2021 CESM OCEAN MODEL WORKING GROUP MEETING

Characterizing the Oceanic Mesoscale Flow by Coarse-graining

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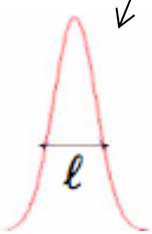
Our (LES / PDE) Approach

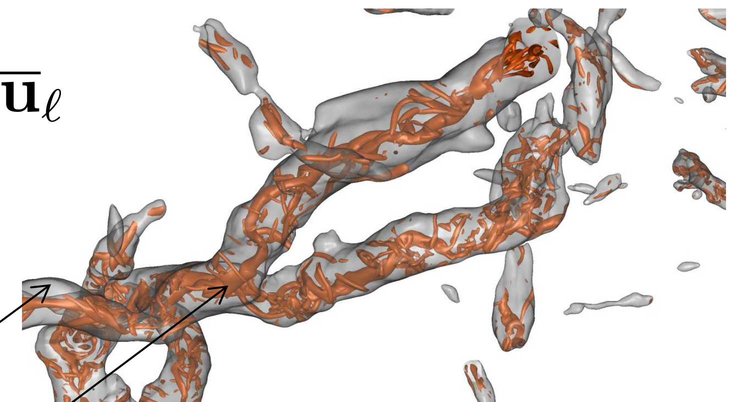
LES literature, Leonard (1974), Germano (1992), Eyink (1994), Meneveau et al. (1994), Ecke, Chen, Ouellette, …

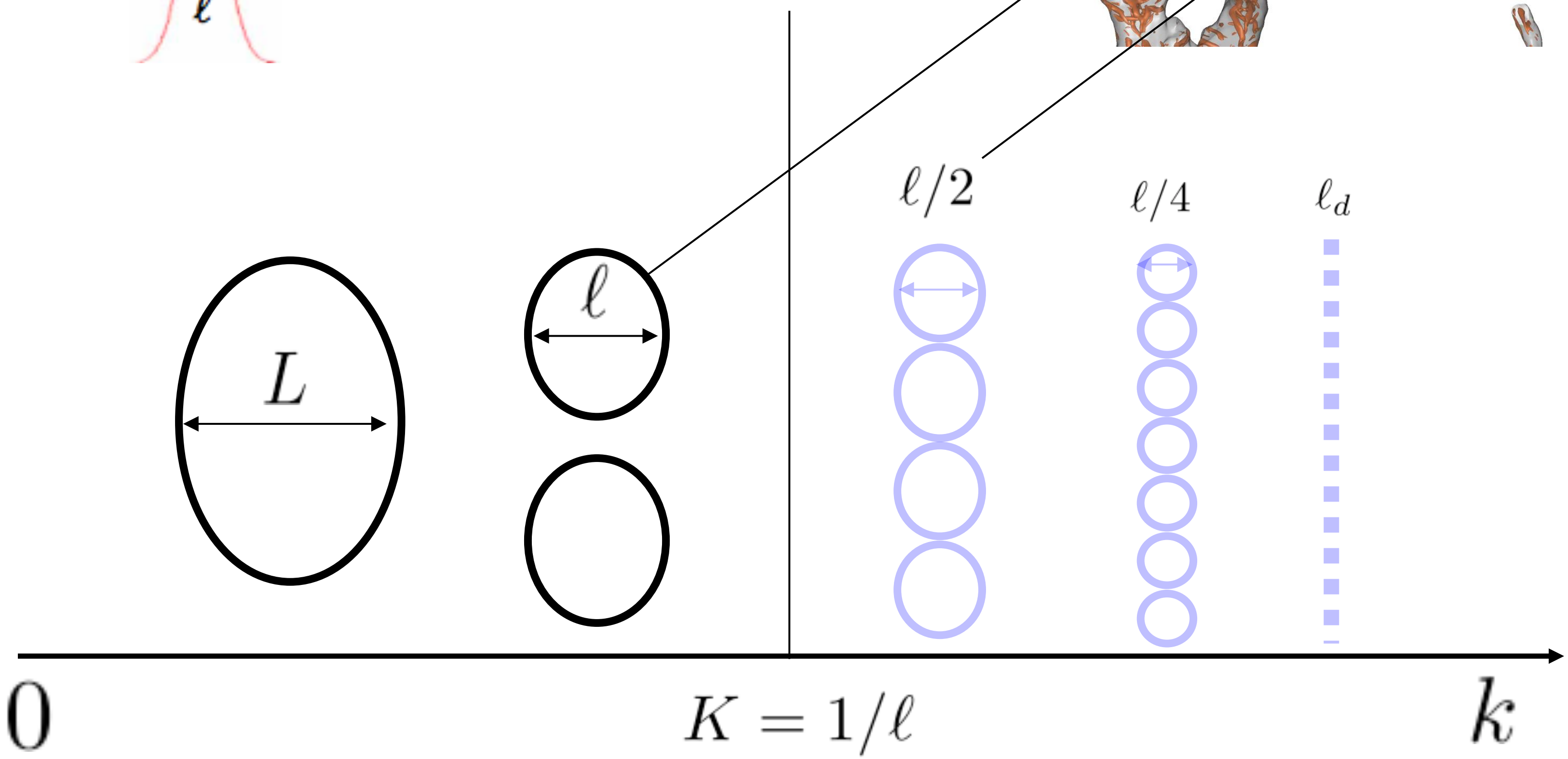
Coarse-graining (Filtering)

$$\overline{\mathbf{u}}_\ell(\mathbf{x}) = \int d\mathbf{r}\; G_\ell(\mathbf{r})\mathbf{u}(\mathbf{x}+\mathbf{r})$$



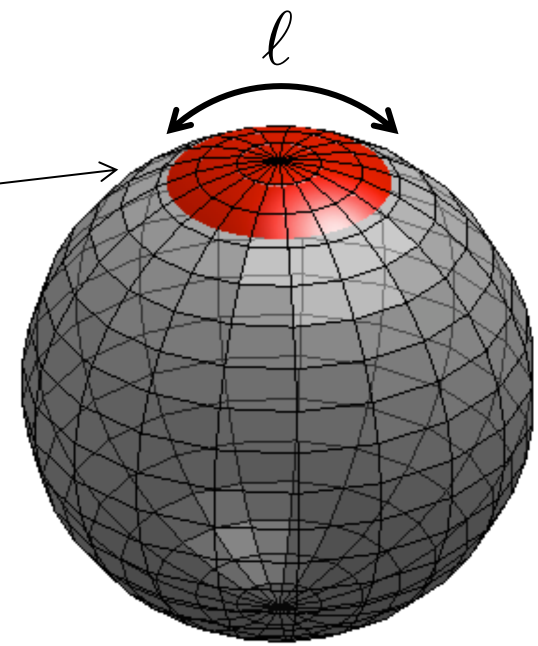
$\nabla \times \overline{\mathbf{u}}_\ell$





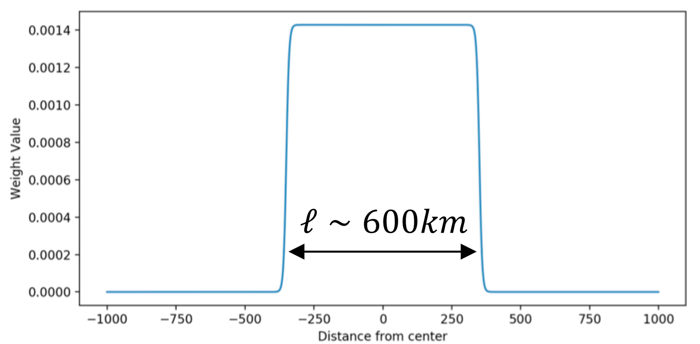
Additional Problems on the Sphere

Aluie, H. (2019). Convolutions on the sphere: Commutation with differential operators. *GEM-International Journal on Geomathematics*, *10*(1), 9.

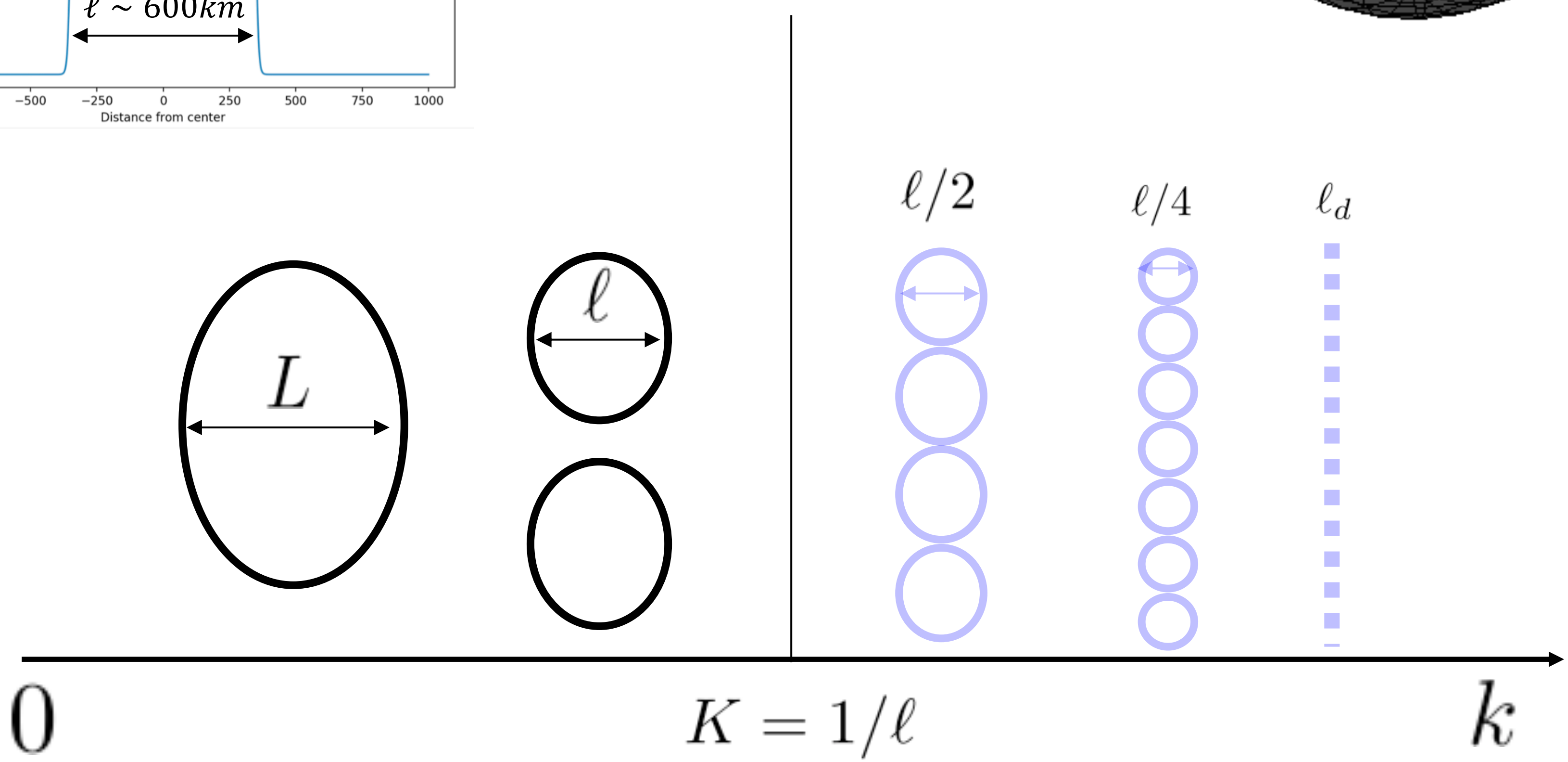


Coarse-graining (Filtering)

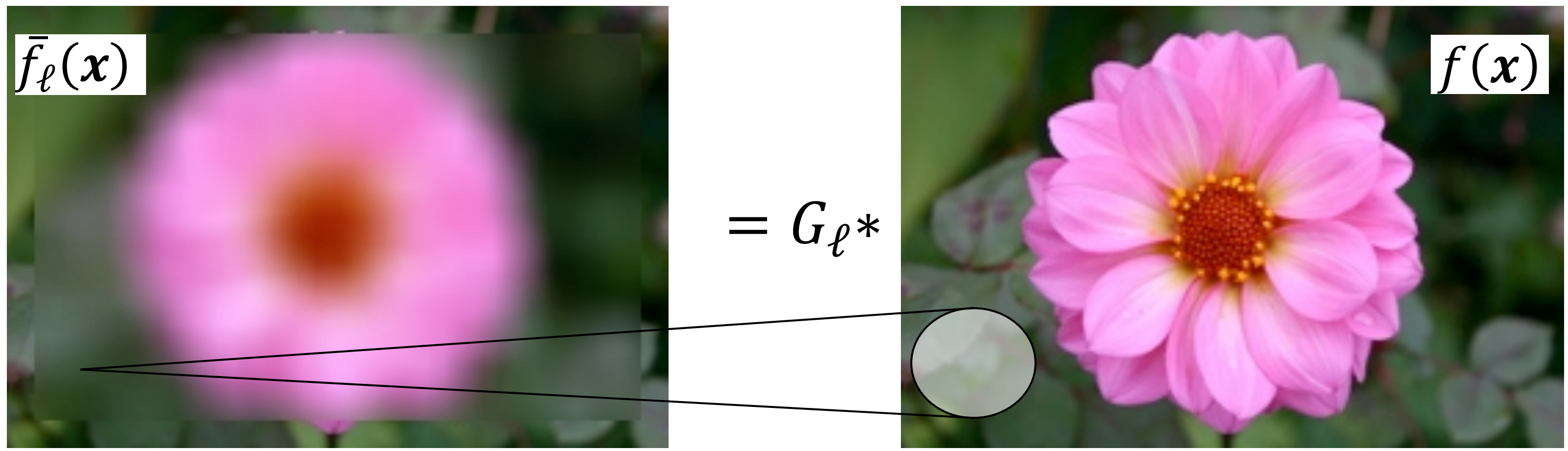
$$\overline{\mathbf{u}}_\ell(\mathbf{x}) = G_\ell * \mathbf{u}$$

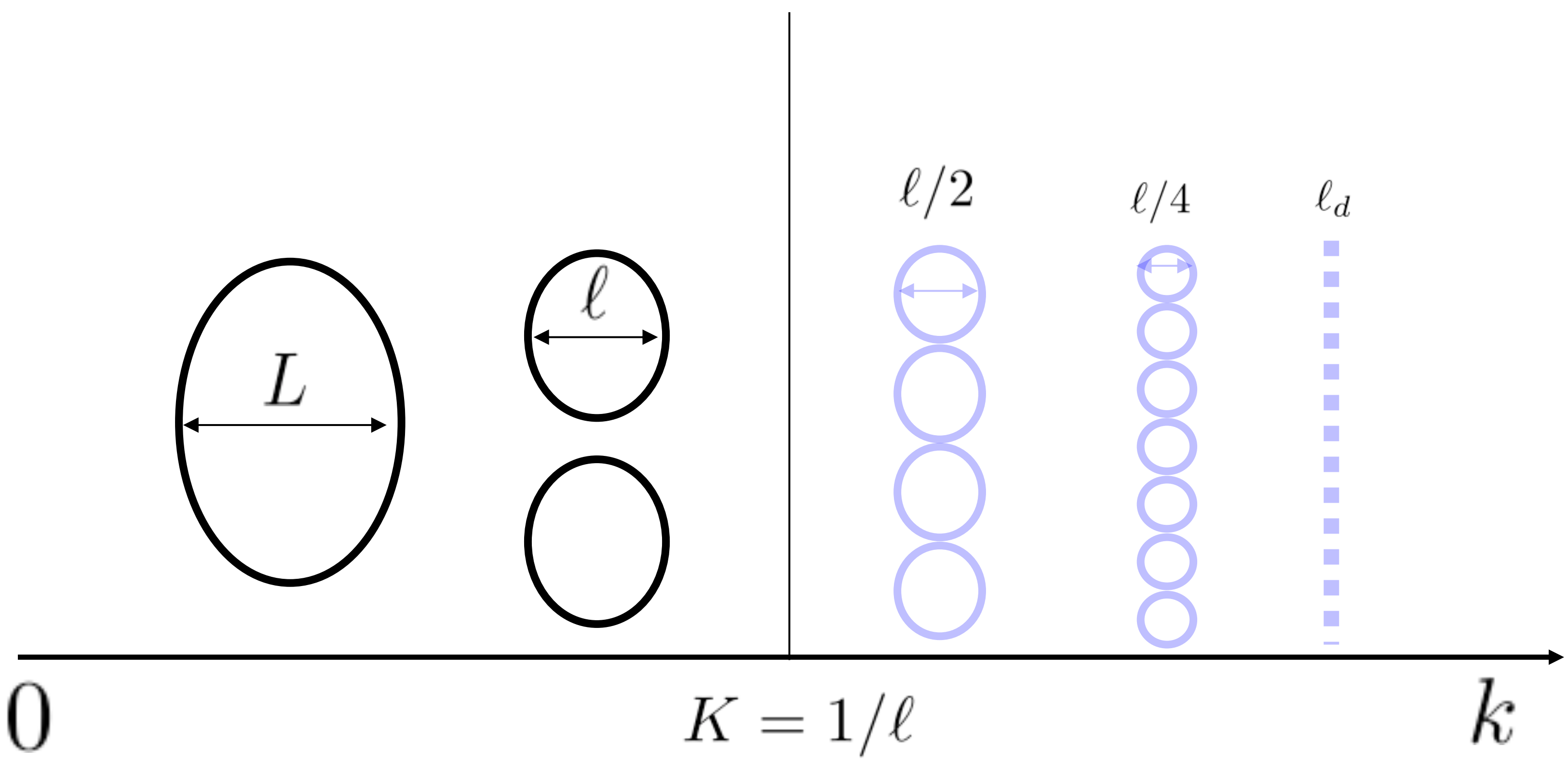


$$G_\ell(\boldsymbol{r}) = \frac{A}{2}\left(1 - \tanh\left(10\left(\frac{|\boldsymbol{r}|}{\ell/2} - 1\right)\right)\right)$$



example of coarse field





Datasets Analyzed:

Observations, AVISO:

Level 4 (L4) post-processed dataset of geostrophic currents

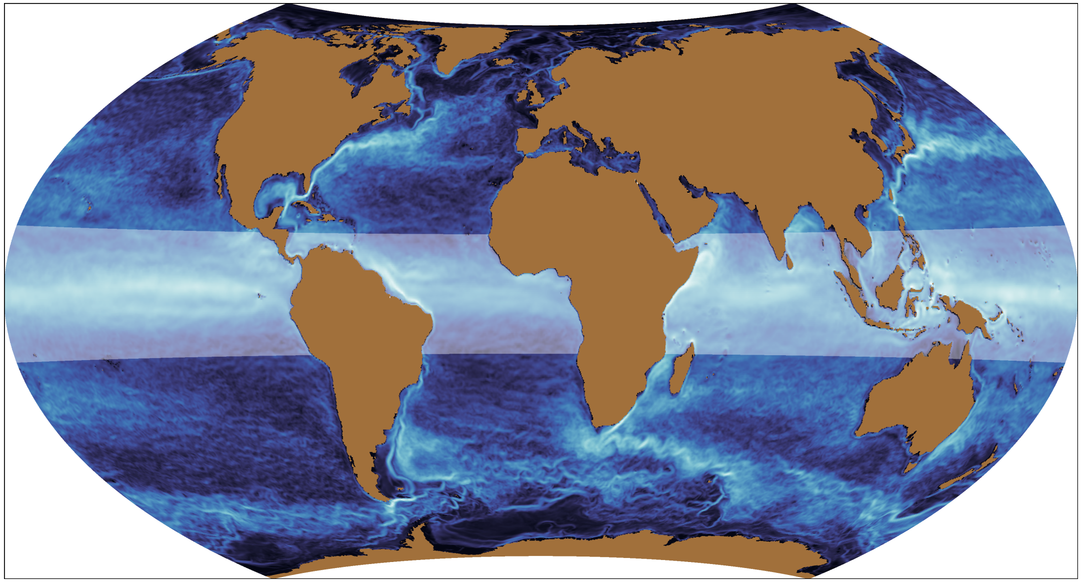
Gridded at a resolution of **0.25º × 0.25º**

Spanning the time window covering the period **from 2010-2018**

Ref:

Prod. ID: SEALEVEL_GLO_PHY_L4_REP_OBSERVATIONS_008_047

Pujol, et al. *Ocean Science* 12.5 (2016): 1067-1090.



Model, NEMO:

Weakly coupled **ocean-atm**. assimilation + forecasting system

Gridded at a resolution of **0.25º × 0.25º**

Spanning the time window covering the period from **2016-2019**

Ref:

Prod. ID: GLOBAL_ANALYSISFORECAST_PHY_CPL_001_015

Hewitt, et al. *Geoscientific Model Development* 4.2 (2011): 223-253.

Methods:

- **We only consider surface layer,** $\boldsymbol{u} = (u_{lat}, v_{long})$
- **We consider the geostrophic velocity components**
- **Average over geographical regions, [15º: 90º] North & [15º: 90º] South of Equator**
- **Continents are treated as zero velocity**

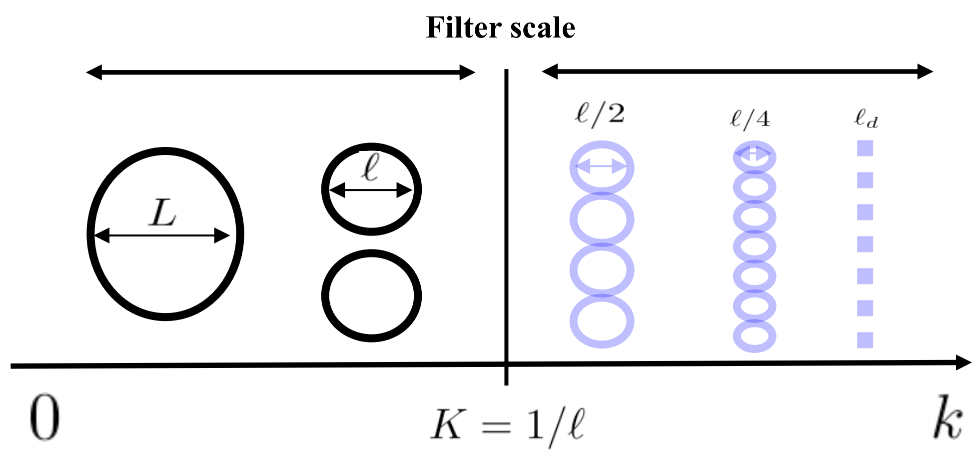
Coarse-graining the **Total** kinetic **Energy:**

Total Energy: $E = \frac{1}{2}\rho\,|\boldsymbol{u}|^2$

$\frac{1}{2}\,\rho|\overline{\boldsymbol{u}}_\ell|^2$

$\frac{1}{2}\,\rho(\,\overline{|\boldsymbol{u}_\ell^2|}\, - |\overline{\boldsymbol{u}}_\ell|^2)$

Coarse Energy, larger than the filter scale



Fine Energy, smaller than the filter scale

A natural choice as fine kinetic energy looks more like: $\frac{1}{2}\,\rho(\,|\boldsymbol{u}^2| - |\overline{\boldsymbol{u}}_\ell|^2)$, but this quantity is not positive definite!
[Vreman, Geurts, & Kuerten, JFM, 1994; Eyink & Aluie, PoF, 2009]

Jensen's inequality tells us: $\mathrm{E}[f(x)] \geq f(\mathrm{E}[x])$ for any convex $f(\mathrm{x})$

So in our case:

1] $f(\mathrm{u}) = u^2$ is convex
2] $\overline{\boldsymbol{u}} = G_\ell * \boldsymbol{u}$ with $G_\ell \geq 0$ is a weighted average

Hence,

$$\frac{1}{2}\,\rho(\,\overline{|\boldsymbol{u}_\ell^2|}\, - |\overline{\boldsymbol{u}}_\ell|^2) \geq 0$$

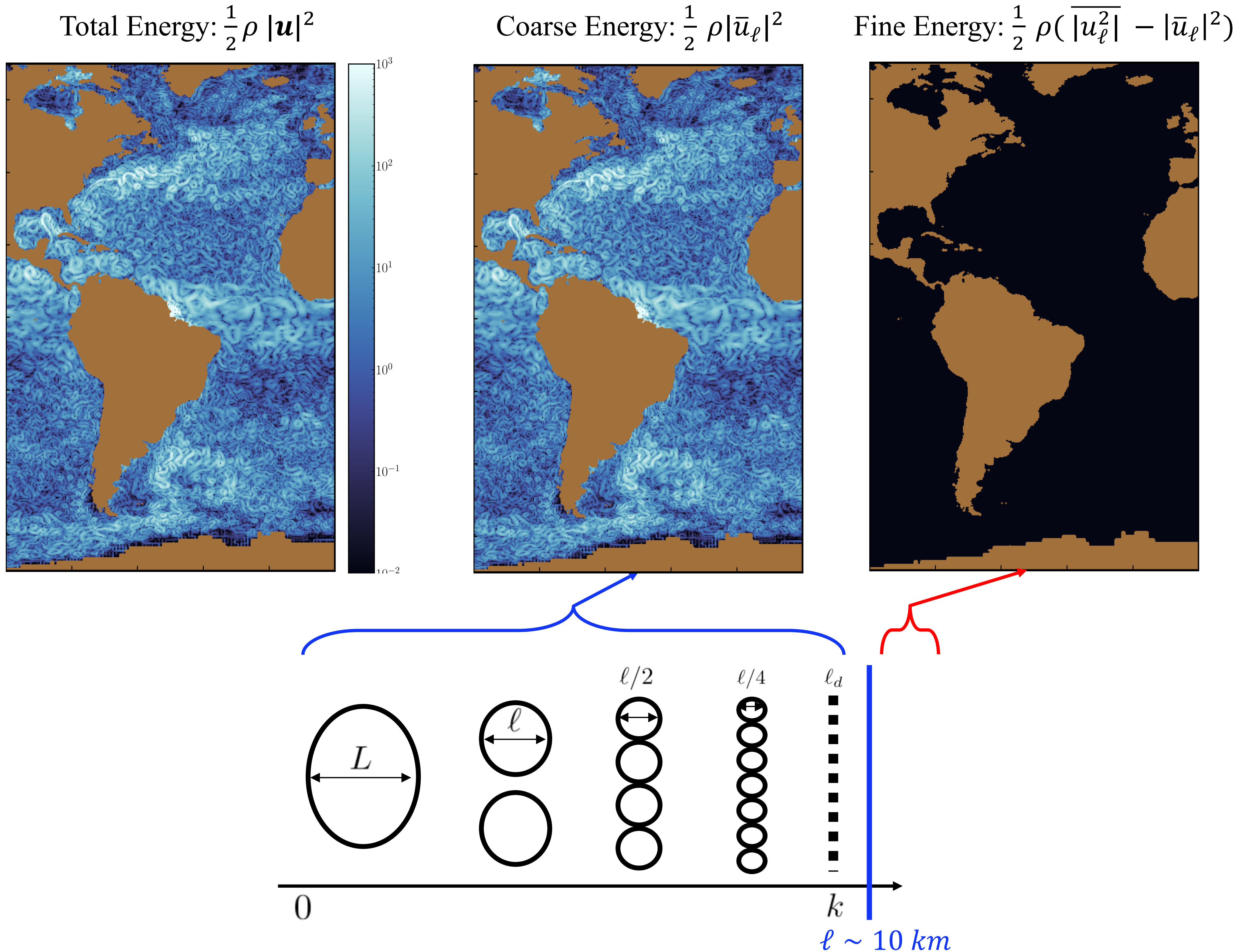
Moreover, defining the spatial average as, $\{\,...\} = 1/A\int d^2\boldsymbol{r}\,(\,...)$, we have, $\{\mathrm{G}_\ell\} = 1$, hence:

$$\{\overline{|\boldsymbol{u}_\ell^2|}\} = \{\boldsymbol{u}^2\}$$

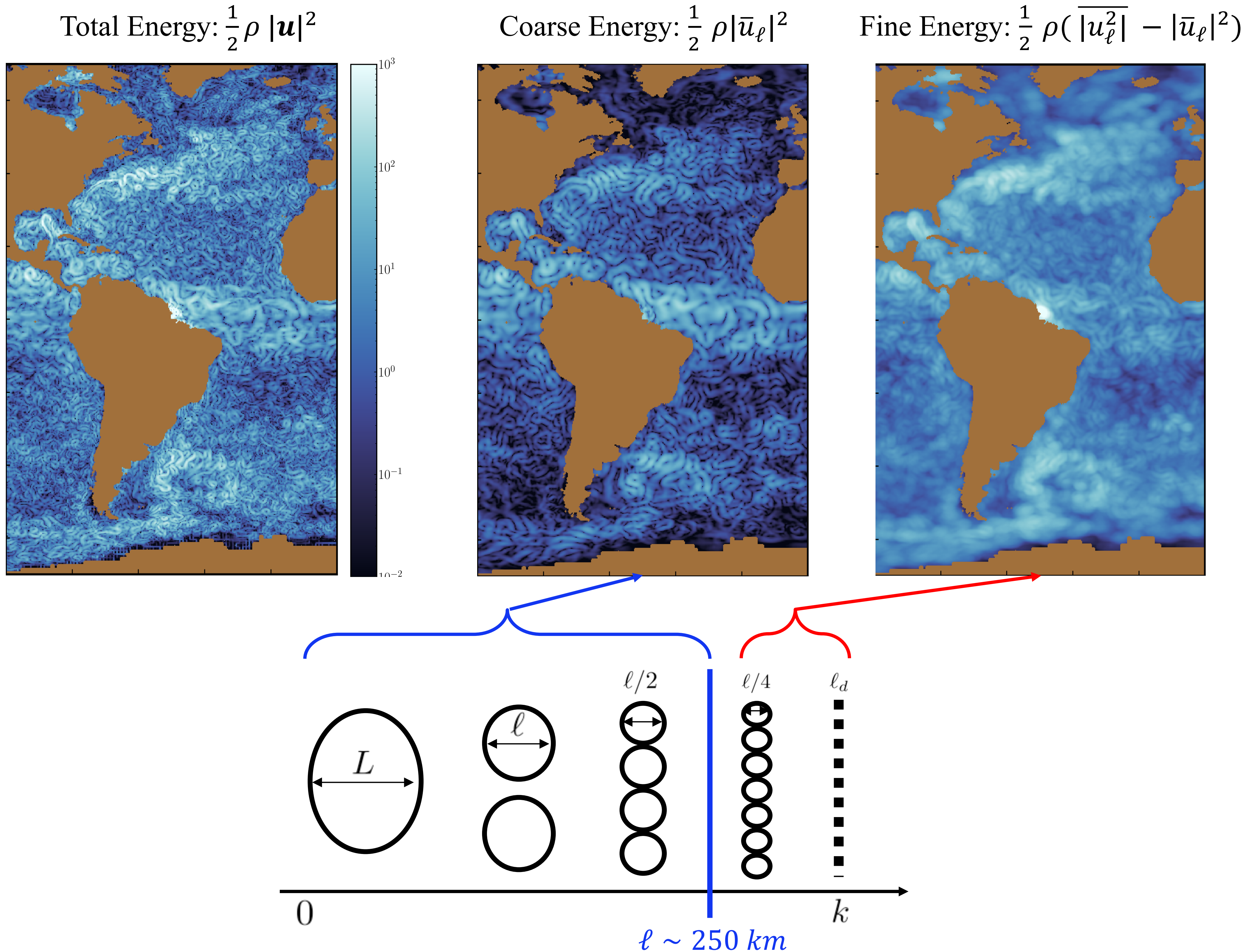
$$\frac{1}{2}\,\rho(\,\{\overline{|\boldsymbol{u}_\ell^2|}\} - \{|\overline{\boldsymbol{u}}_\ell|^2\}) + \frac{1}{2}\,\rho\{|\overline{\boldsymbol{u}}_\ell|^2\} = \frac{1}{2}\rho\,\{|\boldsymbol{u}|^2\}$$

Fine + **Coarse** = **Total**

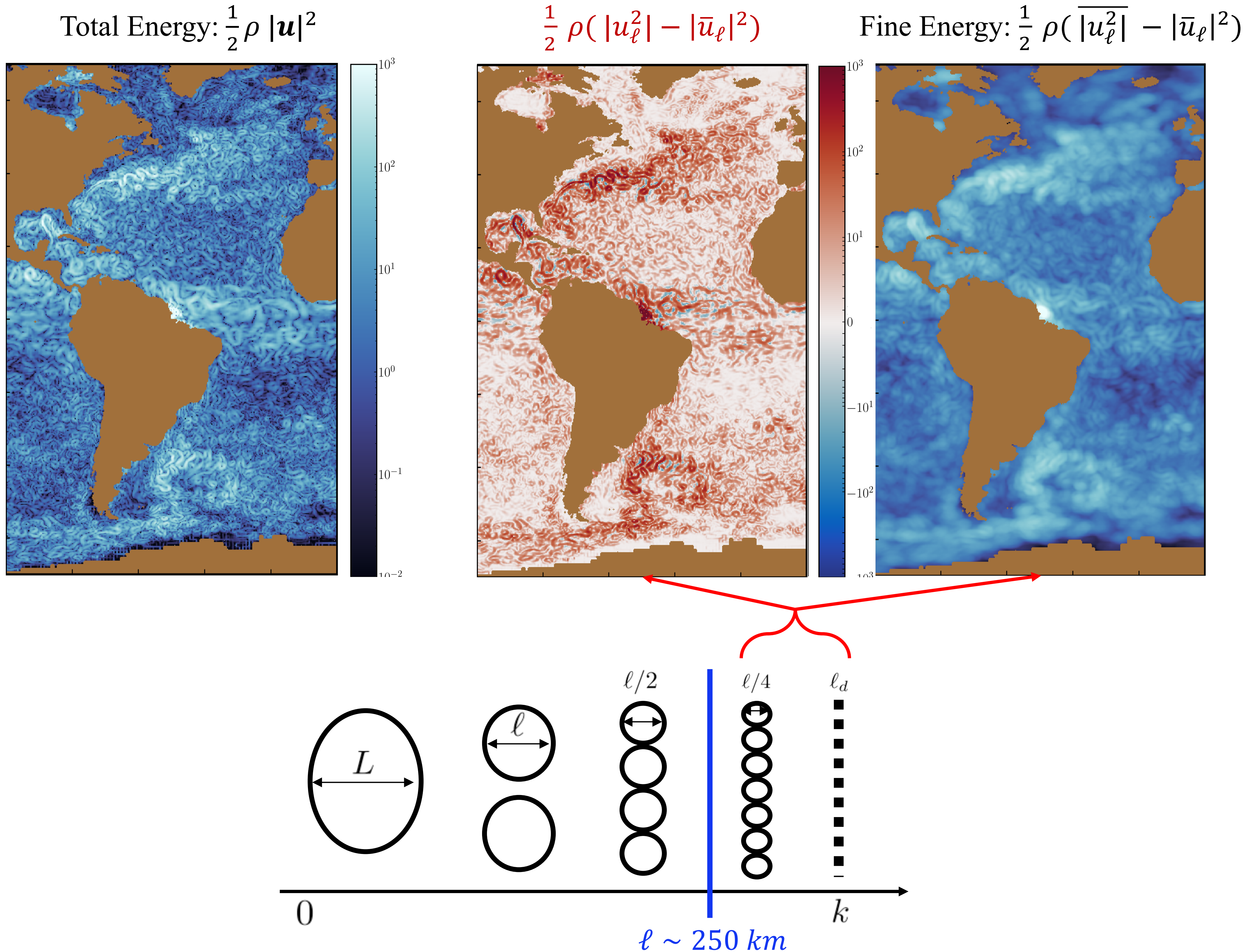
[Sadek & Aluie, Phys Rew Fluids, 2018]

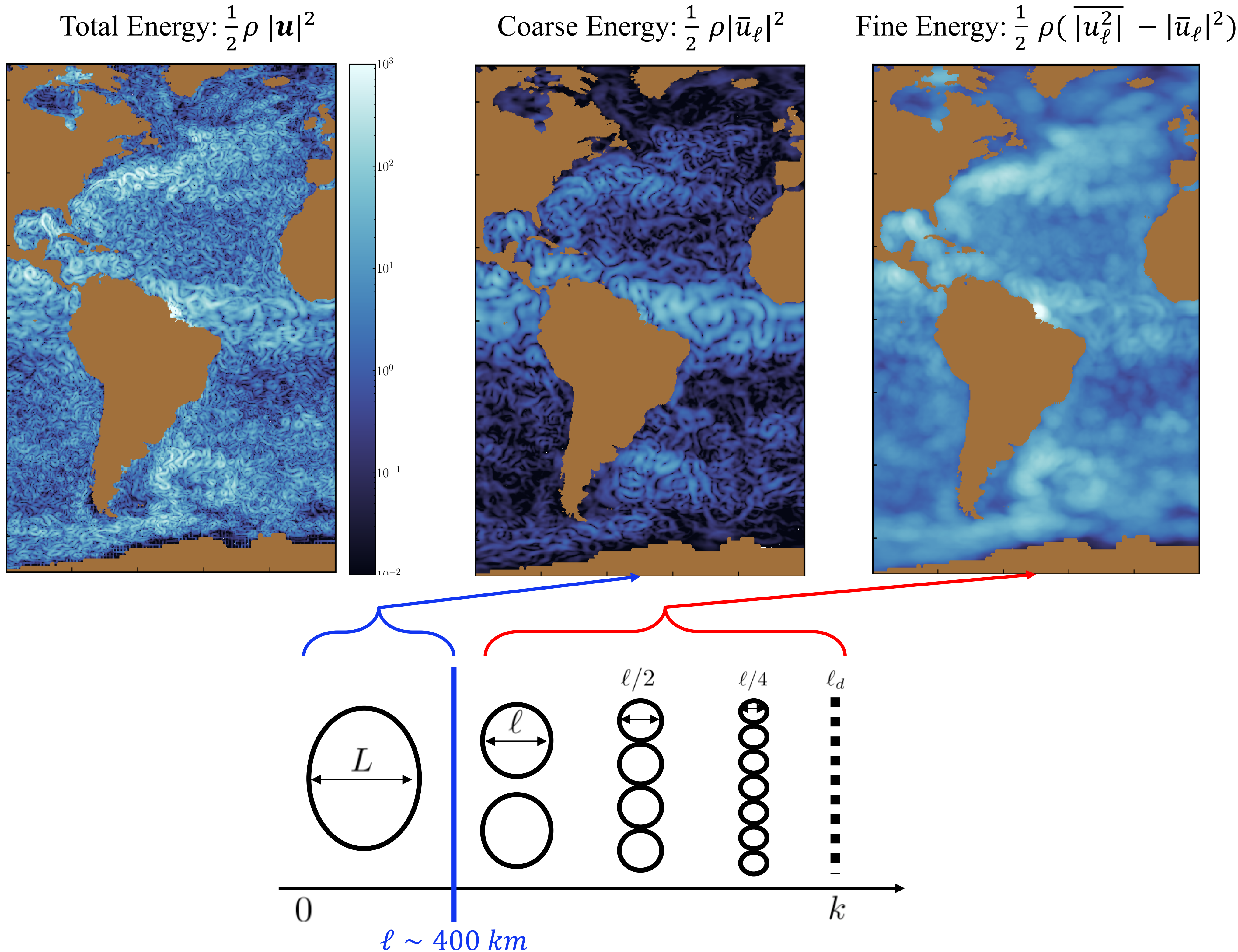


Total Energy: $\frac{1}{2}\rho\,|\boldsymbol{u}|^2$
Coarse Energy: $\frac{1}{2}\,\rho|\bar{u}_\ell|^2$
Fine Energy: $\frac{1}{2}\,\rho(\,\overline{|u_\ell^2|}\, - |\bar{u}_\ell|^2)$
10^3
10^2
10^1
10^0
10^{-1}
10^{-2}
L
ℓ
$\ell/2$
$\ell/4$
ℓ_d
0
k
$\ell \sim 10\ km$

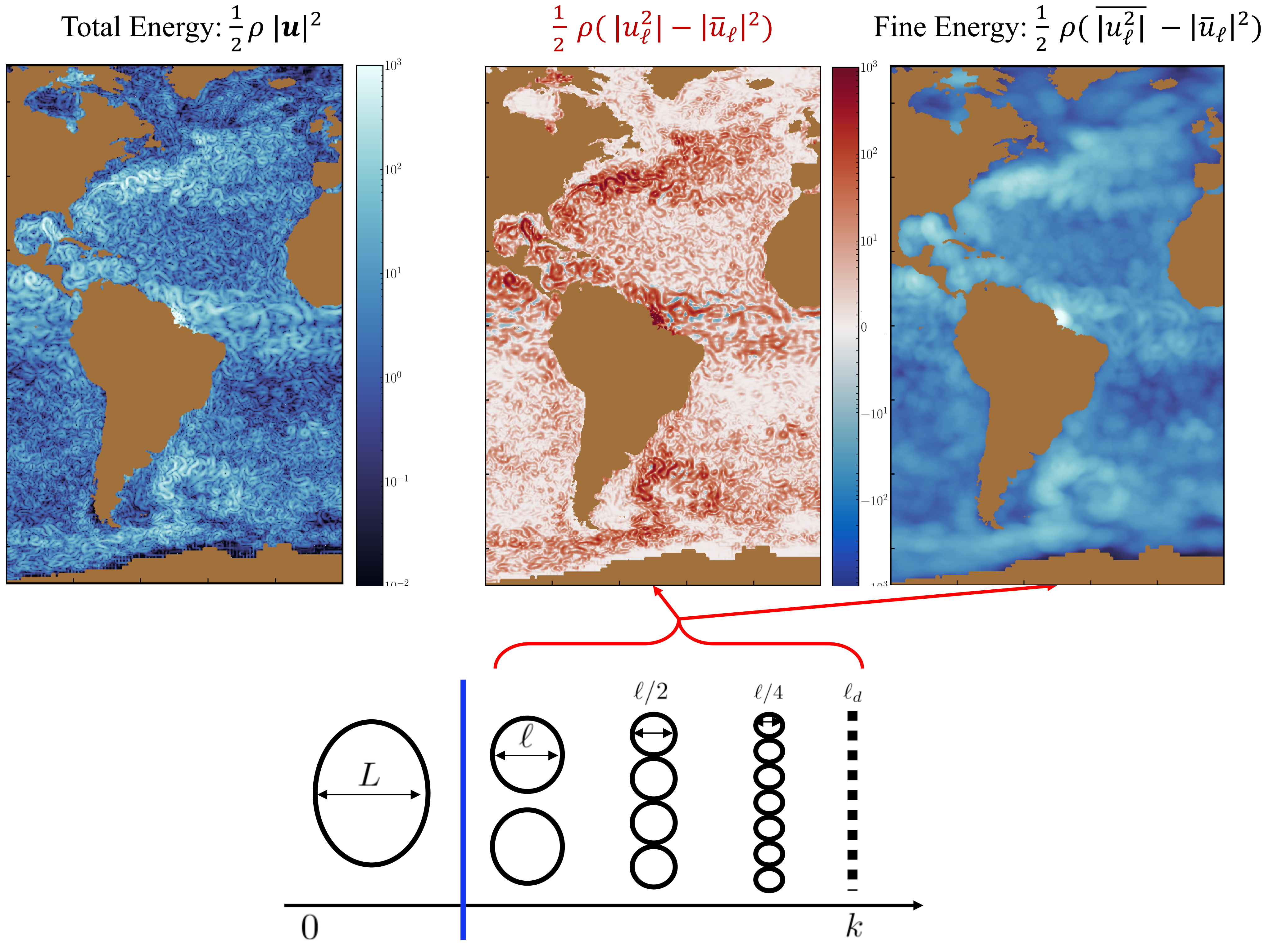


Total Energy: $\frac{1}{2}\rho\,|\boldsymbol{u}|^2$
Coarse Energy: $\frac{1}{2}\,\rho|\bar{u}_\ell|^2$
Fine Energy: $\frac{1}{2}\,\rho(\,\overline{|u_\ell^2|}\,-|\bar{u}_\ell|^2)$
10^3
10^2
10^1
10^0
10^{-1}
10^{-2}
L
ℓ
$\ell/2$
$\ell/4$
ℓ_d
0
k
$\ell \sim 250\ km$


Total Energy: $\frac{1}{2}\rho\,|\boldsymbol{u}|^2$
$\frac{1}{2}\,\rho(\,|u_\ell^2| - |\bar{u}_\ell|^2)$
Fine Energy: $\frac{1}{2}\,\rho(\,\overline{|u_\ell^2|}\, - |\bar{u}_\ell|^2)$
10^{3}
10^{2}
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{3}
10^{2}
10^{1}
0
-10^{1}
-10^{2}
L
ℓ
$\ell/2$
$\ell/4$
ℓ_d
0
k
$\ell \sim 250\ km$



Total Energy: $\frac{1}{2}\rho\,|\boldsymbol{u}|^2$
Coarse Energy: $\frac{1}{2}\,\rho|\bar{u}_\ell|^2$
Fine Energy: $\frac{1}{2}\,\rho(\,\overline{|u_\ell^2|}\, - |\bar{u}_\ell|^2)$
10^3
10^2
10^1
10^0
10^{-1}
10^{-2}
L
ℓ
$\ell/2$
$\ell/4$
ℓ_d
0
k
$\ell \sim 400\ km$

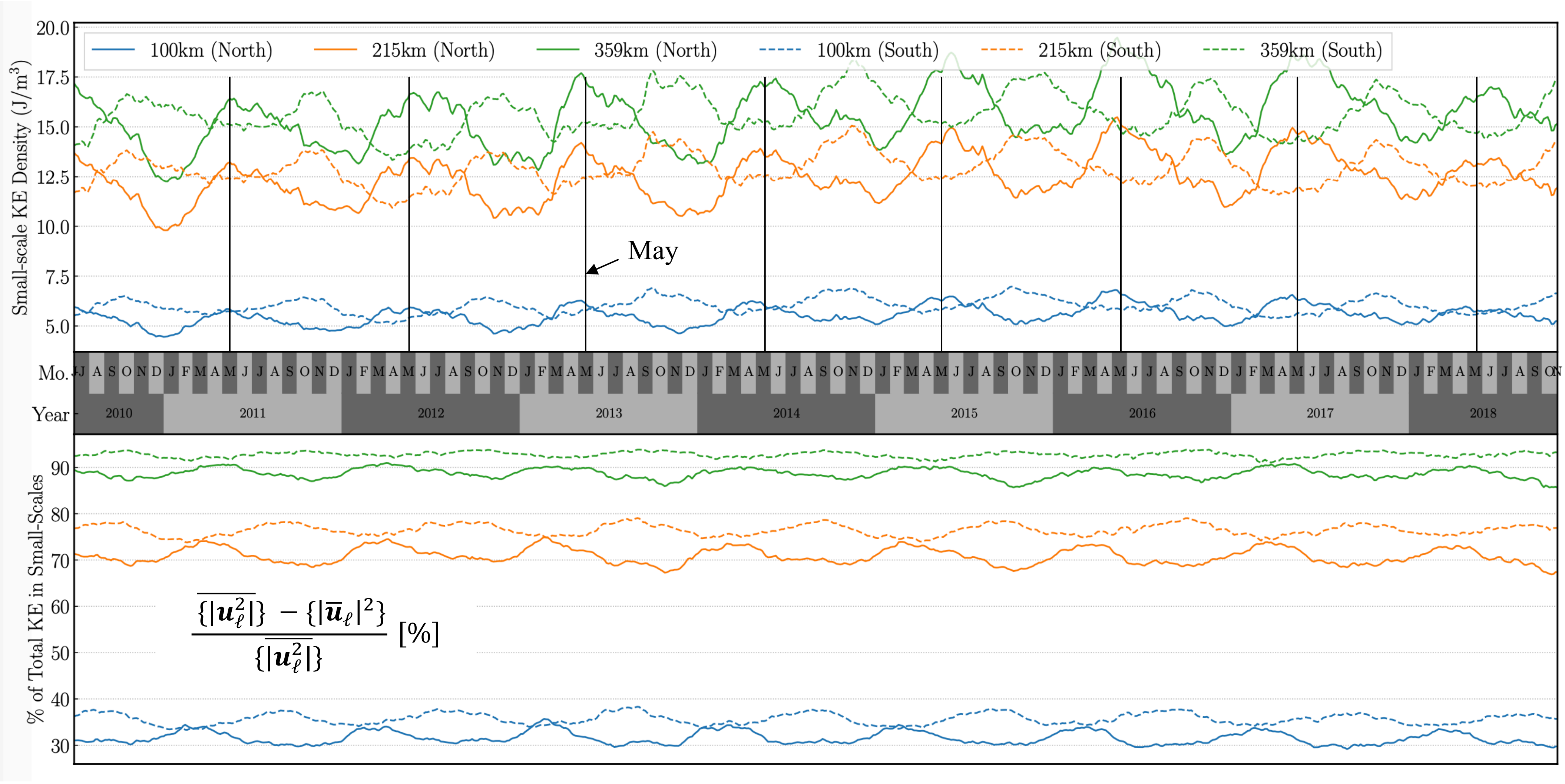


Total Energy: $\frac{1}{2}\rho\,|\boldsymbol{u}|^2$
$\frac{1}{2}\,\rho(\,|u_\ell^2| - |\bar{u}_\ell|^2)$
Fine Energy: $\frac{1}{2}\,\rho(\,\overline{|u_\ell^2|}\; - |\bar{u}_\ell|^2)$
L
ℓ
$\ell/2$
$\ell/4$
ℓ_d
0
k

Seasonality of Fine kinetic Energy

$$\frac{1}{2}\rho(\{\overline{|\boldsymbol{u}_\ell^2|}\} - \{|\overline{\boldsymbol{u}}_\ell|^2\})$$

Scale-decomposition in different regions: [15º:90º] North / [15º:90º] South



- Most (~70 %) of the energy in contained between 100-400 km
- The percentage of Fine-Energy at North of the Eq. is systematically lower wrt South of Eq.

Extracting the Spectrum

Sadek and Aluie, *Phys. Rev. Fluids* (2018)

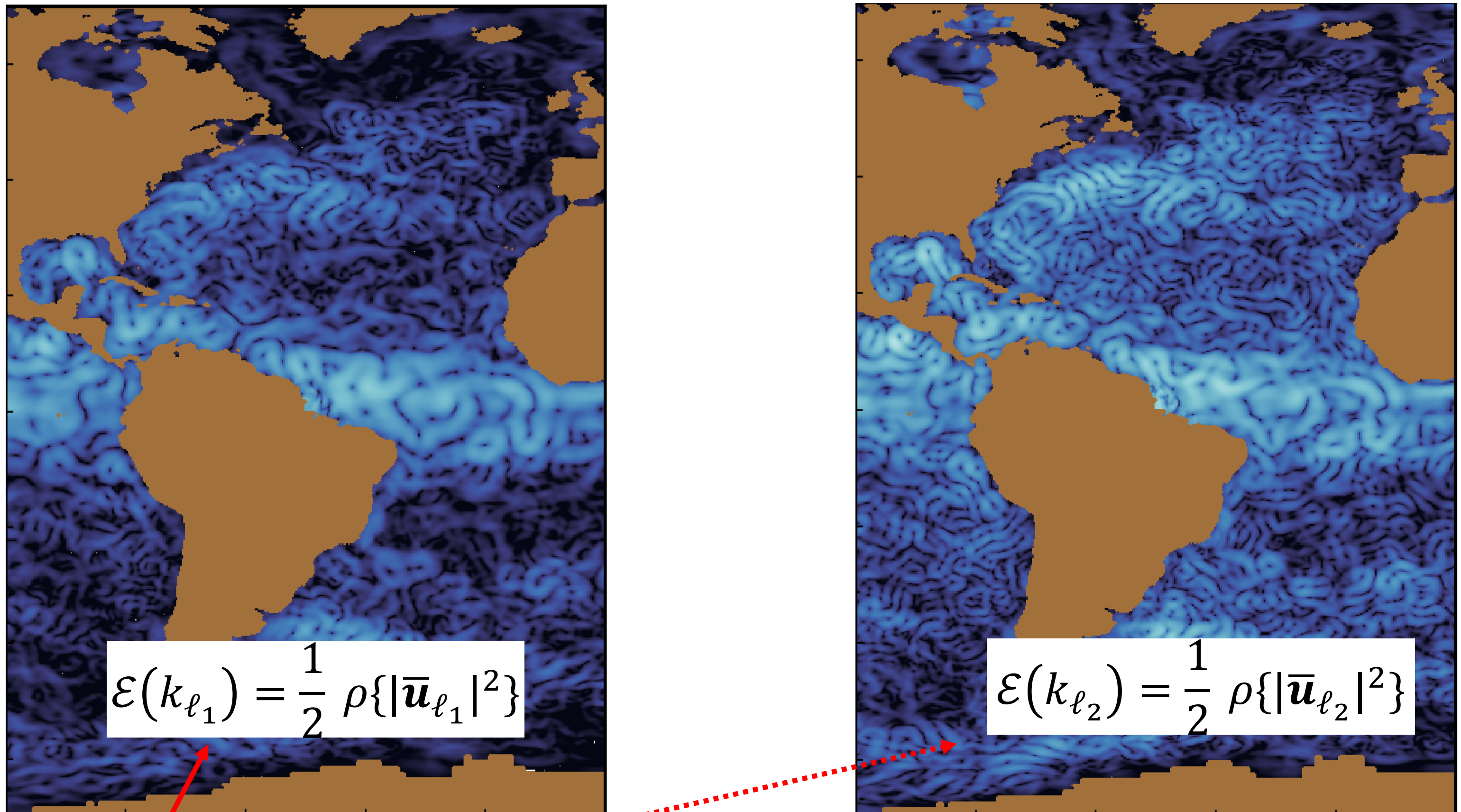
Measuring the spectrum is important

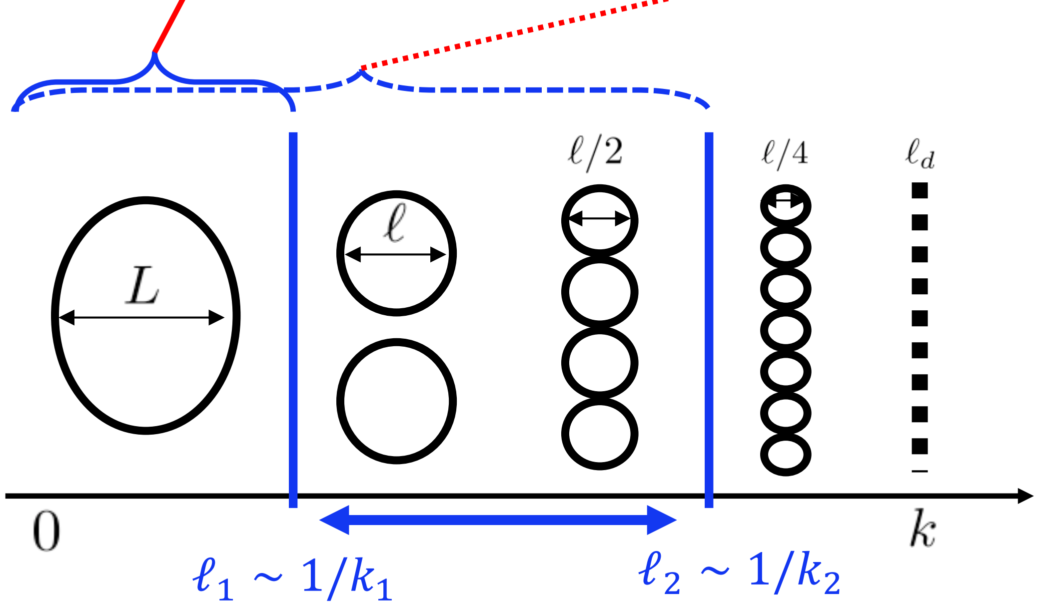
1. Quantifies the energy content of different spatial scales

2. Valuable information into cascade ranges, dissipation, turbulence intensity, upscale/downscale transfer (QG, 2D vs 3D, etc…) [Kolmogorov, Fjortoft, Charney, Salmon, Rhines, ...]

3. Topological structure of the flow. Power-law slopes are intimately related to the smoothness/roughness/fractal nature of fields.

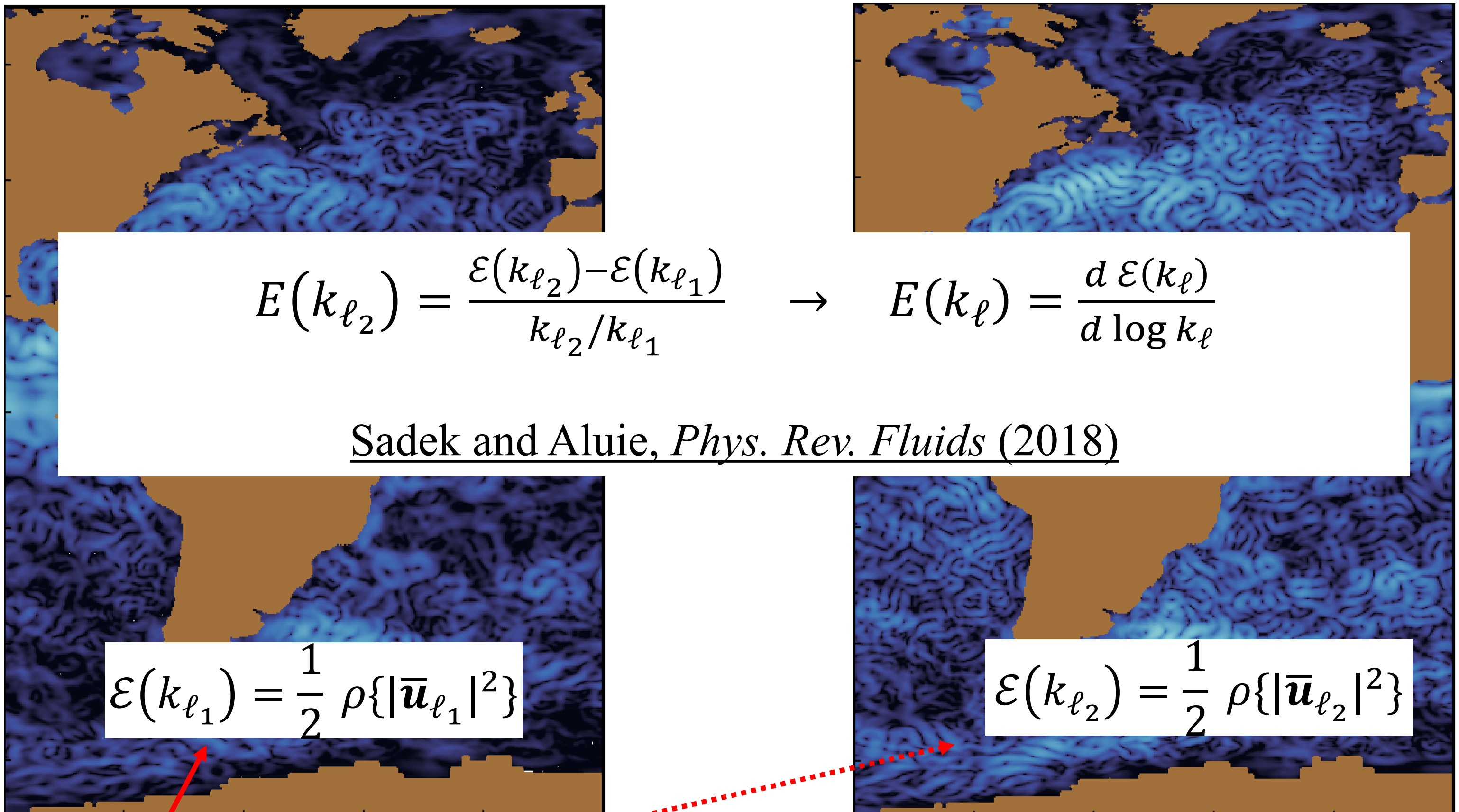
Extracting the Spectrum





- By subtracting, infer energy content at different scales
- Information on location and geometrical structure of that energy

Extracting the Spectrum

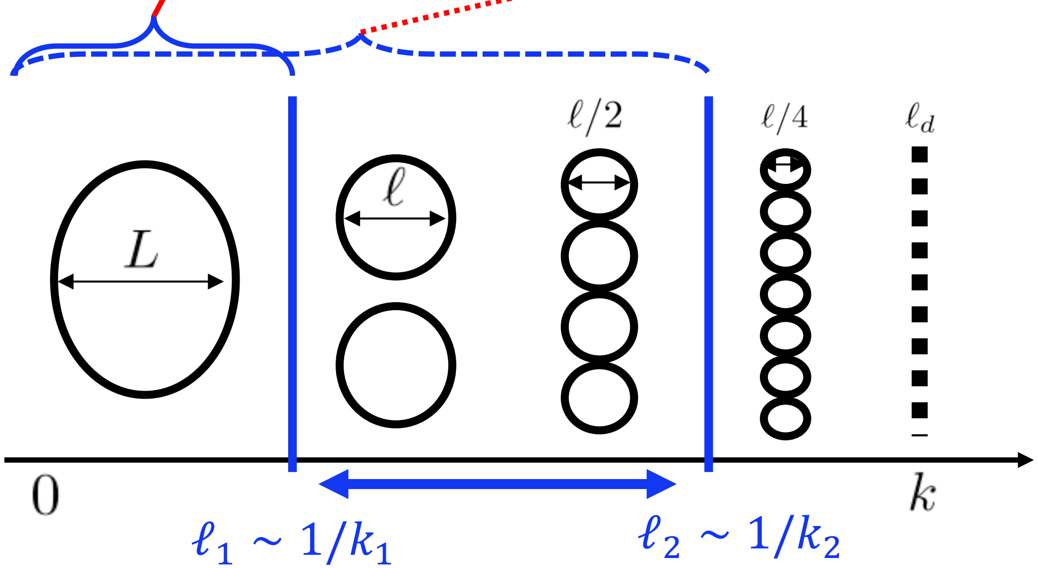


$$E(k_{\ell_2}) = \frac{\mathcal{E}(k_{\ell_2}) - \mathcal{E}(k_{\ell_1})}{k_{\ell_2}/k_{\ell_1}} \rightarrow E(k_\ell) = \frac{d\,\mathcal{E}(k_\ell)}{d \log k_\ell}$$

Sadek and Aluie, *Phys. Rev. Fluids* (2018)

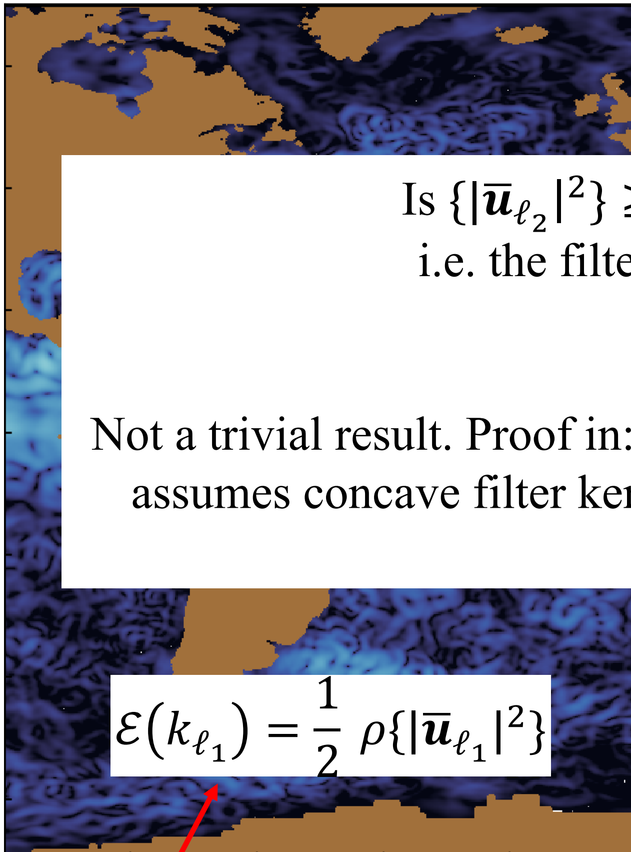
$$\mathcal{E}(k_{\ell_1}) = \frac{1}{2}\rho\{|\overline{\boldsymbol{u}}_{\ell_1}|^2\}$$

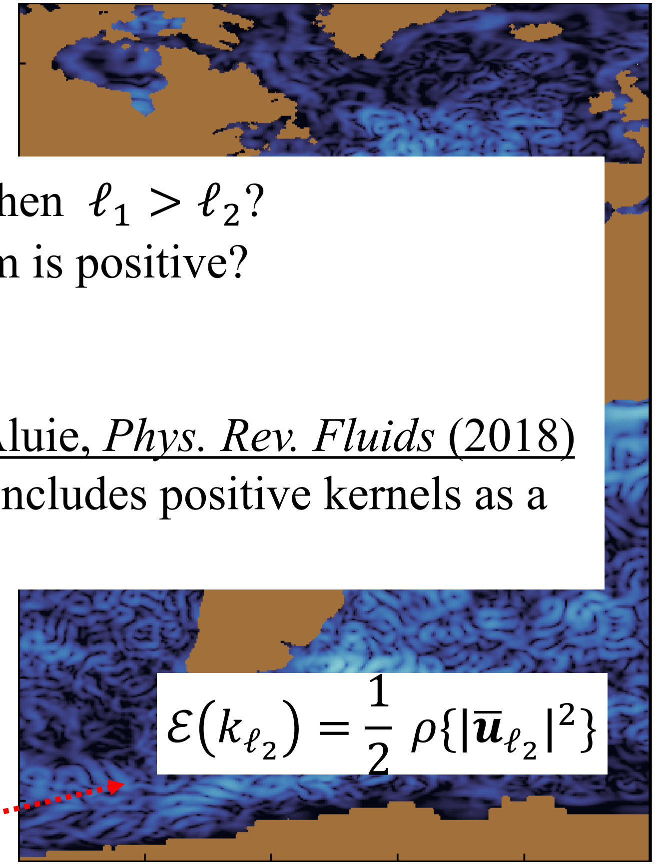
$$\mathcal{E}(k_{\ell_2}) = \frac{1}{2}\rho\{|\overline{\boldsymbol{u}}_{\ell_2}|^2\}$$



- By subtracting, infer energy content at different scales
- Information on location and geometrical structure of that energy

Extracting the Spectrum



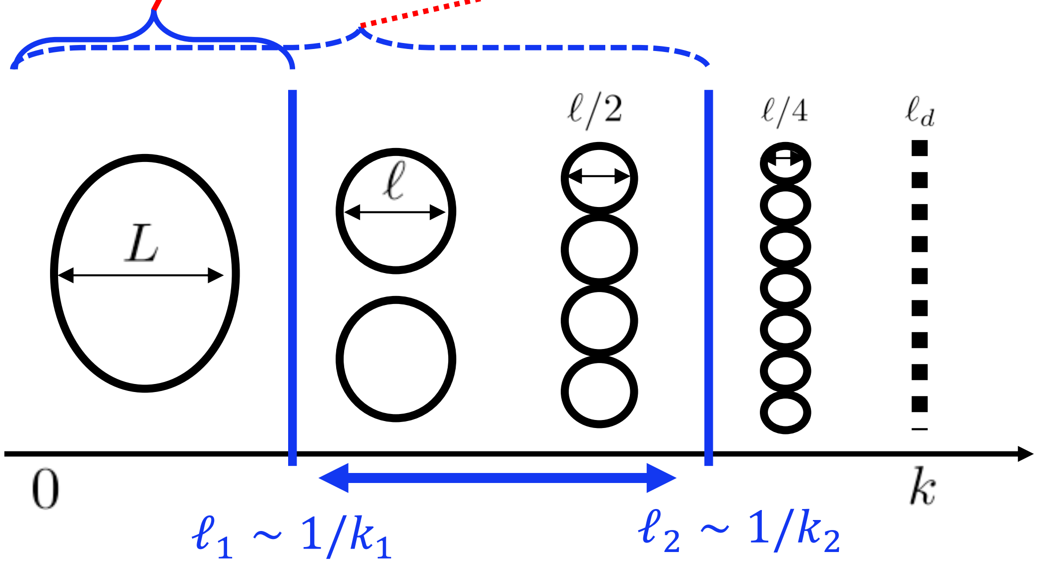


Is $\{|\overline{\boldsymbol{u}}_{\ell_2}|^2\} \geq \{|\overline{\boldsymbol{u}}_{\ell_1}|^2\}$ when $\ell_1 > \ell_2$?
i.e. the filtering spectrum is positive?
Yes!

Not a trivial result. Proof in: Sadek and Aluie, *Phys. Rev. Fluids* (2018)
assumes concave filter kernels, which includes positive kernels as a subset)

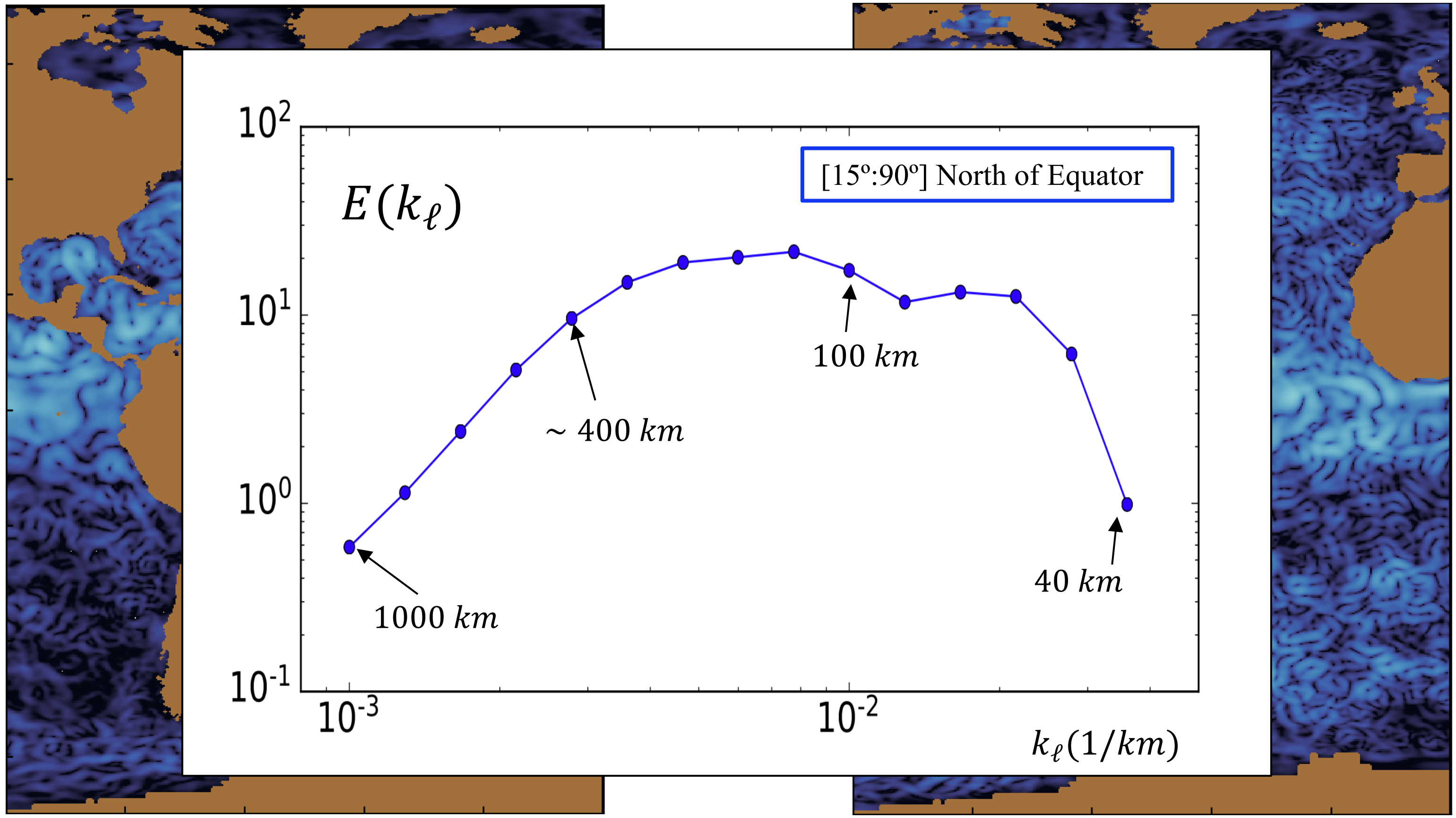
$$\mathcal{E}(k_{\ell_1}) = \frac{1}{2}\,\rho\{|\overline{\boldsymbol{u}}_{\ell_1}|^2\}$$

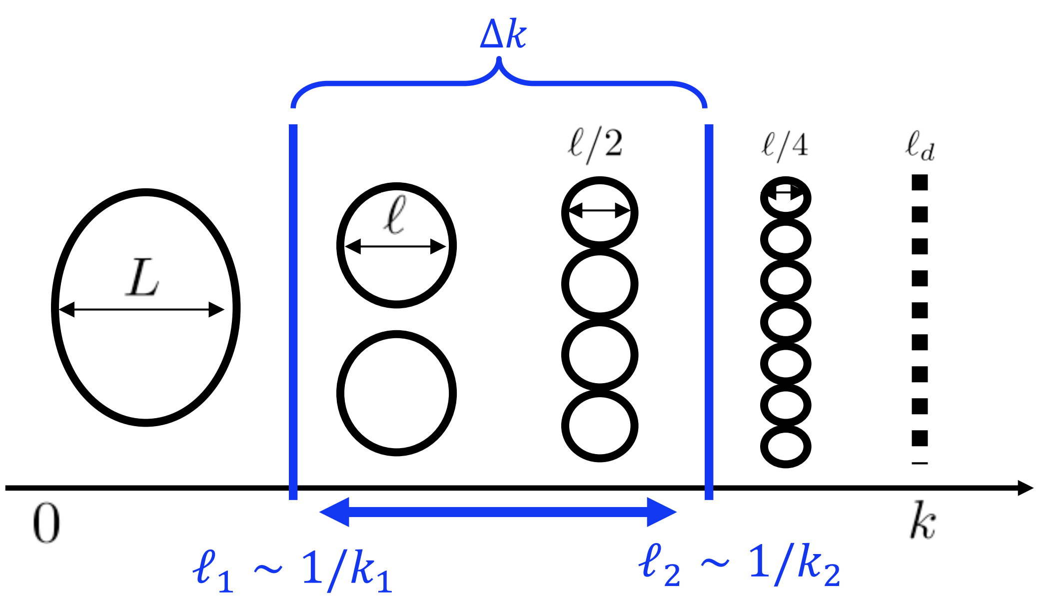
$$\mathcal{E}(k_{\ell_2}) = \frac{1}{2}\,\rho\{|\overline{\boldsymbol{u}}_{\ell_2}|^2\}$$



- By subtracting, infer energy content at different scales
- Information on location and geometrical structure of that energy

Extracting the Spectrum



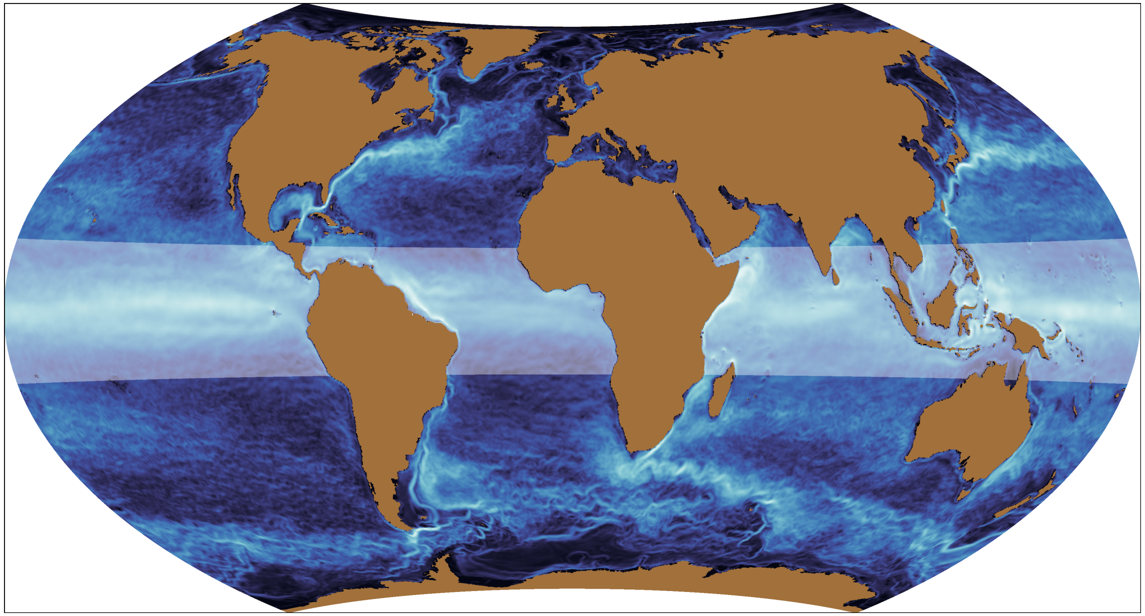


- By subtracting, infer energy content at different scales
- Information on location and geometrical structure of that energy

Classical Mean-Eddy decomposition

What length-scales exist in a "mean" flow?

Reynolds Averaging: relies on *ensemble/time* averaging to decompose the *mean* from the *fluctuating* components of a field



Time averaged velocity field

$$\langle \boldsymbol{u} \rangle (\boldsymbol{x}) = \frac{1}{T}\int_{t_0}^{t_0+T} \boldsymbol{u}(\boldsymbol{x},t)\, dt$$

Eddy/fluctuating velocity field

$$u'(\boldsymbol{x},t) = \boldsymbol{u}(\boldsymbol{x},t) - \langle \boldsymbol{u} \rangle (\boldsymbol{x})$$

In terms of kinetic energy:

Time averaged Kinetic Energy

$$\langle E \rangle(\boldsymbol{x}) = \frac{1}{2}\rho|\langle \boldsymbol{u} \rangle|^2(\boldsymbol{x})$$

Fluctuating Kinetic Energy

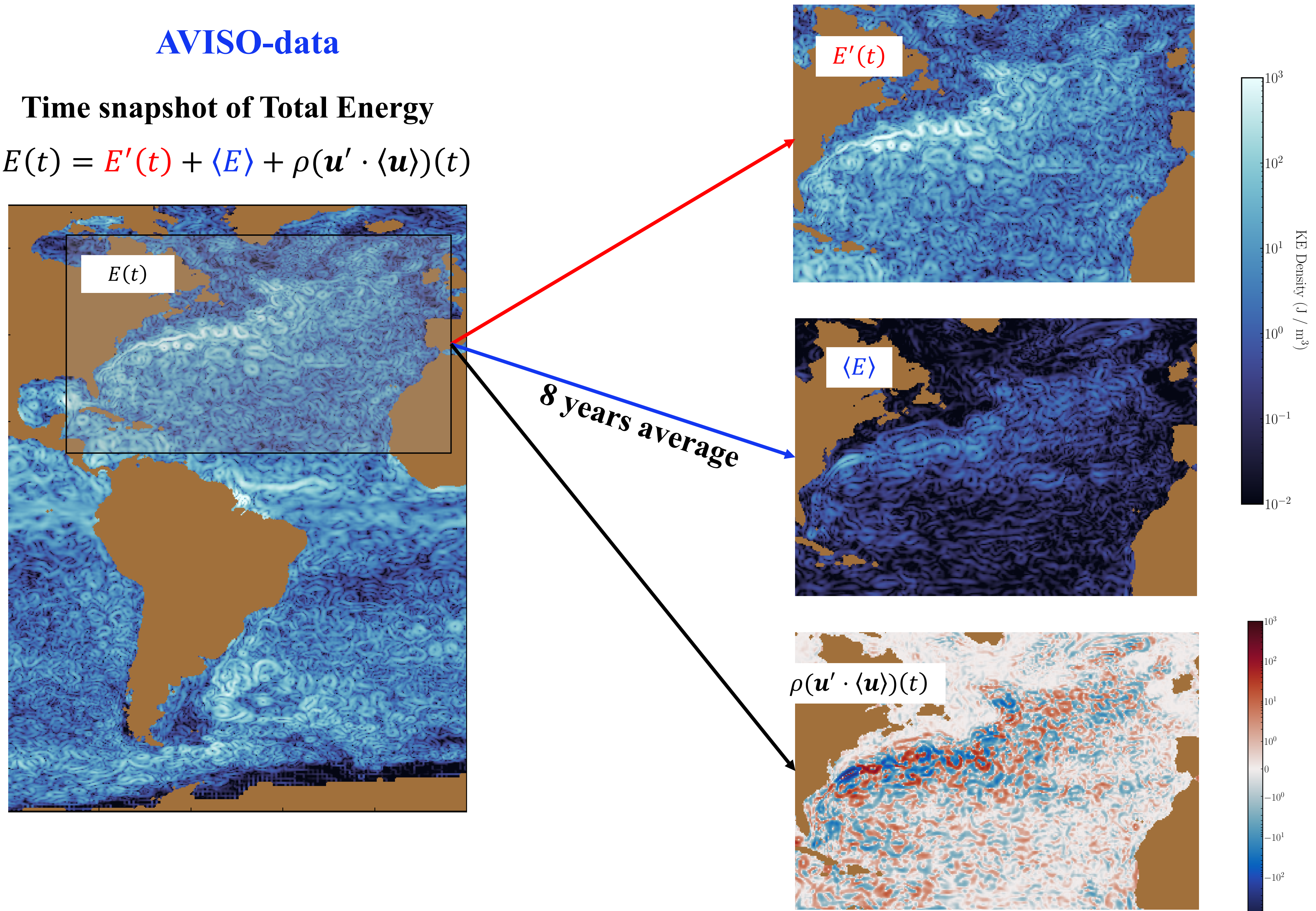
$$E'(\boldsymbol{x},t) = \frac{1}{2}\rho|u'|^2(\boldsymbol{x},t)$$

Classical Mean-Eddy decomposition

AVISO-data

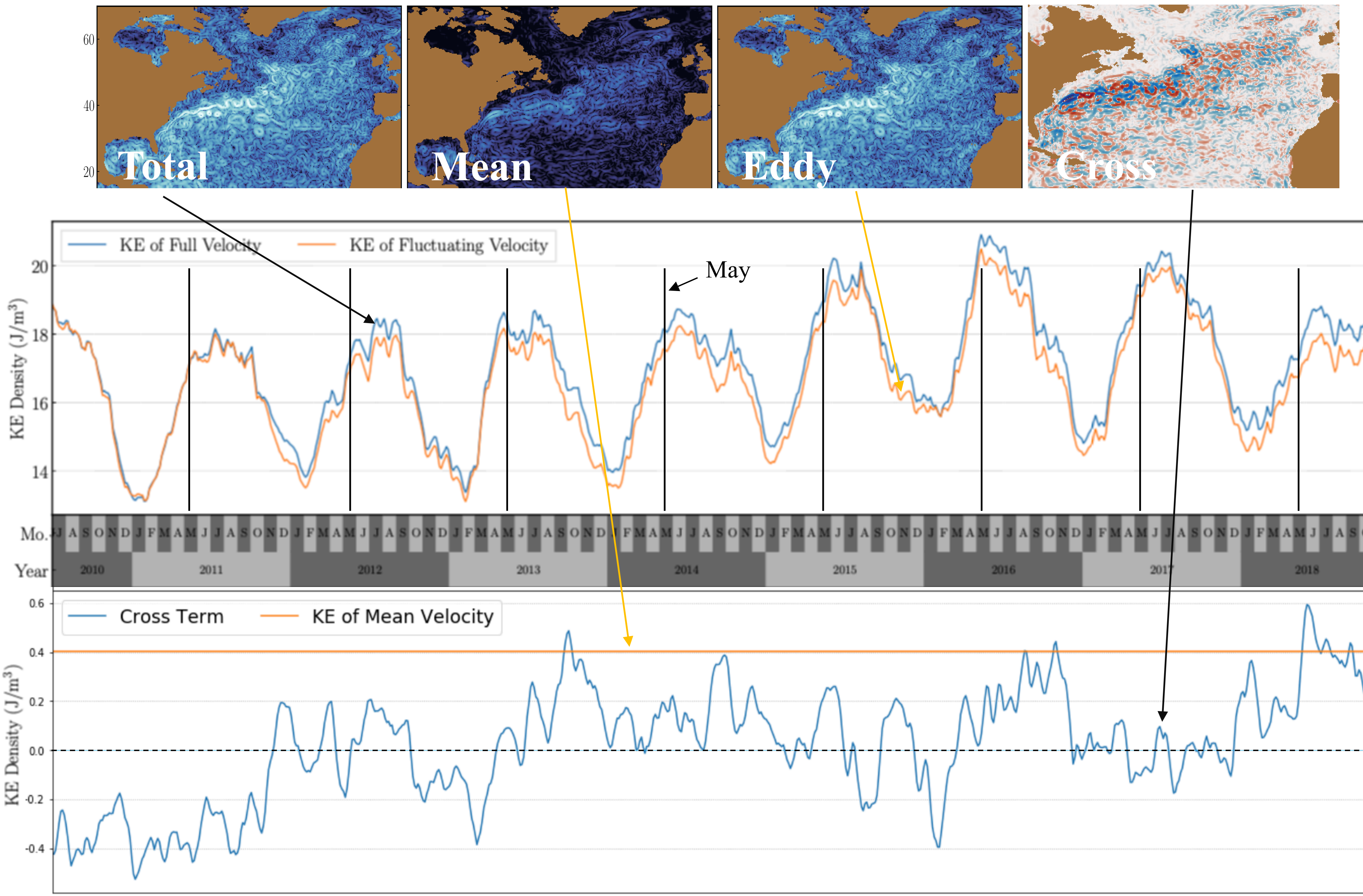
Time snapshot of Total Energy

$$E(t) = E'(t) + \langle E \rangle + \rho(\boldsymbol{u}' \cdot \langle \boldsymbol{u} \rangle)(t)$$



Seasonality time decomposed Kinetic Energy

[15º:90º] North



Energy spectra of time-averaged fields

Moving window time-averaged velocity field, (window of size τ):

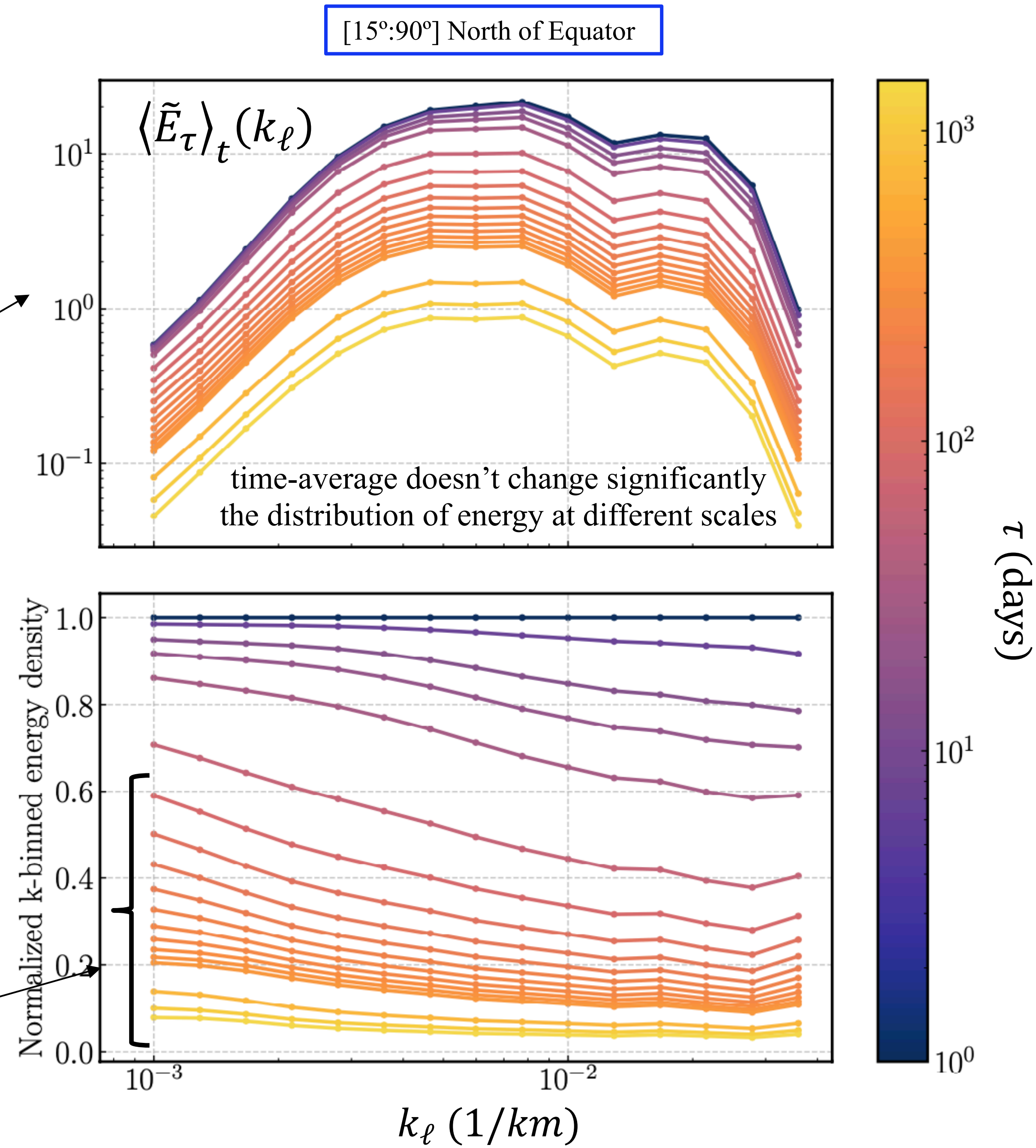
$$\tilde{\boldsymbol{u}}_\tau(\boldsymbol{x},t) = \frac{1}{\tau}\int_{t-\tau/2}^{t+\tau/2} \boldsymbol{u}(\boldsymbol{x},t')dt'$$

Energy spectrum of the window-averaged velocity field:

Normalized Energy Spectra, (to the non time-averaged $\tau = 0$ spectrum)

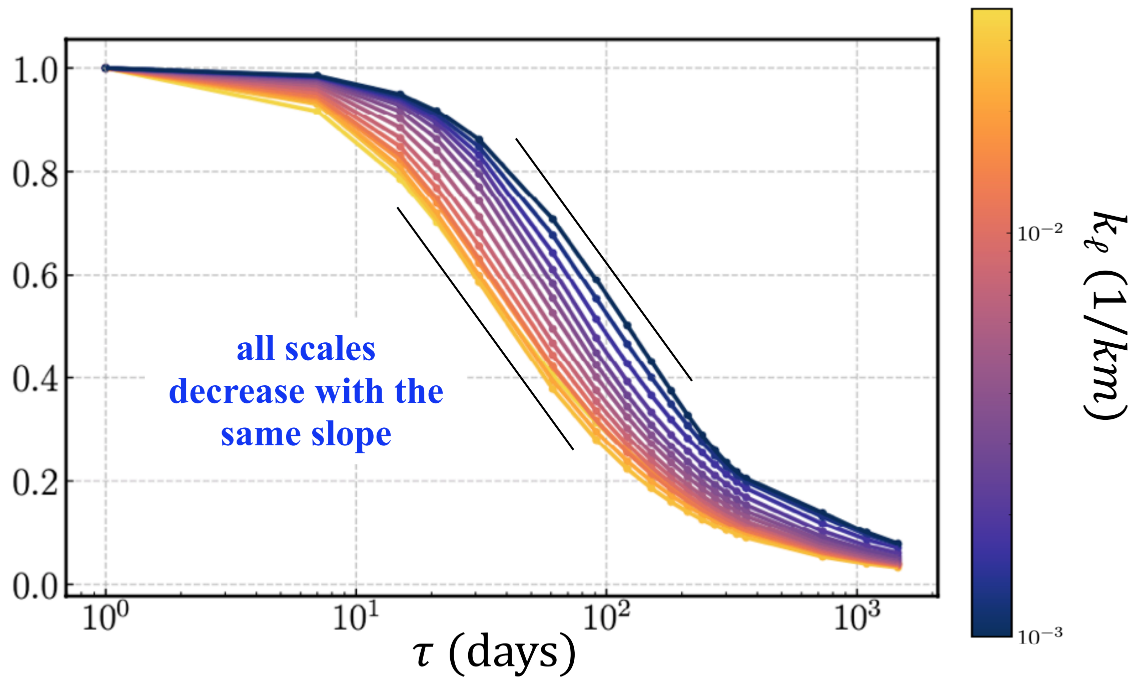
$$\frac{\langle \tilde{E}_\tau \rangle_t}{\langle \tilde{E}_{\tau=0} \rangle_t}(k_\ell)$$

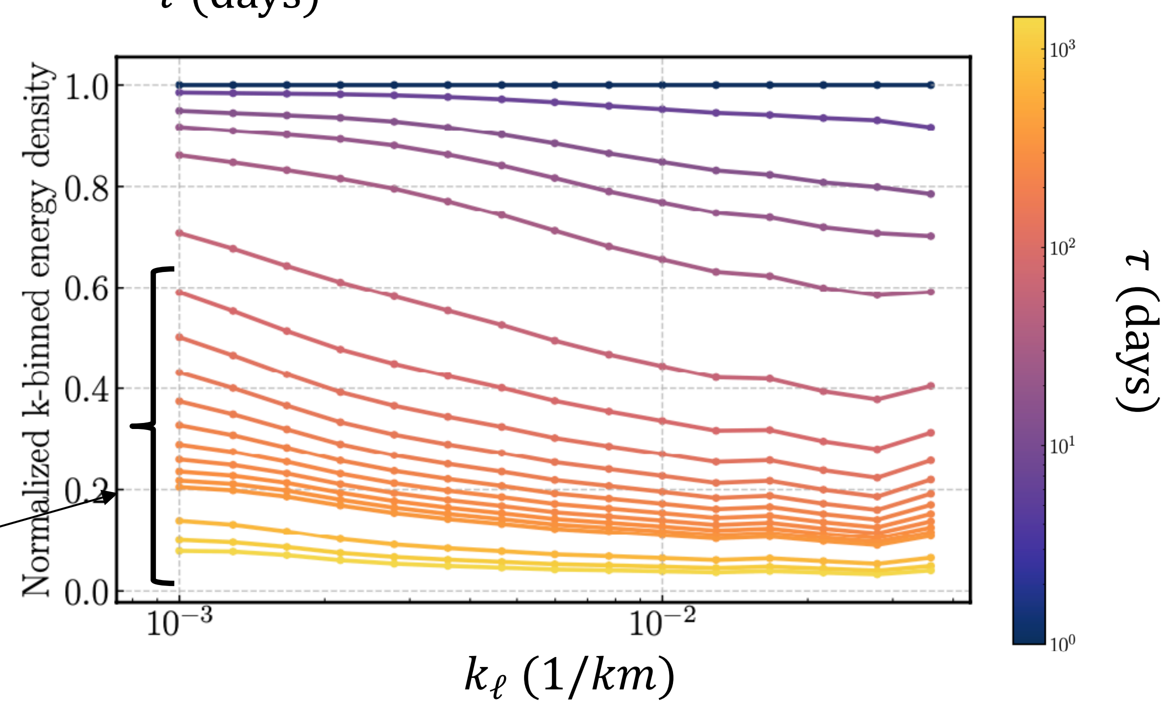
From τ ~ 30 days all scales decrease simultaneously



Energy spectra of time-averaged fields

[15º:90º] North of Equator





Normalized Energy Spectra, (to the non time-averaged $\tau = 0$ spectrum)

$$\frac{\langle \tilde{E}_\tau \rangle_t}{\langle \tilde{E}_{\tau=0} \rangle_t}(k_\ell)$$

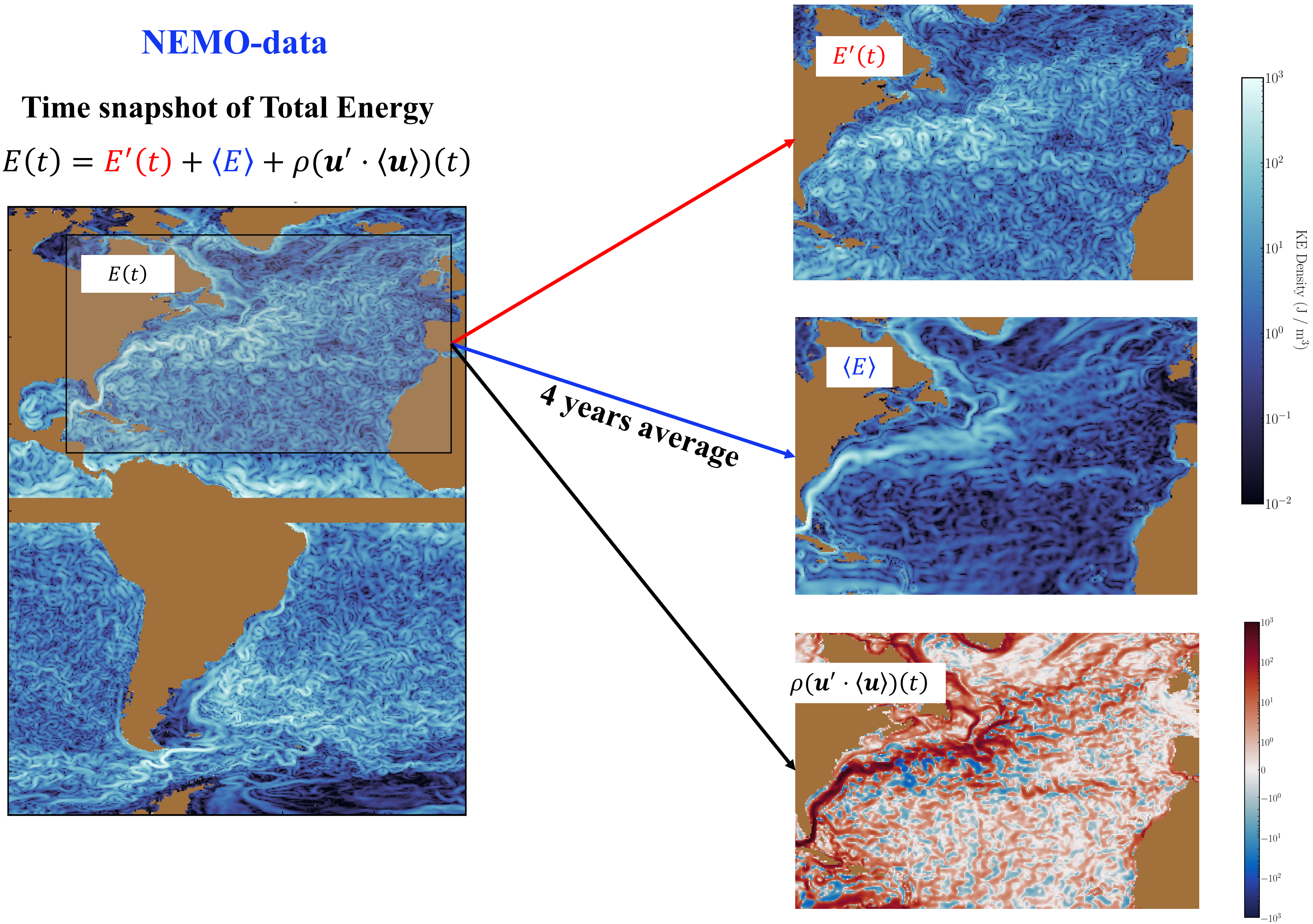
From $\tau \sim 30$ days all scales decrease simultaneously

Classical Mean-Eddy decomposition

NEMO-data

Time snapshot of Total Energy

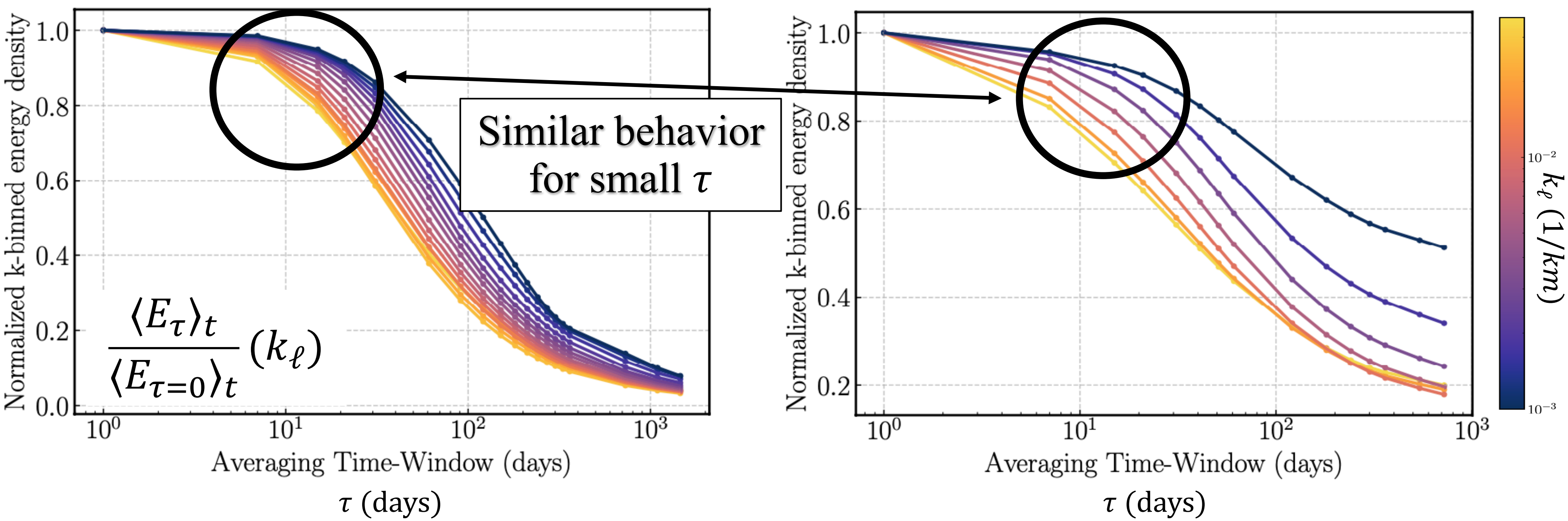
$$E(t) = E'(t) + \langle E \rangle + \rho(\boldsymbol{u}' \cdot \langle \boldsymbol{u} \rangle)(t)$$



Spatio-temporal decomposition of observation and model data

AVISO   NEMO

[15º:90º] North of Equator

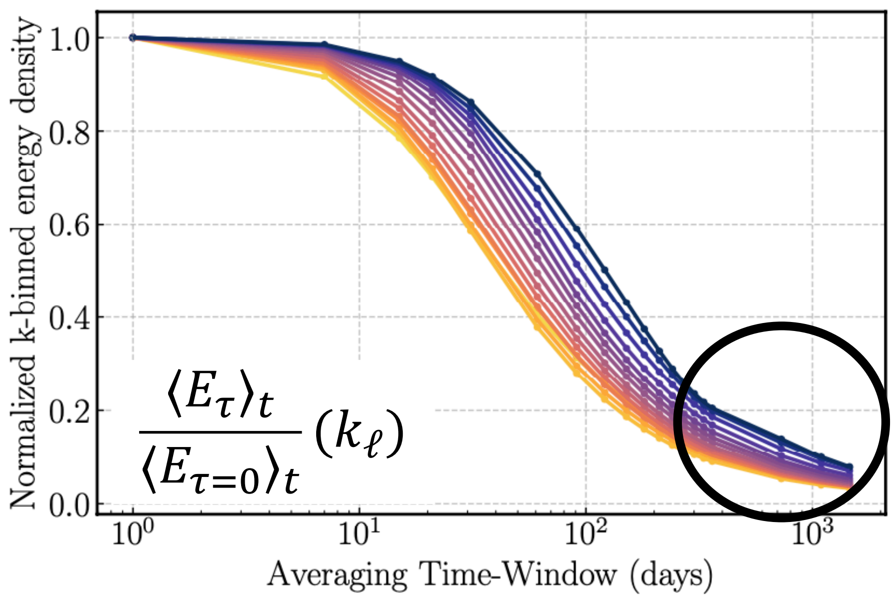


Spatio-temporal decomposition of observation and model data

AVISO

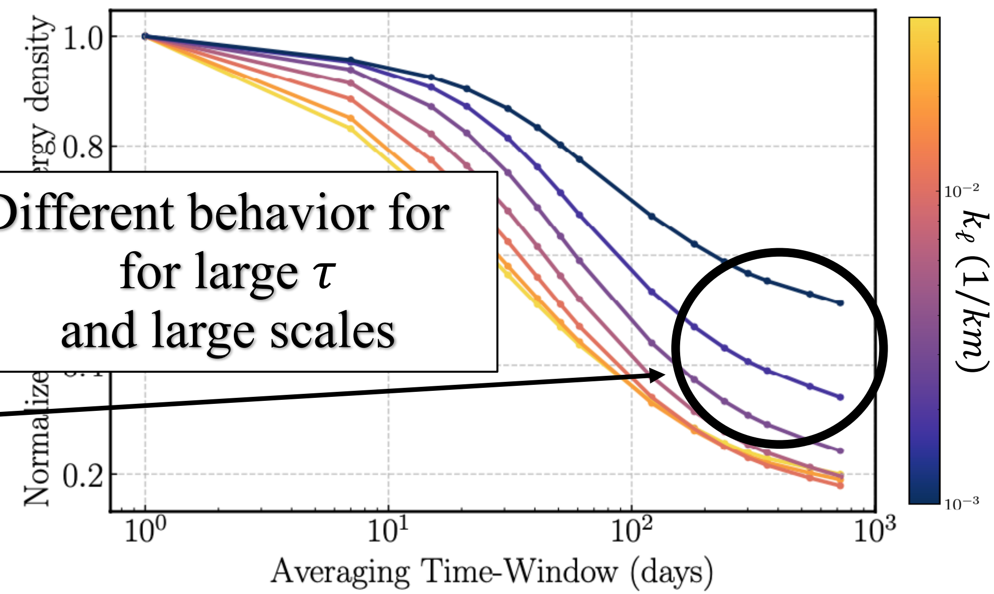
NEMO

[15º:90º] North of Equator



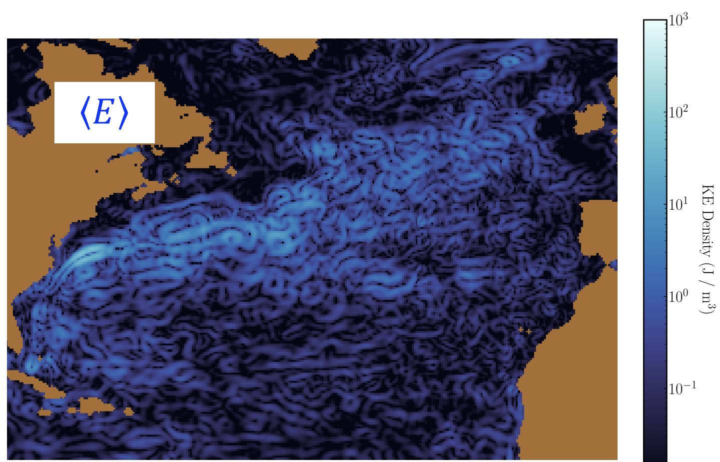
$$\frac{\langle E_\tau \rangle_t}{\langle E_{\tau=0} \rangle_t}(k_\ell)$$

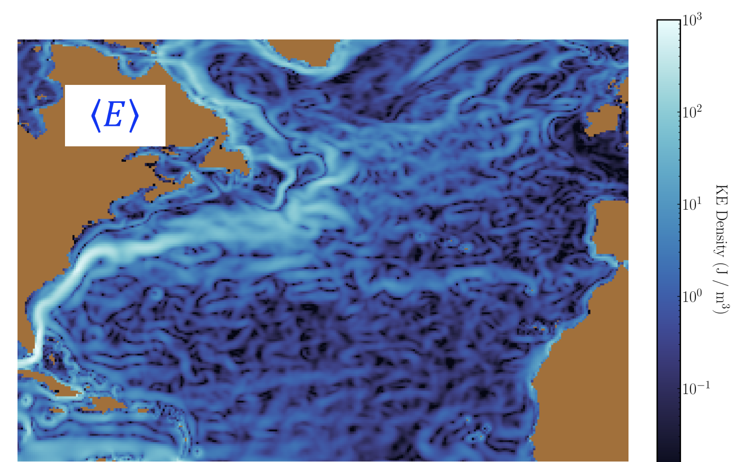
τ (days)



Different behavior for
for large τ
and large scales

τ (days)



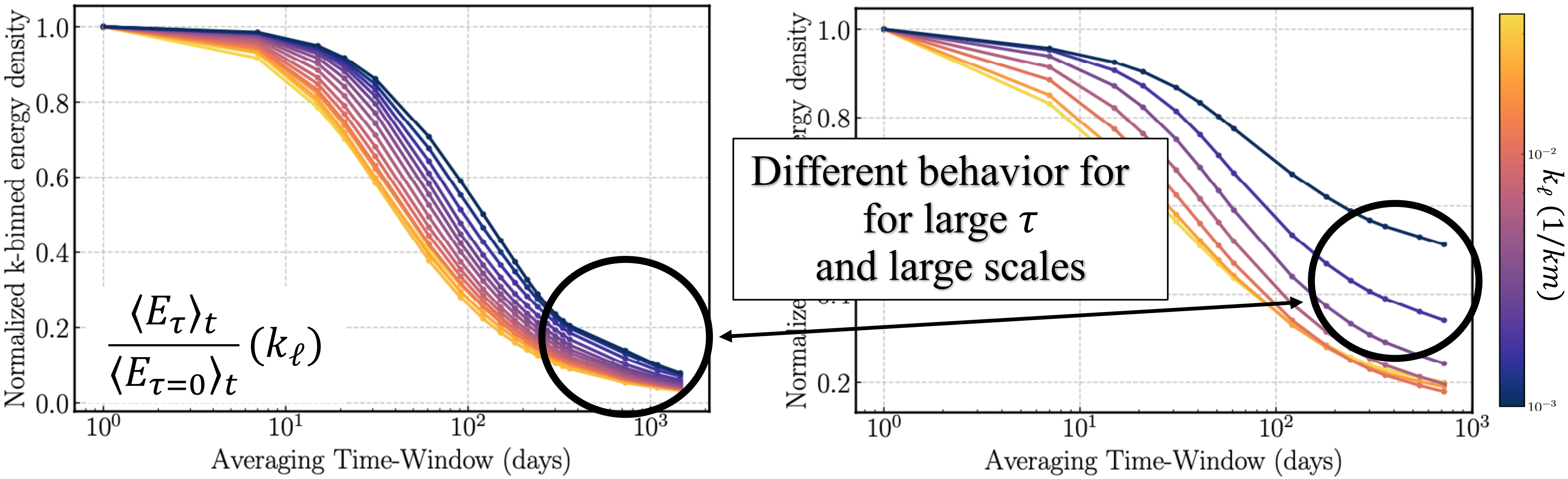


Spatio-temporal decomposition of observation and model data

AVISO

NEMO

[15º:90º] North of Equator



τ (days) τ (days)

- The large-scales of NEMO geostrophic velocity have a longer decorrelation-time compared to the AVISO observed geostrophic velocity

- This can be an effect coming from a longer correlation in the forcing used by the model

1] lack of forcing from sub-mesoscales in NEMO

2] problems with atmospheric forcing

- Other ideas?

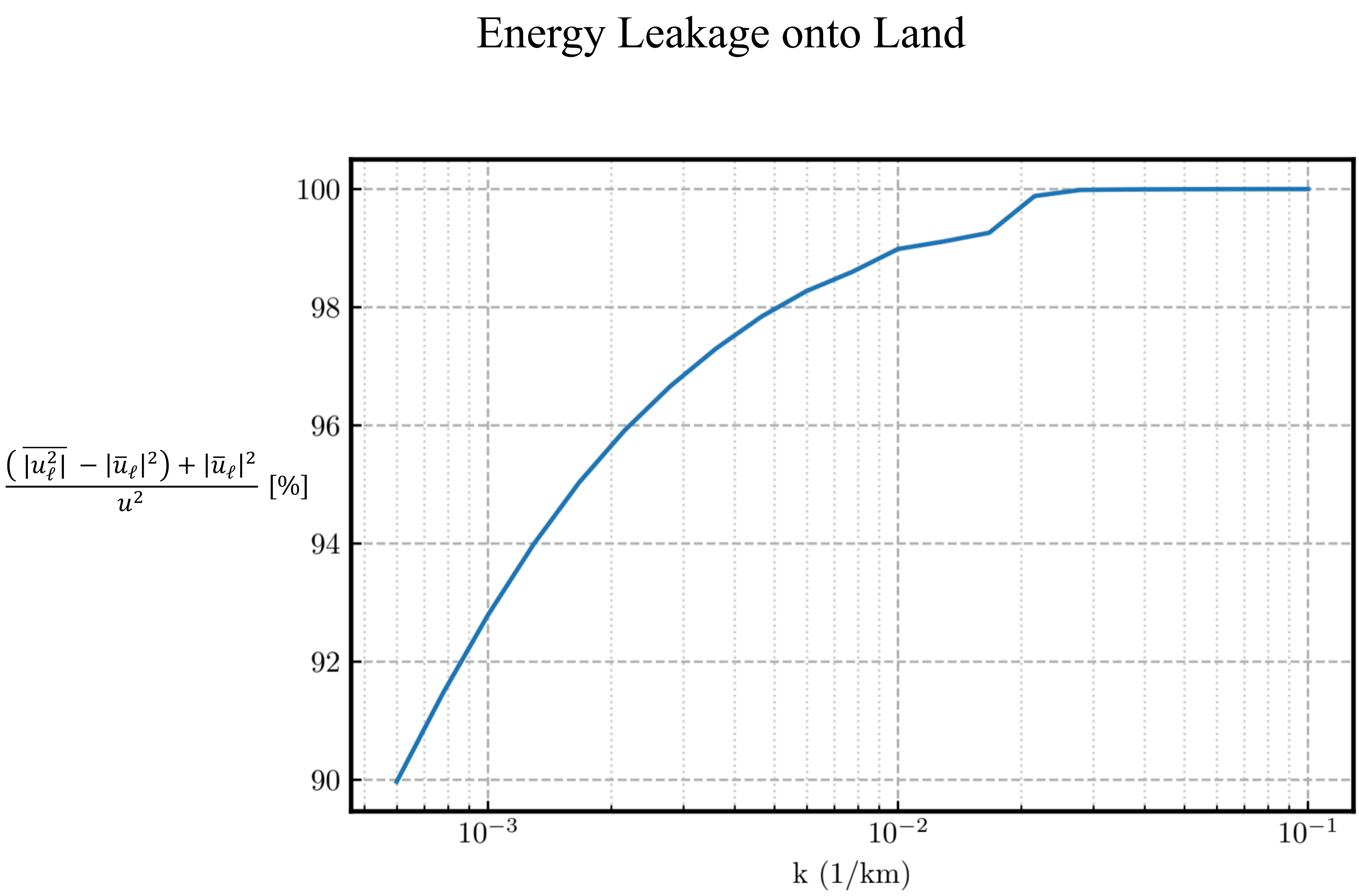
Conclusion

1. Coarse-graining allows to achieve a precise and regional/local scale-decomposition of the velocity structures in the oceanic currents (Not possible with other methods)

   To our knowledge this is the first time the energy spectrum of the global ocean is calculated, and we found that 70% of the geostrophic Kinetic Energy is contained between 100-400 Km.

2. Spatio-temporal decomposition highlights differences between observations and model data

3. Studying seasonality of the energetics at different scales we can measure time-delays energy picks which are hints of the energy transfer mechanisms, and it can teach us about the scales where the main forcing contribution is acting


Energy Leakage onto Land
$\frac{\left(\overline{|u_\ell^2|} - |\bar{u}_\ell|^2\right) + |\bar{u}_\ell|^2}{u^2}$ [%]
100
98
96
94
92
90
10^{-3}
10^{-2}
10^{-1}
k (1/km)