Implementing CLUBB+MF in CAM

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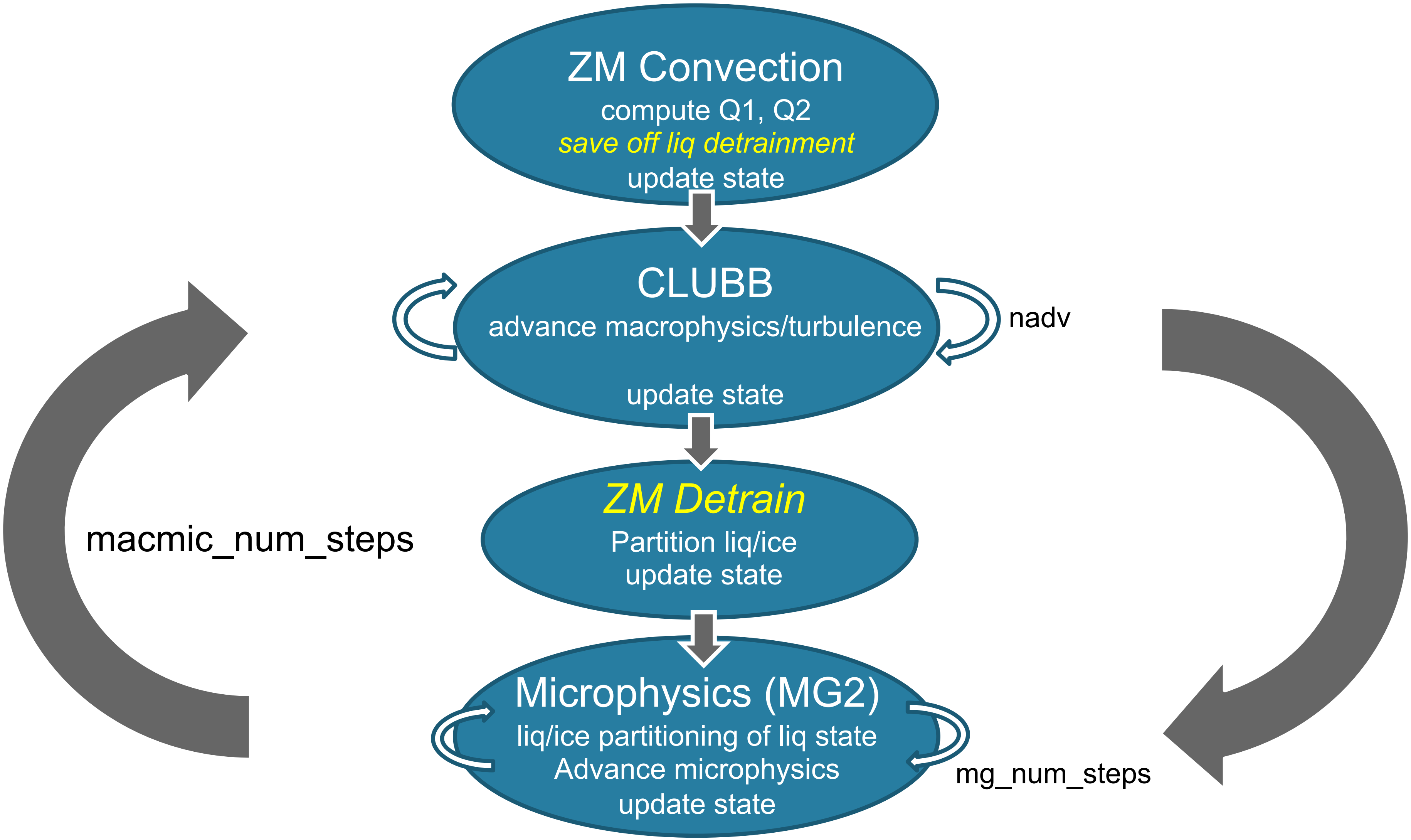
Moist Convection in CAM6



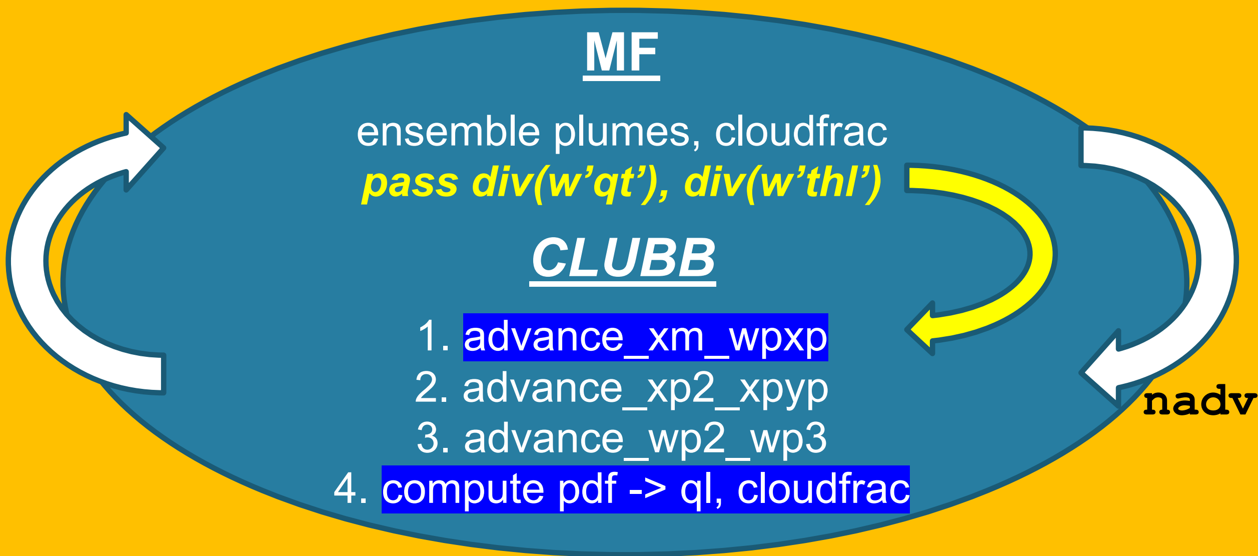
Coupling CAM to CLUBB

clubb_intr.F90

Convert CAM variables (T,qv,ql) to CLUBB variables:

$$\theta_l = \left(T - Lq_l/c_p\right)/\Pi$$

$$q_t = q_v + q_l$$



Convert CLUBB variables to CAM variables

$$T^{t+\Delta t} = f\left(\theta_l{}^{t+\Delta t}, q_l{}^{t+\Delta t}\right)$$

$$q_v{}^{t+\Delta t} = q_t{}^{t+\Delta t} + q_l{}^{t+\Delta t}$$

Update State

Surface Driven Convection

(Chinet 2003; Siebisma et al. 2007; Suselj et al. 2013)

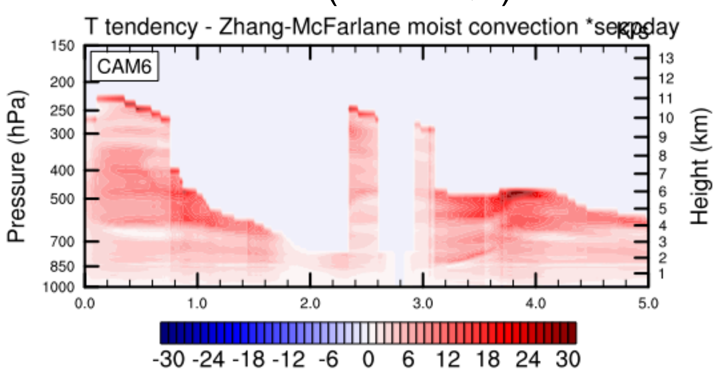
- Convective velocity scale $w_* = (g\beta Q_* z_*)^{1/3}$
  - $\theta_{v*} = \overline{w'\theta_v'}_{sfc}/w_*$
  - $q_{t*} = \overline{w'q_t'}_{sfc}/w_*$
  - $\sigma_w/w_*, \sigma_{q_t}/q_{t*}, \sigma_{\theta_v}/\theta_{v*}$ = constants from LES
  - Initialize ensemble: $w_n(z=0)$ from n samples between $1.5 * \sigma_w - 3.0 * \sigma_w$
  - $\theta_{v,n}, q_{t,n}$ from assumed covariances $C(w,\theta_v)$, $C(w,q_t)$ = constants from LES

Integrate steady-state plume equations:

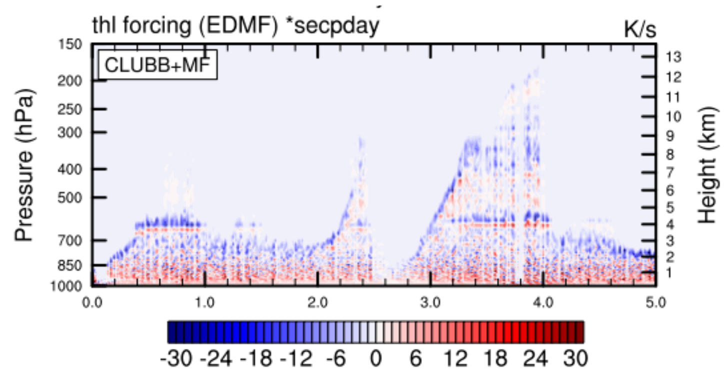
$$\frac{\partial \varphi_{u_n}}{\partial z} = \varepsilon_{u_n}(\overline{\varphi} - \varphi_{u_n}) + \frac{S_{\varphi,u_n}}{w_{u_n}},$$

$$\frac{1}{2}\frac{\partial w^2_{u_n}}{\partial z} = a_w B_{u_n} - b_w \varepsilon_{u_n} w^2_{u_n},$$

ZM (Conv. QE)



Surface Driven



How to integrate MFs into CLUBB

CLUBB
turbulence

$$\frac{\partial \overline{r_t}}{\partial t} = -\overline{w}\frac{\partial \overline{r_t}}{\partial z} - \frac{\partial \overline{w'r_t'}}{\partial z} + \left.\frac{\partial \overline{r_t}}{\partial t}\right|_{forcing}$$

Differences from traditional EDMF

- ED is prognostic, high-order (i.e., CLUBB)
- ED and MF fluxes are not summed together prior to solving
- Rather the divergence of fluxes are taken separately and tendencies summed

How to integrate MFs into CLUBB

CLUBB turbulence

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Handled by dycore

MF-plume ensemble

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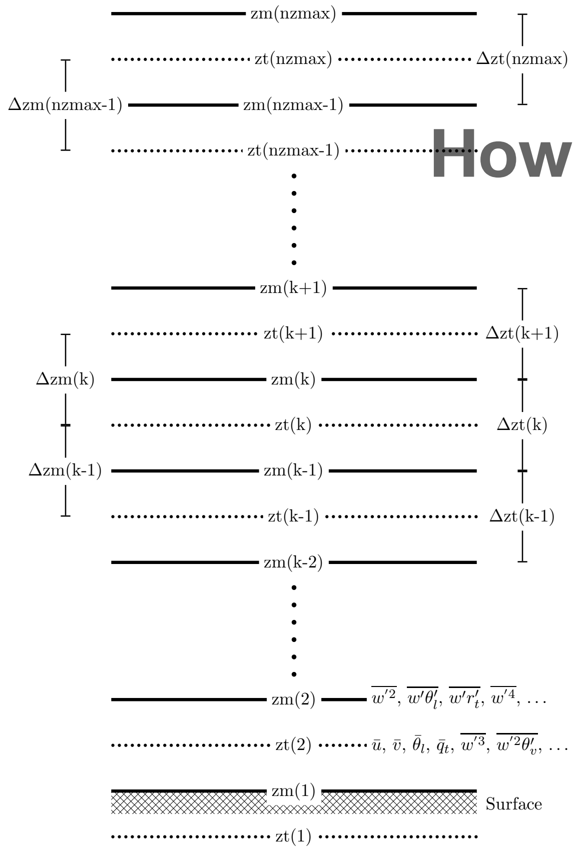
How to integrate MFs into CLUBB



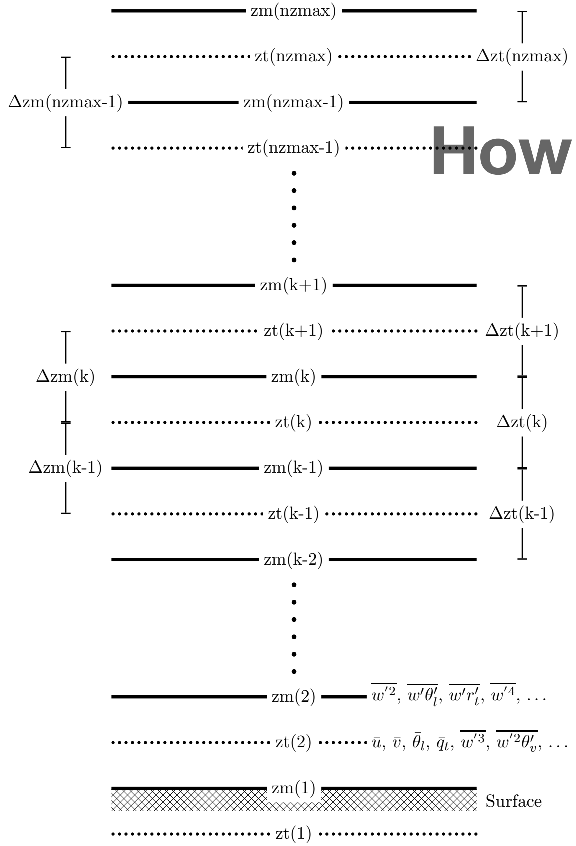
$$\begin{bmatrix} L_{3,k_{xm}} & L_{4,k_{xm}} & L_{5,k_{xm}} & 0 & 0 & 0 & \dots \\ L_{2,k_{wpxp}} & L_{3,k_{wpxp}} & L_{4,k_{wpxp}} & L_{5,k_{wpxp}} & 0 & 0 & \dots \\ L_{1,k_{xm}} & L_{2,k_{xm}} & L_{3,k_{xm}} & L_{4,k_{xm}} & L_{5,k_{xm}} & 0 & \dots \\ 0 & L_{1,k_{wpxp}} & L_{2,k_{wpxp}} & L_{3,k_{wpxp}} & L_{4,k_{wpxp}} & L_{5,k_{wpxp}} & \dots \\ \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\ \dots & L_{1,k_{xm}} & L_{2,k_{xm}} & L_{3,k_{xm}} & L_{4,k_{xm}} & L_{5,k_{xm}} & 0 \\ \dots & 0 & L_{1,k_{wpxp}} & L_{2,k_{wpxp}} & L_{3,k_{wpxp}} & L_{4,k_{wpxp}} & L_{5,k_{wpxp}} \\ \dots & 0 & 0 & L_{1,k_{xm}} & L_{2,k_{xm}} & L_{3,k_{xm}} & L_{4,k_{xm}} \\ \dots & 0 & 0 & 0 & L_{1,k_{wpxp}} & L_{2,k_{wpxp}} & L_{3,k_{wpxp}} \end{bmatrix} \times \begin{bmatrix} \overline{r_t}^{t+\Delta t}_{k=1} \\ \overline{w'r_t'}^{t+\Delta t}_{k=1} \\ \overline{r_t}^{t+\Delta t}_{k=2} \\ \overline{w'r_t'}^{t+\Delta t}_{k=2} \\ \overline{r_t}^{t+\Delta t}_{k=3} \\ \overline{w'r_t'}^{t+\Delta t}_{k=3} \\ \vdots \\ \overline{r_t}^{t+\Delta t}_{k=nz-2} \\ \overline{w'r_t'}^{t+\Delta t}_{k=nz-2} \\ \overline{r_t}^{t+\Delta t}_{k=nz-1} \\ \overline{w'r_t'}^{t+\Delta t}_{k=nz-1} \\ \overline{r_t}^{t+\Delta t}_{k=nz} \\ \overline{w'r_t'}^{t+\Delta t}_{k=nz} \end{bmatrix} = \begin{bmatrix} R_{k_{xm}} \\ R_{k_{wpxp}} \\ R_{k_{xm}} \\ R_{k_{wpxp}} \\ R_{k_{xm}} \\ R_{k_{wpxp}} \\ \vdots \\ R_{k_{xm}} \\ R_{k_{wpxp}} \\ R_{k_{xm}} \\ R_{k_{wpxp}} \\ R_{k_{xm}} \\ R_{k_{wpxp}} \end{bmatrix}$$

Related pairs of means and fluxes are solved simultaneously: 5-band diagonal matrix

$$\frac{\overline{r_t}^{t+\Delta t}}{\Delta t} + \frac{\partial}{\partial z}\overline{w'r_t'}^{t+\Delta t} = \frac{\overline{r_t}^{t}}{\Delta t} + \left.\frac{\partial \overline{r_t}}{\partial t}\right|_{forcing}$$

$$\frac{\overline{w'r_t'}^{t+\Delta t}}{\Delta t} + \overline{w}\frac{\partial}{\partial z}\overline{w'r_t'}^{t+\Delta t} + \frac{\partial}{\partial z}\left(a_1\frac{\overline{w'^3}}{\overline{w'^2}}\overline{w'r_t'}^{t+\Delta t}\right) + \overline{w'^2}\frac{\partial \overline{r_t}^{t+\Delta t}}{\partial z}$$
$$+ (1-C_7)\overline{w'r_t'}^{t+\Delta t}\frac{\partial \overline{w}}{\partial z} + \frac{C_6}{\tau}\overline{w'r_t'}^{t+\Delta t} - \frac{\partial}{\partial z}\left[(K_{w6}+\nu_6)\frac{\partial}{\partial z}\overline{w'r_t'}^{t+\Delta t}\right]$$
$$= \frac{\overline{w'r_t'}^{t}}{\Delta t} + (1-C_7)\frac{g}{\theta_0}\overline{r_t'\theta_v'}^{t}$$

How to integrate MFs into CLUBB



$$
\begin{bmatrix}
L_{3,k_{xm}} & L_{4,k_{xm}} & L_{5,k_{xm}} & 0 & 0 & 0 & \cdots \\
L_{2,k_{wpxp}} & L_{3,k_{wpxp}} & L_{4,k_{wpxp}} & L_{5,k_{wpxp}} & 0 & 0 & \cdots \\
L_{1,k_{xm}} & L_{2,k_{xm}} & L_{3,k_{xm}} & L_{4,k_{xm}} & L_{5,k_{xm}} & 0 & \cdots \\
0 & L_{1,k_{wpxp}} & L_{2,k_{wpxp}} & L_{3,k_{wpxp}} & L_{4,k_{wpxp}} & L_{5,k_{wpxp}} & \cdots \\
\ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\
\cdots & L_{1,k_{xm}} & L_{2,k_{xm}} & L_{3,k_{xm}} & L_{4,k_{xm}} & L_{5,k_{xm}} & 0 \\
\cdots & 0 & L_{1,k_{wpxp}} & L_{2,k_{wpxp}} & L_{3,k_{wpxp}} & L_{4,k_{wpxp}} & L_{5,k_{wpxp}} \\
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\end{bmatrix}
\times
\begin{bmatrix}
\overline{r_t}^{t+\Delta t}_{k=1} \\
\overline{w'r_t'}^{t+\Delta t}_{k=1} \\
\overline{r_t}^{t+\Delta t}_{k=2} \\
\overline{w'r_t'}^{t+\Delta t}_{k=2} \\
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\vdots \\
\overline{r_t}^{t+\Delta t}_{k=nz-2} \\
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\overline{r_t}^{t+\Delta t}_{k=nz-1} \\
\overline{w'r_t'}^{t+\Delta t}_{k=nz-1} \\
\overline{r_t}^{t+\Delta t}_{k=nz} \\
\overline{w'r_t'}^{t+\Delta t}_{k=nz}
\end{bmatrix}
=
\begin{bmatrix}
R_{k_{xm}} \\
R_{k_{wpxp}} \\
R_{k_{xm}} \\
R_{k_{wpxp}} \\
R_{k_{xm}} \\
R_{k_{wpxp}} \\
\vdots \\
R_{k_{xm}} \\
R_{k_{wpxp}} \\
R_{k_{xm}} \\
R_{k_{wpxp}} \\
R_{k_{xm}} \\
R_{k_{wpxp}}
\end{bmatrix}
$$

Related pairs of means and fluxes are solved simultaneously: 5-band diagonal matrix

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\frac{\overline{w'r_t'}^{t+\Delta t}}{\Delta t} + \overline{w}\frac{\partial}{\partial z}\overline{w'r_t'}^{t+\Delta t} + \frac{\partial}{\partial z}\left(a_1\frac{\overline{w'^3}}{\overline{w'^2}}\overline{w'r_t'}^{t+\Delta t}\right) + \overline{w'^2}\frac{\partial \overline{r_t}^{t+\Delta t}}{\partial z}
$$

$$
+ (1-C_7)\overline{w'r_t'}^{t+\Delta t}\frac{\partial \overline{w}}{\partial z} + \frac{C_6}{\tau}\overline{w'r_t'}^{t+\Delta t} - \frac{\partial}{\partial z}\left[(K_{w6}+\nu_6)\frac{\partial}{\partial z}\overline{w'r_t'}^{t+\Delta t}\right]
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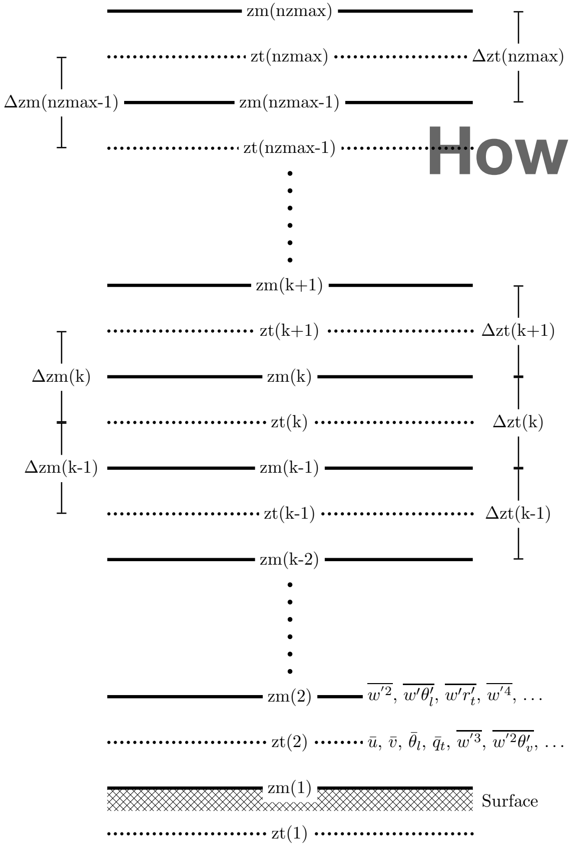
How to integrate MFs into CLUBB



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\ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\
\dots & L_{1,k_{xm}} & L_{2,k_{xm}} & L_{3,k_{xm}} & L_{4,k_{xm}} & L_{5,k_{xm}} & 0 \\
\dots & 0 & L_{1,k_{wpxp}} & L_{2,k_{wpxp}} & L_{3,k_{wpxp}} & L_{4,k_{wpxp}} & L_{5,k_{wpxp}} \\
\dots & 0 & 0 & L_{1,k_{xm}} & L_{2,k_{xm}} & L_{3,k_{xm}} & L_{4,k_{xm}} \\
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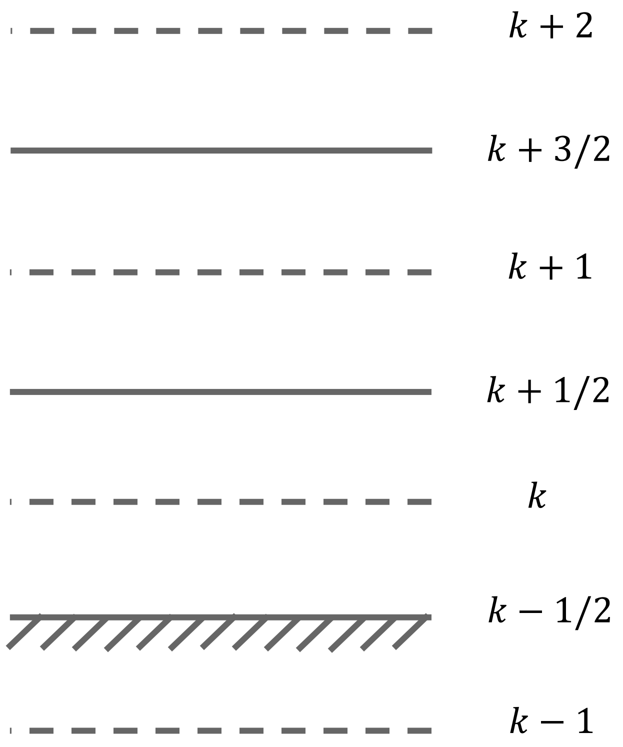
$$

Divergence Operator
(integrated forward in time by CLUBB)

$$\left(\frac{d\bar{X}}{dt}\right)_{forcing} = \left(\frac{dF}{dz}\right)_k = \frac{1}{\bar{\rho}_k \Delta z_k}\left[\tilde{\bar{\rho}}_{k+1/2}\boldsymbol{F}_{\boldsymbol{k+1/2}} - \tilde{\bar{\rho}}_{k-1/2}\boldsymbol{F}_{\boldsymbol{k-1/2}}\right]$$

Options for the Fluxes

- Centered Differences
- Partial Upwind (**pupwind*)
- Upwind



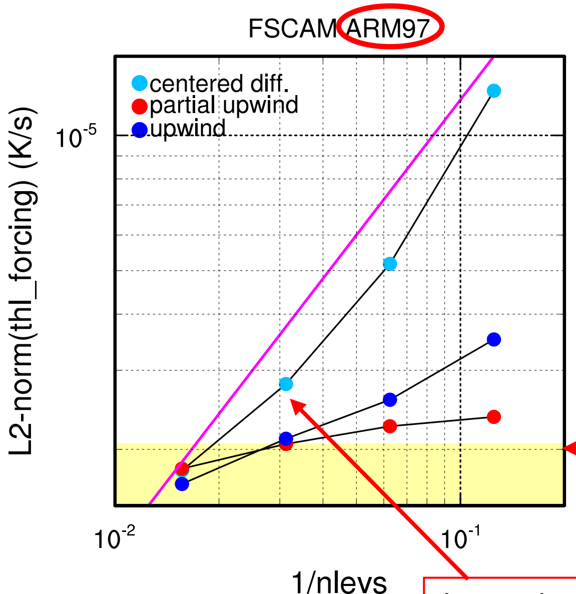
Convergence Experiments (in dz) – ARM97

$$L_2(t,\hat{x},x_{ref}) = \frac{\sum_{k=1}^{nlevs}\sqrt{\left(\hat{x}(t,k) - x_{ref}(t,k)\right)^2 * \Delta p(k)}}{\sum_{k=1}^{nlevs}\Delta p(k)}$$

$\hat{x} = x$ interpolated to x_{ref} grid

x_{ref} = solution for 128 level grid

$$x = -\frac{\partial F}{\partial z}$$



Time-average $U(t)$

Large absolute error for 32levs

Uncertainty in the reference solution:

$U(t) = max\left[L_2(t,x_{ref0},x_{ref1}), L_2(t,x_{ref0},x_{ref2}), L_2(t,x_{ref1},x_{ref2})\right]$

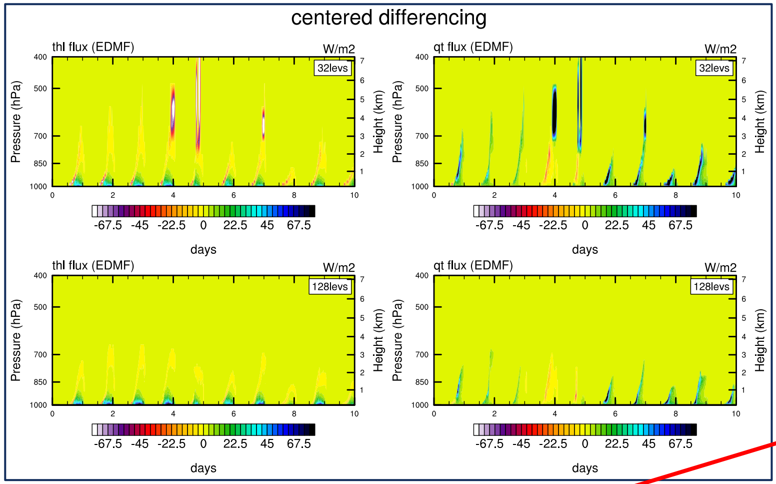
x_{ref0} = ref soln. for centered diff.
x_{ref1} = ref soln. for partial upwind
x_{ref2} = ref. soln. for upwind

Convergence Experiments (in dz) – ARM97

$Mc(\theta_{l_{MF}} - \theta_{l_{ENV}})$ $Mc(q_{t_{MF}} - q_{t_{ENV}})$

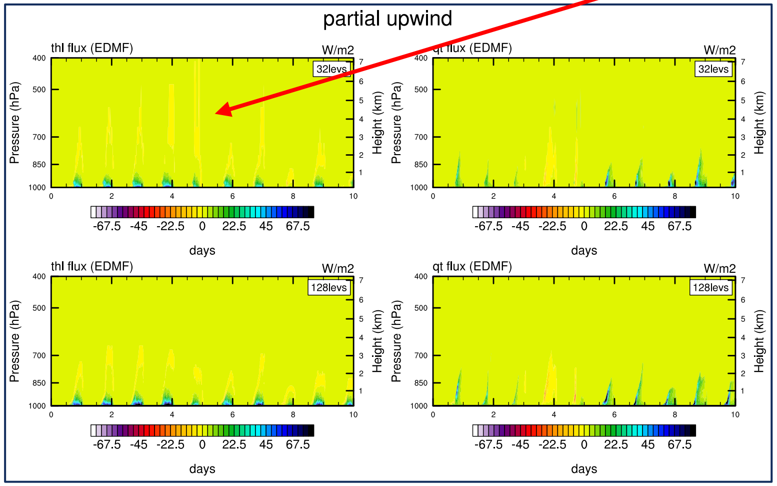
32 levs

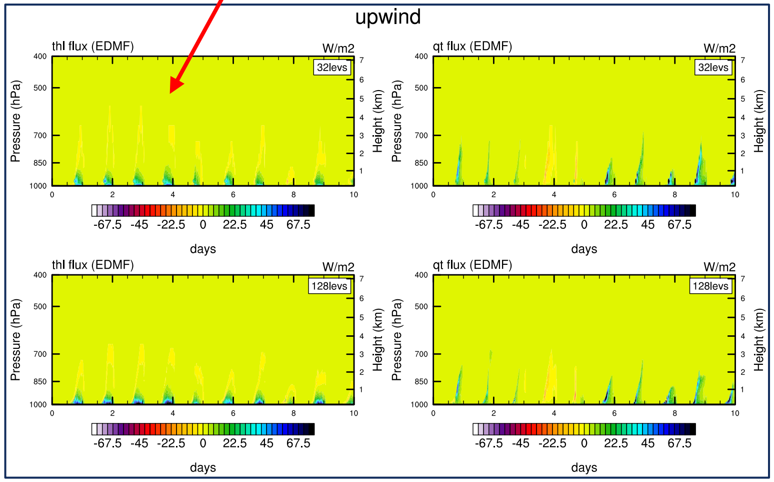
128 levs



Negative fluxes are oddly high up in the troposphere

They look much better for upwind





Current Implementation

Energy conservation, with E the vertically integrated energy:

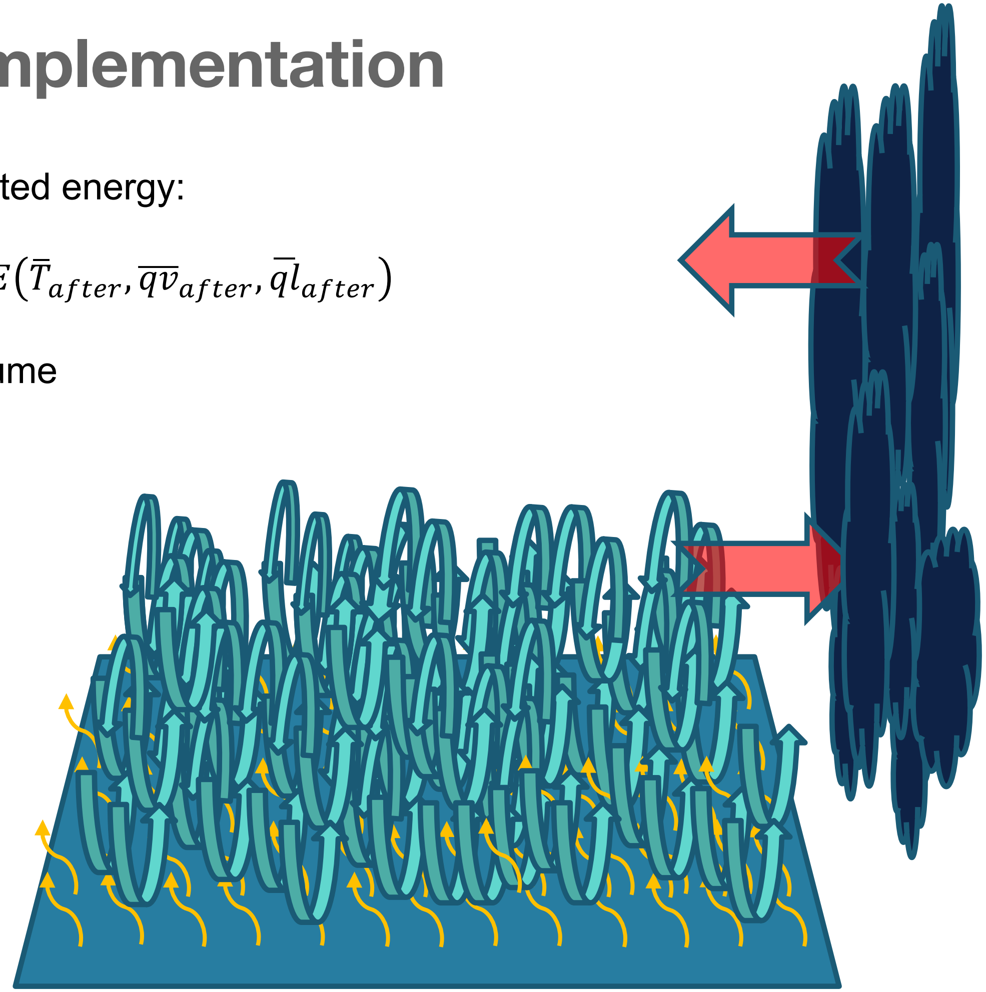
$$E\left(\bar{T}_{before}, \overline{qv}_{before}, \bar{q}l_{before}\right) + dt * sfcfluxes = E\left(\bar{T}_{after}, \overline{qv}_{after}, \bar{q}l_{after}\right)$$

CLUBB is assumed to occupy the entire grid volume

$$\bar{X}_{after} = \bar{X}_{CLUBB}$$

With Boundary Conditions

$$\overline{w'X'}_{clubb,sfc} = sfcfluxes$$



2-Volume Problem

Energy conservation, with E the vertically integrated energy:

$$E(\bar{T}_{before}, \overline{qv}_{before}, \bar{q}l_{before}) + dt * sfcfluxes = E(\bar{T}_{after}, \overline{qv}_{after}, \bar{q}l_{after})$$

Conserving energy with two volumes requires:

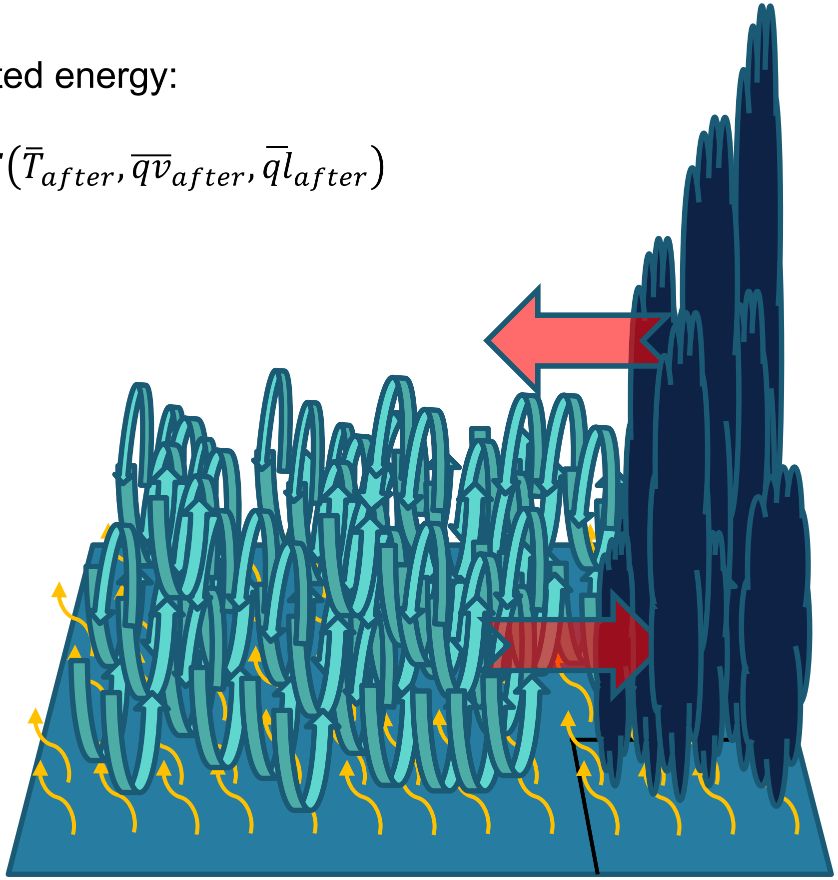
$$\bar{X}_{after} = a\bar{X}_a + b\bar{X}_b$$

$$a + b = 1$$

and Boundary Conditions for each volume:

$$\overline{w'X'}_{b,sfc} = sfcfluxes$$

$$\left[\sum_n w_{a,n} * (X_{a,n} - \bar{X}_b)\right]_{a,sfc} = sfcfluxes$$



Where we are now

- CLUBB+MF v0 is on cam_development
  - Surface driven convection (draws from an assumed w-pdf at sfc)
  - Plume profiles from a stochastically entraining plume model
  - Steady-state w-equation
  - Simple auto-conversion microphysics (Suselj et al 2019)

- CLUBB+MF v1.X
  - Energy conservation fix
  - Switched to upwind fluxes
  - Evaporation of rain, downdrafts & cold pool param (Suselj et al 2019)
  - Monte-Carlo sampling of the surface w-pdf (use CLUBB pdf?)
  - Experimental 2-volume framework (?)