Diagnosing the energy budget of mesoscale eddies in an idealized model

Nora Loose, Ian Grooms, Scott Bachman, Malte Jansen

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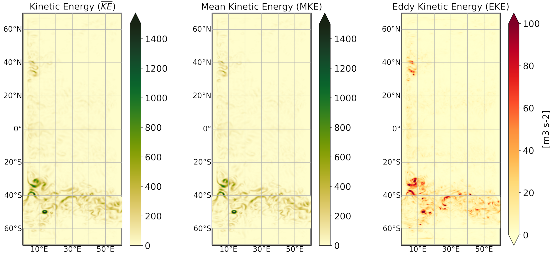
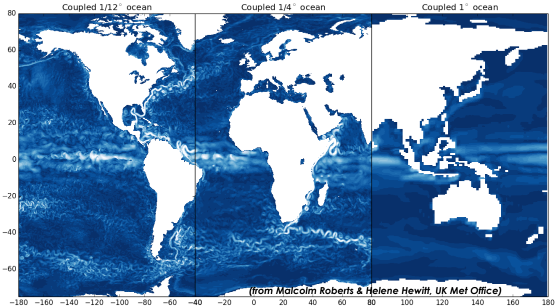
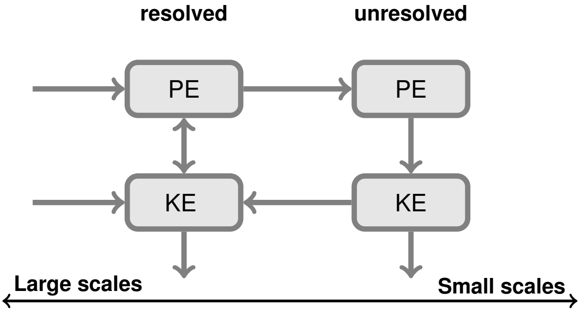


Motivation & Goals





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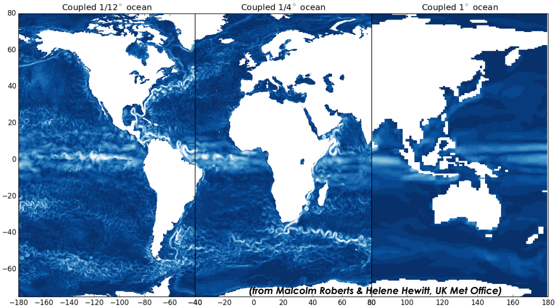
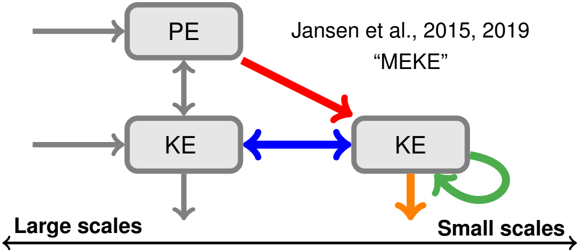
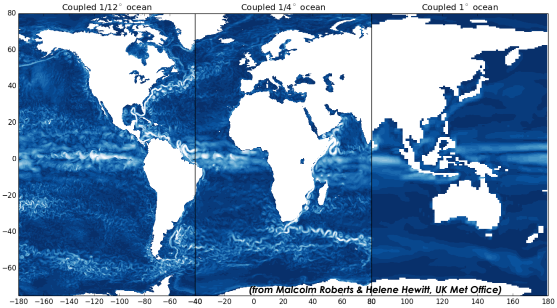


$$\partial_t E = \dot{E}_{GM} + \dot{E}_{v_4} - \dot{E}_{v_2} - \dot{E}_{diss} - \nabla \cdot (\bar{\mathbf{u}} E + (\kappa \nabla E))$$



Recent advances in mesoscale eddy parameterizations based on energy budgets

(e.g., Eden & Greatbatch, 2008; Marshall & Adcroft, 2010; Jansen et al., 2015, 2019; Mak et al., 2018)



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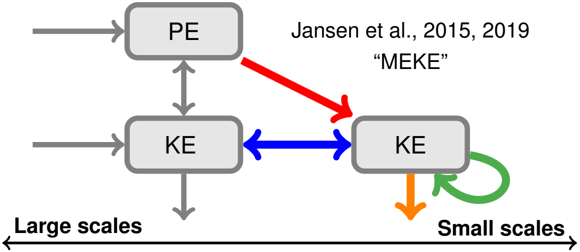

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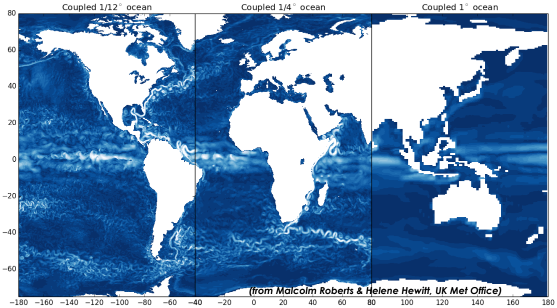
CPT goal: Evaluate/Improve energy budget-based eddy parameterizations

Motivation & Goals



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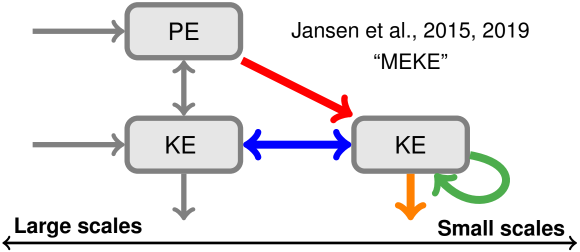

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CPT goal: Evaluate/Improve energy budget-based eddy parameterizations

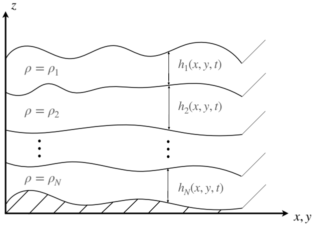
This talk: Diagnose energy budget of mesoscale eddies

- in **stacked shallow water** model (NeverWorld2)
- with a **spatial filtering** approach
- in a thickness-weighted average (**TWA**) framework

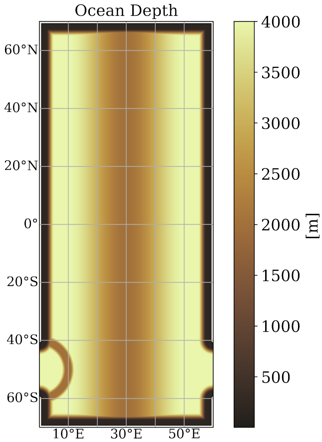
NeverWorld2

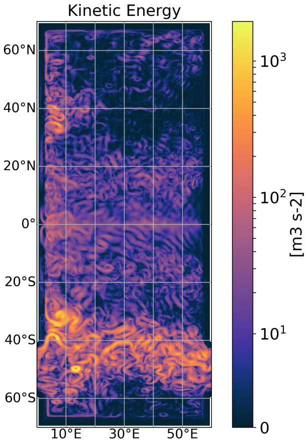
- MOM6 simulation in stacked shallow water mode
- 15 layers, (1/16)° resolution

Stacked shallow water model



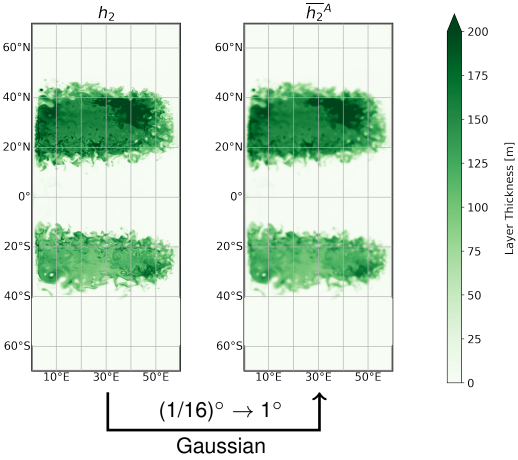
Schematic: Courtesy of **Neeraja Bhamidipati**





vertially integrated
1 snapshot (5-day average)

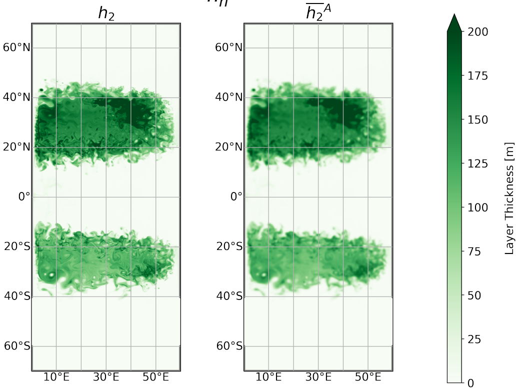
Spatial Filtering



Spatial Filtering

Spatial ("area") filter:

$\overline{h}_n^A$



$(1/16)^\circ \rightarrow 1^\circ$

Gaussian

area integral preserving filter

$$\int h_n \, dx \, dy = \int \overline{h}_n^A \, dx \, dy$$

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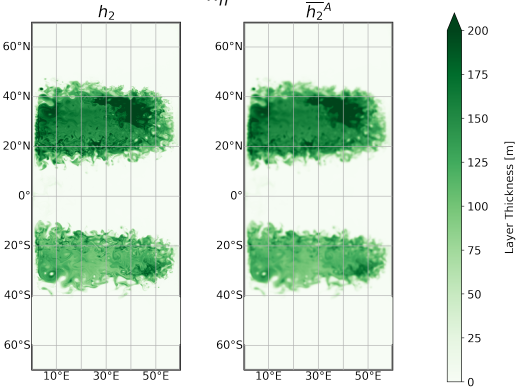


$(1/16)^\circ \rightarrow 1^\circ$

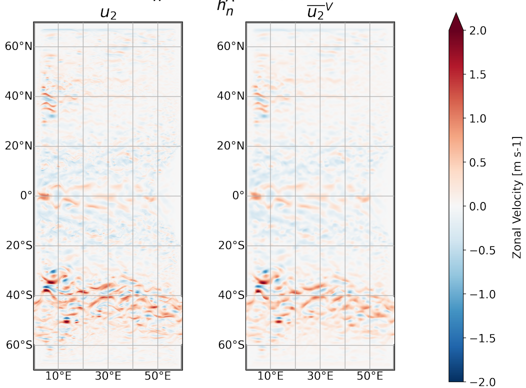
Gaussian

area integral preserving filter

$$\int h_n \, dx \, dy = \int \overline{h_n}^A \, dx \, dy$$

Spatial & thickness-weighted ("volume") filter:

$$\overline{u_n}^V = \frac{\overline{h_n \cdot u_n}^A}{\overline{h_n}^A}$$ (TWA, cf. Young, 2012)



$(1/16)^\circ \rightarrow 1^\circ$

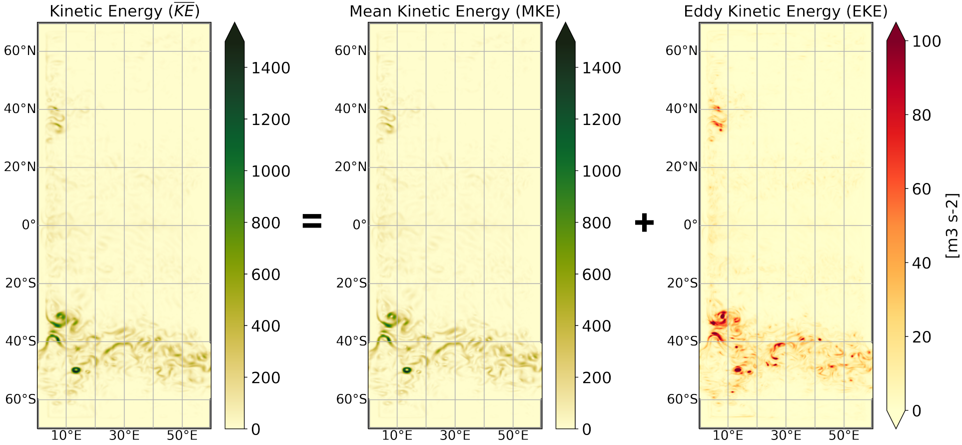
Gaussian

volume integral preserving filter

$$\int h_n \cdot u_n \, dx \, dy = \int \overline{h_n}^A \cdot \overline{u_n}^V \, dx \, dy$$

Eddy Kinetic Energy (EKE)

$$(\mathrm{EKE})_n = \overline{(\mathrm{KE})}_n - (\mathrm{MKE})_n = \frac{\overline{h_n|\mathbf{u}_n|^2}^A}{2} - \frac{\overline{h}_n^A|\overline{\mathbf{u}}_n^V|^2}{2}$$



$(1/16)^\circ \rightarrow (1/2)^\circ$, vertically integrated, 1 snapshot

Deriving the EKE budget

$$(\mathrm{EKE})_n = \overline{(\mathrm{KE})}_n^A - (\mathrm{MKE})_n$$

Stacked shallow water equations:

$$\partial_t h_n = -\nabla \cdot (h_n \mathbf{u}_n) \qquad (\textbf{Continuity equation})_n$$

$$\partial_t \mathbf{u}_n + \mathbf{u}_n \cdot \nabla \mathbf{u}_n + \mathbf{f} \times \mathbf{u}_n = -\frac{1}{\rho_1} \nabla p_n + \mathbf{F}_n \qquad (\textbf{Velocity equation})_n$$

KE equation: $h_n \mathbf{u}_n \cdot (\text{Velocity equation})_n + \frac{|\mathbf{u}_n|^2}{2} (\text{Continuity equation})_n$

$$\partial_t \frac{h_n |\mathbf{u}_n|^2}{2} + \nabla \cdot \left(\mathbf{u}_n \frac{h_n |\mathbf{u}_n|^2}{2} \right) = -\frac{1}{\rho_1} h_n \mathbf{u}_n \cdot \nabla p_n + h_n \mathbf{u}_n \cdot \mathbf{F}_n$$

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TWA equations:

$$\partial_t \overline{h_n}^A = -\nabla \cdot (\overline{h}_n^A \overline{\mathbf{u}}_n^V) + 0 \qquad \textbf{(TWA Continuity equation)}_n$$

$$\partial_t \overline{\mathbf{u}}_n^V + \overline{\mathbf{u}}_n^V \cdot \nabla \overline{\mathbf{u}}_n^V + \overline{\mathbf{f} \times \mathbf{u}_n}^V = -\frac{1}{\rho_1} \nabla \overline{p}_n^A + \overline{\mathbf{F}}^V - \frac{1}{\rho_1} \overline{\nabla p_n'}^V - \frac{1}{\overline{h}_n^A} \nabla \cdot \left[\overline{h}_n^A \overline{\mathbf{u}_n'' \mathbf{u}_n''}^V \right] \qquad \textbf{(TWA Velocity equation)}_n$$

Deriving the EKE budget

$$(\mathrm{EKE})_n = \overline{(\mathrm{KE})}_n^A - (\mathrm{MKE})_n$$

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MKE equation: $\overline{h_n \mathbf{u}_n}^A \cdot (\text{TWA Velocity equation})_n + \frac{|\overline{\mathbf{u}}_n^V|^2}{2} (\text{TWA Continuity equation})_n$

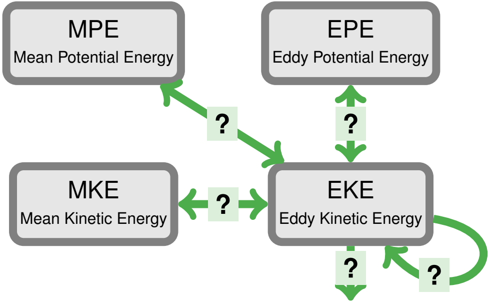
$$\partial_t \frac{\overline{h}_n^A |\overline{\mathbf{u}}_n^V|^2}{2} + \nabla \cdot \left(\overline{\mathbf{u}}_n^V \frac{\overline{h}_n^A |\overline{\mathbf{u}}_n^V|^2}{2} \right) + \overline{\mathbf{u}}_n^V \cdot \overline{\mathbf{f} \times h_n \mathbf{u}_n}^A = -\frac{1}{\rho_1} \overline{h_n \mathbf{u}_n}^A \cdot \nabla \overline{p}_n^A + \overline{h_n \mathbf{u}_n}^A \cdot \overline{\mathbf{F}}_n^V - \frac{1}{\rho_1} \overline{h_n \mathbf{u}_n}^A \cdot \overline{\nabla p_n'}^V - \overline{\mathbf{u}}_n^V \cdot \nabla \cdot (\overline{h}_n^A \overline{\mathbf{u}_n'' \mathbf{u}_n''}^V)$$

EKE budget

$$(\text{EKE})_n = \left[\frac{\overline{h_n|\mathbf{u}_n|^2}^A}{2} - \frac{\overline{h_n}^A|\overline{\mathbf{u}_n}^V|^2}{2}\right]$$

$$\partial_t(\text{EKE})_n + \nabla\cdot\left(\overline{\mathbf{u}_n\frac{h_n|\mathbf{u}_n|^2}{2}}^A - \overline{\mathbf{u}_n}^V\frac{\overline{h_n}^A|\overline{\mathbf{u}_n}^V|^2}{2}\right)$$

$$= \underbrace{-\frac{1}{\rho_1}\left[\overline{h_n\mathbf{u}_n\cdot\nabla p_n}^A - \overline{h_n\mathbf{u}_n}^A\cdot\nabla\overline{p}_n^A\right] + \frac{1}{\rho_1}\overline{h_n\mathbf{u}_n}^A\cdot\overline{\nabla p_n'}^V}_{-\frac{1}{\rho_1}\left[\overline{h_n\mathbf{u}_n\cdot\nabla p_n}^A - \overline{\mathbf{u}}_n^V\cdot\overline{h_n\nabla p_n}^A\right]} + \overline{\mathbf{u}}_n^V\cdot\overline{\mathbf{f}\times h_n\mathbf{u}_n}^A + \overline{\mathbf{u}_n}^V\cdot\nabla\cdot\left(\overline{h_n}^A\overline{\mathbf{u}_n''\mathbf{u}_n''}^V\right) + \overline{h_n\mathbf{u}_n\cdot\mathbf{F}_n}^A - \overline{h_n\mathbf{u}_n}^A\cdot\overline{\mathbf{F}_n}^V$$



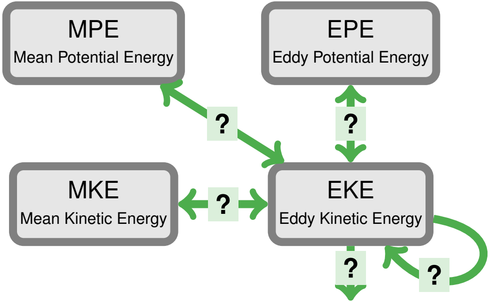
Goal: Characterize the EKE budget terms

EKE budget

$$(\mathrm{EKE})_n = \left[\frac{\overline{h_n|\mathbf{u}_n|^2}^A}{2} - \frac{\overline{h_n}^A|\overline{\mathbf{u}_n}^V|^2}{2}\right]$$

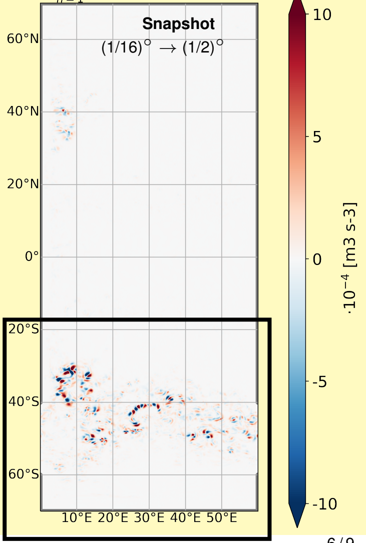
$$\partial_t(\mathrm{EKE})_n + \nabla\cdot\left(\overline{\mathbf{u}_n\frac{h_n|\mathbf{u}_n|^2}{2}}^A - \overline{\mathbf{u}_n}^V\frac{\overline{h_n}^A|\overline{\mathbf{u}_n}^V|^2}{2}\right)$$

$$= \underbrace{-\frac{1}{\rho_1}\left[\overline{h_n\mathbf{u}_n\cdot\nabla p_n}^A - \overline{h_n\mathbf{u}_n}^A\cdot\nabla\overline{p}_n^A\right] + \frac{1}{\rho_1}\overline{h_n\mathbf{u}_n}^A\cdot\overline{\nabla p_n'}^V}_{-\frac{1}{\rho_1}\left[\overline{h_n\mathbf{u}_n\cdot\nabla p_n}^A - \overline{\mathbf{u}}_n^V\cdot\overline{h_n\nabla p_n}^A\right]} + \overline{\mathbf{u}}_n^V\cdot\overline{\mathbf{f}\times h_n\mathbf{u}_n}^A + \overline{\mathbf{u}_n}^V\cdot\nabla\cdot(\overline{h}$$

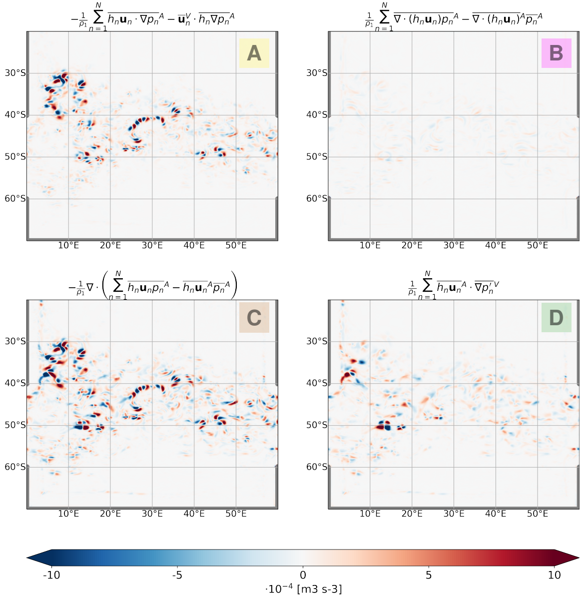


Goal: Characterize the EKE budget terms

$$-\frac{1}{\rho_1}\sum_{n=1}^{N}\overline{h_n\mathbf{u}_n\cdot\nabla p_n}^A - \overline{\mathbf{u}}_n^V\cdot\overline{h_n\nabla p_n}^A$$



Characterizing the EKE budget terms: An example



Snapshots

Decomposition: **A** = **B** + **C** + **D**

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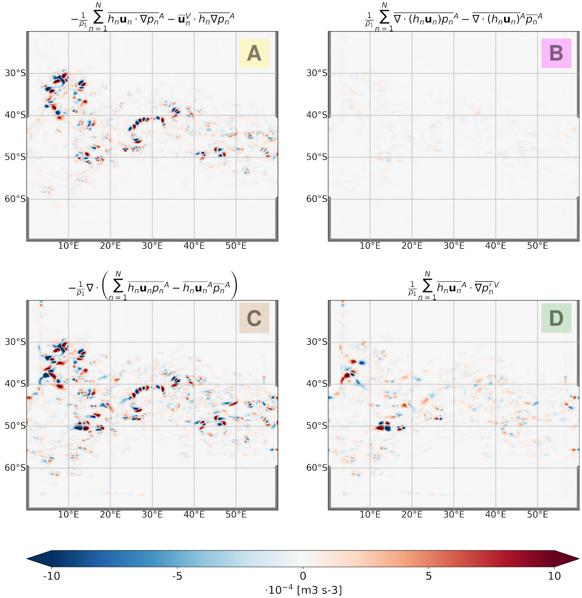
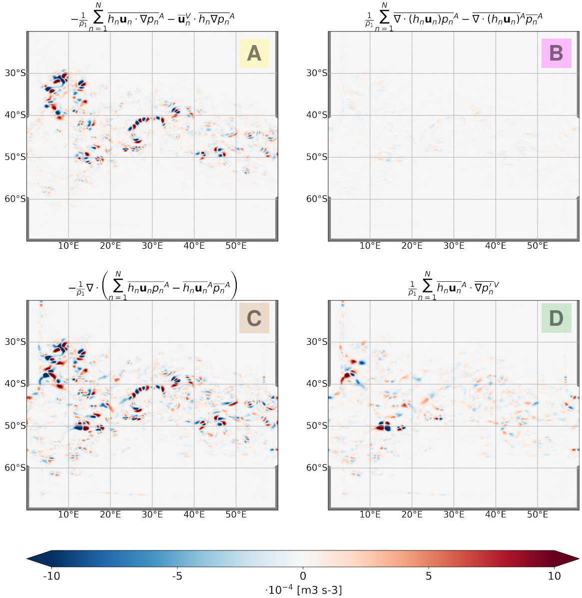


Snapshots

Decomposition: **A** = **B** + **C** + **D**

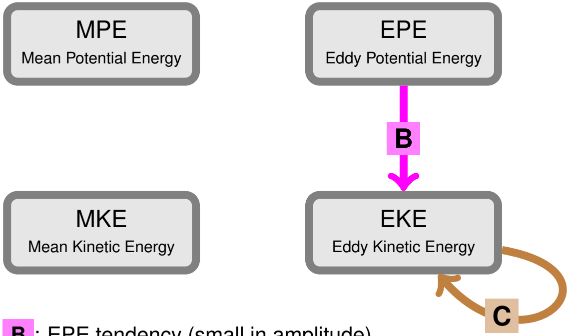
B: EPE tendency (small in amplitude)

Characterizing the EKE budget terms: An example



Snapshots

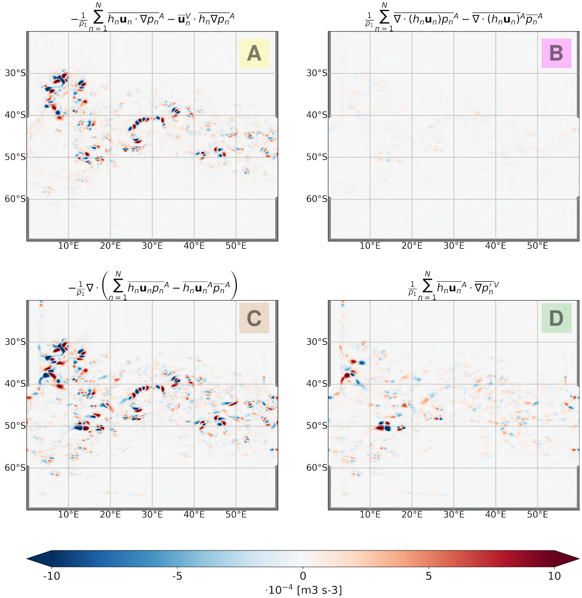
Decomposition: A = B + C + D



B: EPE tendency (small in amplitude)

C: EKE transport

Characterizing the EKE budget terms: An example



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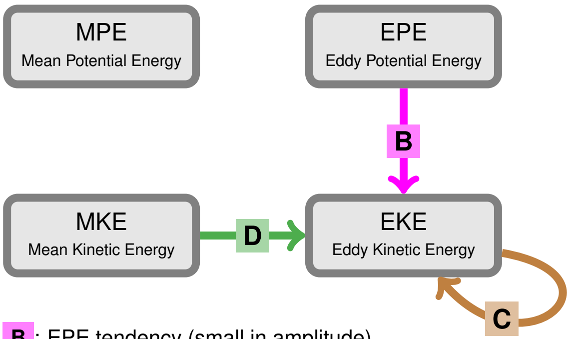


B : EPE tendency (small in amplitude)

C : EKE transport

D : MKE-EKE conversion associated with eddy form stress

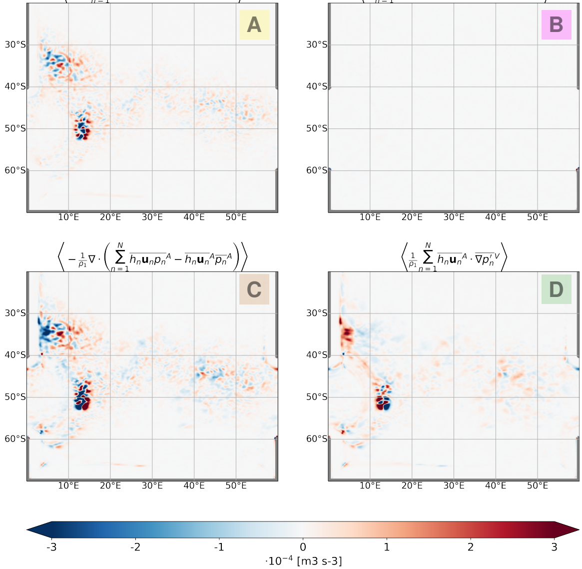
Characterizing the EKE budget terms: An example

$$\left\langle -\frac{1}{\rho_1}\sum_{n=1}^{N} \overline{h_n\mathbf{u}_n\cdot\nabla p_n}^A - \overline{\mathbf{u}}_n^V\cdot\overline{h_n\nabla p_n}^A \right\rangle$$

$$\left\langle \frac{1}{\rho_1}\sum_{n=1}^{N} \overline{\nabla\cdot(h_n\mathbf{u}_n)p_n}^A - \overline{\nabla\cdot(h_n\mathbf{u}_n)}^A\,\overline{p_n}^A \right\rangle$$

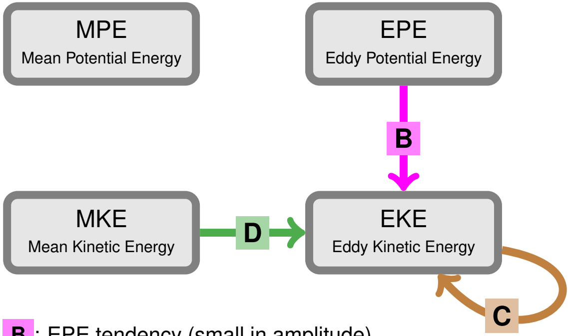
$$\left\langle -\frac{1}{\rho_1}\nabla\cdot\left(\sum_{n=1}^{N}\overline{h_n\mathbf{u}_np_n}^A - \overline{h_n\mathbf{u}_n}^A\,\overline{p_n}^A\right)\right\rangle$$

$$\left\langle \frac{1}{\rho_1}\sum_{n=1}^{N}\overline{h_n\mathbf{u}_n}^A\cdot\overline{\nabla p'_n}^V\right\rangle$$



Time mean

Decomposition: **A** = **B** + **C** + **D**



B: EPE tendency (small in amplitude)

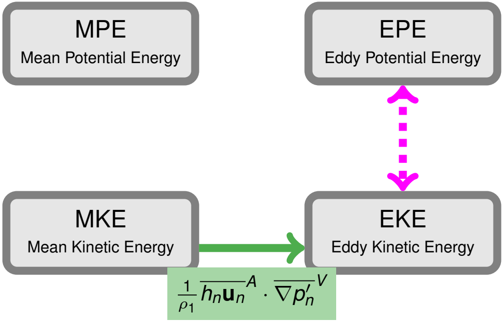
C: EKE transport

D: MKE-EKE conversion associated with eddy form stress

Conversions between Energy Reservoirs

TWA

stacked shallow water equations



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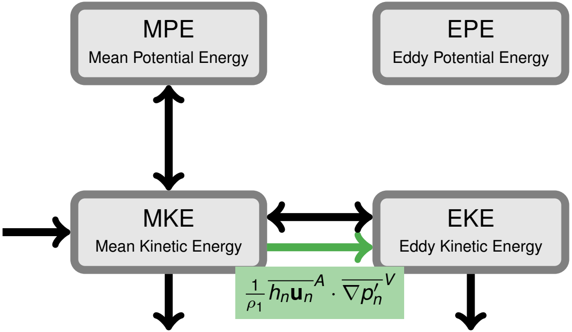

- Eddy form stress: in **velocity** equation
- Associated energy transfer: in **KE** budget

Conversions between Energy Reservoirs

TWA

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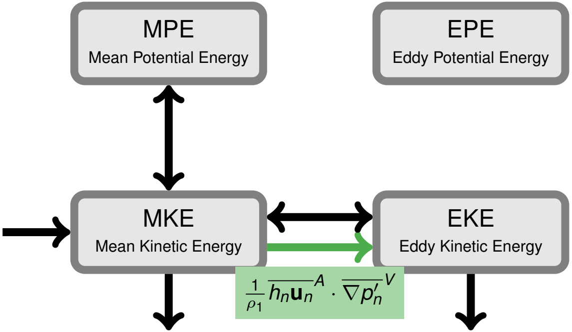
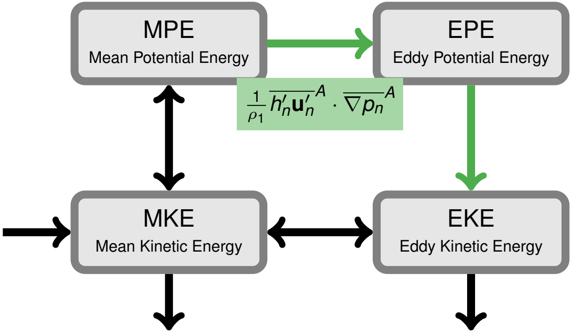


non-TWA

stacked shallow water equations



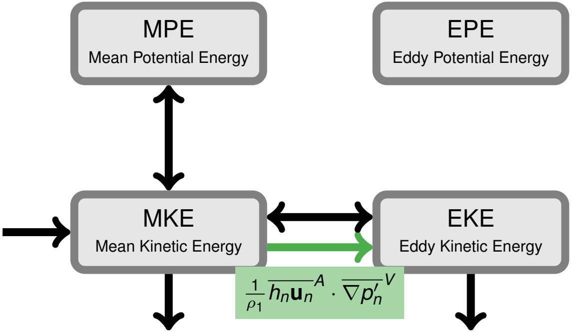
- Eddy form stress: in **velocity** equation
- Associated energy transfer: in **KE** budget

- Eddy form stress: in **continuity** equation
- Associated energy transfer: in **PE** budget

Conversions between Energy Reservoirs

TWA

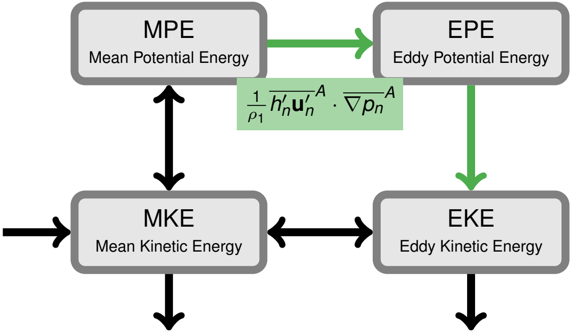
stacked shallow water equations



- Eddy form stress: in **velocity** equation
- Associated energy transfer: in **KE** budget

non-TWA

stacked shallow water equations



- Eddy form stress: in **continuity** equation
- Associated energy transfer: in **PE** budget
- Framework currently used for energy-budget based eddy parameterizations

Summary & Future Work

Work in progress:

- Diagnosing & characterizing the EKE budget terms in stacked shallow water model
- Comparing TWA and non-TWA EKE budgets

Next steps:

1. Evaluate energy-budget based parameterizations.
2. What carries over to general coordinate ocean models?