Vertical structure of tracer diffusivity in an idealized basin circulation model

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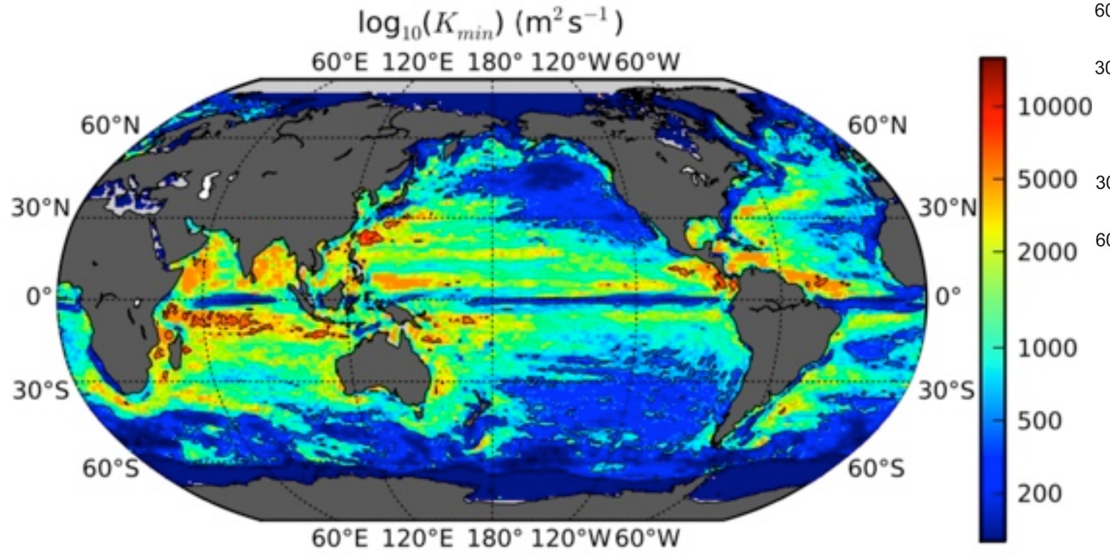
Observations of mesoscale mixing rate

Parameterization for eddy fluxes:

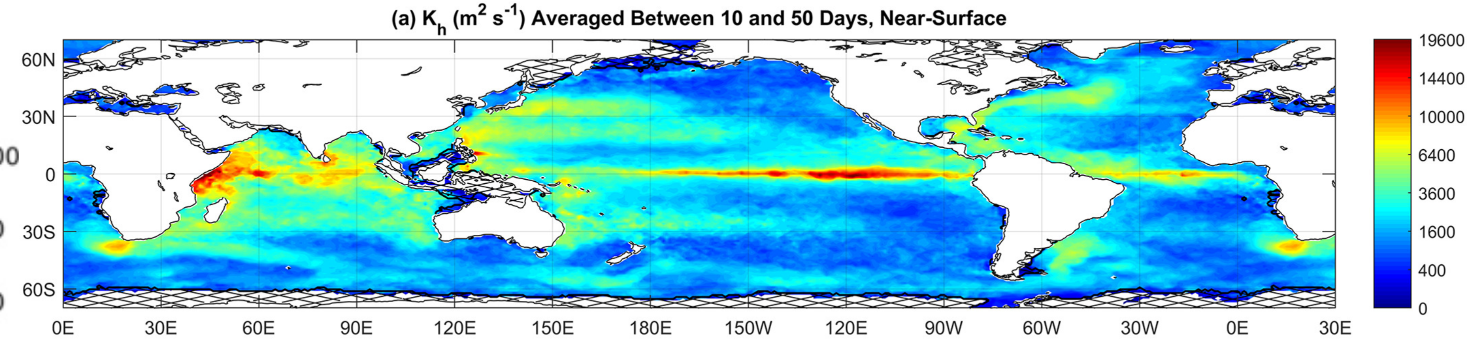
$$\overline{\boldsymbol{u}'\tau'} = -\mathbf{K}\nabla\bar{\tau}$$

Tracer-based method



Abernathey et al. (2013)

Lagrangian method



Roach et al. (2018)

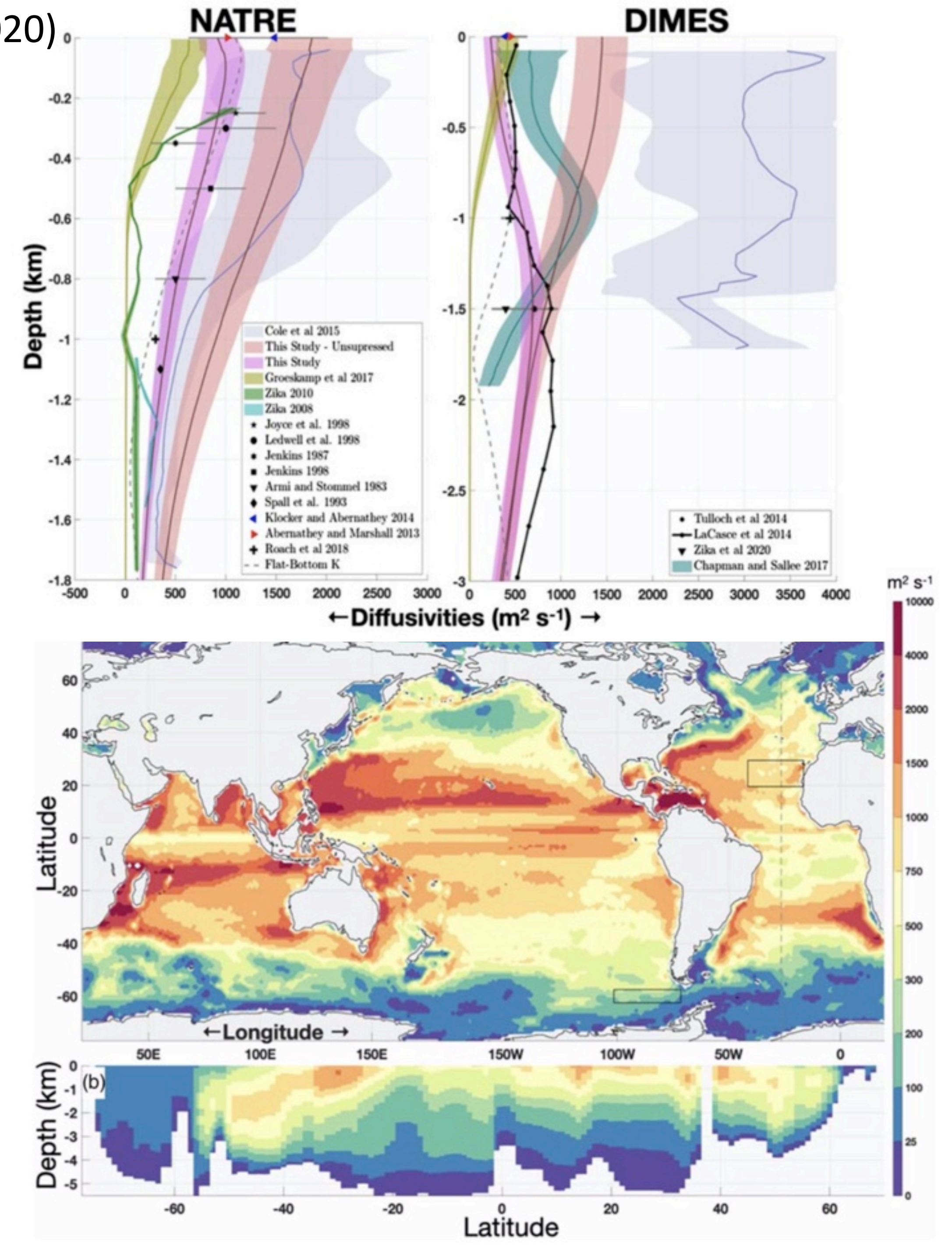
Groeskamp et al. (2020)

Vertical structure of diffusivity

Based on the suppressed mixing length theory (Ferrari and Nikurashin, 2010), Groeskamp et al., 2020 proposed an estimate of full-depth diffusivity,

$$K(x, y, z) = \Gamma \phi(z) \sqrt{2\mathrm{EKE}_0} L_d \times \min(S^x, S^y),$$

where $\phi(z)$ is a **vertical structure** function of the first "**surface mode**" (LaCasce 2017), EKE_0 is the EKE at surface, L_d is the deformation radius of the the first "surface mode". S^x and S^y are the **suppression factors** accounting for eddy propagation relative to mean flow, and Γ is the mixing efficiency.

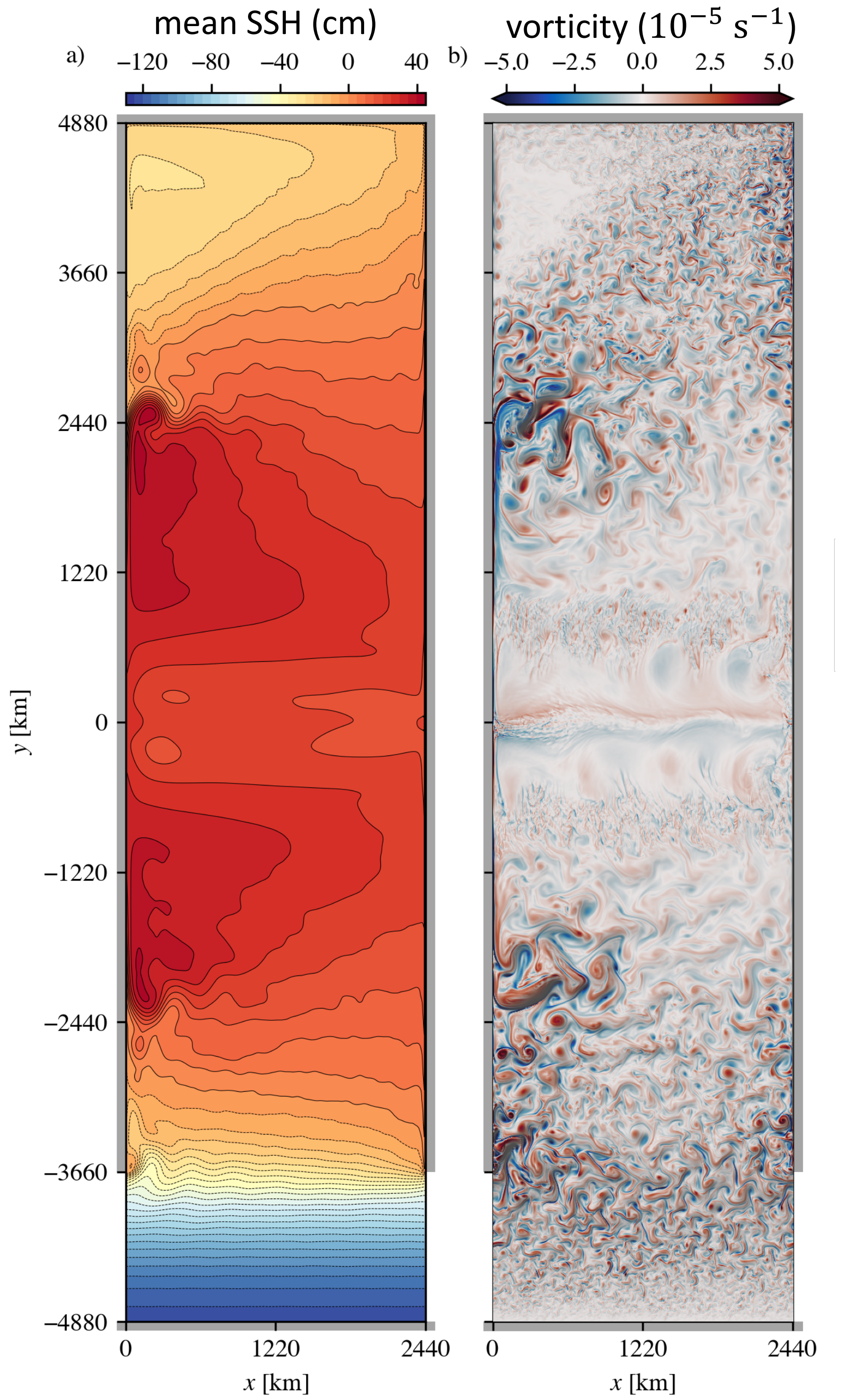


Questions:

Does mixing length theory work over full water column?
Can the vertical structure of EKE be reconstructed from hydrography?
How does this theory apply to the entire diffusivity tensor?

Numerical Model

- This model is an idealized configuration of the MITgcm (Marshall et al. 1997a,b; Campin et al. 2020) used by previous studies (e.g., Wolfe et al. 2008; Wolfe and Cessi 2009, 2010, 2011)
- A two-hemisphere basin on an **equatorial *β*-plane** with a **flat bottom**
- Extent of the domain: 2440 km in zonal direction, 9880 km in meridional direction and a uniform depth of 2440 m
- The southernmost eighth of the domain is **zonally reentrant**
- **5.4 km** horizontal resolution; **20** vertical levels
- Forcing: zonally uniform **zonal winds** and a relaxation to a zonally uniform **surface temperature distribution**

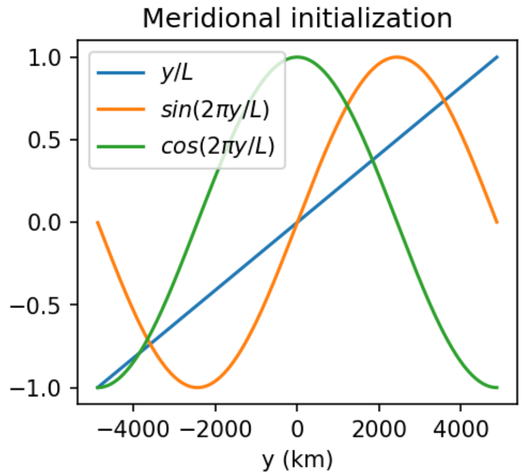


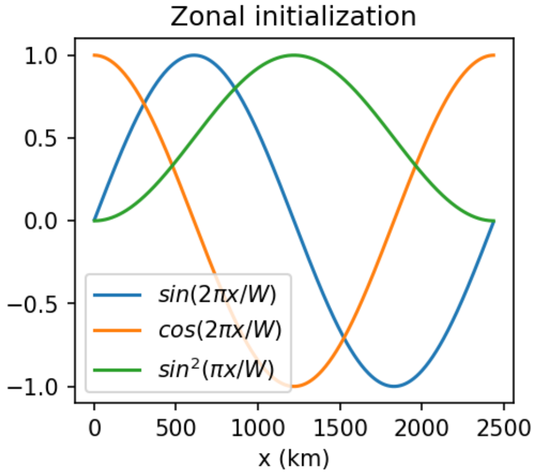
Diagnosing **K**

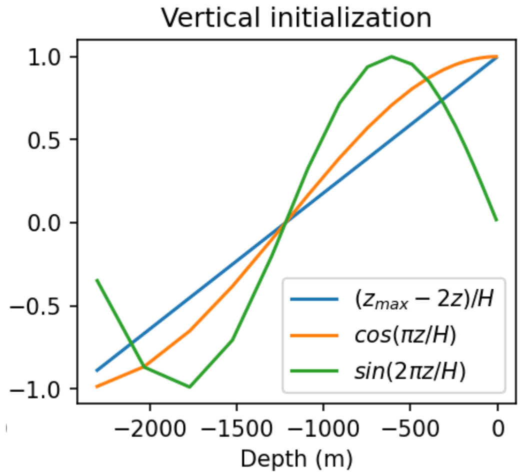
- **9** components of diffusivity tensor **K** are diagnosed using the tracer inversion method of Bachman et al. (2015)
- Advect a total of **27 passive tracers** with **9 initial conditions**, each set relaxed to their initial conditions with **3 different relaxation rates** (1 year, 3 years, and 6 years)
- Solve for diffusivity tensor **K** in a least-squares sense:

$$K_{ij} = -\overline{u_i'\tau_\alpha'}\left[\partial_j\bar{\tau}_\alpha\right]^\dagger$$

where $(i,j) = (x,y,z)$, $\alpha = 0,\ldots,26$ is the tracer number, $\overline{(\cdot)}$ is **a 20-year** and **152-km** spatial average, and $(\cdot)^\dagger$ indicates the **Moore-Penrose pseudoinverse**



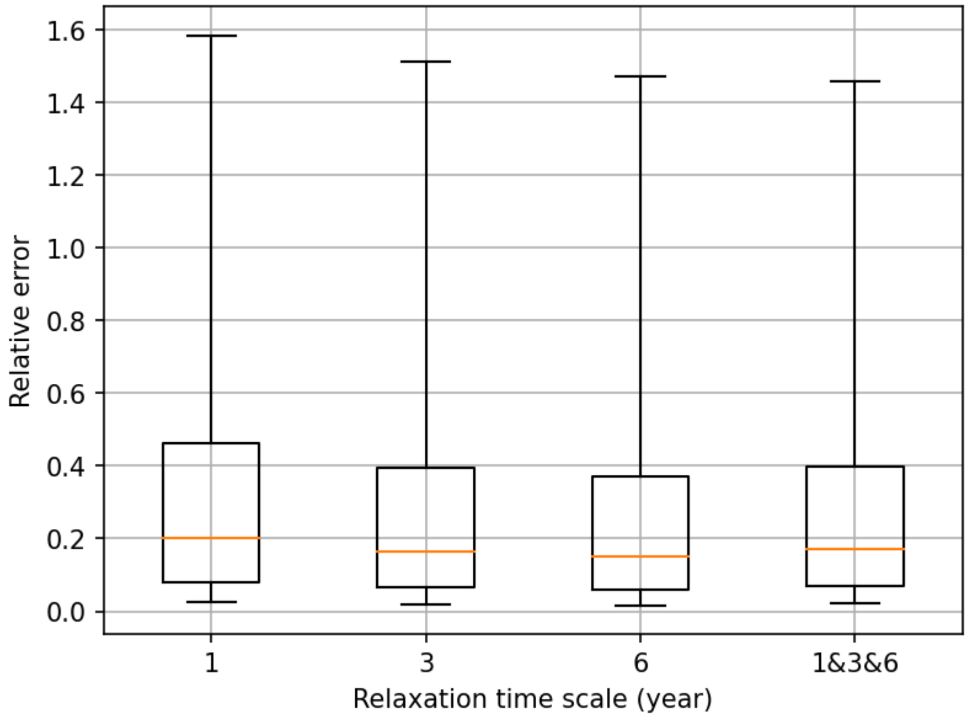


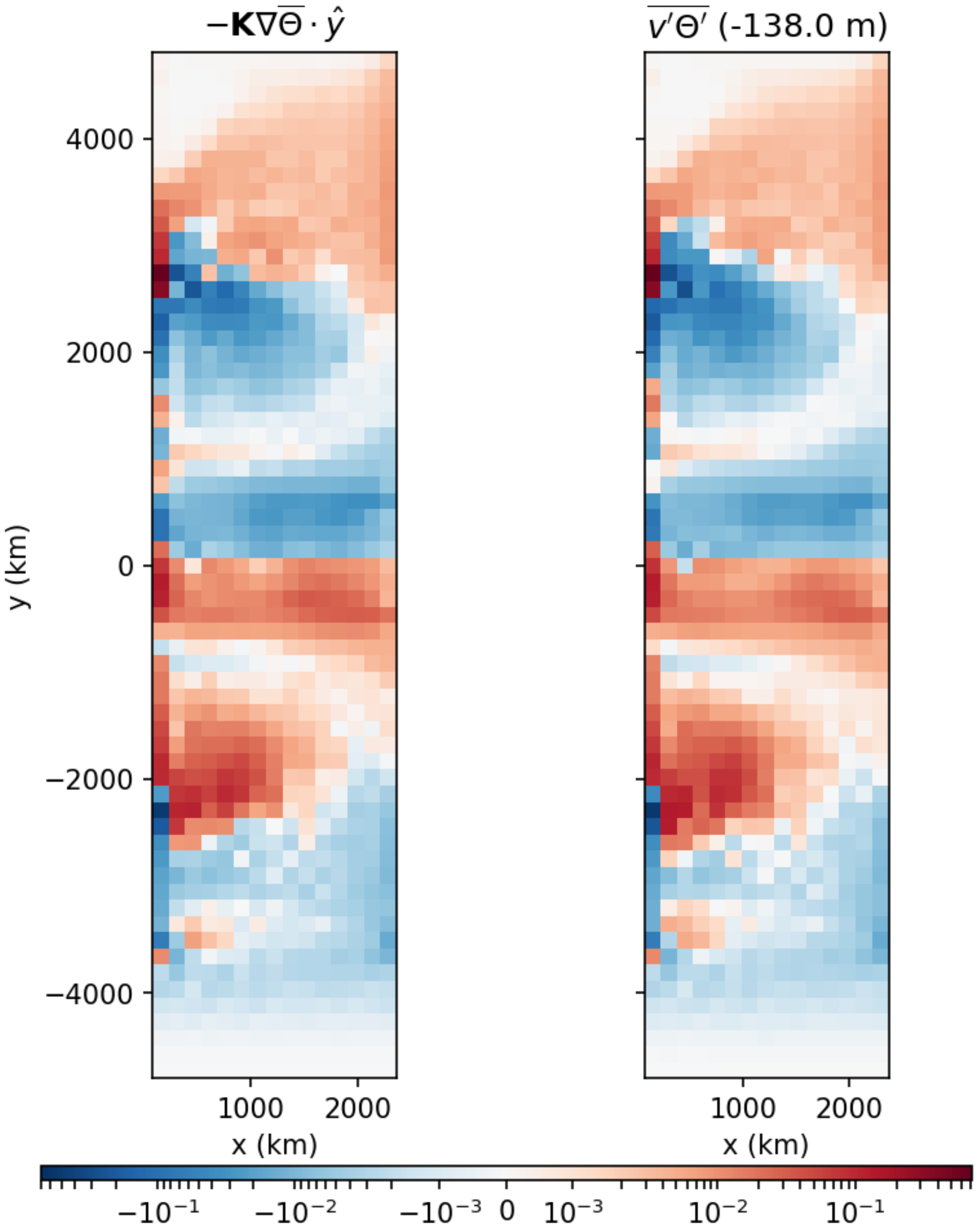


Reconstruction of buoyancy flux is excellent

- Relative error of reconstructed flux:

$$\varepsilon = \frac{\left|\overline{\boldsymbol{u}'\theta'} + \mathbf{K}\nabla\bar{\theta}\right|}{\left|\overline{\boldsymbol{u}'\theta'}\right|}$$





Eigenvalues of symmetric part of $\mathbf{K}$

- $\mathbf{K} = \mathbf{S} + \mathbf{A}$ where

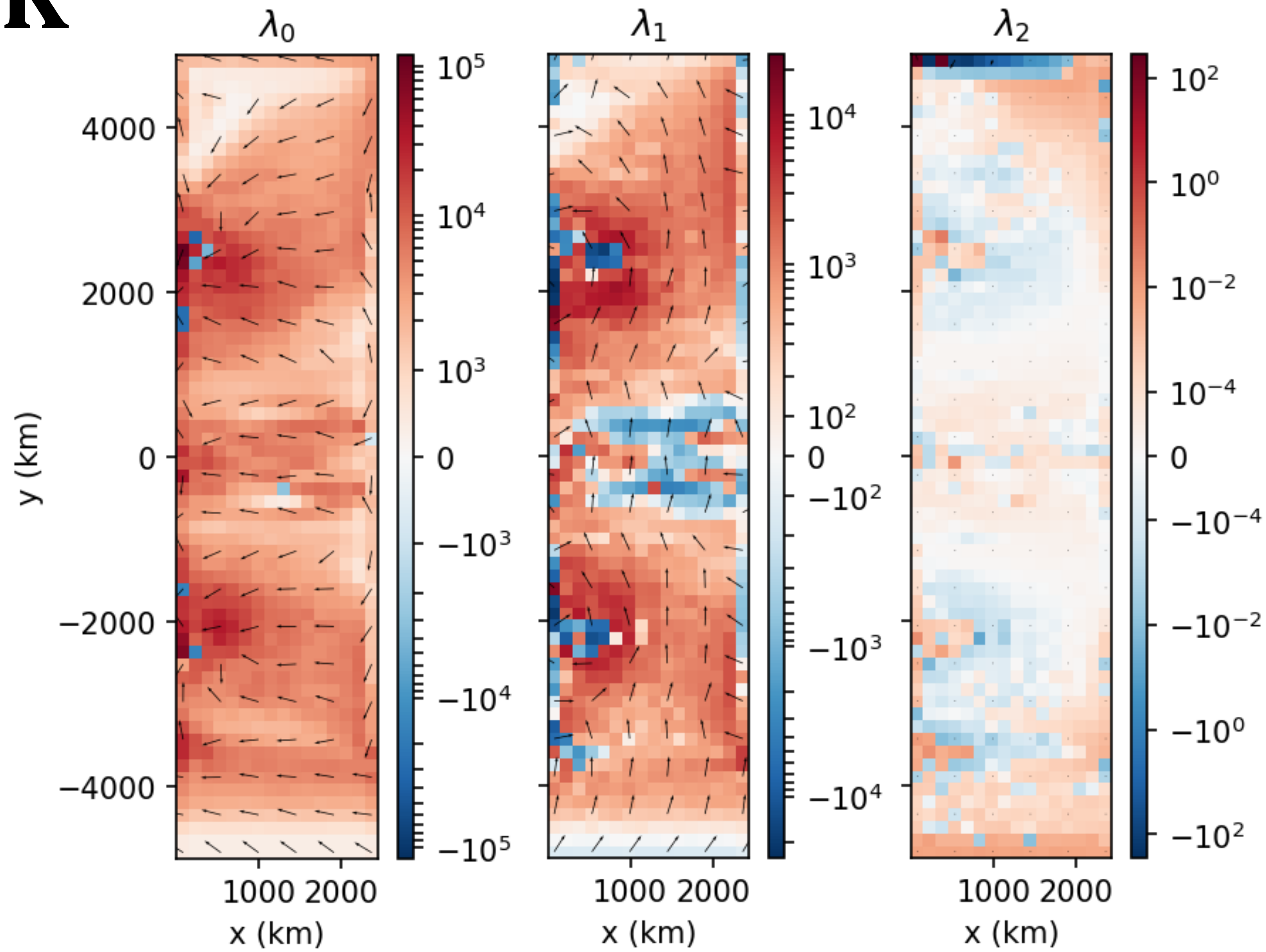
$$\mathbf{S} = \frac{\mathbf{K} + \mathbf{K}^T}{2}, \mathbf{A} = \frac{\mathbf{K} - \mathbf{K}^T}{2}$$

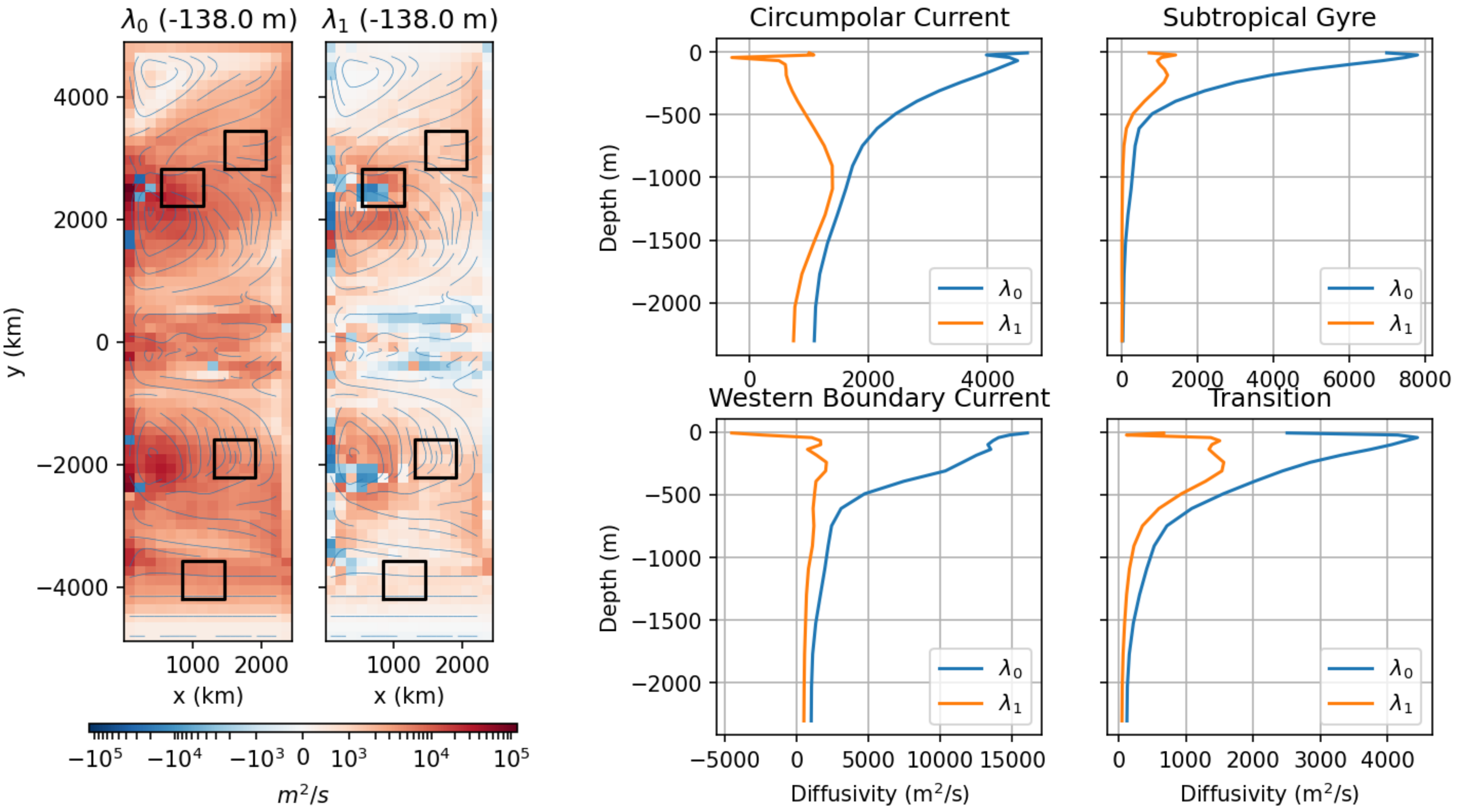
- The **symmetric** tensor $\mathbf{S}$ is diffusive and can be decomposed as

$$\mathbf{S} \cdot \mathbf{v}_i = \lambda_i \cdot \mathbf{v}_i$$

where λ_i $(i = 0, 1, 2)$ are the eigenvalues of $\mathbf{S}$, representing the diffusivity along eigenvectors $\mathbf{v}_i$

Eigenvalues in m² s⁻¹ (colors) and eigenvectors (arrows) at 138 m depth





λ_0 (-138.0 m)
λ_1 (-138.0 m)
y (km)
x (km)
m^2/s
Circumpolar Current
Subtropical Gyre
Western Boundary Current
Transition
Depth (m)
Diffusivity (m^2/s)
λ_0
λ_1

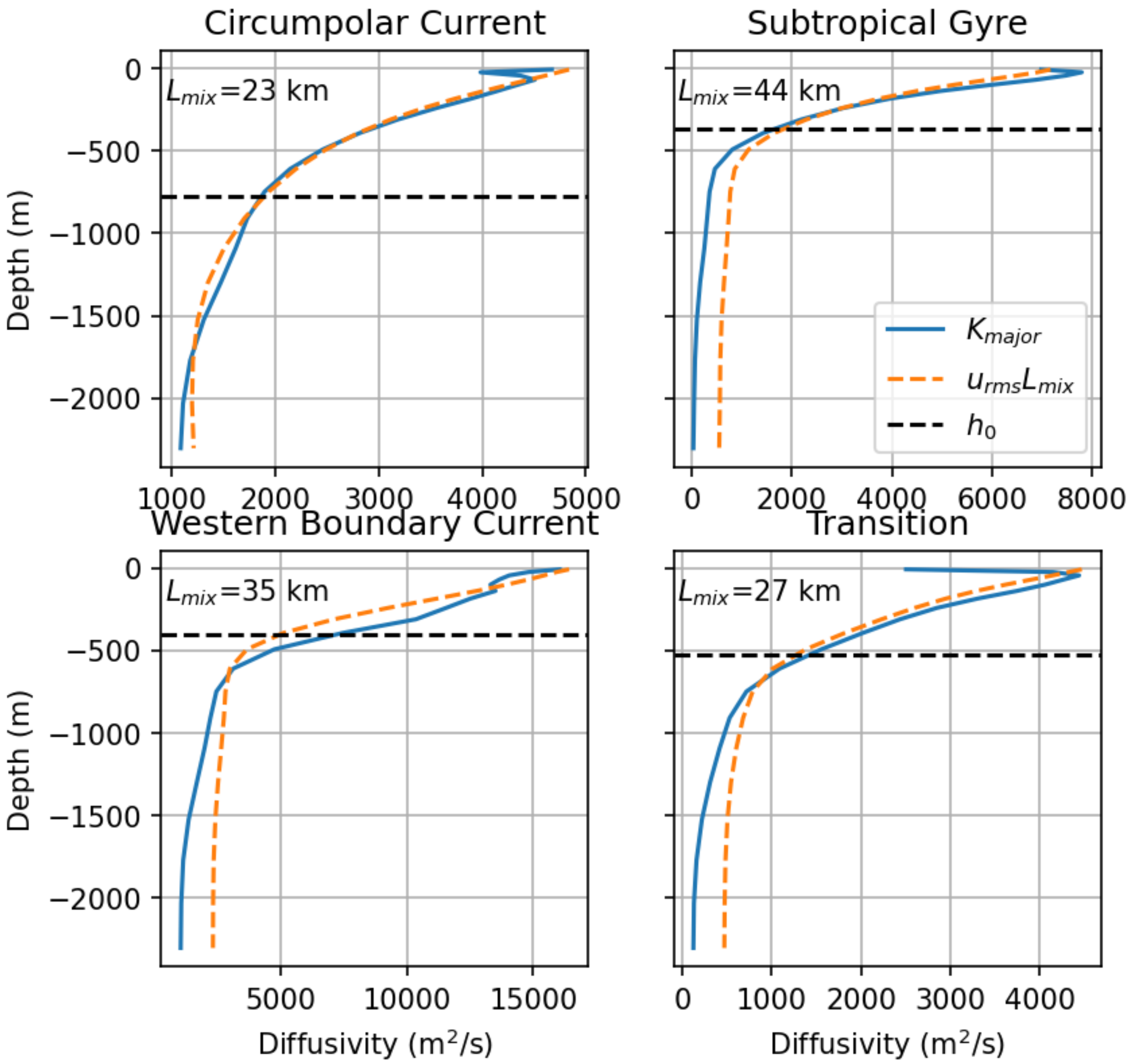
Major diffusivity (λ_0)

- Mixing length theory:

$$K \sim u_{rms} L_{mix}$$

- u_{rms} is diagnosed from model, and L_{mix} is assumed to be constant over depth and obtained by least squares fitting.
- The scale height of stratification h_0:

$$h_0 = -\frac{\int_{-H}^{0} z N^2 dz}{\int_{-H}^{0} N^2 dz}$$



Minor diffusivity (λ_1)

- Suppressed mixing length theory (Ferrari and Nikurashin, 2010; Klocker et al., 2012):

Mixing efficiency (from fit)

$$K = \frac{\Gamma L_{mix} u_{rms}}{1 + \frac{4\pi^2 \tau_\gamma^2}{L_{mix}^2}(\overline{U} - c_w)^2}$$

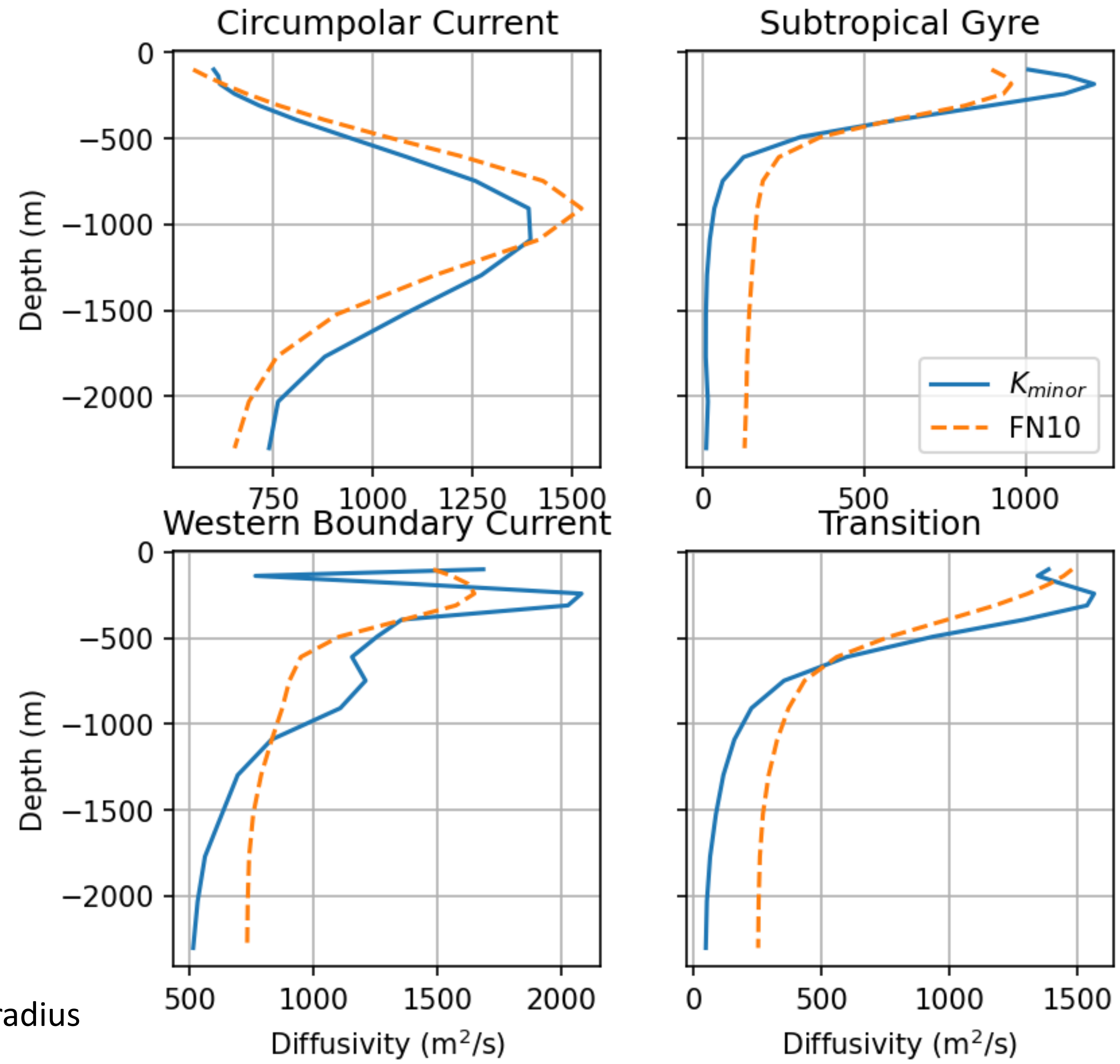
decorrelation time scale (from fit)

mean flow

where $c_w = \overline{U}^z - \beta L_d^2$ (Klocker and Marshall, 2014)

Depth-averaged mean flow

deformation radius



“Surface mode” used by Groeskamp et al. (2020)

- Assuming a wave-like solution to the linear QG PV equation yields an equation for the vertical structure

$$\frac{d}{dz}\left(\frac{f_0^2}{N^2}\frac{d\phi}{dz}\right)+\frac{\phi}{L_d^2}=0$$

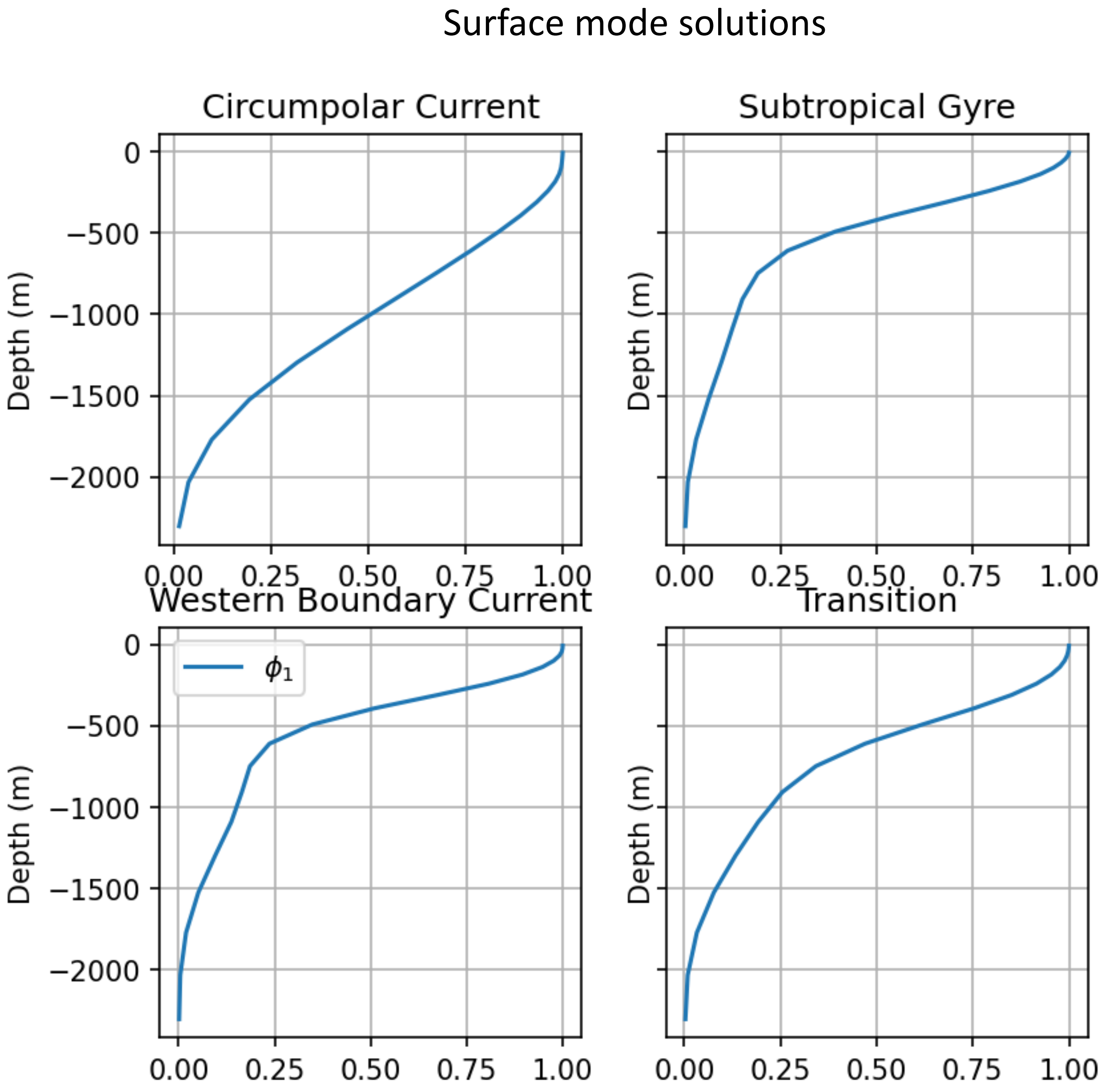
- Traditional solution assumes **a rigid lid** and **flat bottom:**

$$\frac{d\phi}{dz}=0 \text{ at } \mathrm{z}=0,-\mathrm{H}.$$

- “Surface mode” solution (de La Lama et al. 2016; LaCasce 2017) assumes a **rough bottom:**

$$\frac{d\phi}{dz}=0 \text{ at } \mathrm{z}=0, \qquad \phi=0 \text{ at } \mathrm{z}=-\mathrm{H}$$

- Vertical structure of current meter data is found to be similar as the **first ”surface mode”** (de La Lama et al. 2016).



SQG solution

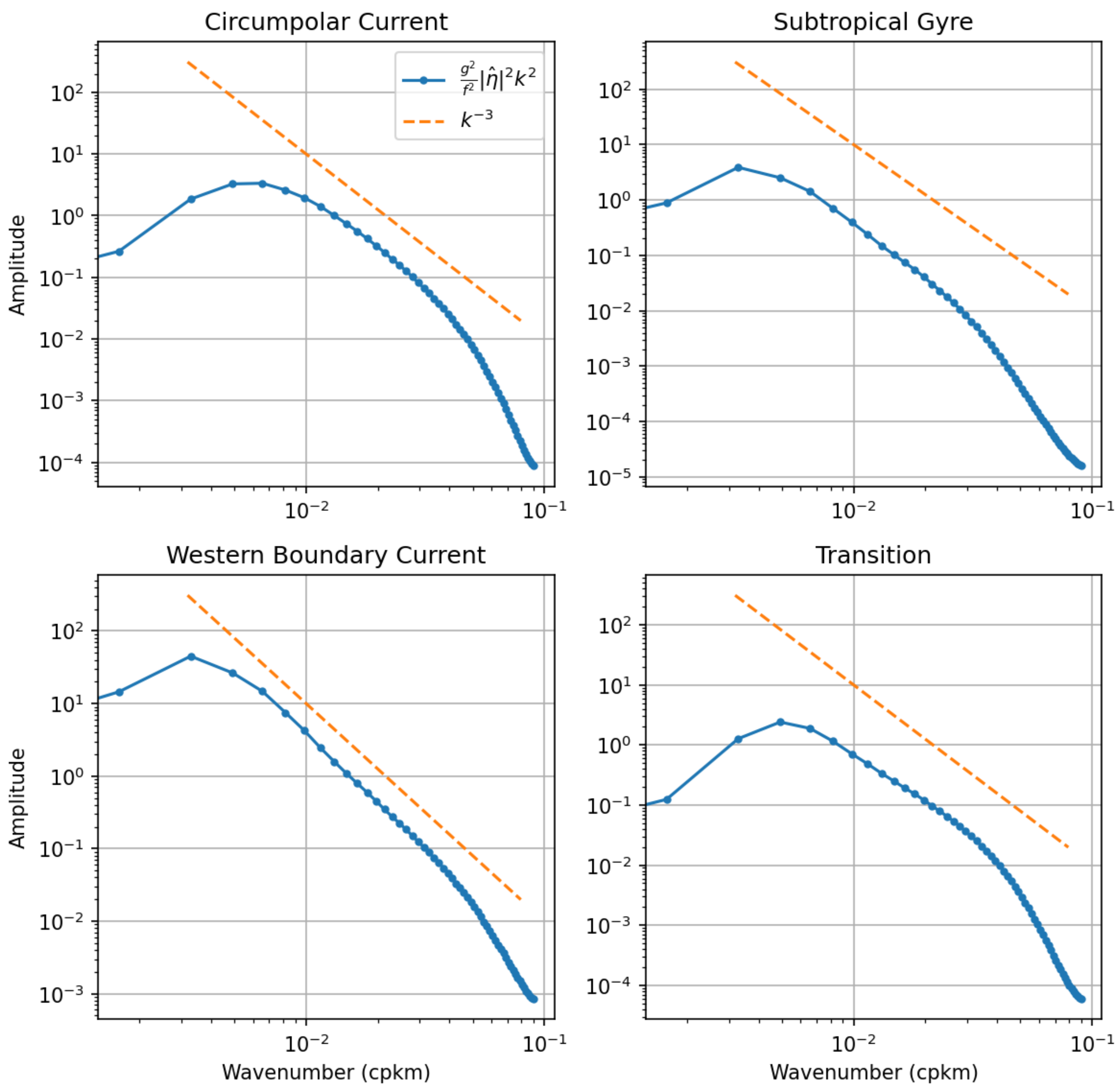
- Eddy streamfunction ψ can be decomposed into an **interior solution**, ψ_{int}, with zero buoyancy anomaly on the boundaries and a **surface solution**, ψ_{sur}, with zero interior PV anomaly (Lapeyre and Klein, 2006)
- ψ_{sur} is driven by the surface buoyancy anomaly, described by the **surface quasigeostrophic (SQG)** dynamics (Held et al., 1995)
- ψ_{sur} is solved numerically in Fourier space,

$$\frac{\partial}{\partial z}\left(\frac{f_0^2}{N^2}\frac{\partial \hat{\psi}_{sur}}{\partial z}\right) - \kappa^2\,\hat{\psi}_{sur} = 0,$$

$$\hat{\psi}\,\Big|_{z=0} = \frac{g}{f_0}\hat{\eta}, \qquad \frac{\partial \hat{\psi}}{\partial z}\Big|_{z=-H} = 0,$$

where $\kappa^2 = k^2 + l^2$.

EKE spectrum from SSH spectrum (600 km × 600 km window)



Reconstruction of EKE

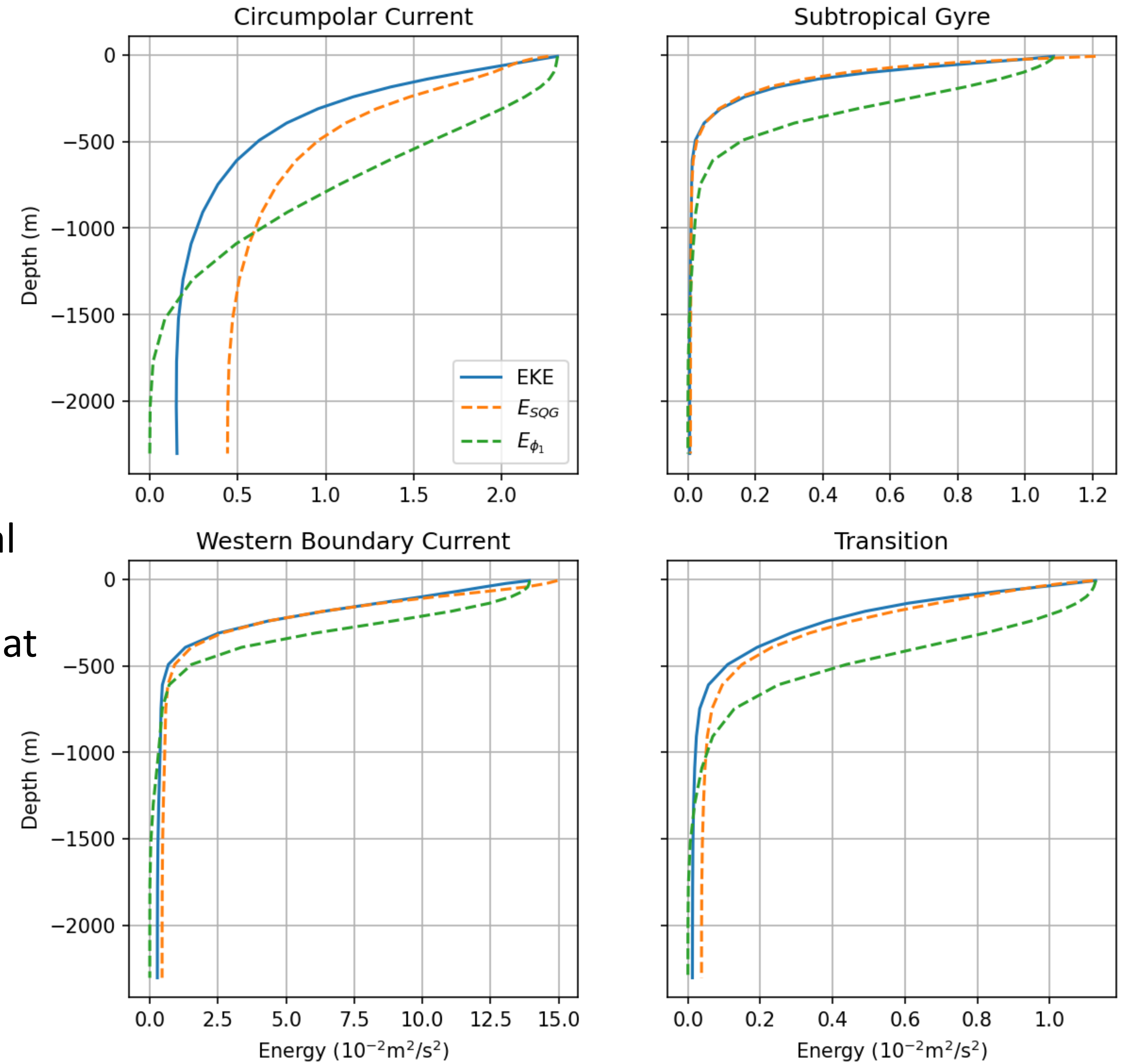
- EKE from the first "surface mode" (Groeskamp et al., 2020):

$$E_{\phi_1} = \phi_1(z)^2 EKE_0,$$

where ϕ_1 is the normalized vertical structure function of the first "surface mode", EKE_0 is the EKE at surface.

- EKE from SQG method:

$$E_{SQG} = \frac{1}{2}\sum_{k,l} \kappa^2 \left|\hat{\psi}(\boldsymbol{k}, z)\right|^2$$



Reconstruction of major diffusivity

- Estimate with "surface mode":

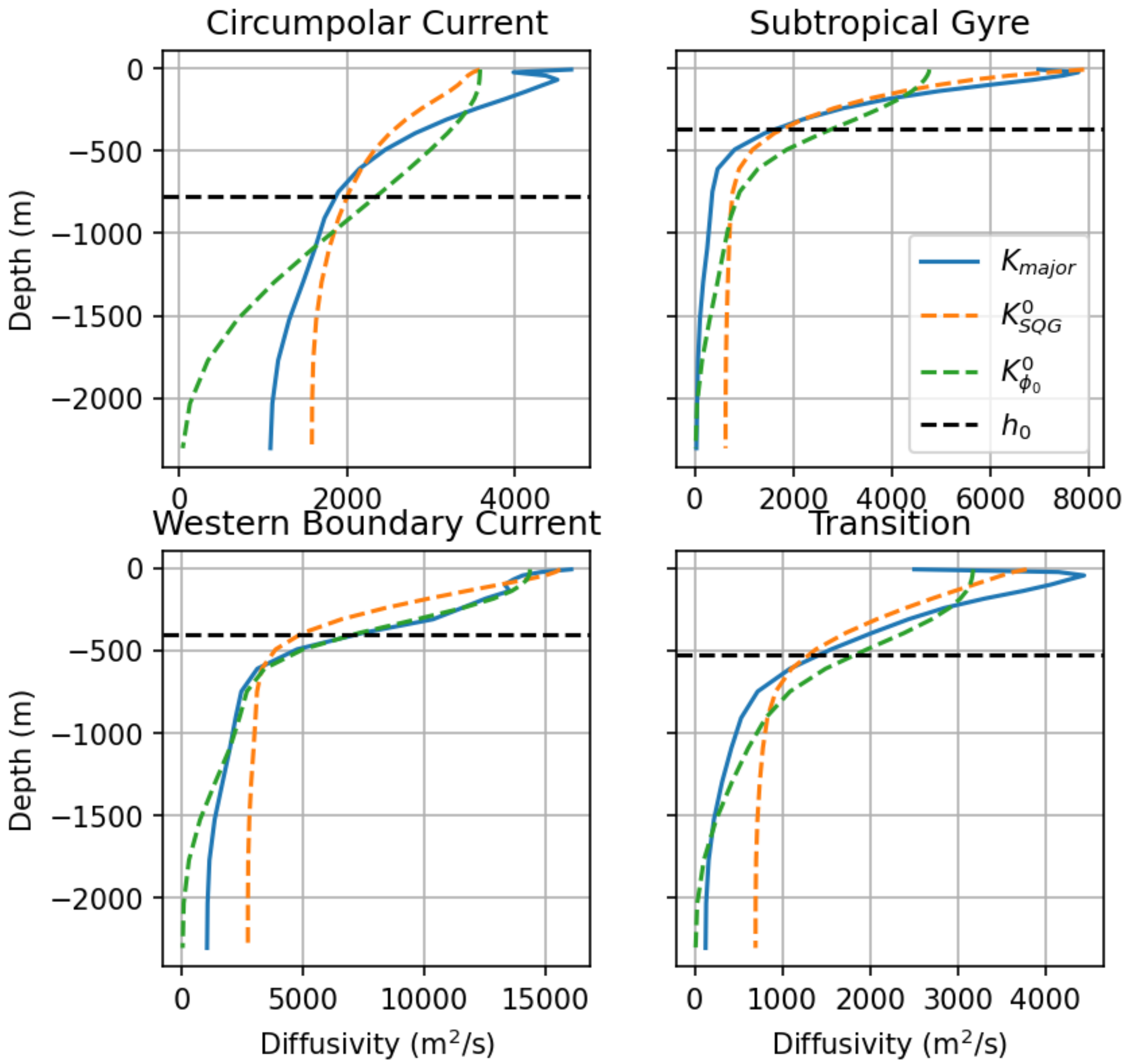
from fit

$$K^0_{\phi_1} = L_{\phi_1}\phi_1(z)^2\sqrt{2EKE_0}$$

- Estimate with SQG method:

from fit

$$K^0_{SQG} = L_{SQG}\sqrt{2E_{SQG}}$$



Reconstruction of minor diffusivity

- Estimate with "surface mode":

from fit → $\Gamma L_{\phi_1}\phi_1(z)$

$$K_{\phi_1}^1 = \frac{\Gamma L_{\phi_1}\phi_1(z)}{1+\frac{4\pi^2\tau_\gamma^2}{L_{\phi_1}^2}(\bar{U}-c_w)^2}\sqrt{2EKE_0}$$

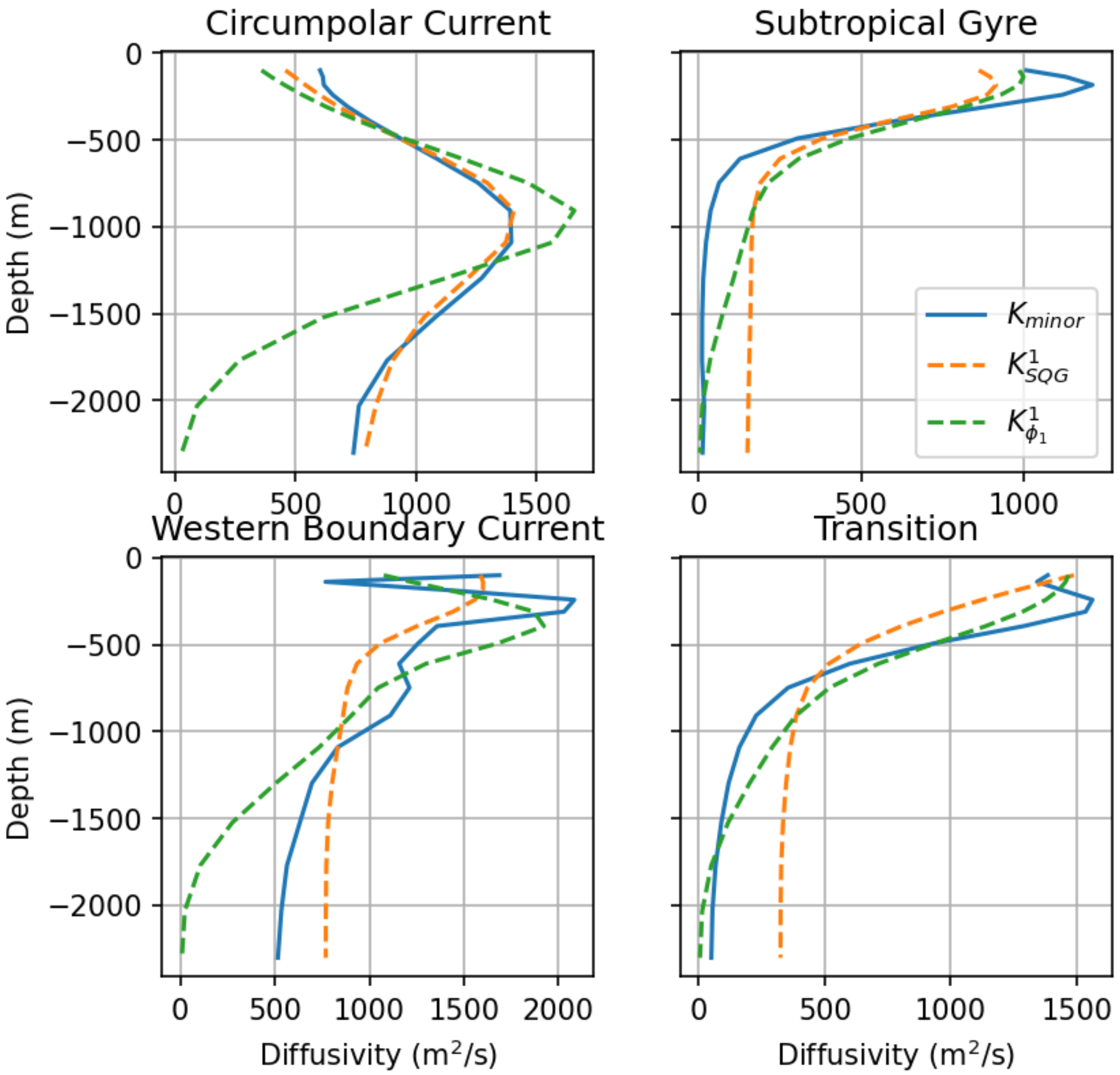
from fit → τ_γ

- Estimate with SQG method:

from fit → ΓL_{SQG}

$$K_{SQG}^1 = \frac{\Gamma L_{SQG}}{1+\frac{4\pi^2\tau_\gamma^2}{L_{SQG}^2}(\bar{U}-c_w)^2}\sqrt{2E_{SQG}}$$

from fit → τ_γ



Summary

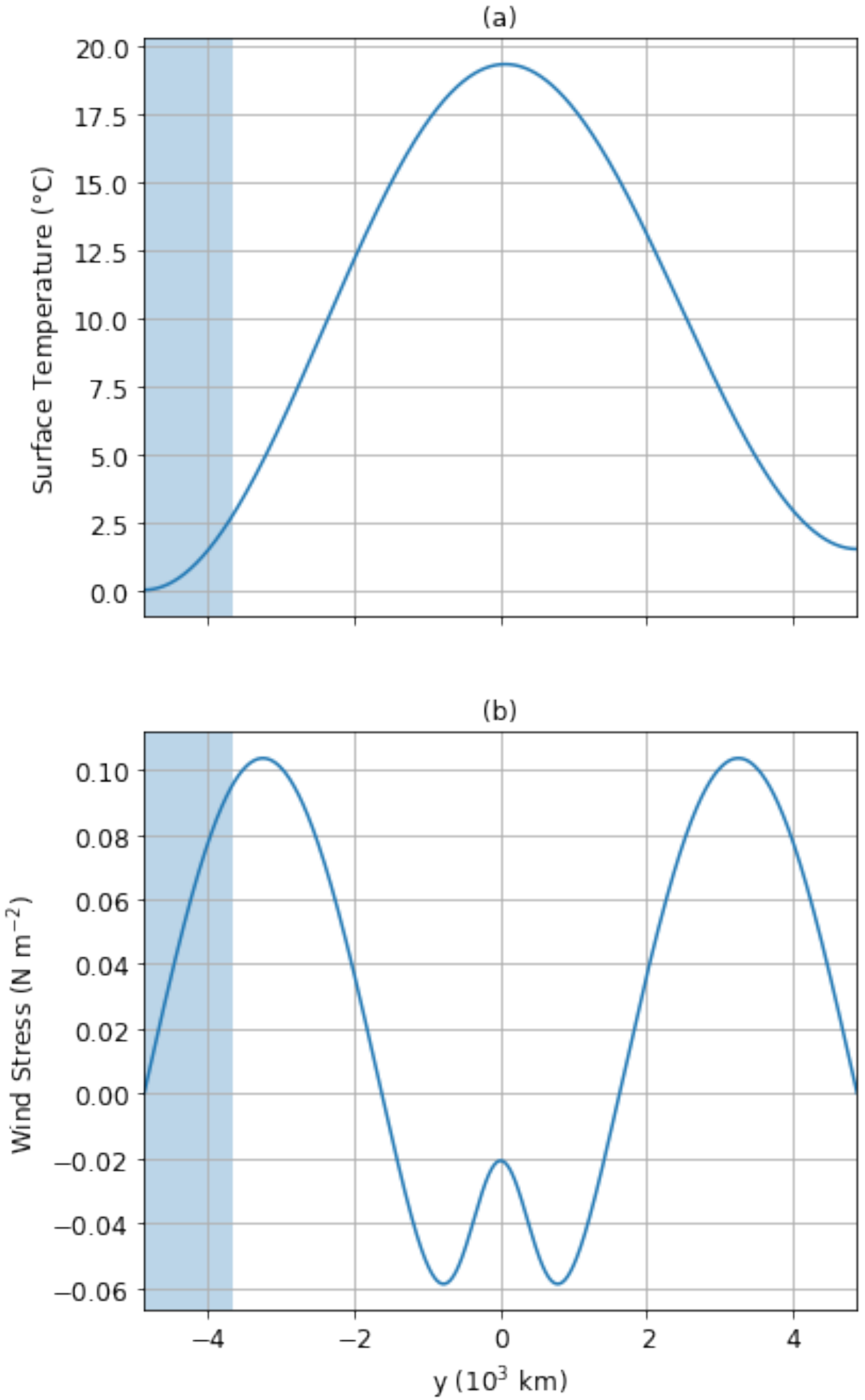
- Mixing length theory assuming **constant mixing length over depth** works well for the **major diffusivity** in the upper levels, but often overestimates the diffusivity at depth away from the channel.
- The **minor diffusivity** is consistent with the suppressed mixing length theory, in which the diffusivity is **modulated by eddy propagation relative to the mean flow**.
- Vertical structure of **EKE** is better reconstructed by the **SQG method** than the first "**surface mode**".
- SQG method performs better than the first "surface mode" in reconstructing the **diffusivities** in upper levels; the former (latter) tends to overestimate (underestimate) the diffusivities in the deep levels.

Future work

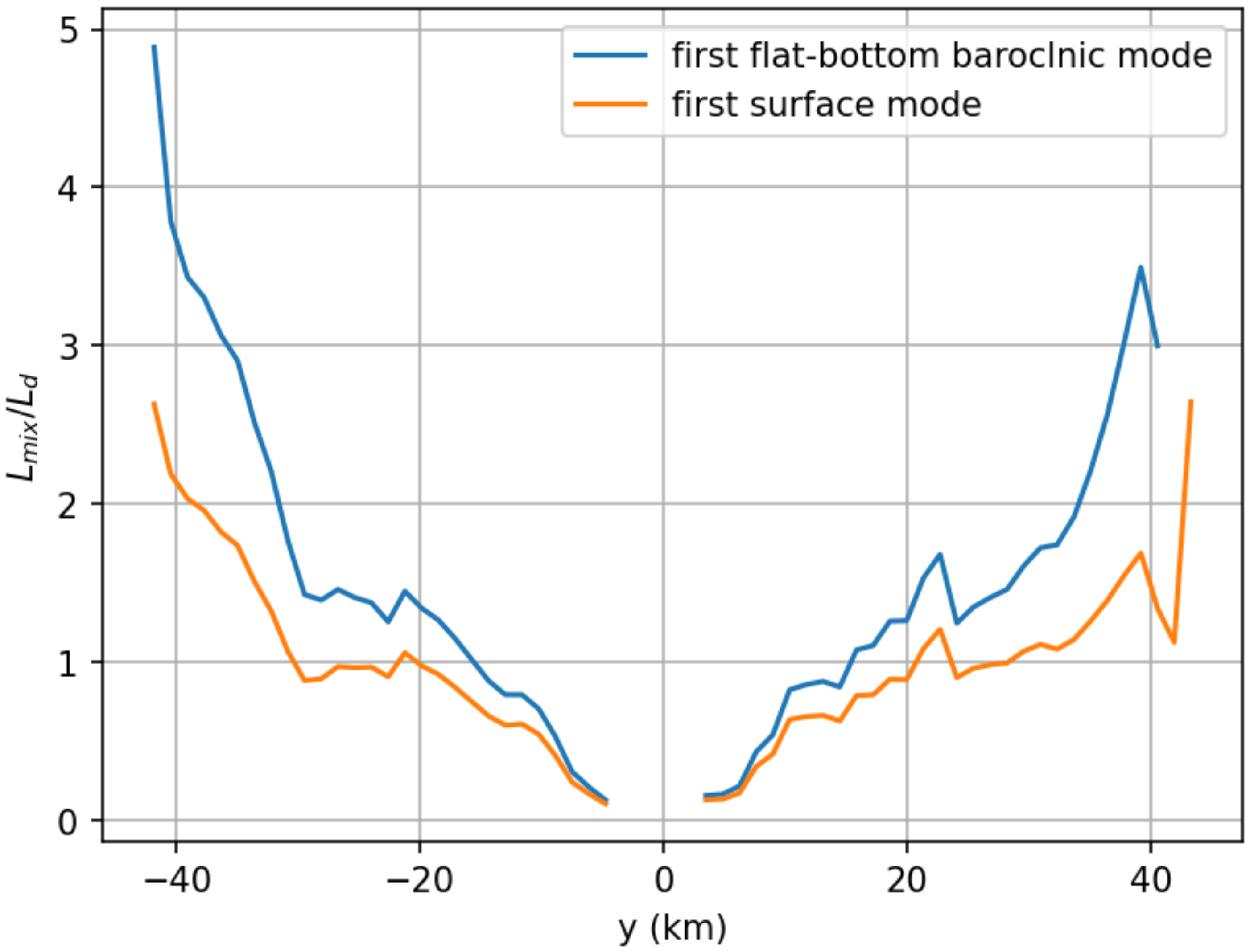
- The current model has a flat bottom. We are planning to add topography to further compare the reconstruction with the ”surface mode” and SQG method.
- Determination of the **mixing length** and **decorrelation time scale** is still an open question.

Extra slides

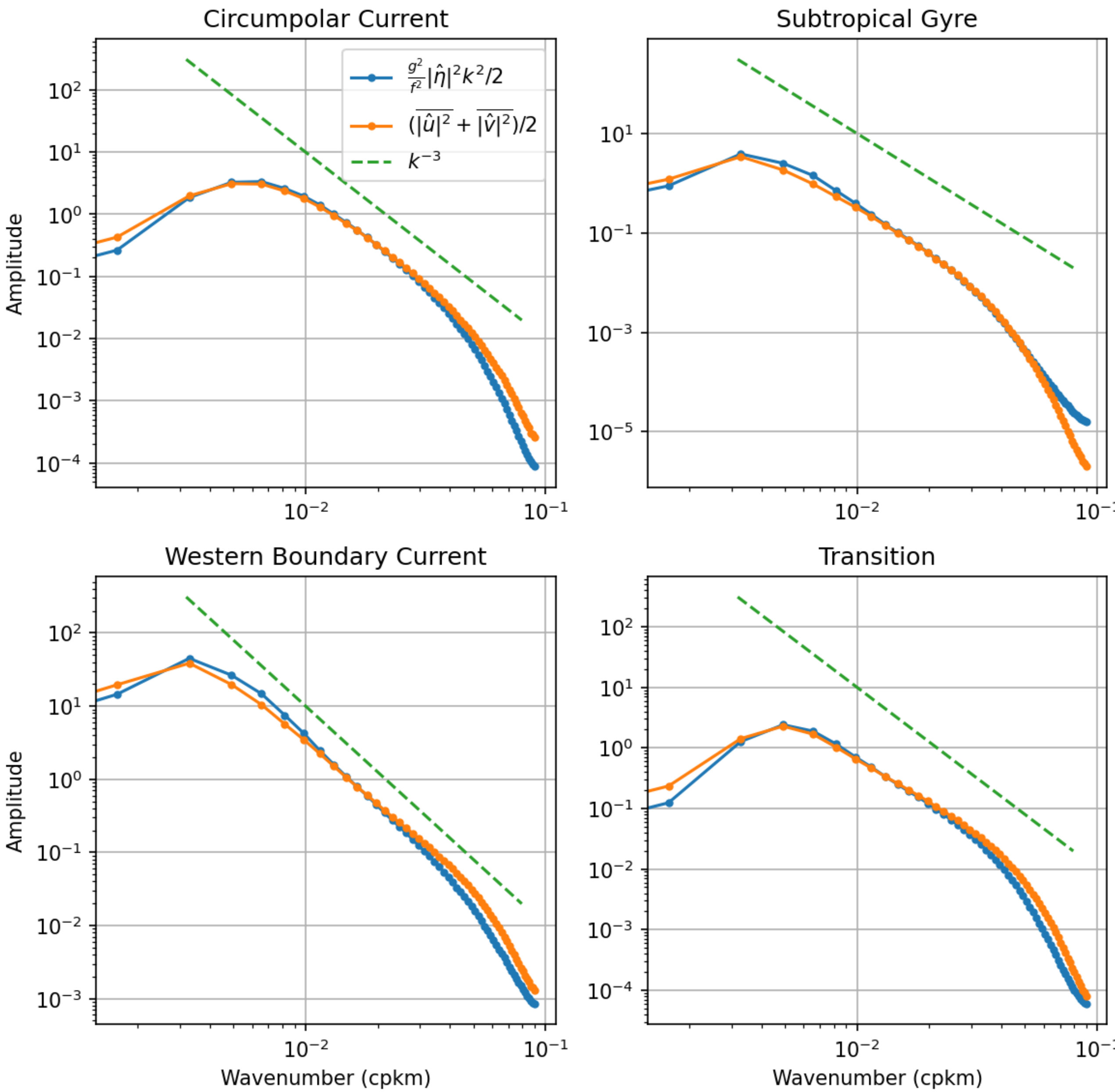
Forcings (extra slide)



Mixing length (extra slide)



Energy spectrum (extra slide)



Minor diffusivity (λ_1) (extra slide)

- Suppressed mixing length theory (Ferrari and Nikurashin, 2010; Klocker et al., 2012):

$$K = \frac{\Gamma L_{mix} u_{rms}}{1 + \frac{4\pi^2 \tau_\gamma^2}{L_{mix}^2} (\overline{U} - c_w)^2}$$

mean flow

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Depth-averaged mean flow

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