Towards a potential-vorticity based mesoscale closure scheme

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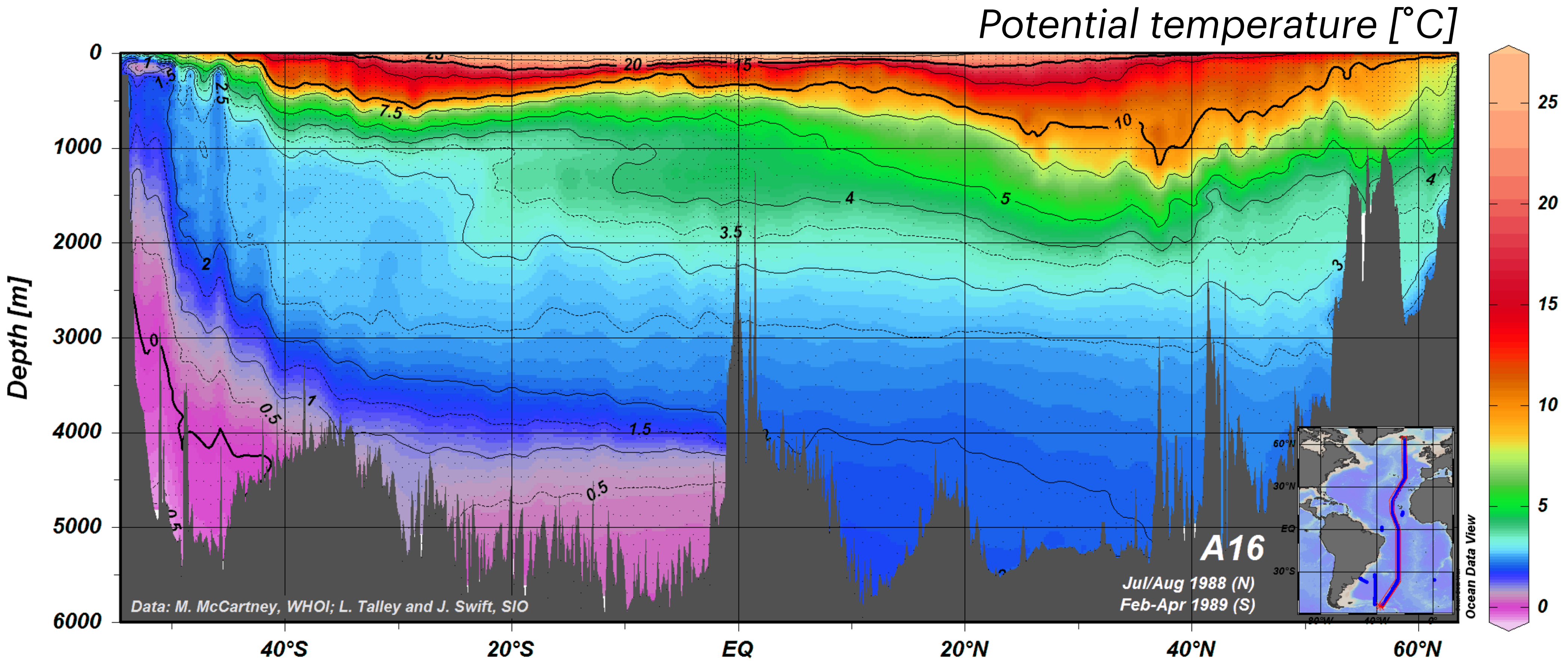
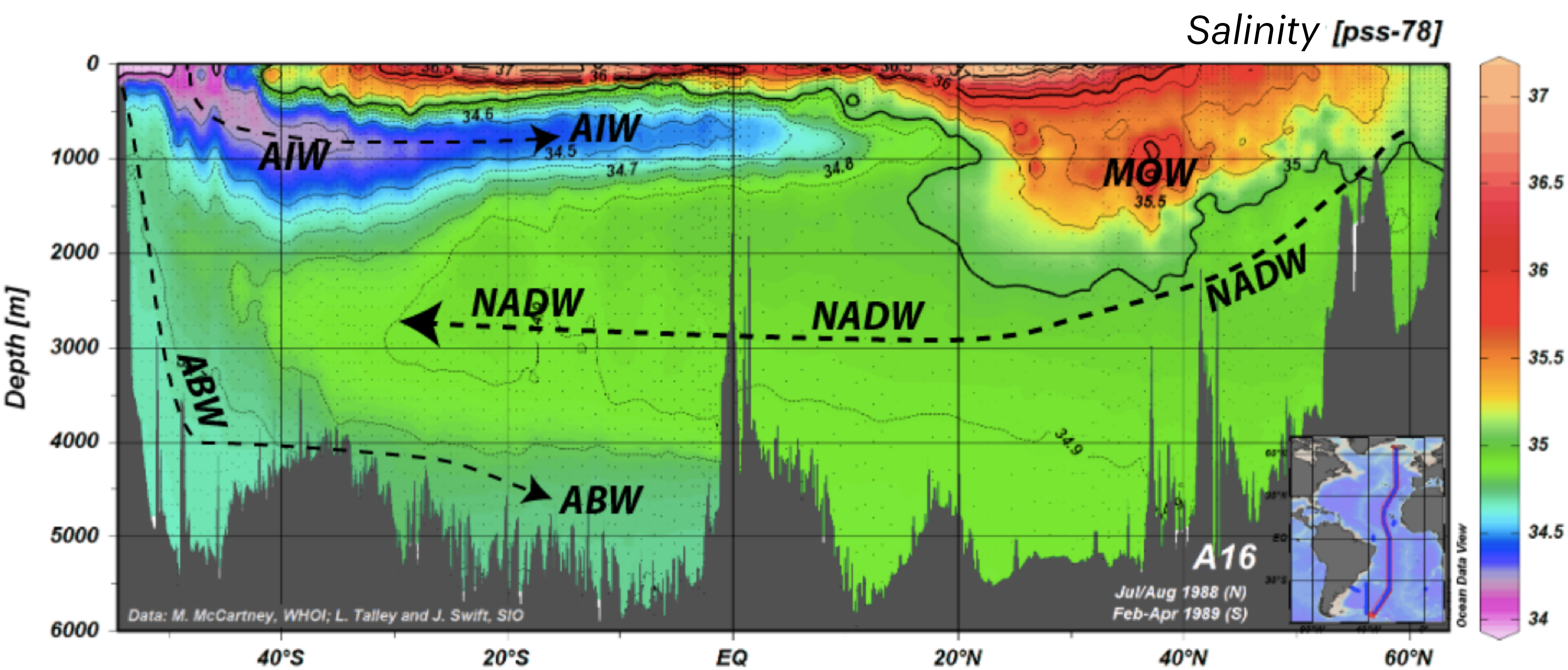
The stratified ocean

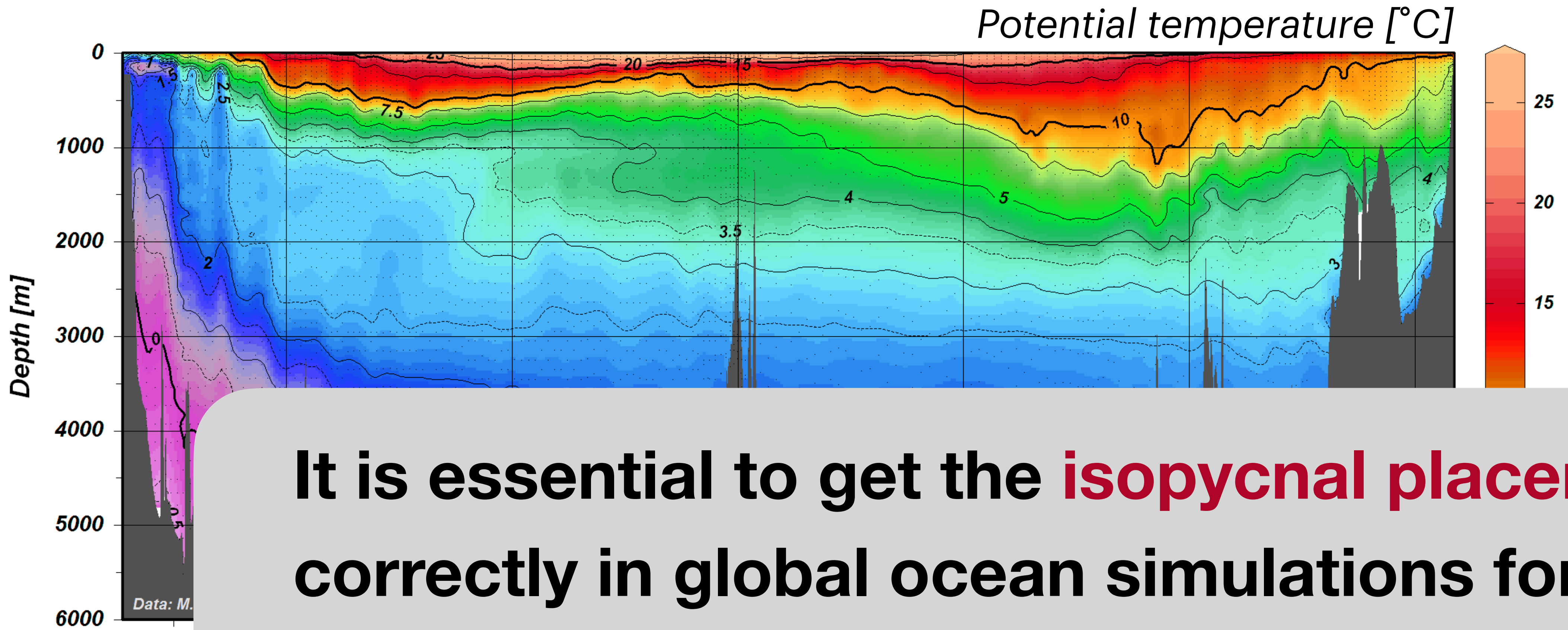




Figures taken from the World Ocean Circulation Experiment (WOCE)

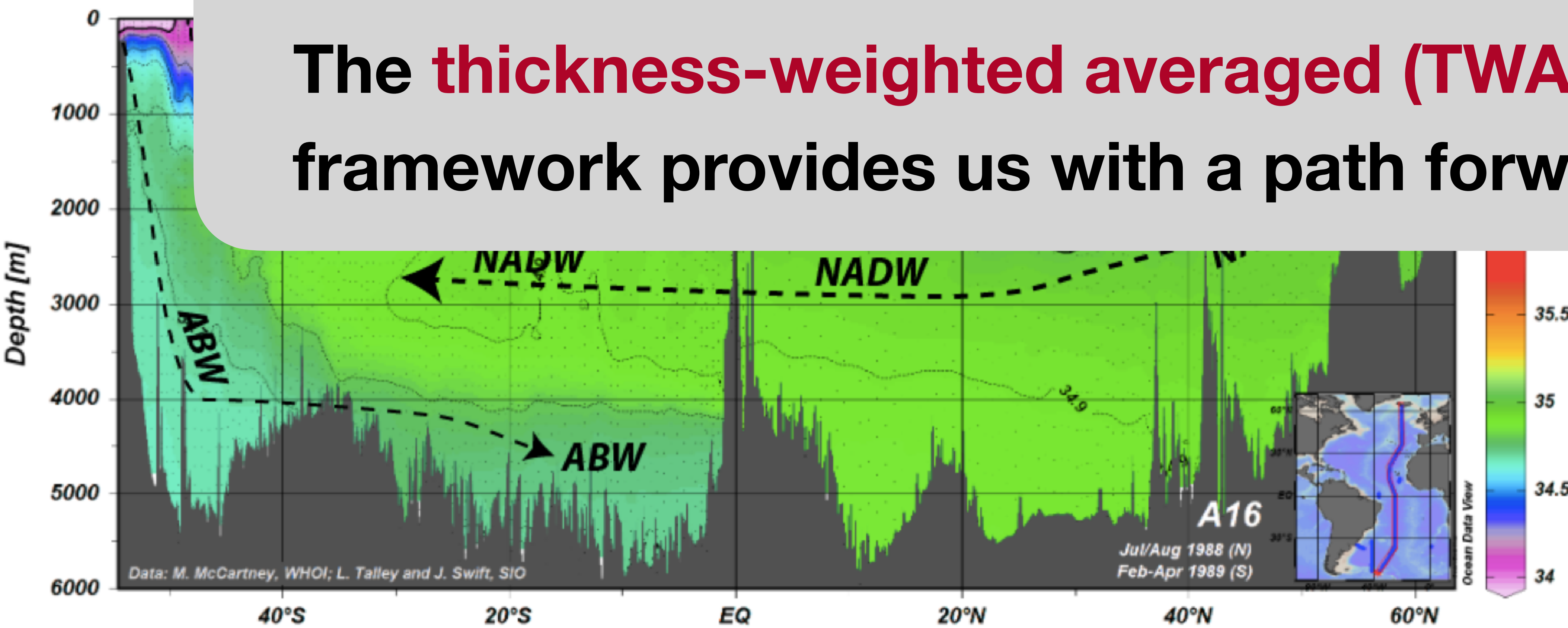
The ocean interior is stratified and quasi-adiabatic, so much so that we infer the global ocean circulation from tracer distribution along isopycnals.

The stratified ocean

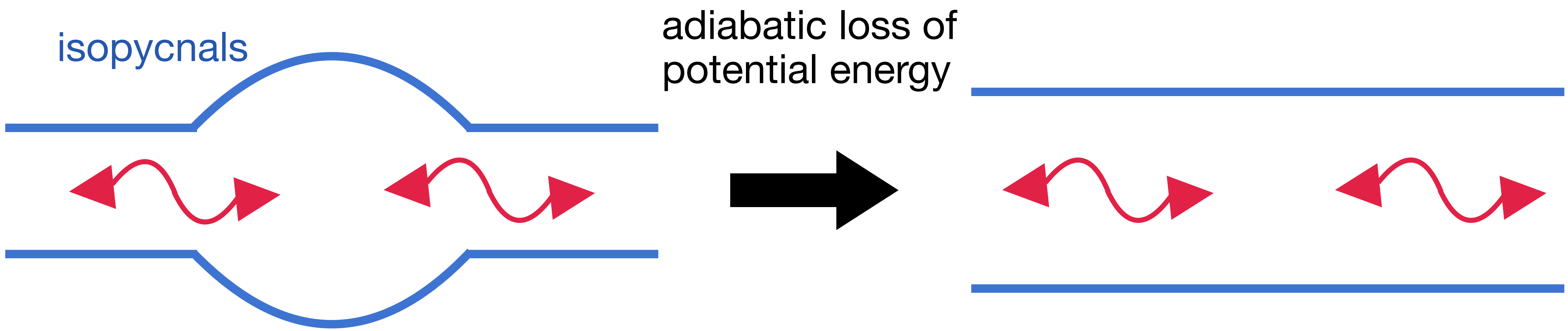


It is essential to get the **isopycnal placement** correctly in global ocean simulations for estimating global heat and tracer transport.

The **thickness-weighted averaged (TWA)** framework provides us with a path forward.

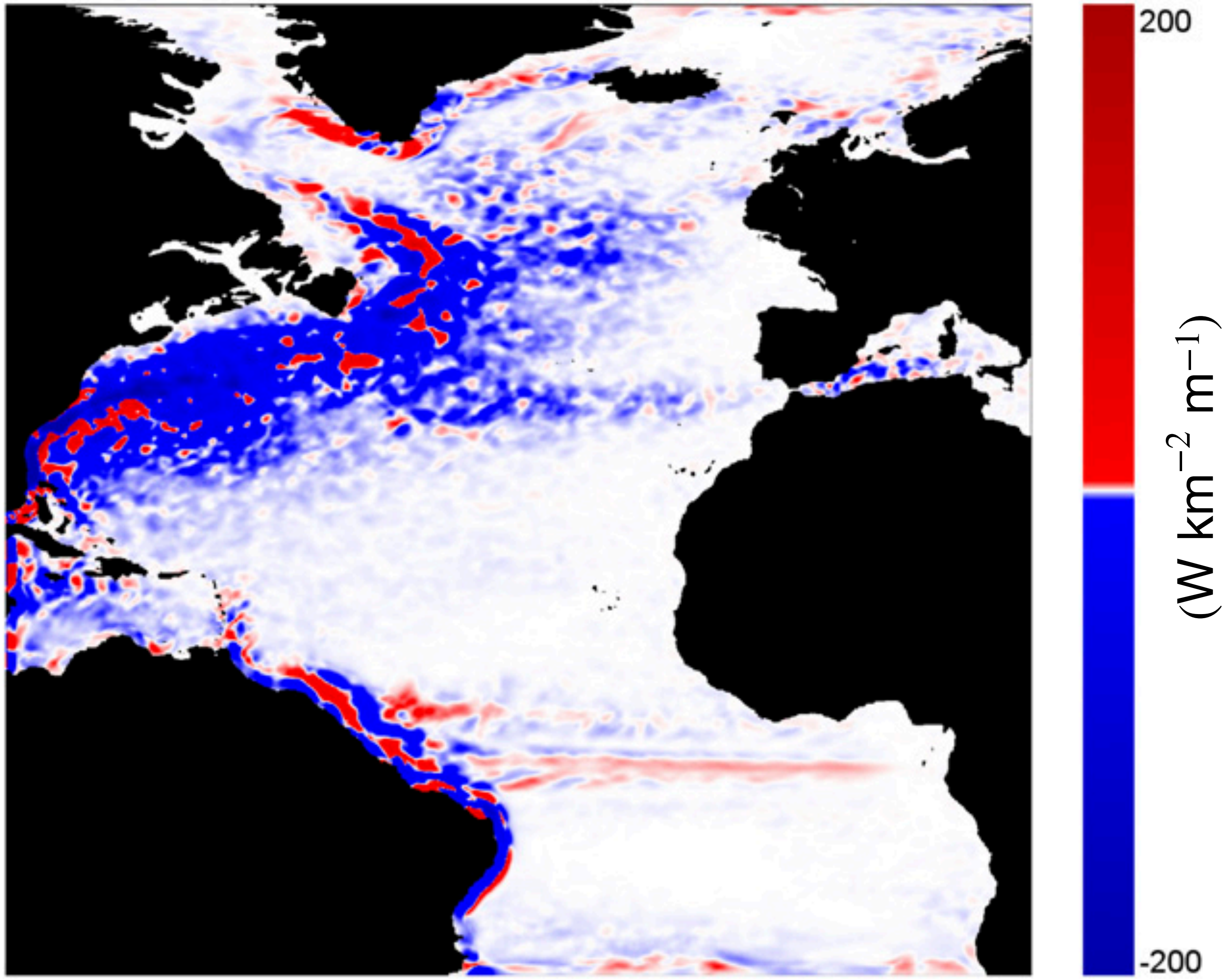


Figures taken from the World Ocean Circulation Experiment (WOCE)



- The Gent-McWilliams (GM) skew diffusivity diffuses the isopycnal thickness in a similar manner to how baroclinic instability would if resolved.
- The Redi diffusivity represents the enhanced tracer stirring along isopycnals due to eddies.

- **GM and Redi should be related to one another.**
- **Can we capture the full eddy feedback and not just the release of available potential energy?**



Aluie et al. (2018)

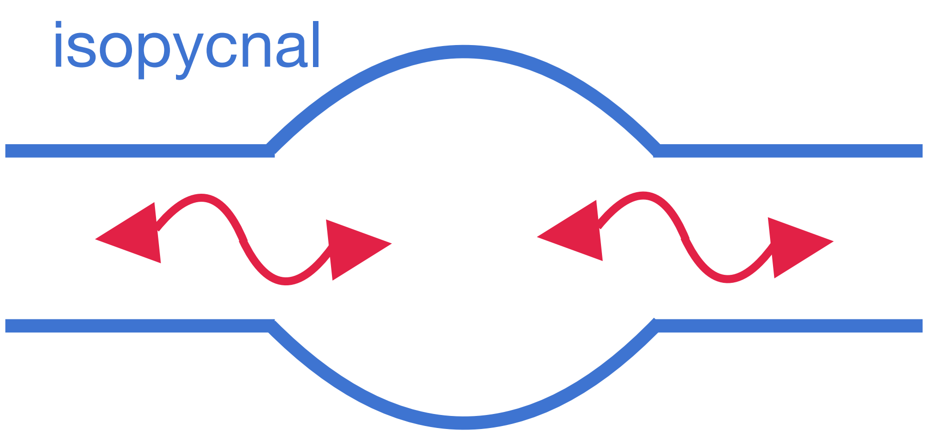
- Employing a coarse-graining method, Aluie et al. (2018) examined the direction of kinetic energy cascade from a $0.1°$ model simulation.
- Blue shadings indicate the eddies fluxing kinetic energy back into the mean flow.

Mesoscale eddies energize the Gulf Stream.
Can we say more on how and where?

$$\hat{u}_{\tilde{t}} + \hat{u}\hat{u}_{\tilde{x}} + \hat{v}\hat{u}_{\tilde{y}} + \hat{\varpi}\hat{u}_{\tilde{b}} - f\hat{v} + \overline{m}_{\tilde{x}} = -\overline{\mathbf{e}}_1 \cdot (\tilde{\nabla} \cdot \mathbf{E}) + \hat{\mathcal{X}}$$

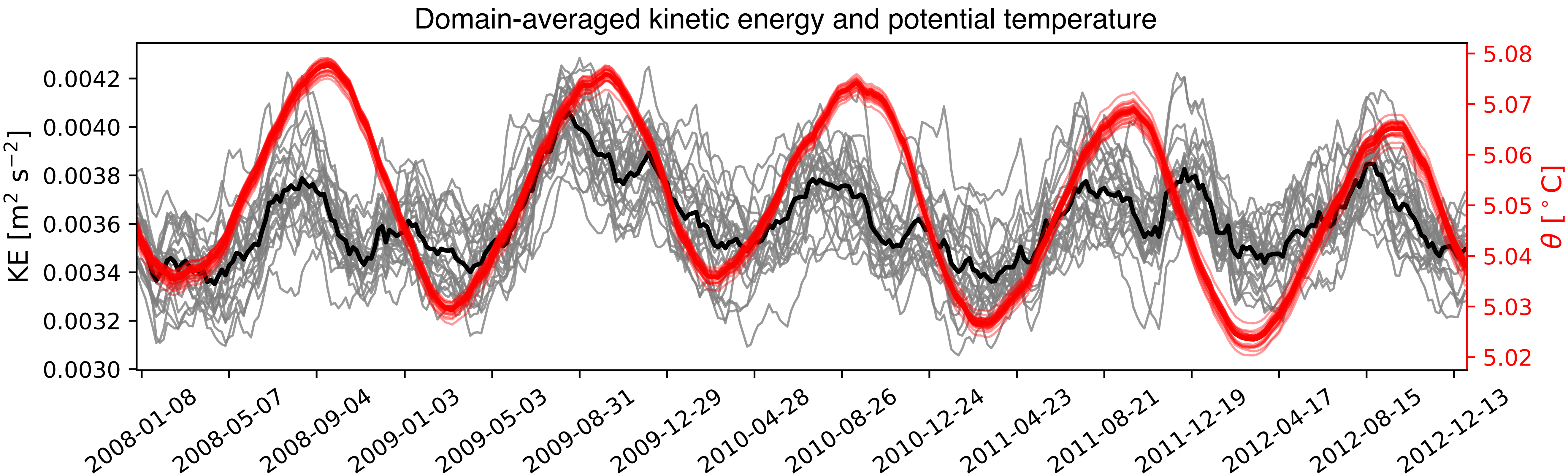
$$\hat{v}_{\tilde{t}} + \hat{u}\hat{v}_{\tilde{x}} + \hat{v}\hat{v}_{\tilde{y}} + \hat{\varpi}\hat{v}_{\tilde{b}} + f\hat{u} + \overline{m}_{\tilde{y}} = -\overline{\mathbf{e}}_2 \cdot (\tilde{\nabla} \cdot \mathbf{E}) + \hat{\mathcal{Y}}$$

- $\hat{\mathbf{u}}$ $\left(= \dfrac{\overline{\sigma \mathbf{u}}}{\overline{\sigma}} \right)$: the thickness-weighted averaged (TWA) velocity.
- $\overline{(\ \cdot\ )}$ is the ensemble mean.
- $\sigma\ (= \zeta_{\tilde{b}})$: the isopycnal thickness.
- ϖ : the diapycnal velocity.
- $m\ (= \phi - b\zeta)$: the Montgomery potential.



Young (2012); Ringler et al. (2017)

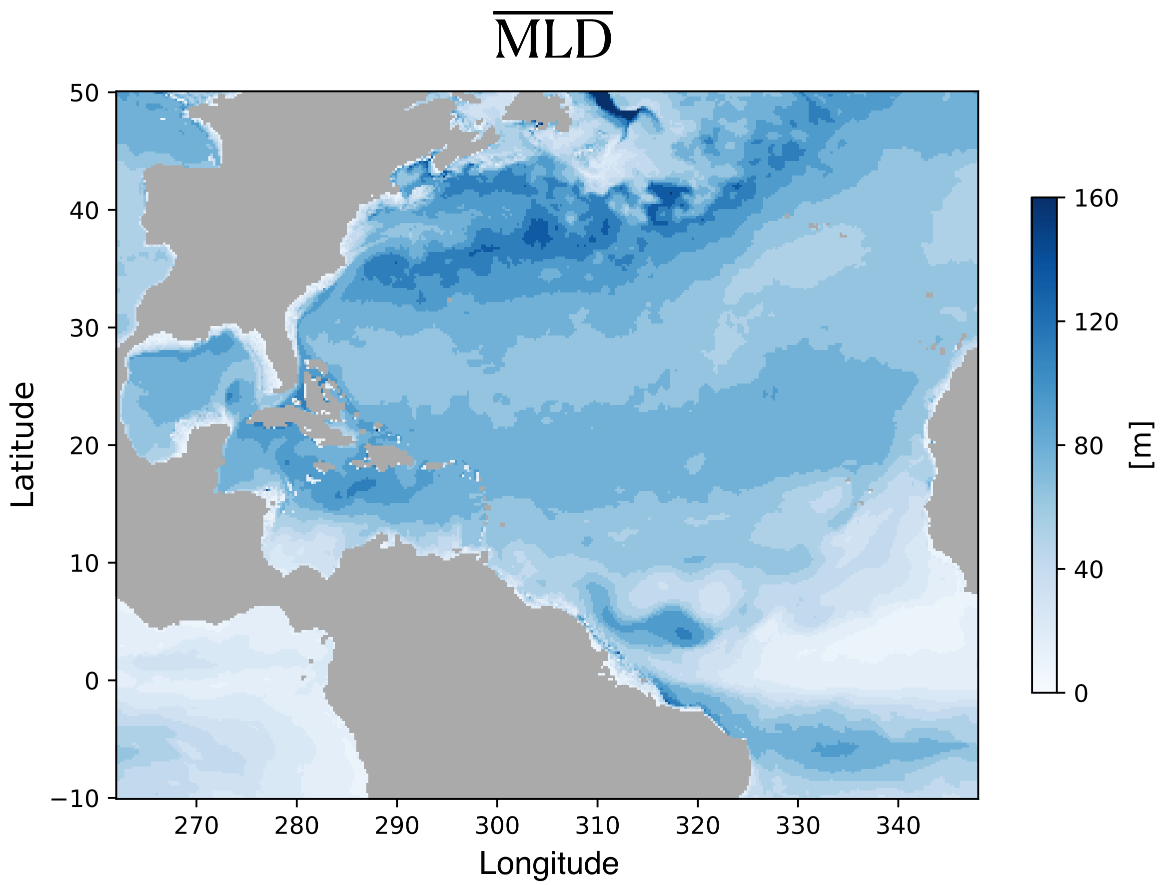
A 24-member ensemble simulation

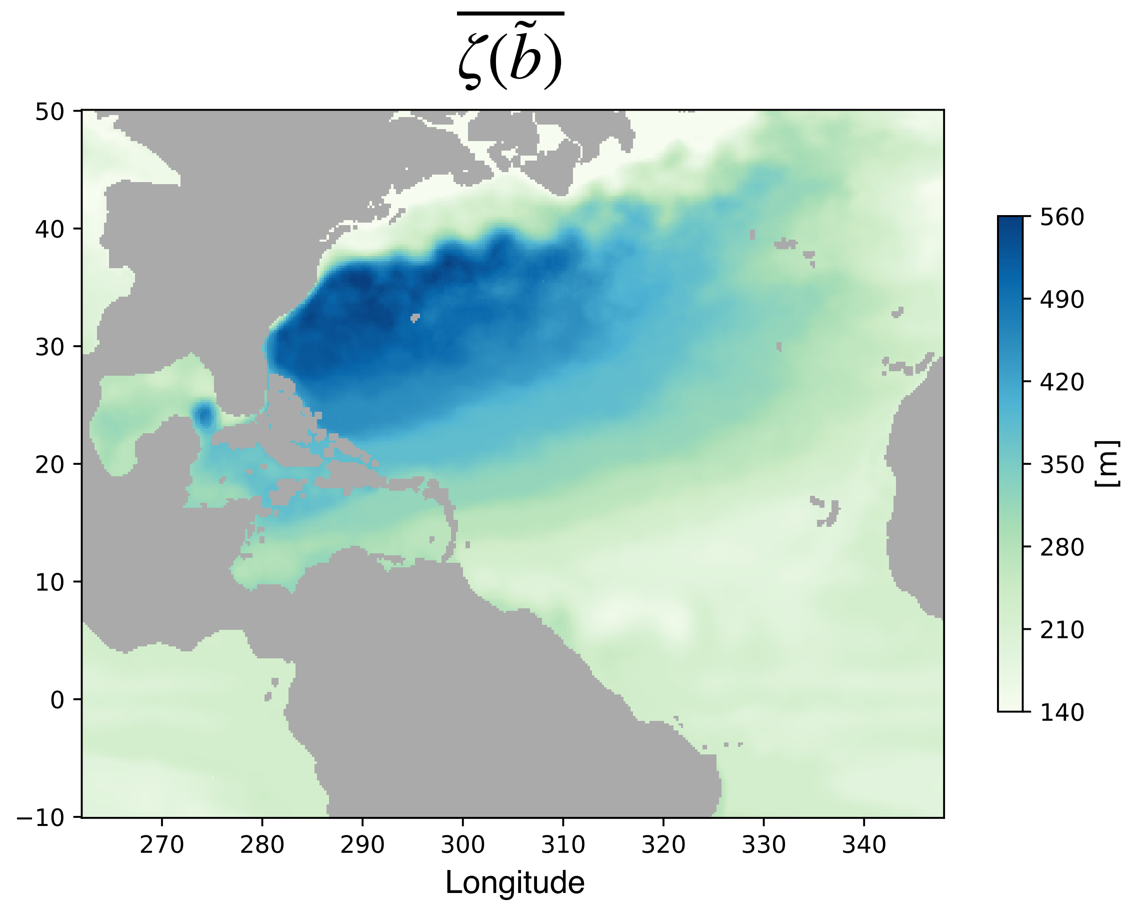


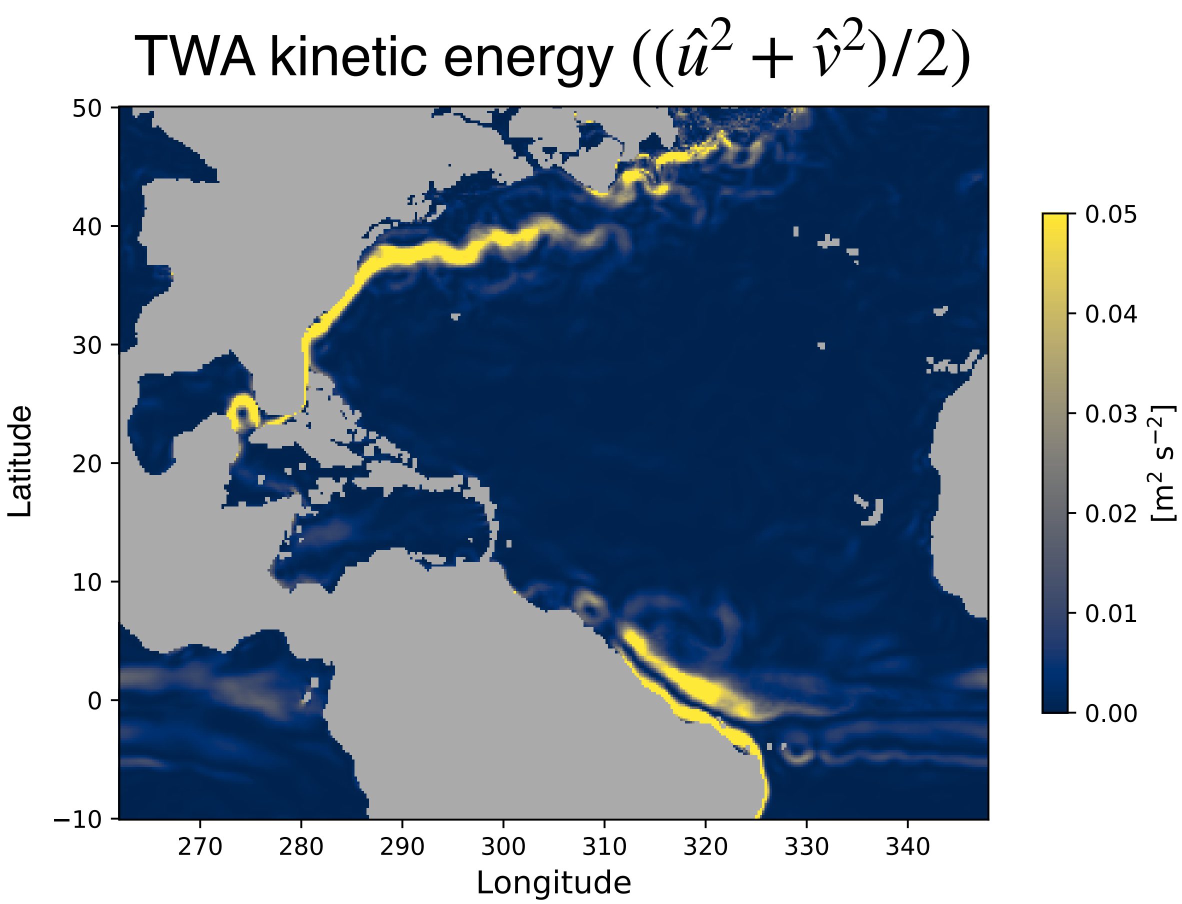
Thin lines: each ensemble member
Thick lines: ensemble mean

- **No. of ensemble members**: 24.
- **Resolution**: $1/12°$; **Duration**: 50 years (1963-2012).
- **Model**: MITgcm; **Basin**: North Atlantic.
- **Surface boundary condition**: partially air-sea coupled.
- **Lateral boundary condition**: relaxation and radiation conditions.

Results on January 3, 2008







- Focus on an isopycnal whose ensemble-mean depth ($\bar{\zeta}$) is below the ensemble-mean mixed-layer depth ($\overline{\mathrm{MLD}}$).
- The isopycnal shoals drastically across the separated Gulf Stream.

$$\hat{u}_{\tilde{t}} + \hat{u}\hat{u}_{\tilde{x}} + \hat{v}\hat{u}_{\tilde{y}} + \hat{\varpi}\hat{u}_{\tilde{b}} - f\hat{v} + \overline{m}_{\tilde{x}} = \boxed{-\overline{\mathbf{e}}_1 \cdot (\tilde{\nabla} \cdot \mathbf{E})} + \hat{\mathcal{X}}$$

$$\hat{v}_{\tilde{t}} + \hat{u}\hat{v}_{\tilde{x}} + \hat{v}\hat{v}_{\tilde{y}} + \hat{\varpi}\hat{v}_{\tilde{b}} + f\hat{u} + \overline{m}_{\tilde{y}} = \boxed{-\overline{\mathbf{e}}_2 \cdot (\tilde{\nabla} \cdot \mathbf{E})} + \hat{\mathcal{Y}}$$

- The net eddy feedback onto the (TWA) mean fields are encapsulated in the Eliassen-Palm flux $(\mathbf{E})$ divergence.
- For an eddy closure, it shifts the focus from the buoyancy equation (GM) to the momentum equations.

If we can parametrize $\overline{\mathbf{e}} \cdot (\nabla \cdot \mathbf{E})$, we have a physically consistent eddy closure scheme which represents the eddy buoyancy & momentum fluxes.

Can we parametrize the Eliassen-Palm flux divergence?

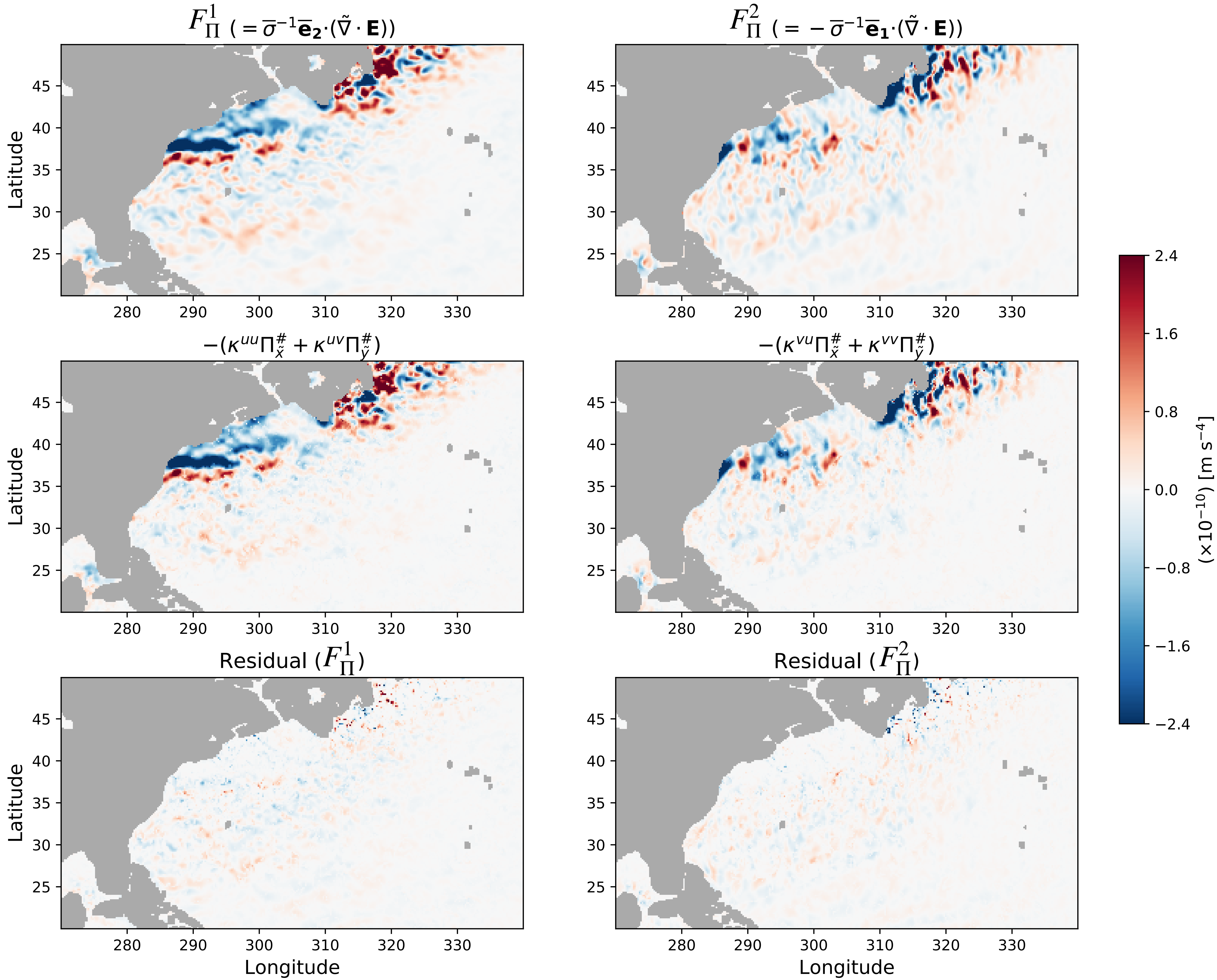
- The Eliassen-Palm flux divergence is directly related to the eddy Ertel potential vorticity (PV) flux ($\mathbf{F}^{\Pi}$).
- This primes us to connect the Eliassen-Palm flux divergence to the large-scale Ertel PV.
- We relate the eddy Ertel PV flux to the local-gradient flux of the mean Ertel PV ($\Pi^{\#}$) via an anisotropic eddy diffusivity tensor ($\mathbf{K}$).

$$\underbrace{\begin{pmatrix} \widehat{u''\theta''} & \widehat{v''\theta''} \\ \widehat{u''s''} & \widehat{v''s''} \\ F^{\Pi 1} & F^{\Pi 2} \end{pmatrix}}_{\mathbf{F}} = -\underbrace{\begin{pmatrix} \hat{\theta}_{\tilde{x}} & \hat{\theta}_{\tilde{y}} \\ \hat{s}_{\tilde{x}} & \hat{s}_{\tilde{y}} \\ \Pi^{\#}_{\tilde{x}} & \Pi^{\#}_{\tilde{y}} \end{pmatrix}}_{\mathbf{G}} \cdot \underbrace{\begin{pmatrix} \kappa^{uu} & \kappa^{vu} \\ \kappa^{uv} & \kappa^{vv} \end{pmatrix}}_{\mathbf{K}}$$

Redi

GM
+
Eddy momentum flux

Can we parametrize the Eliassen-Palm flux divergence?



- The Eliassen-Palm flux divergence, which is directly related to the eddy PV flux, encapsulates the net eddy feedback onto the mean flow.
- The eddy PV flux can be related to the TWA field as an active tracer.
- The 2×2 diffusivity tensor $\mathbf{K}$, which provides a closure for the eddy PV flux, single-handedly includes the information of eddy momentum fluxes in addition to bringing the GM and Redi diffusivities together.
- For a prognostic closure scheme, we would need to inform the "κ"s via the (resolved?) TWA field.

(**Extra slide**) Defining the buoyancy coordinate for a realistic EOS

With a realistic equation of state (EOS) the vertical coordinate cannot “naively” be defined by potential density (ρ_θ) as the pressure anomaly $(\phi = \int - g\rho_0^{-1}(\rho_\theta - \rho_0)d\zeta)$ does not translate to a body force in buoyancy coordinates, i.e. $\nabla_\text{h}\phi \neq \tilde{\nabla}_\text{h}m$. We, therefore, argue for the use of in-situ density *anomaly* $\delta\ (= \rho - \underset{\sim}{\rho}(\zeta))$ where ρ is the in-situ density and $\underset{\sim}{\rho}$ is a function of only depth. The buoyancy coordinate can then be defined as $\tilde{b} = -g\rho_0^{-1}\delta$ which removes a large portion of compressibility; the iso-surfaces of $\tilde{b}$ become close to neutral surfaces. The formulation of $\tilde{b}$ is analogous to where the buoyancy reduces to $\tilde{b} = -g\rho_0^{-1}(\rho - \rho_0)$ for a linear EOS.

$$\hat{u}_{\tilde{t}} + \hat{u}\hat{u}_{\tilde{x}} + \hat{v}\hat{u}_{\tilde{y}} + \hat{\varpi}\hat{u}_{\tilde{b}} - f\hat{v} + \overline{m}_{\tilde{x}} = -\bar{\mathbf{e}}_1 \cdot (\tilde{\nabla} \cdot \mathbf{E}) + \hat{\mathcal{X}}$$

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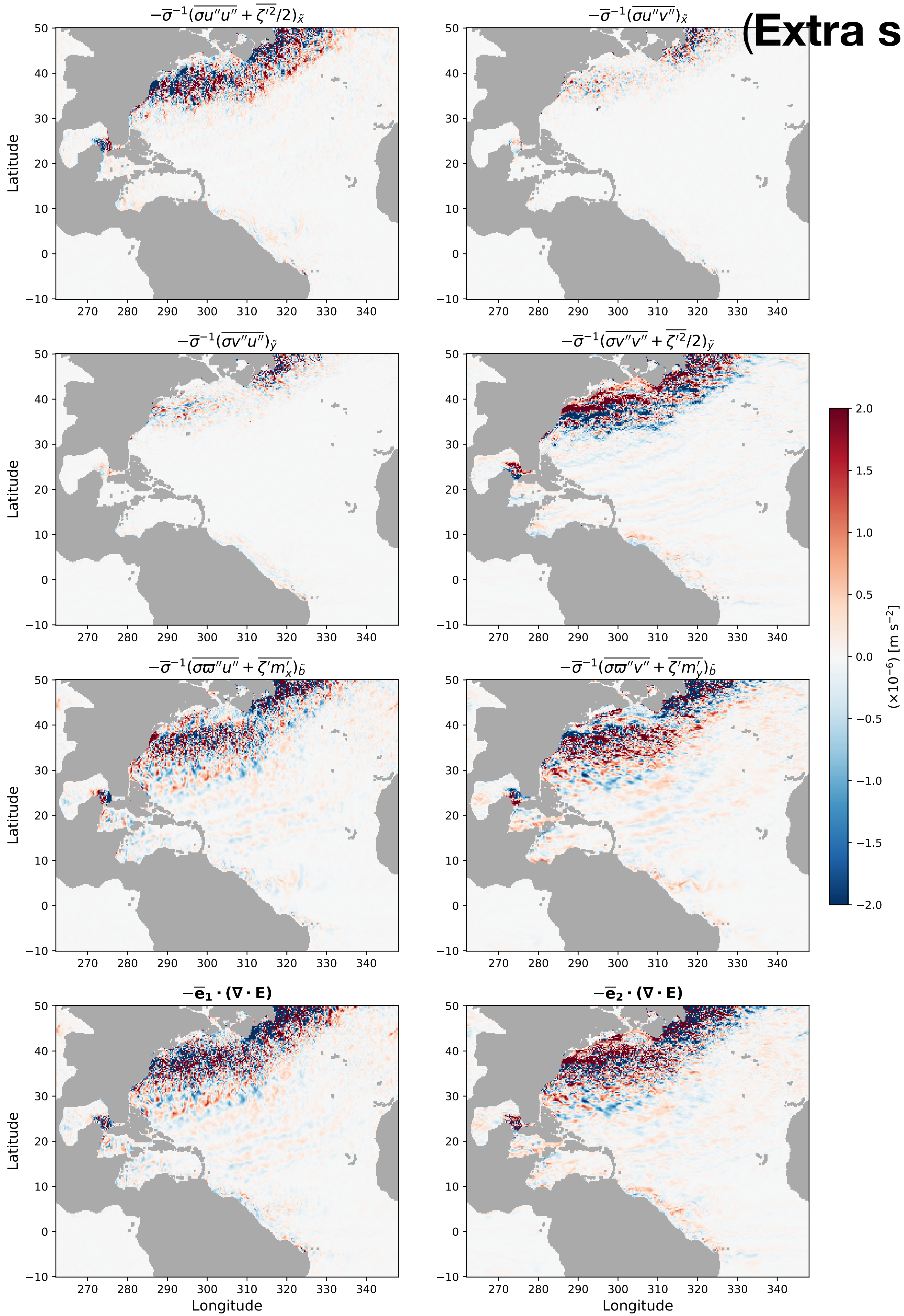
$$\mathbf{E} = \begin{pmatrix} \widehat{u''u''} + \frac{1}{2\overline{\sigma}}\overline{\zeta'^2} & \widehat{u''v''} & 0 \\ \widehat{v''u''} & \widehat{v''v''} + \frac{1}{2\overline{\sigma}}\overline{\zeta'^2} & 0 \\ \widehat{\varpi''u''} + \frac{1}{\overline{\sigma}}\overline{\zeta' m'_{\tilde{x}}} & \widehat{\varpi''v''} + \frac{1}{\overline{\sigma}}\overline{\zeta' m'_{\tilde{y}}} & 0 \end{pmatrix}$$

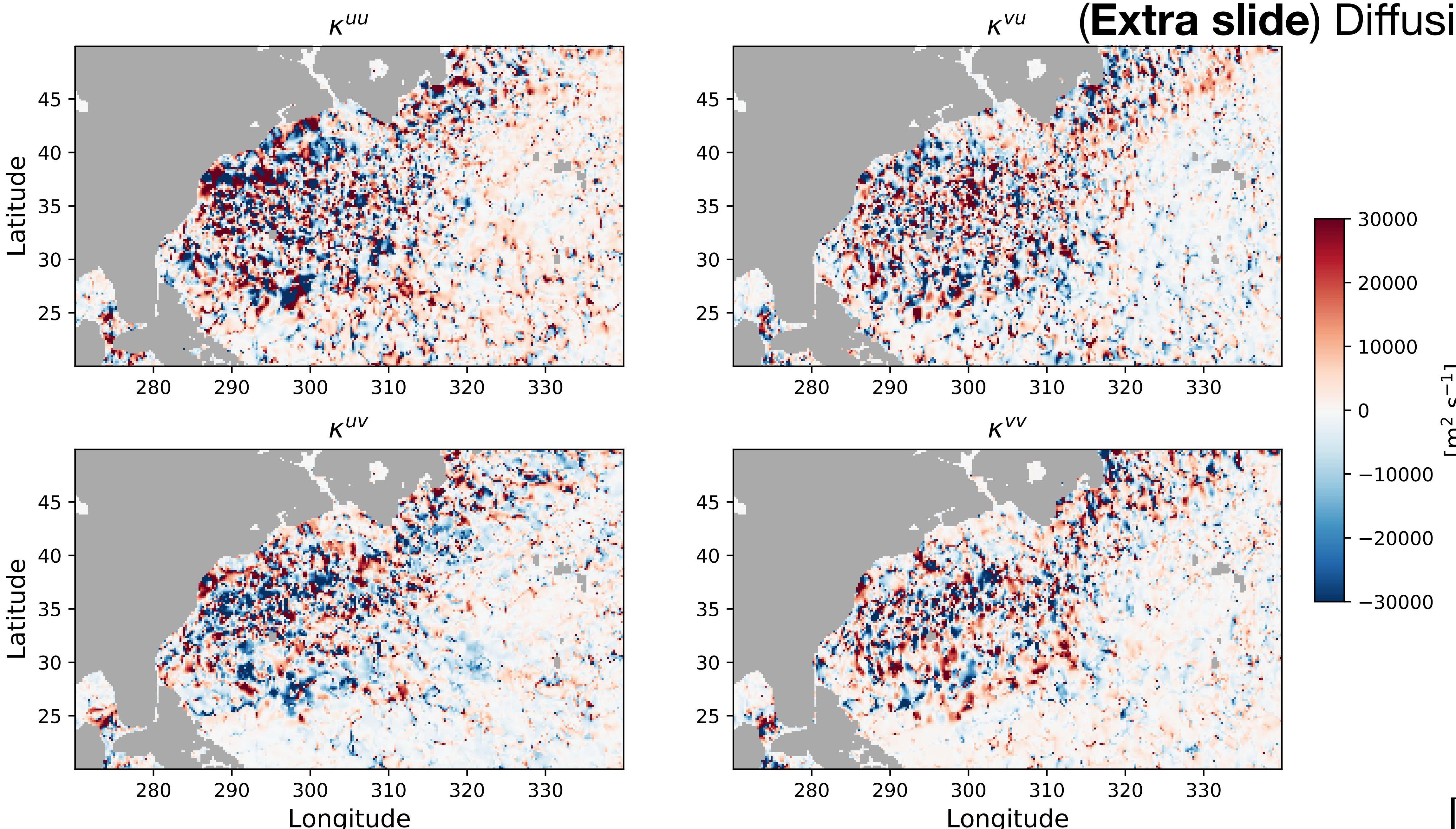
- $\mathbf{u}''\ (= \mathbf{u} - \hat{\mathbf{u}})$: the eddy velocity.
- $(\,\cdot\,)'\ (= (\,\cdot\,) - \overline{(\,\cdot\,)})$: the residual from the ensemble mean.

(Extra slide) E-P flux divergence on Jan. 3, 2008

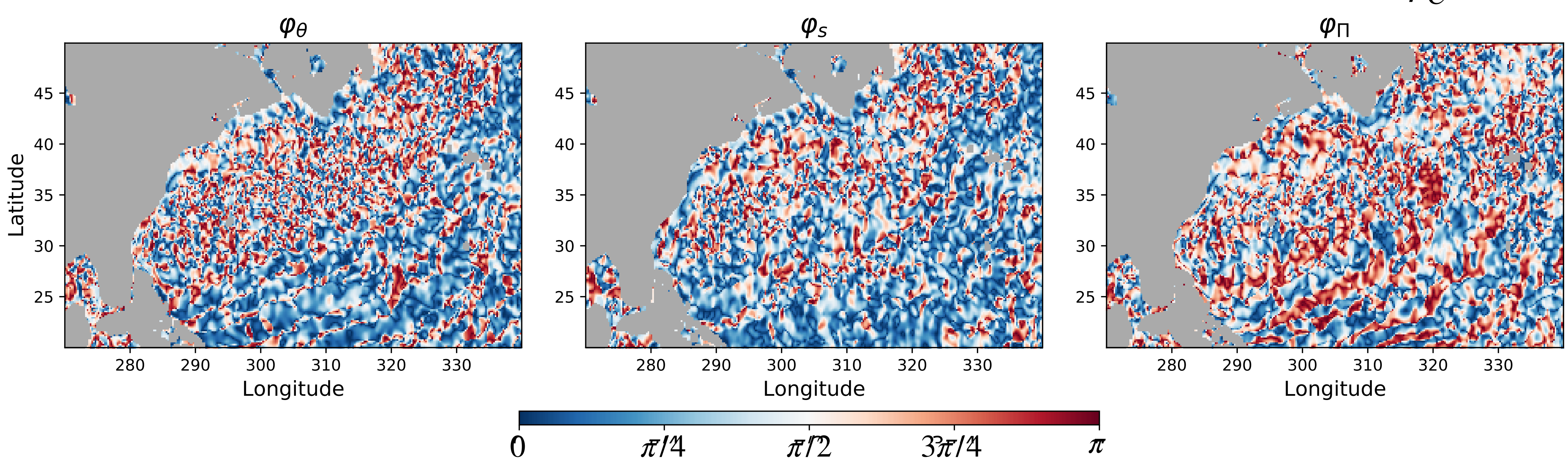


- Each column is laid out so that the sum of the first three rows sum up to the E-P flux divergence shown in the bottom row.
- The terms associated with eddy momentum flux and baroclinic instability tend to cancel each other out.

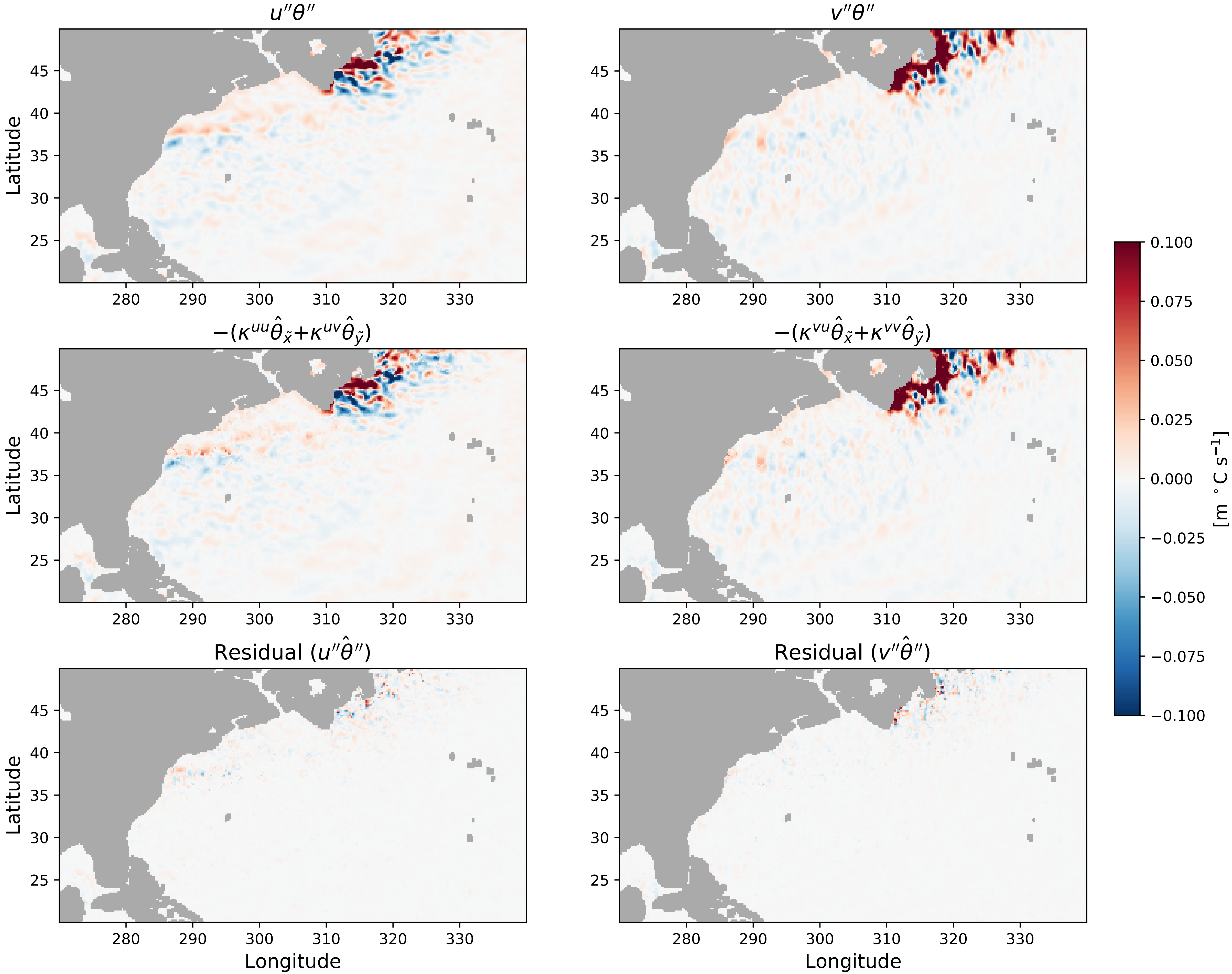
(**Extra slide**) Diffusivity tensor



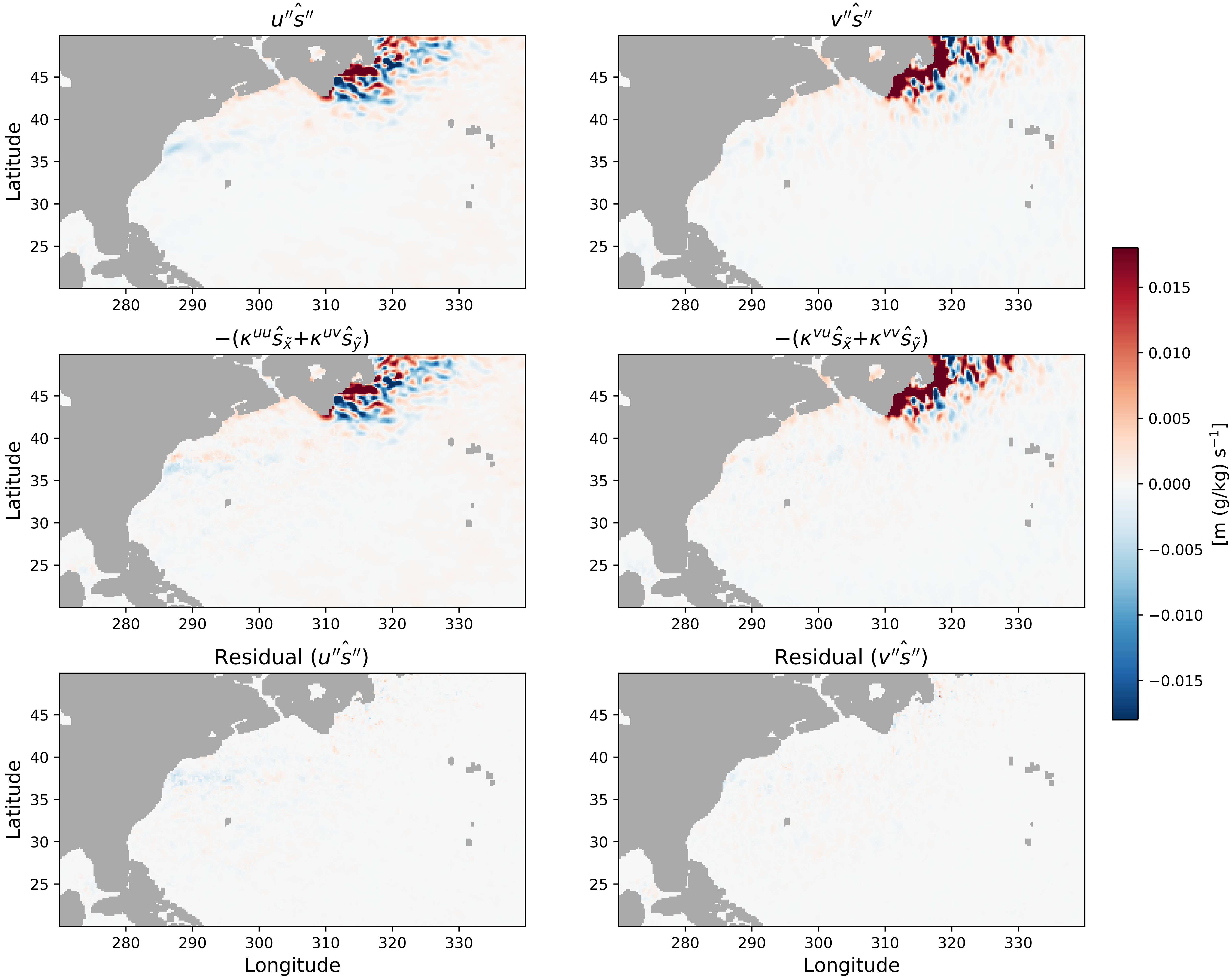
$$\varphi_C = \arccos\left[\frac{\mathbf{F}_C \cdot \mathbf{G}_C}{|\mathbf{F}_C||\mathbf{G}_C|}\right]$$



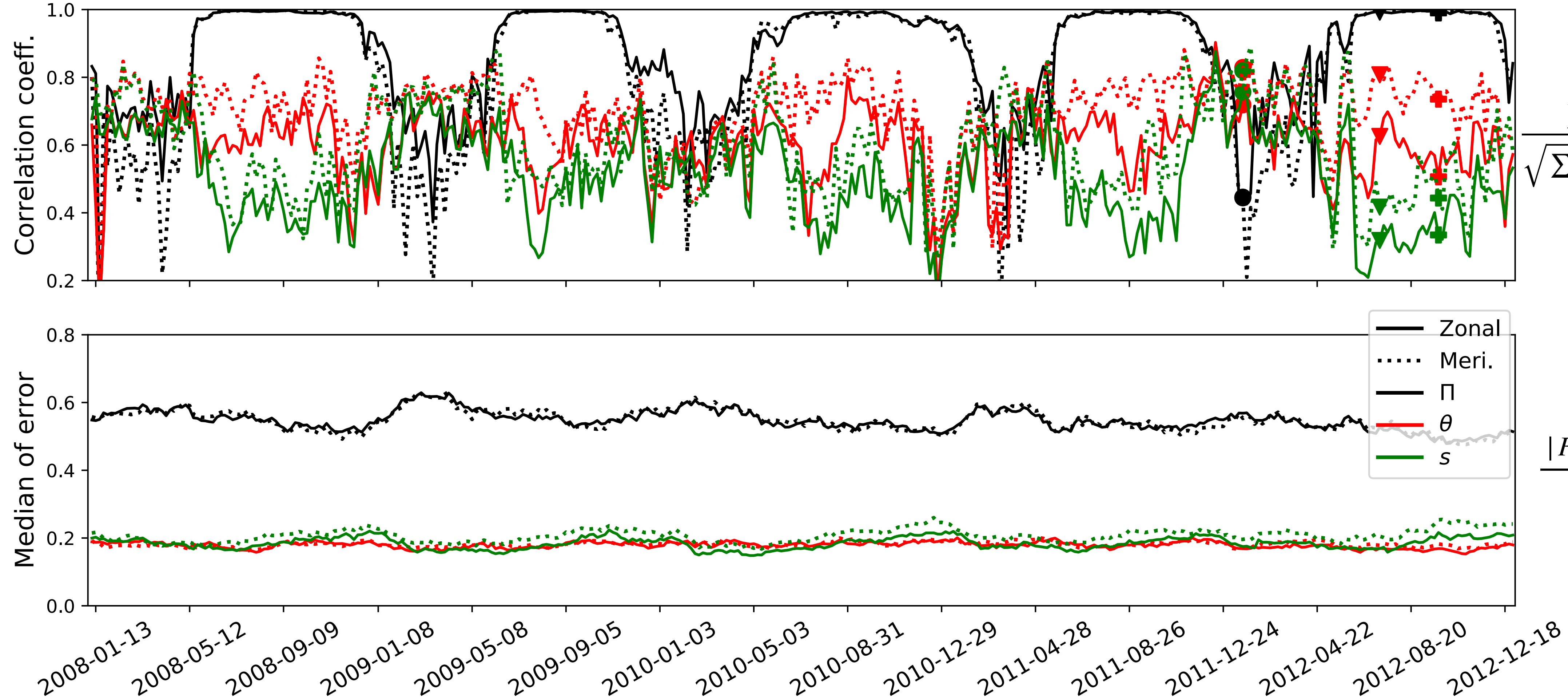
(**Extra slide**) Temperature parametrization



(**Extra slide**) Salinity parametrization



(**Extra slide**) Correlation and error of parametrization



$$\frac{\sum[(F_C-\langle F_C\rangle)(F_C^{\text{param}}-\langle F_C^{\text{param}}\rangle)]}{\sqrt{\sum(F_C-\langle F_C\rangle)^2}\sqrt{\sum(F_C^{\text{param}}-\langle F_C^{\text{param}}\rangle)^2}}$$

$$\frac{|F_C-F_C^{\text{param}}|}{|F_C|};\ F_C^{\text{param}}=G_C\cdot K_C$$