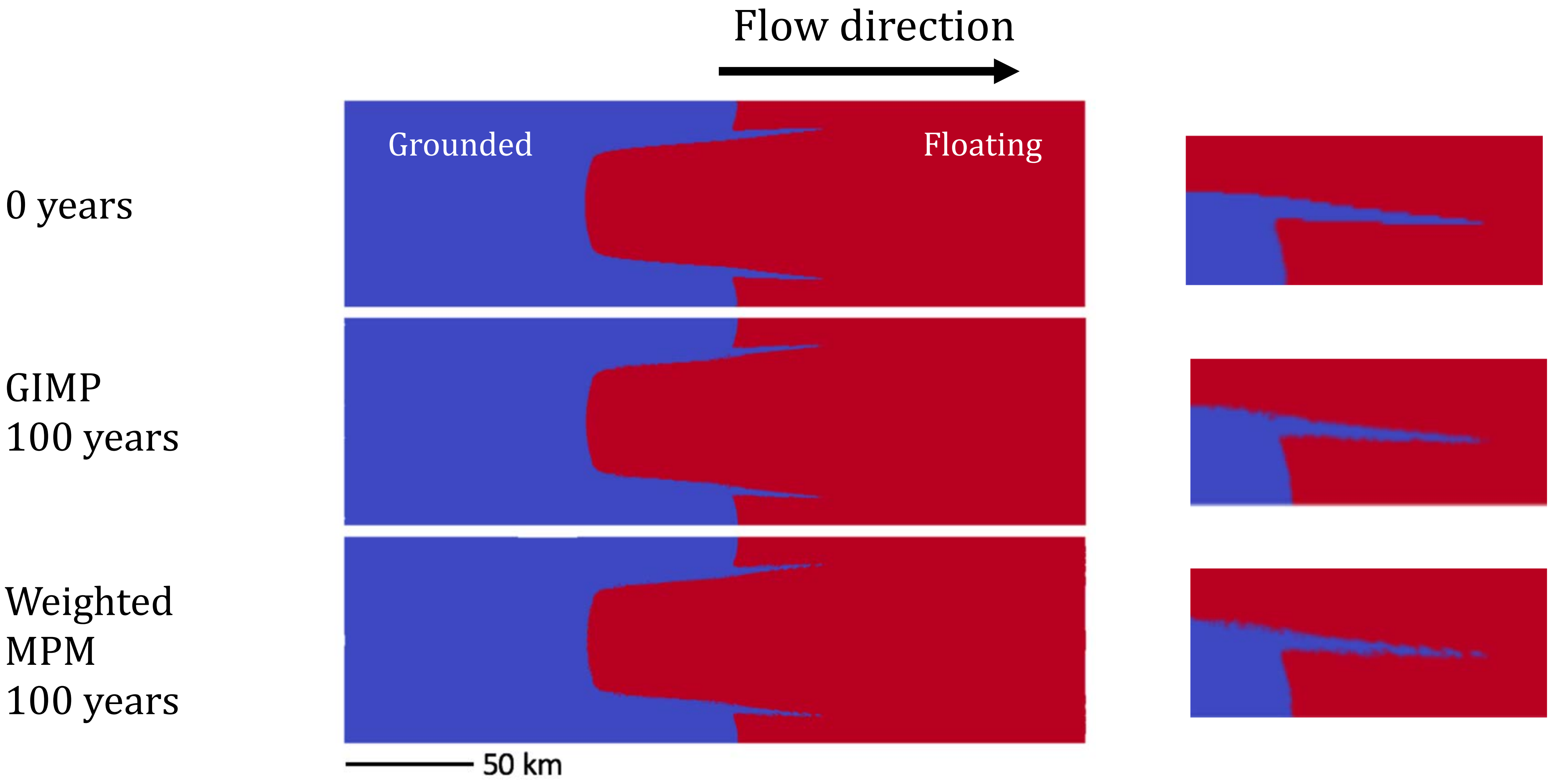
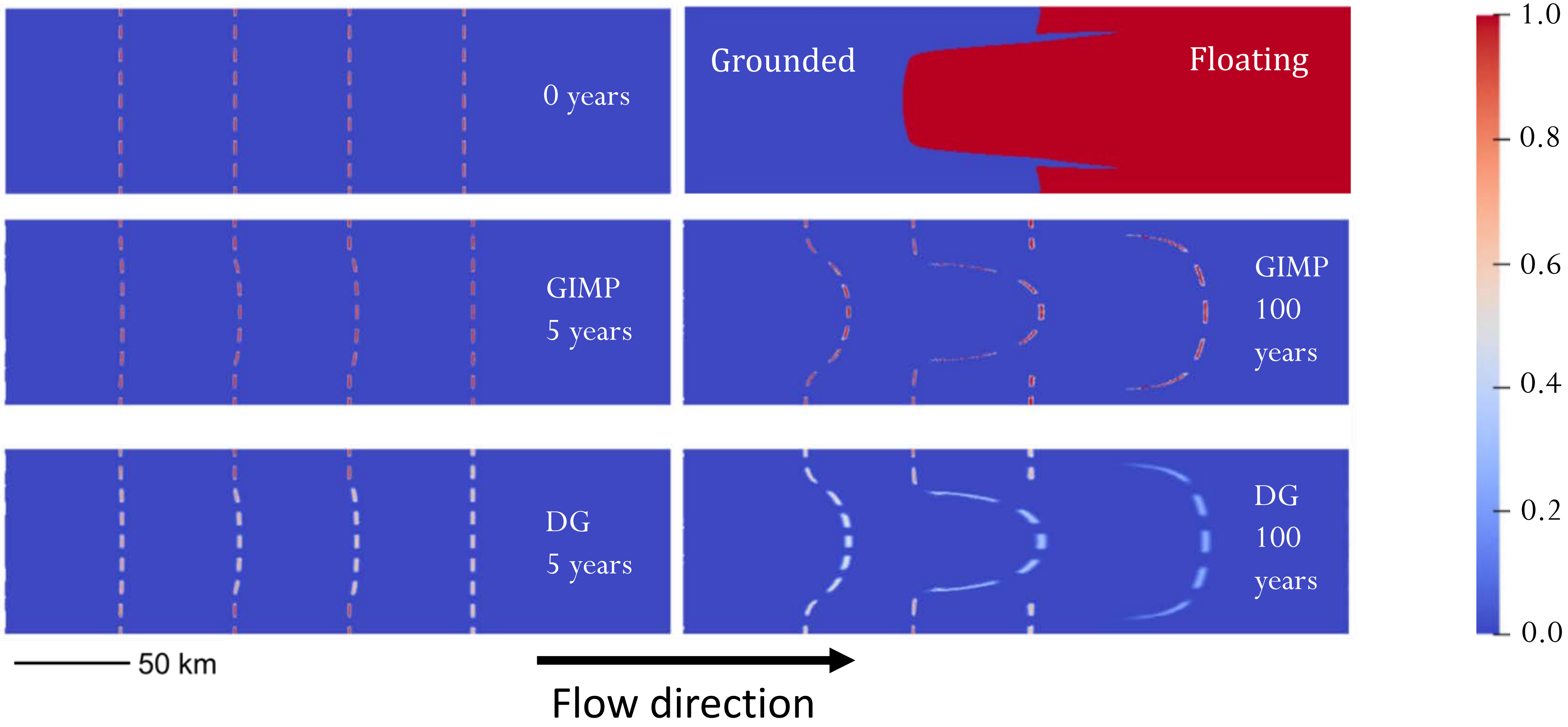
MISMIP+: Grounding Line at Steady State



The GIMP method can hold the steady state grounding line well for 100 years

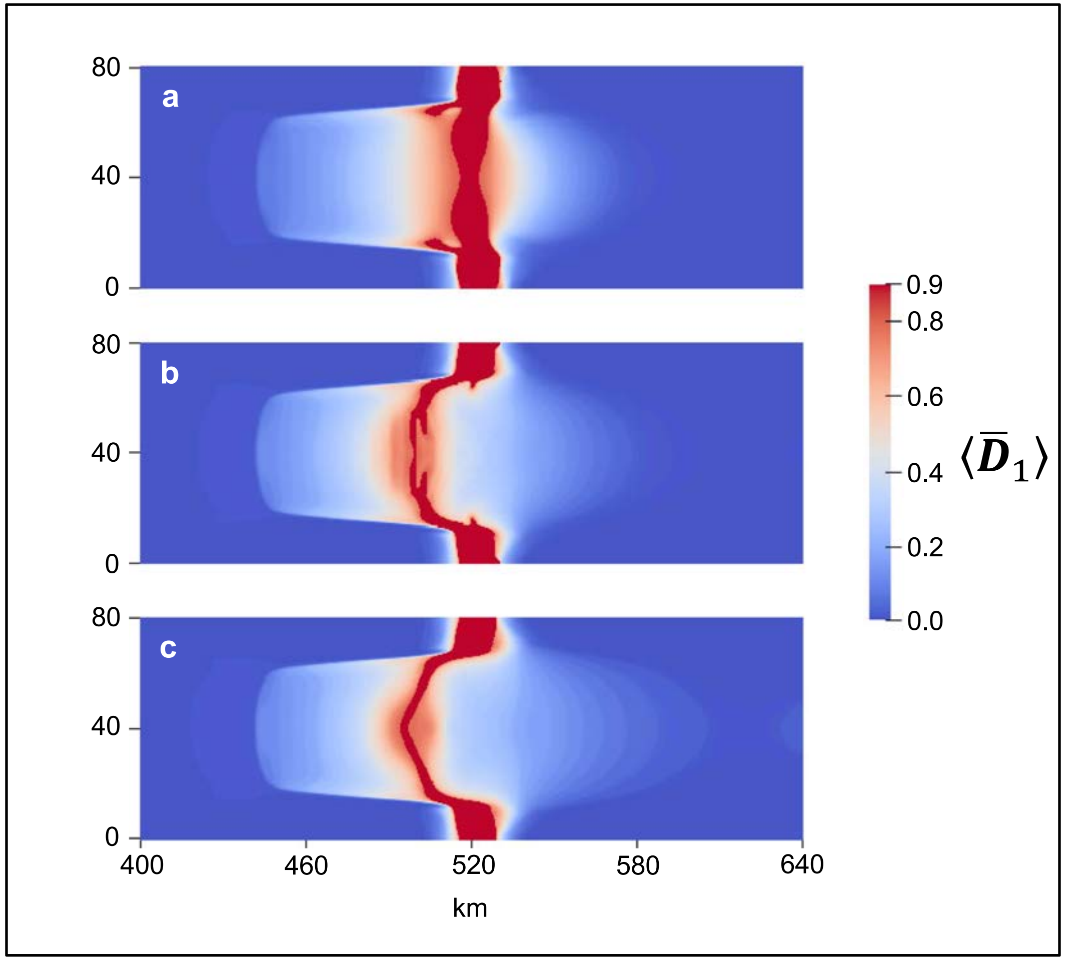
MISMIP+: Advection of Scalar Variable – GIMP Versus DG methods



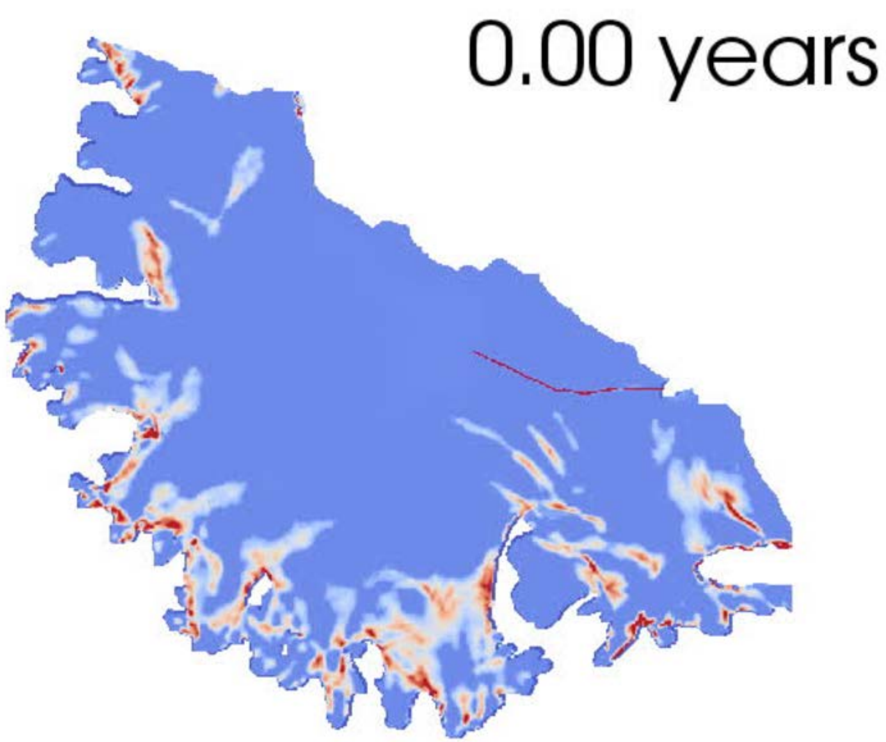
The GIMP method causes less numerical dissipation after 100 years

Simulation of Ice Fracture with the GIMP Method

MISMIP+



Larsen C ice shelf calving



The GIMP method shows promise in simulating rift propagation in ice shelves

Huth, A., Smith, B. and Duddu R.(2020) A generalized interpolation material point method for shallow shelf approximation of ice flow, to be submitted.

Conclusion

1. The GIMP method with SSA formulation provides a powerful approach to model flow and fracture of ice shelves

2. Future work is focused on implementing anisotropic creep damage models and tuning model parameters to reproduce observations better.