

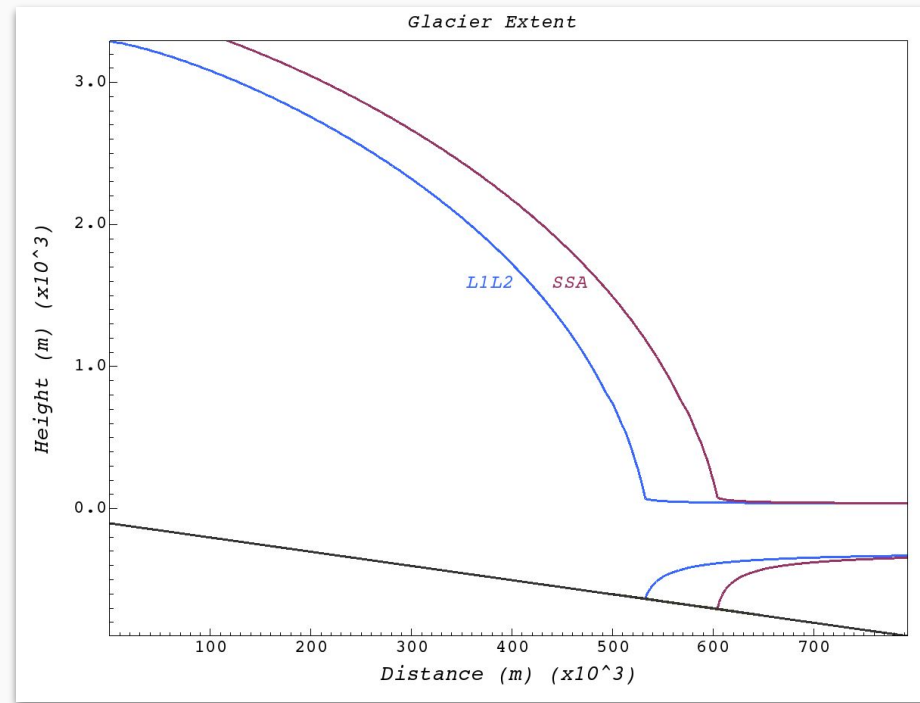
Understanding the implications of using the Shallow Shelf Approximation for Marine Ice Sheets

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Background

- The MISMIP3D intercomparison project showed a nontrivial difference in steady-state grounding line positions produced from models using SSA and higher order models like L1L2 (Schoof & Hindmarsh), first-order (Blatter-Pattyn) or the Full-Stokes models with identical spatial resolution and discretization
- Runs using BISICLES produced grounding line positions at $\sim 530\text{km}$ and $\sim 610\text{km}$ respectively



“The initial grounding lines of the shallow-shelf approximation (SSA) models were around 80 km downstream from the Stokes models, but the grounding line only moved about 20 km in the perturbation experiment. That left an obvious question entirely unanswered: in a realistic simulation with the model parameters chosen to match geometry and velocity derived from observations, and thus with prescribed initial conditions, does the SSA provide a good approximation to the Stokes model?”

Asay-Davis, X. S. et al.: Experimental design for three interrelated marine ice sheet and ocean model intercomparison projects: MISMIP v.3 (MISMIP +), ISOMIP v.2 (ISOMIP +) and MISOMIP v.1 (MISOMIP1), *Geosci. Model Dev.*, 9, 2471–2497, <https://doi.org/10.5194/gmd-9-2471-2016>, 2016

The experiment

1. Generate steady-state L1L2 “observations” for initialization
2. Determine parameters by solving an inversion problem to initialize SSA model
3. Attempt reproduction of L1L2 steady-state using SSA model
4. Introduce perturbation to observe dynamic response

The experiment

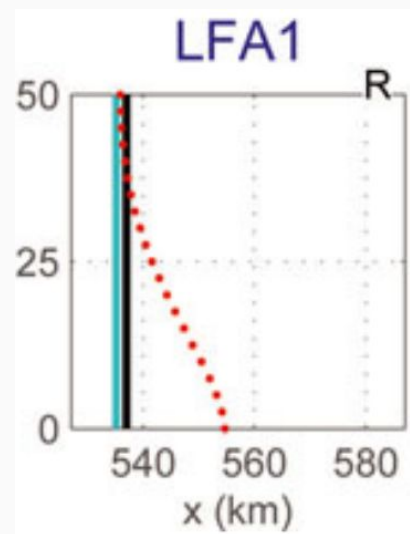
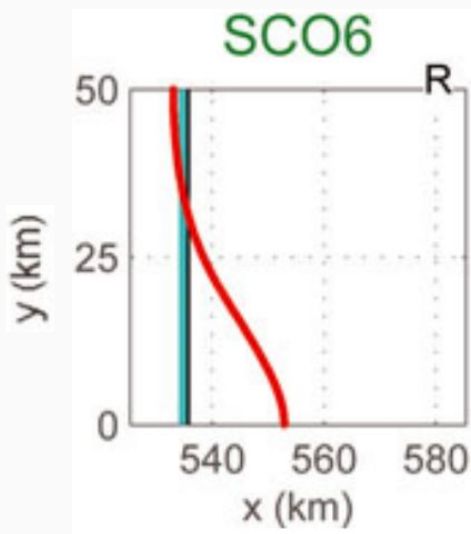
1. **Generate steady-state L1L2 “observations” for initialization**
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Justification for L1L2

- Why use L1L2 as observations?
- The L1L2 MISMIP3D results behaved similarly to the Full Stokes model and thus provides a good starting point

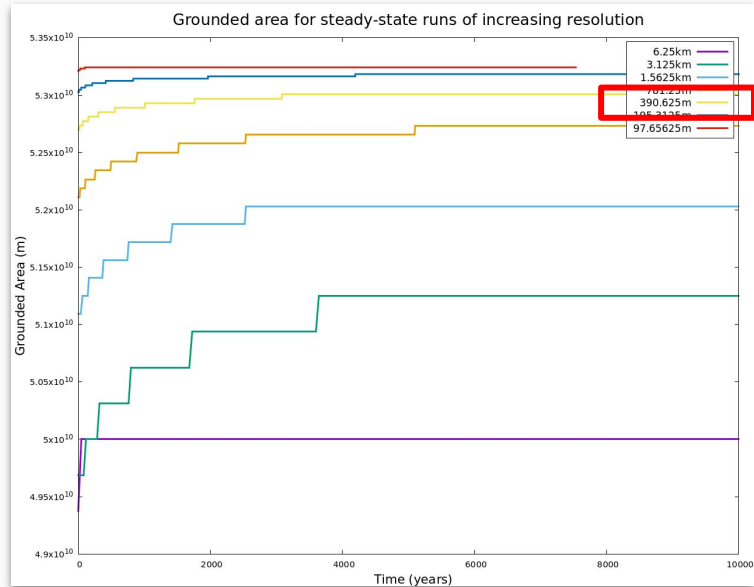
SCO6: L1L2 run using BISICLES

LFA1: FS run using Elmer/Ice
Favier and others (2012)

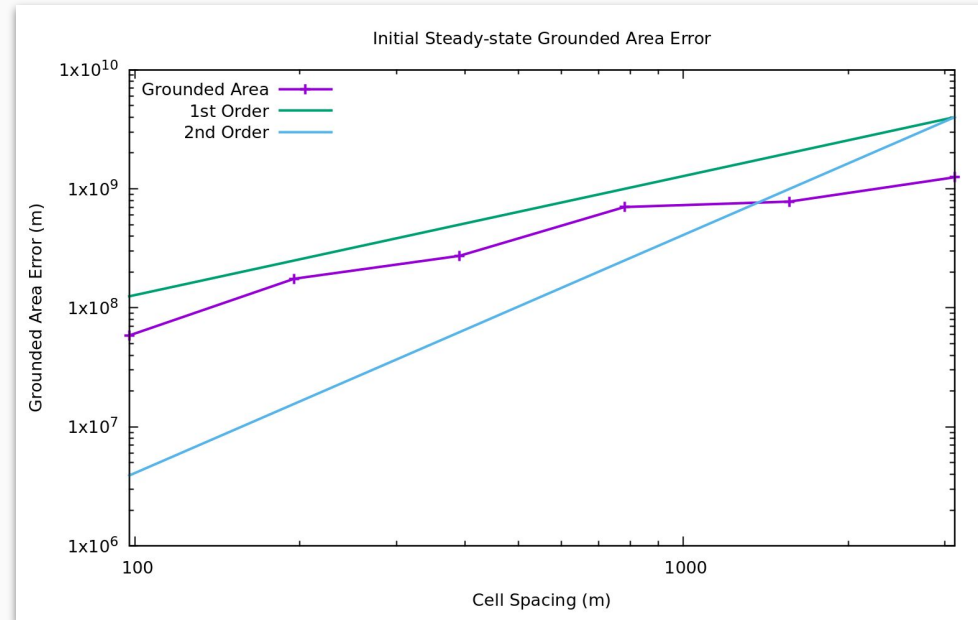


Generating "Observations"

- Multiple runs were generated using the same problem specification as MISMIP3D with increasing resolution to find a solution that converges at an acceptable refinement
- Used the Weertman sliding law



- The error converged at first order rates



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Inverse Problem

- We follow the approach used in Cornford et al. (2015)
- Our goal is to choose a basal friction and viscosity multiplier that minimize the objective function
- The objective is the mismatch between observed and modeled velocity as well as a Tikhonov penalty function
- C and ϕ are chosen using the conjugate gradient method such that the difference in velocity is minimized

Objective Function

$$J = J_m + J_p$$

Mismatch Function

$$J_m = \frac{1}{2} \int_{\Omega_V} \alpha_u^2(x, y) (|\mathbf{u}| - |\mathbf{u}_o|)^2 d\Omega$$

Tikhonov Penalty Function

$$J_p = \frac{\alpha_C^2}{2} \int_{\Omega_V} |\nabla C|^2 d\Omega + \frac{\alpha_\phi^2}{2} \int_{\Omega_V} |\nabla \phi|^2 d\Omega$$

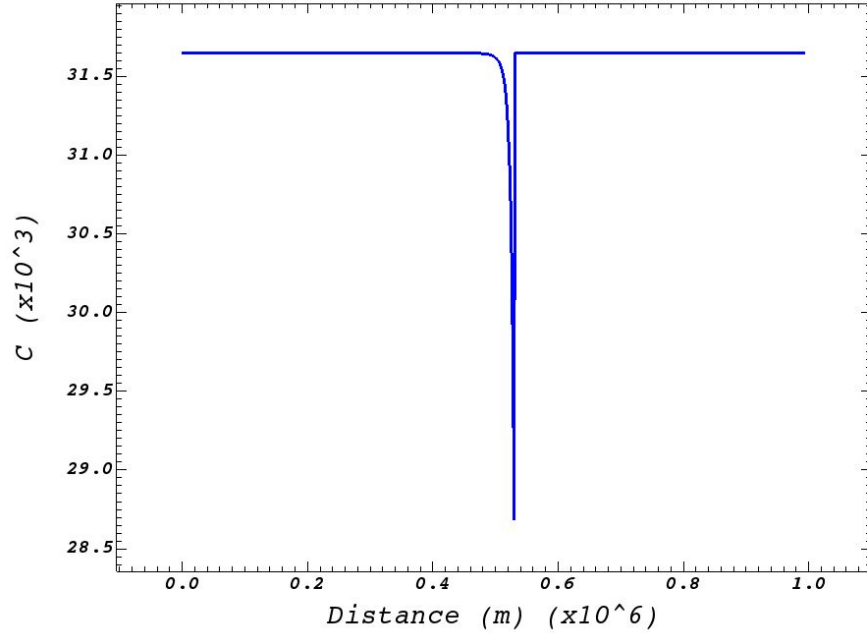
Optimization scheme

- With two variables to adjust, we have three different ways in which $C(x,y)$ and $\phi(x,y)$ can be optimized to find a u that minimizes the objective function
- True/True
 - Both basal friction and rheology were optimized
- True/False
 - Only basal friction is optimized. Rheology kept at a constant value of 1.
- False/True
 - Only rheology is optimized. Basal friction is kept at a constant value of 31651.76 Pa/(m/s)^{1/3}

Optimization Scheme		
	$C(x,y)$	$\phi(x,y)$
True/True	✓	✓
True/False	✓	✗
False/True	✗	✓

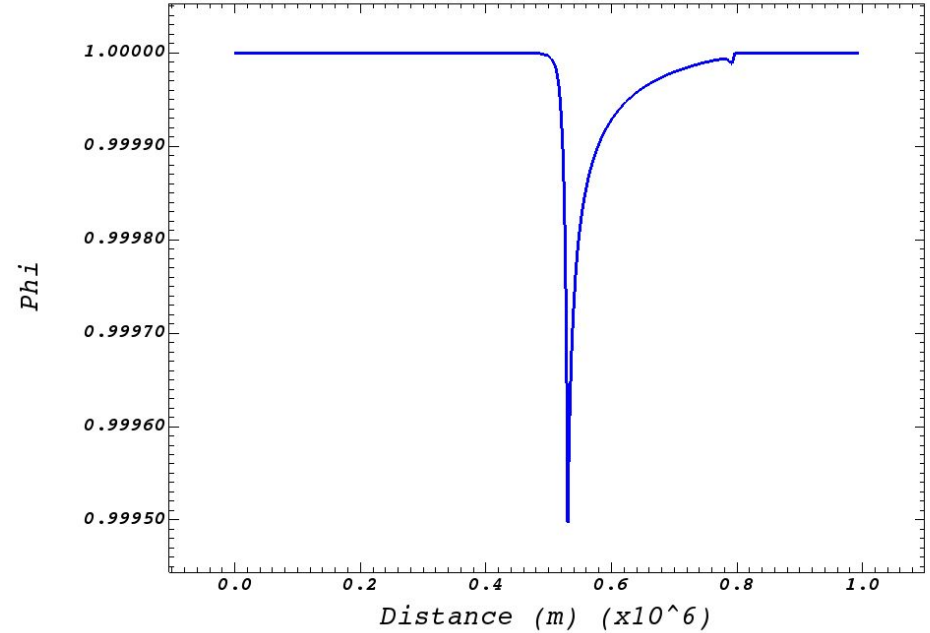
Optimized: Basal Friction and Viscosity Coefficients (True/True)

Basal Friction Coefficient for True/True



- Steep spike at the grounding line

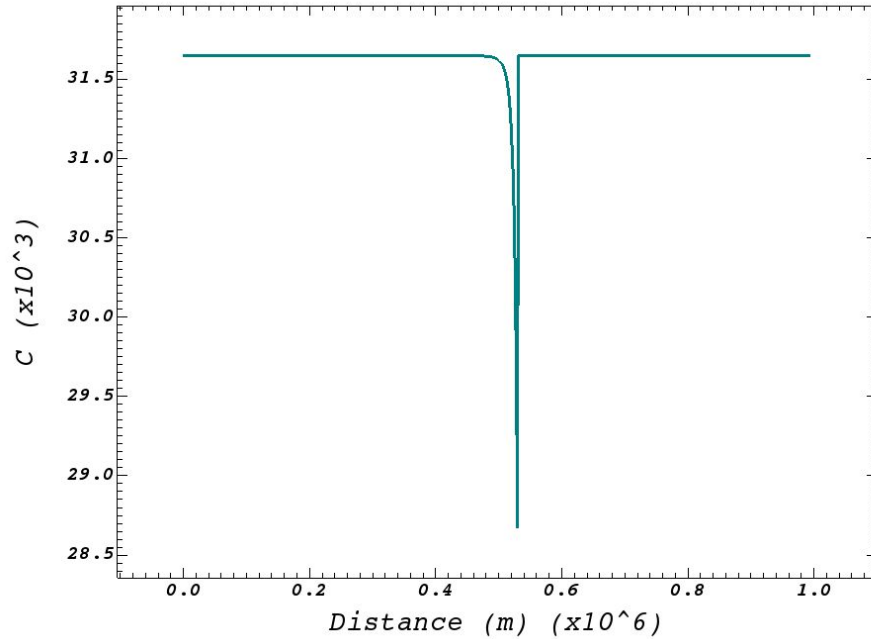
Viscosity Coefficient for True/True



- Infinitesimal change in phi

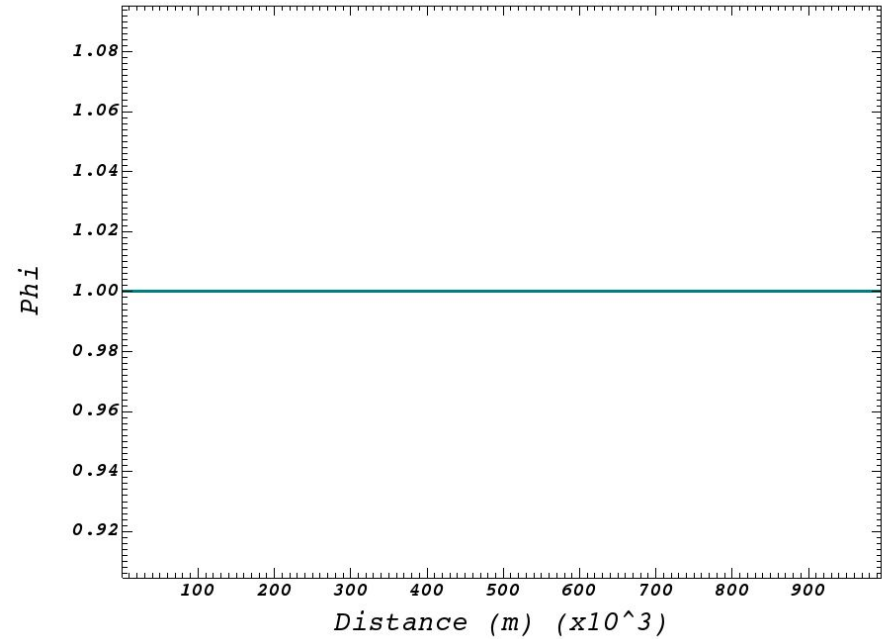
Optimized: Only Basal Friction Coefficient (True/False)

Basal Friction Coefficient for True/False



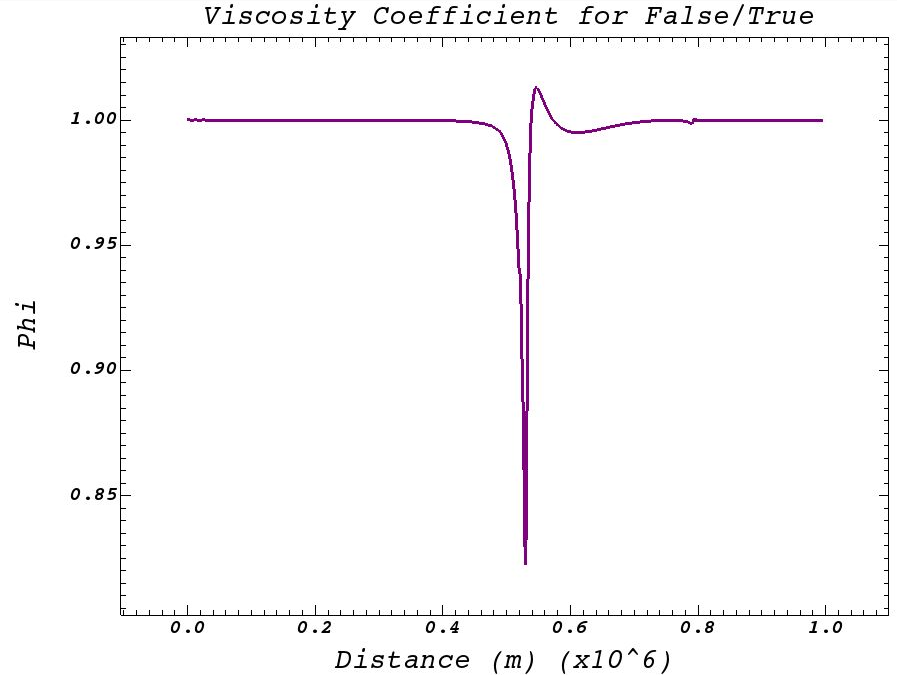
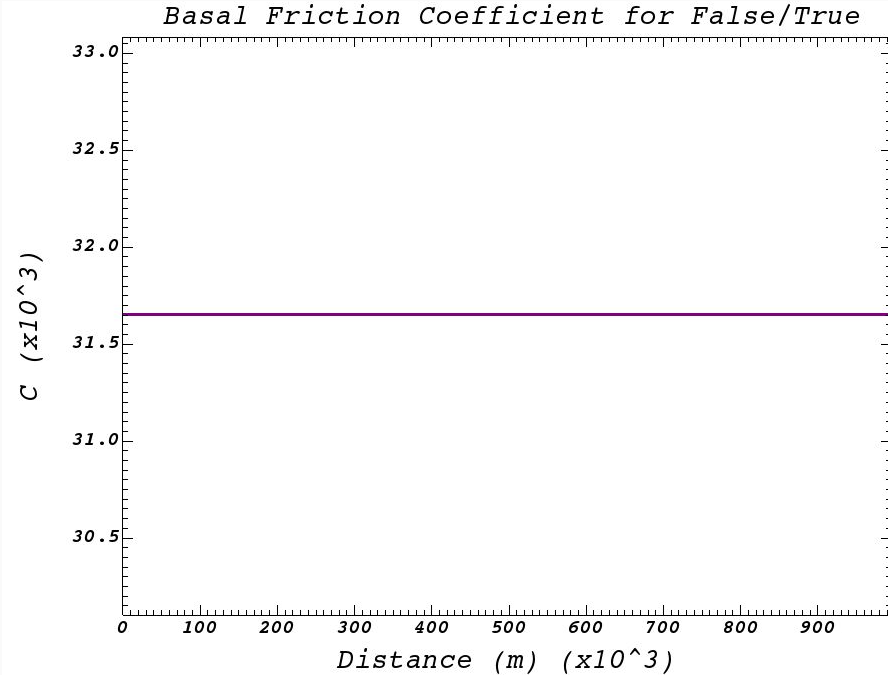
- Similar spike at the grounding line

Viscosity Coefficient for True/False



- Constant value of $\phi = 1$

Optimized: Only Viscosity Coefficient (False/True)



- Constant value of $C = 31651.76 \text{ Pa}/(\text{m/s})^{1/3}$

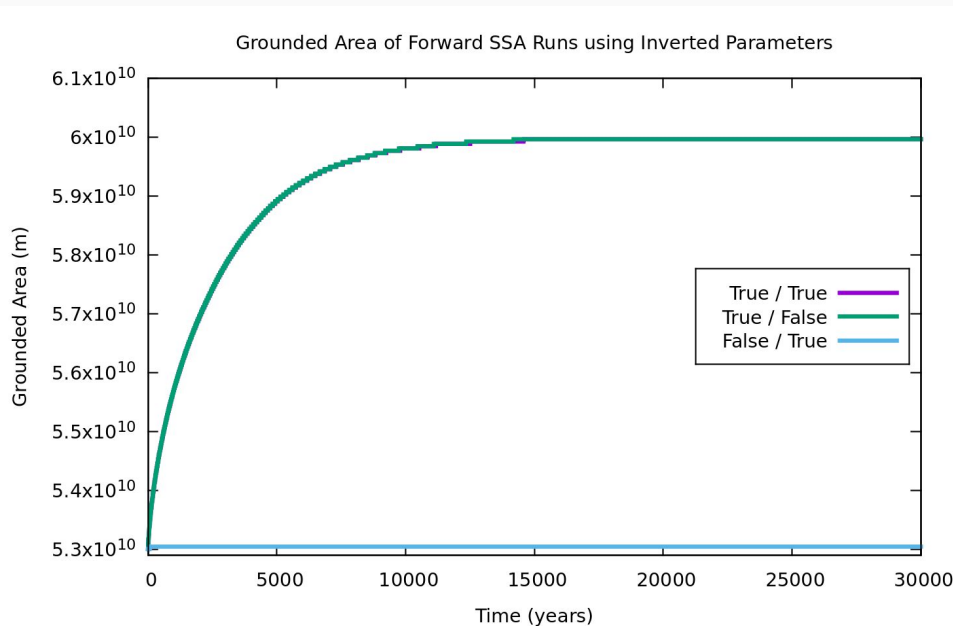
- The SSA model can't represent vertical shearing

The experiment

1. Generate steady-state L1L2 “observations” for initialization
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3. **Attempt to reproduce L1L2 steady-state using SSA model**
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Stable Steady-state?

- With C and ϕ solved for, it's expected that the forward runs using SSA should reproduce the L1L2 stable steady-state



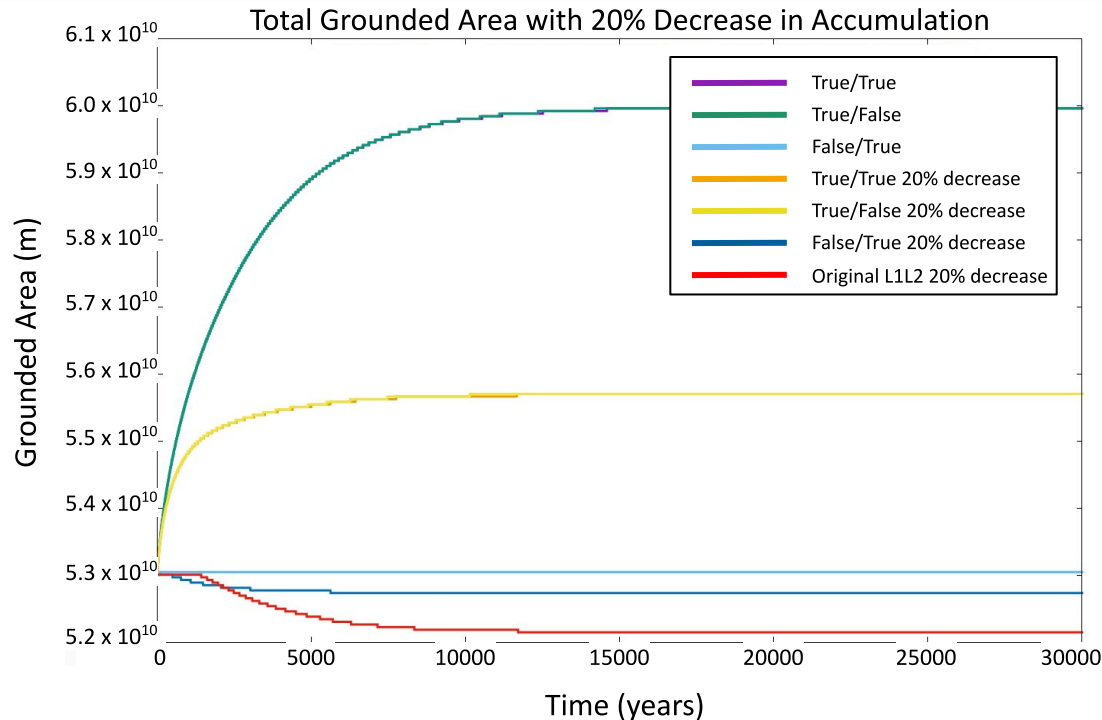
- The methods where basal friction was optimized failed in reproducing the steady-state due to how narrow the slippery spot was which made it unstable to small perturbations
- The method where only rheology was optimized was the only method that was able to reproduce the L1L2 stable steady-state

The experiment

1. Generate steady-state L1L2 “observations” for initialization
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4. **Introduce perturbation to observe dynamic response**

Dynamic response due to perturbation

- Decreasing accumulation by 20%, we observe the dynamic response due to the perturbation for each case including the original L1L2 model



- The total grounded area decreases as expected for all 3 cases as well as the original L1L2 case
- The approaches with basal friction optimized were unsuccessful, which was unsurprising since they initially failed to maintain a steady-state
- The dynamic response of False/True does not match the response of the L1L2 observations

Summary

- With how narrow the basal friction values were for both the True/True case and True/False case, it's hard to force the SSA GL to match the L1L2 stable state without falling off the slippery spot
- Matching the observations using only $\varphi(x,y)$, however, was more successful and we were able to reproduce the steady-state
- Although the determined $\varphi(x,y)$ reproduced the stable steady-state, it was unsuccessful in predicting the dynamic response in part due to the time-independent nature of φ
- Overall, by treating MISMIP3D like a real-world problem, we were unable to get SSA to behave as expected. We may see different behavior if we were to include a different sliding law or if buttressing was present.

Future work

- We are continuing this experiment with MISMIP+ set up which includes ice shelf buttressing and may be more effective in reproducing the stable steady-state grounding line positions for all three optimization methods
- Amundsen Sea Embayment has Thwaites and Pine Island, these may be similar to the MISMIP3D and MISMIP+ problems respectively