



# *Missing Enthalpy Flux in CAM*

**Peter Hjort Lauritzen**

Atmospheric Modeling and Prediction (AMP) Section  
Climate and Global Dynamics (CGD) Laboratory  
National Center for Atmospheric Research (NCAR)

**CESM workshop**  
**June, 15, 2021**



# Acknowledgments



## Physics-Dynamics Coupling in Earth System Models (19w5153)

### Organizers

Nicholas Kevlahan (McMaster University)

Peter Lauritzen (National Center for Atmospheric Research)

October 13-19, 2019

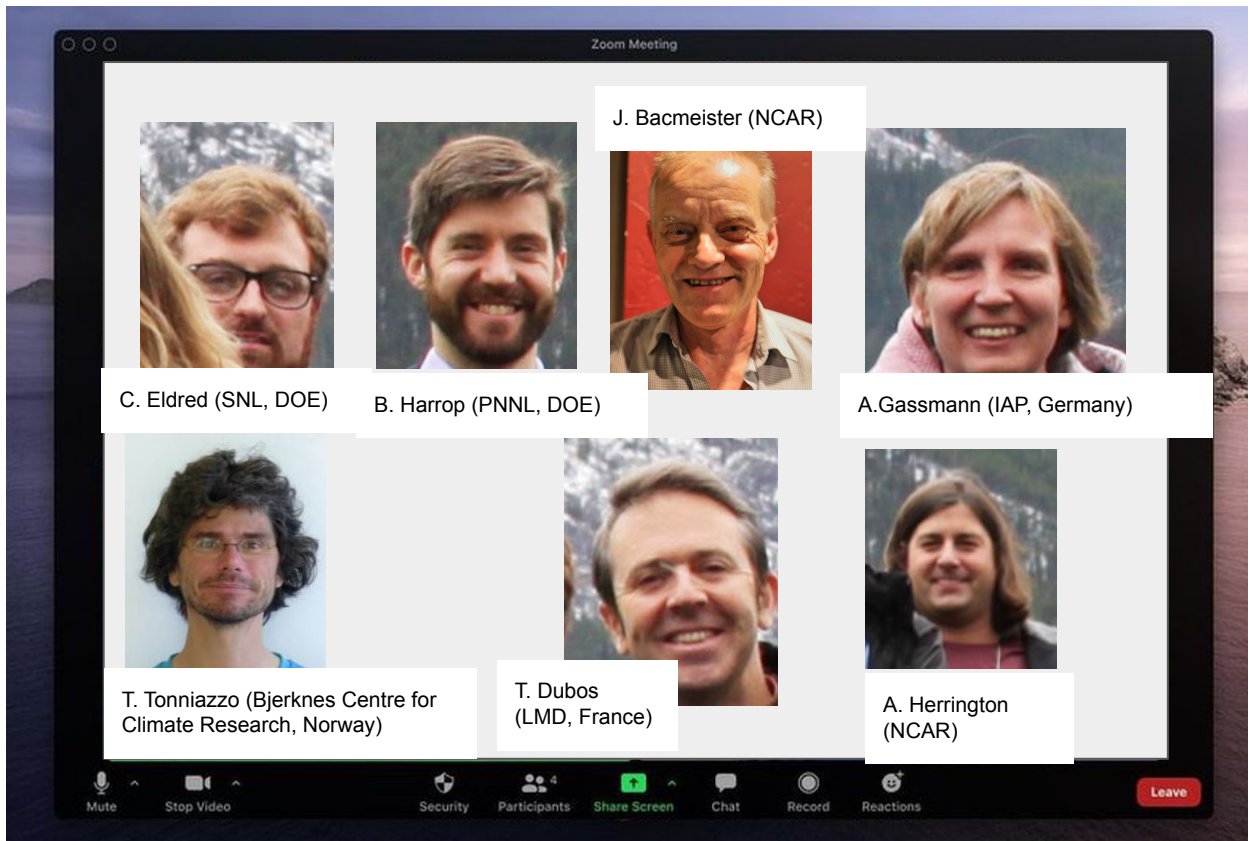


Reconciling and improving formulations for  
thermodynamics and conservation principles in Earth  
system models

P.H. Lauritzen<sup>1</sup>, N.K.-R. Kevlahan<sup>2</sup>, A.J. Adcroft<sup>3</sup>, A.S. Donahue<sup>4</sup>, T. Dubos<sup>5</sup>, C. Eldred<sup>6</sup>, A. Gassmann<sup>7</sup>, B.E. Harrop<sup>8</sup>, A.R. Herrington<sup>1</sup>, C. Jablonowski<sup>9</sup>, H. Johansen<sup>15</sup>, W. Large<sup>1</sup>, V.E. Larson<sup>10</sup>, F. Lemarié<sup>6</sup>, D.A. Randall<sup>11</sup>, P.J. Rasch<sup>8</sup>, K. Roy<sup>12</sup>, B. Shipway<sup>13</sup>, R. Tailleux<sup>14</sup>, H. Wan<sup>8</sup>



# What is the total energy equation for the atmosphere?



Reconciling and improving formulations for thermodynamics and conservation principles in Earth system models

P.H. Lauritzen<sup>1</sup>, N.K.-R. Kevlahan<sup>2</sup>, A.J. Adcroft<sup>3</sup>, A.S. Donahue<sup>4</sup>, T. Dubos<sup>5</sup>, C. Eldred<sup>6</sup>, A. Gassmann<sup>7</sup>, B.E. Harrop<sup>8</sup>, A.R. Herrington<sup>1</sup>, C. Jablonowski<sup>9</sup>, H. Johansen<sup>15</sup>, W. Large<sup>1</sup>, V.E. Larson<sup>10</sup>, F. Lemarié<sup>6</sup>, D.A. Randall<sup>11</sup>, P.J. Rasch<sup>8</sup>, K. Roy<sup>12</sup>, B. Shipway<sup>13</sup>, R. Tailleux<sup>14</sup>, H. Wan<sup>8</sup>



# Total energy equation



Assume:

- Primitive equations (hydrostatic, shallow atmosphere, ideal gas)
- Assume model top pressure is constant (equations could also be derived for constant volume models!)
- All components of moist air have the same temperature and move with the same barycentric velocity
- Assume that water entering the atmosphere (evaporation, snow drift, sea spray) has same temperature as water leaving the atmosphere (dew, liquid and frozen precipitation)

Then it can be shown that the following energy equation holds (assuming ice enthalpy reference state):

$$\frac{\partial}{\partial t} \iiint \rho^{(d)} \left\{ \left( m^{(d)} + m^{(H_2O)} \right) (K + \Phi_s) + c_p^{(d)} T + m^{(H_2O)} c_p^{(ice)} T + m^{(wv)} L_s(T) + m^{(liq)} L_f(T) \right\} dA dz$$
$$= \iint \left\{ c_p^{(ice)} \tilde{T}_{surf} F_{net}^{(H_2O)} + F_{net}^{(H_2O)} (\tilde{K}_{surf} + \Phi_s) + F_{net}^{(wv)} L_s(\tilde{T}_{surf}) + F_{net}^{(liq)} L_f(\tilde{T}_{surf}) + F_{net}^{(turb,rad)} \right\} dA,$$

[see Lauritzen et al., in prep., and CGD seminar from 3/16/2021 <https://www.cgd.ucar.edu/cms/pel/papers/L2021CGD-SEMINAR.pdf> ]

Now also assume that the energy equation is valid for grid mean values in the model.

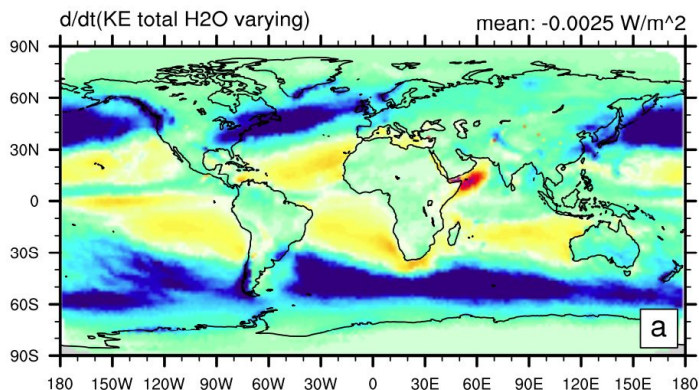


# Total energy equation

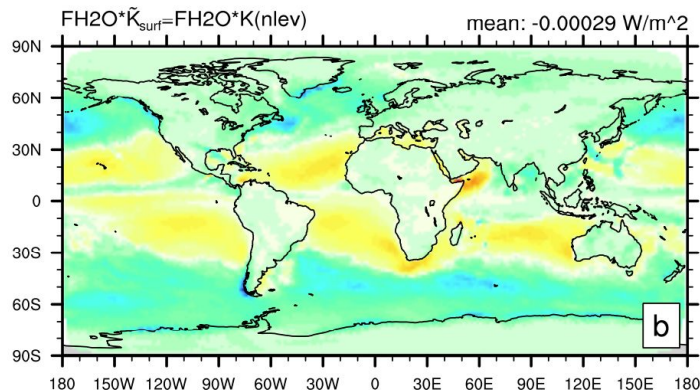
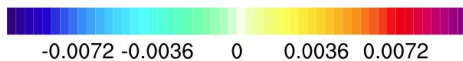
$$\frac{\partial}{\partial t} \iiint \rho^{(d)} \left\{ \left( m^{(d)} + m^{(H_2O)} \right) (K + \Phi_s) + c_p^{(d)} T + m^{(H_2O)} c_p^{(ice)} T + m^{(wv)} L_s(T) + m^{(liq)} L_f(T) \right\} dA dz$$

$$= \iint \left\{ c_p^{(ice)} \tilde{T}_{surf} F_{net}^{(H_2O)} + F_{net}^{(H_2O)} (\tilde{K}_{surf} + \Phi_s) + F_{net}^{(wv)} L_s(\tilde{T}_{surf}) + F_{net}^{(liq)} L_f(\tilde{T}_{surf}) + F_{net}^{(turb,rad)} \right\} dA,$$

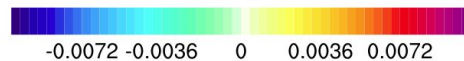
Kinetic energy terms



global min = -0.04616 global max=0.01715



global min = -0.01166 global max=0.007874



Plots (a) and (b) show estimate of 1 year average K terms associated with water flux in/out of the atmospheric column

$$\tilde{K}_{surf} = (\text{lowest level atmosphere winds})^2$$

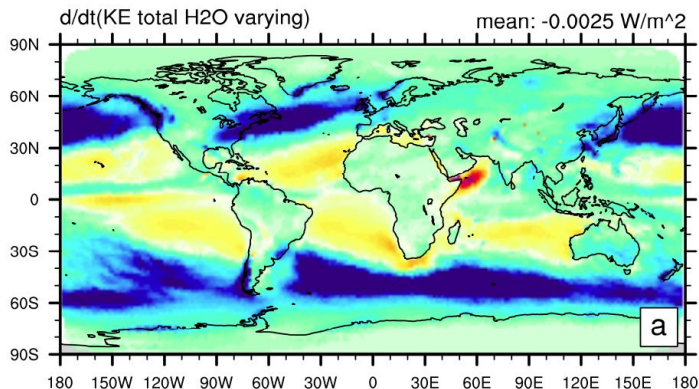
(CAM-SE-CSLAM, 1 degree horizontal resolution, 32 levels, diagnostics computed inline in code)

# Total energy equation

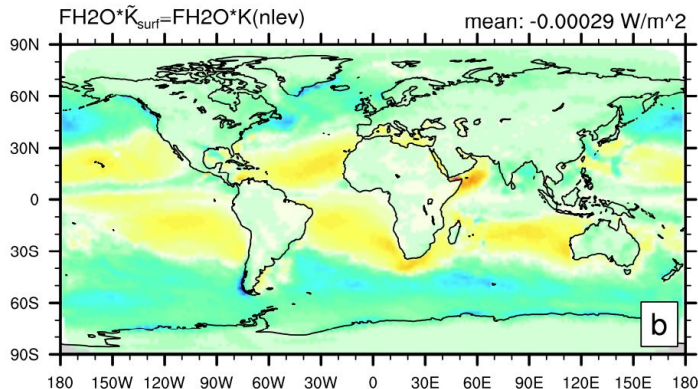
$$\frac{\partial}{\partial t} \iiint \rho^{(d)} \left\{ \left( m^{(d)} + m^{(H_2O)} \right) (K + \Phi_s) + c_p^{(d)} T + m^{(H_2O)} c_p^{(ice)} T + m^{(wv)} L_s (T - T_{surf}) \right\}$$

$$= \iiint \left\{ c_p^{(ice)} \tilde{T}_{surf} F_{net}^{(H_2O)} + F_{net}^{(H_2O)} (\tilde{K}_{surf} + \Phi_s) + F_{net}^{(wv)} L_s (\tilde{T}_{surf}) + F_{net}^{(liq)} L_f (\tilde{T}_{surf}) \right\}$$

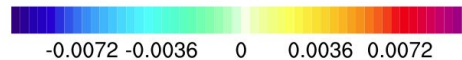
Kinetic energy terms



global min = -0.04616 global max=0.01715



global min = -0.01166 global max=0.007874



- Terms are small
- (a) and (b) do not balance very well.

E.g. when precipitation is produced at a certain level it instantaneously leaves the atmosphere

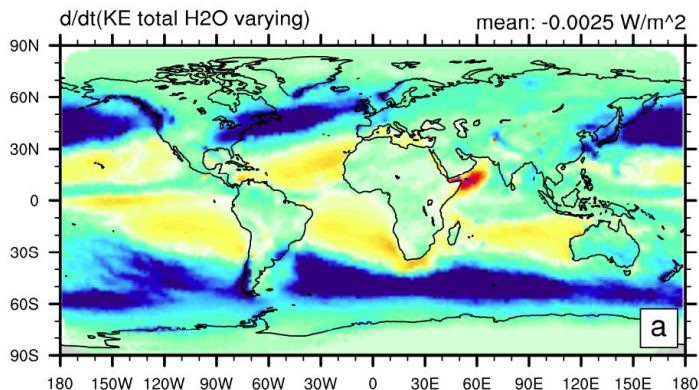
=> has different K than the surface K; we are not rigorously modeling falling precipitation (frictional heating, drag of falling precipitation, etc.)

# Total energy equation

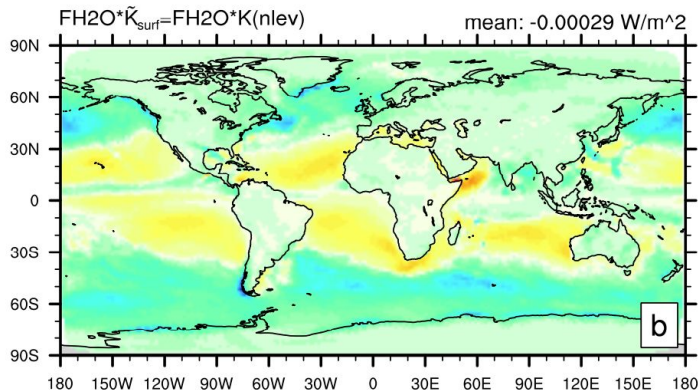
$$\frac{\partial}{\partial t} \iiint \rho^{(d)} \left\{ \left( m^{(d)} + m^{(H_2O)} \right) (K + \Phi_s) + c_p^{(d)} T + m^{(H_2O)} c_p^{(ice)} T + m^{(wv)} L_s(T) + m^{(liq)} L_f(T) \right\} dA dz$$

$$= \iint \left\{ c_p^{(ice)} \tilde{T}_{surf} F_{net}^{(H_2O)} + F_{net}^{(H_2O)} (\tilde{K}_{surf} + \Phi_s) + F_{net}^{(wv)} L_s(\tilde{T}_{surf}) + F_{net}^{(liq)} L_f(\tilde{T}_{surf}) + F_{net}^{(turb,rad)} \right\} dA,$$

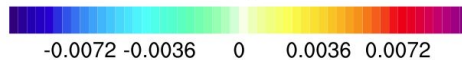
## Kinetic energy terms



global min = -0.04616    global max = 0.01715



global min = -0.01166    global max = 0.007874

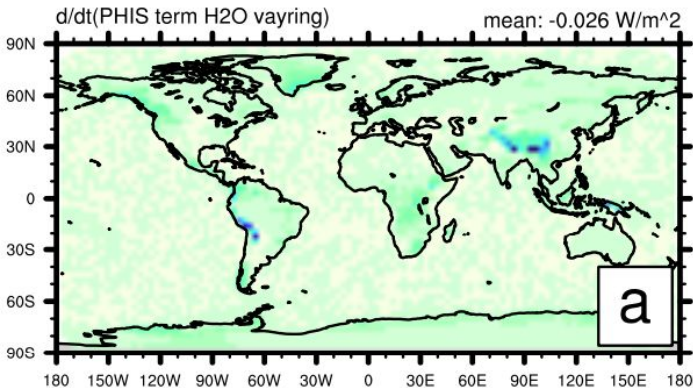




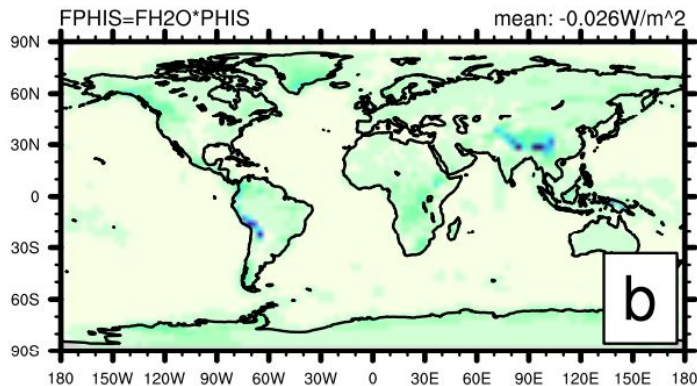
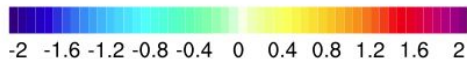
# Total energy equation

$$\frac{\partial}{\partial t} \iiint \rho^{(d)} \left\{ \left( m^{(d)} + \underline{m^{(H_2O)}} \right) (K + \underline{\Phi_s}) + c_p^{(d)} T + m^{(H_2O)} c_p^{(ice)} T + m^{(wv)} L_s(T) + m^{(liq)} L_f(T) \right\} dA dz$$

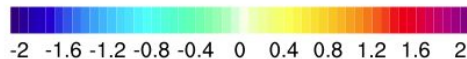
$$= \iint \left\{ c_p^{(ice)} \tilde{T}_{surf} F_{net}^{(H_2O)} + \underline{F_{net}^{(H_2O)}} (\tilde{K}_{surf} + \underline{\Phi_s}) + F_{net}^{(wv)} L_s(\tilde{T}_{surf}) + F_{net}^{(liq)} L_f(\tilde{T}_{surf}) + F_{net}^{(turb,rad)} \right\} dA,$$



global min = -3.49    global max=0.2486



global min = -3.49    global max=0.2486



Plots (a) and (b) show estimate of 1 year average PHIS terms associated with water flux in/out of the atmospheric column (.ne. potential energy but related!)

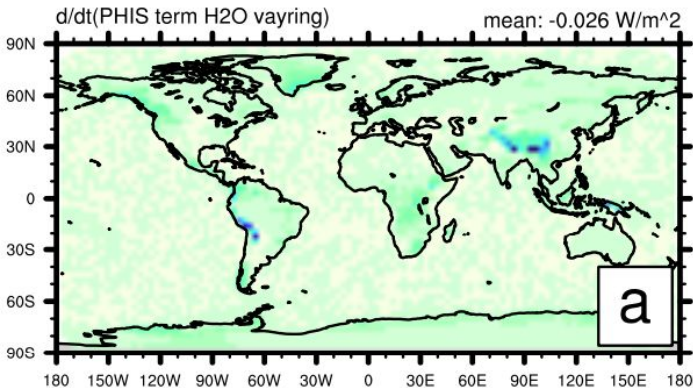
- Exactly match
- Small term!



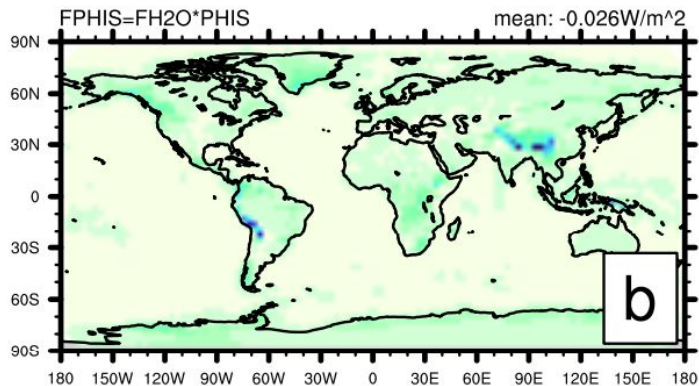
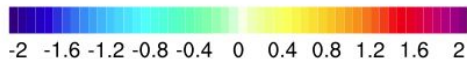
# Total energy equation

$$\frac{\partial}{\partial t} \iiint \rho^{(d)} \left\{ \left( m^{(d)} + m^{(H_2O)} \right) (K + \Phi_s) + c_p^{(d)} T + m^{(H_2O)} c_p^{(ice)} T + m^{(wv)} L_s(T) + m^{(liq)} L_f(T) \right\} dA dz$$

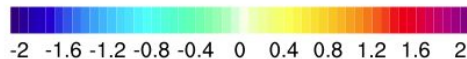
$$= \iint \left\{ c_p^{(ice)} \tilde{T}_{surf} F_{net}^{(H_2O)} + F_{net}^{(H_2O)} (\tilde{K}_{surf} + \Phi_s) + F_{net}^{(wv)} L_s(\tilde{T}_{surf}) + F_{net}^{(liq)} L_f(\tilde{T}_{surf}) + F_{net}^{(turb,rad)} \right\} dA,$$



global min = -3.49    global max=0.2486



global min = -3.49    global max=0.2486





# Total energy equation



$$\frac{\partial}{\partial t} \iiint \rho^{(d)} \left\{ \left( m^{(d)} + m^{(H_2O)} \right) (K + \Phi_s) + c_p^{(d)} T + m^{(H_2O)} c_p^{(ice)} T + m^{(wv)} \underline{L_s(T)} + m^{(liq)} \underline{L_f(T)} \right\} dA dz$$

$$= \iint \left\{ c_p^{(ice)} \tilde{T}_{surf} F_{net}^{(H_2O)} + F_{net}^{(H_2O)} (\tilde{K}_{surf} + \Phi_s) + F_{net}^{(wv)} \underline{L_s(\tilde{T}_{surf})} + F_{net}^{(liq)} \underline{L_f(\tilde{T}_{surf})} + F_{net}^{(turb,rad)} \right\} dA,$$

Assume constant latent heats  $\Leftrightarrow$  heat capacity for all forms of water is the same:

The latent heat formulas for sublimation (solid  $\rightarrow$  water vapor):

$$L_s(T) = L_{s,00} + \left( c_p^{(wv)} - c_p^{(ice)} \right) (T - T_{00}), \text{ where } L_{s,00} \equiv h_{00}^{(wv)} - h_{00}^{(ice)}$$

The latent heat formulas for fusion (solid  $\rightarrow$  liquid):

$$L_f(T) = L_{f,00} + \left( c_p^{(liq)} - c_p^{(ice)} \right) (T - T_{00}), \text{ where } L_{f,00} \equiv h_{00}^{(liq)} - h_{00}^{(ice)}$$

In CAM we use:  $c_p^{(\ell)} = c_p^{(d)}$  for  $\ell \in \mathcal{L}_{H_2O}$ .



# CAM energy equation

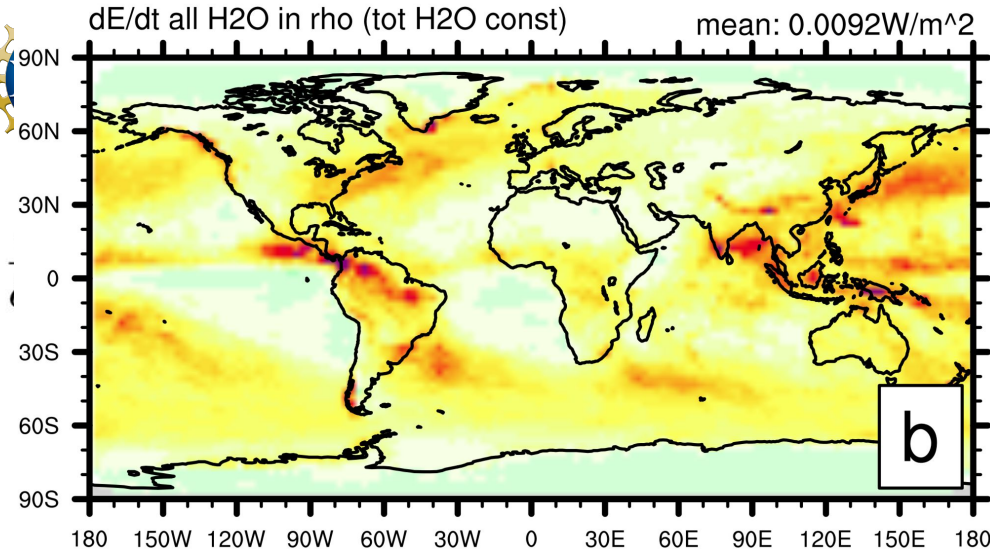


$$\frac{\partial}{\partial t} \iiint \rho^{(d)} \left\{ \left[ m^{(d)} + \underline{m_{t=t^n}^{(H_2O)}} \right] (K + \Phi_s) + c_p^{(d)} T + \underline{m_{t=t^n}^{(H_2O)}} \underline{c_p^{(d)}} T + m^{(wv)} L_{s,00} + m^{(liq)} L_{f,00} \right\} dA dz$$

$$\approx \iint \left\{ L_{s,00} F_{net}^{(wv)} + L_{f,00} F_{net}^{(liq)} + F^{(turb,rad)} \right\} dA. \quad (98)$$

Further simplifications in CAM:

1. Assume  $m^{(H_2O)} = m^{(wv)}$   
i.e. condensates do not contribute to kinetic, internal or geopotential energy (they are mass-less!)
2. Assume that  $\underline{m^{(H_2O)}}$  during physics updates:  $m^{(H_2O)} = m_{t=t^n}^{(H_2O)}$
3. Discard enthalpy flux at the surface  $c_p^{(d)} \tilde{T}_{surf} F_{net}^{(H_2O)}$
4. Use heat capacity of dry air for all forms of water



Plot (b) shows energy imbalance by including all forms of water in pressure/density of air.

=> Assumption 1. is justifiable at 1 degree horizontal resolution!

Further simplifications in CAM:

1. Assume  $m^{(H_2O)} = m^{(wv)}$   
i.e. condensates do not contribute to kinetic, internal or geopotential energy (they are mass-less!)
2. Assume that  $m^{(H_2O)}$  during physics updates:  $m^{(H_2O)} = m_{t=t^n}^{(H_2O)}$
3. Discard enthalpy flux at the surface  $c_p^{(d)} \tilde{T}_{surf} F_{net}^{(H_2O)}$
4. Use heat capacity of dry air for all forms of water





# CAM energy equation



$$\frac{\partial}{\partial t} \iiint \rho^{(d)} \left\{ \left[ m^{(d)} + \underline{m_{t=t^n}^{(H_2O)}} \right] (K + \Phi_s) + c_p^{(d)} T + \underline{m_{t=t^n}^{(H_2O)}} \underline{c_p^{(d)}} T + m^{(wv)} L_{s,00} + m^{(liq)} L_{f,00} \right\} dA dz$$

$$\approx \iint \left\{ L_{s,00} F_{net}^{(wv)} + L_{f,00} F_{net}^{(liq)} + F^{(turb,rad)} \right\} dA. \quad (98)$$

Further simplifications in CAM:

1. Assume  $m^{(H_2O)} = m^{(wv)}$   
i.e. condensates do not contribute to kinetic, internal or geopotential energy (they are mass-less!)
2. Assume that  $\underline{m^{(H_2O)}}$  during physics updates:  $m^{(H_2O)} = m_{t=t^n}^{(H_2O)}$
3. Discard enthalpy flux at the surface  $c_p^{(d)} \tilde{T}_{surf} F_{net}^{(H_2O)}$
4. Use heat capacity of dry air for all forms of water



# CAM energy equation



$$\frac{\partial}{\partial t} \iiint \rho^{(d)} \left\{ \left[ m^{(d)} + m_{t=t^n}^{(H_2O)} \right] (K + \Phi_s) + c_p^{(d)} T + m_{t=t^n}^{(H_2O)} c_p^{(d)} T + m^{(wv)} L_{s,00} + m^{(liq)} L_{f,00} \right\} dA dz$$
$$\approx \iint \left\{ L_{s,00} F_{net}^{(wv)} + L_{f,00} F_{net}^{(liq)} + F^{(turb,rad)} \right\} dA. \quad (98)$$

Consider a physics column (no interaction between columns so equation holds in column!):

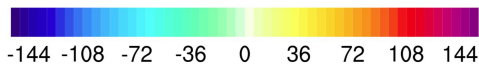
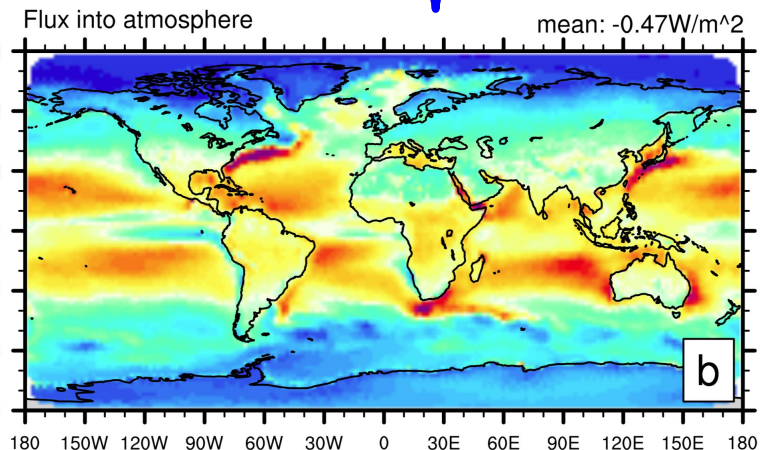
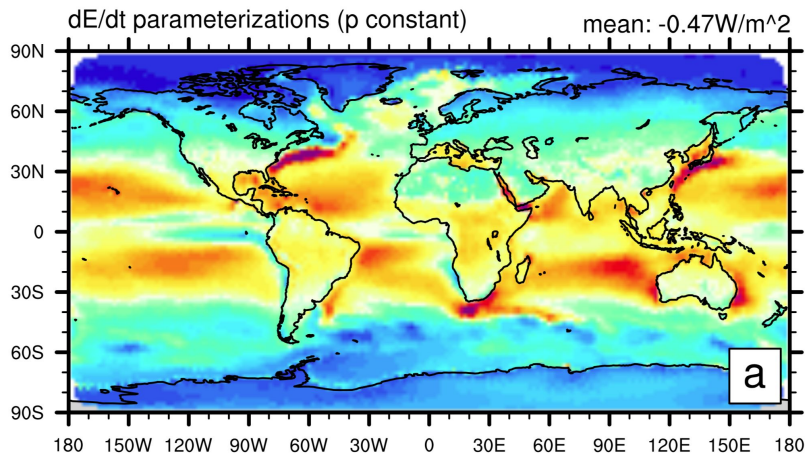
- Each parameterization (in theory) satisfies this equation in CAM (see Figure on next slide)

This system is energetically consistent - **and cleverly chosen to keep energetics simple and let the CAM energy fixer restore global energy conservation for processes that we are not accounting for (e.g. enthalpy of hydrometeors leaving/entering the column, kinetic and geopotential energy associated with hydrometeors leaving/entering the column)**

# CAM energy equation

$$\frac{\partial}{\partial t} \iiint \rho^{(d)} \left\{ \left[ m^{(d)} + m_{t=t^n}^{(H_2O)} \right] (K + \Phi_s) + c_p^{(d)} T + m_{t=t^n}^{(H_2O)} c_p^{(d)} T + m^{(wv)} L_{s,00} + m^{(liq)} L_{f,00} \right\} dA dz$$

$$\approx \iint \left\{ L_{s,00} F_{net}^{(wv)} + L_{f,00} F_{net}^{(liq)} + F^{(turb,rad)} \right\} dA. \quad (98)$$



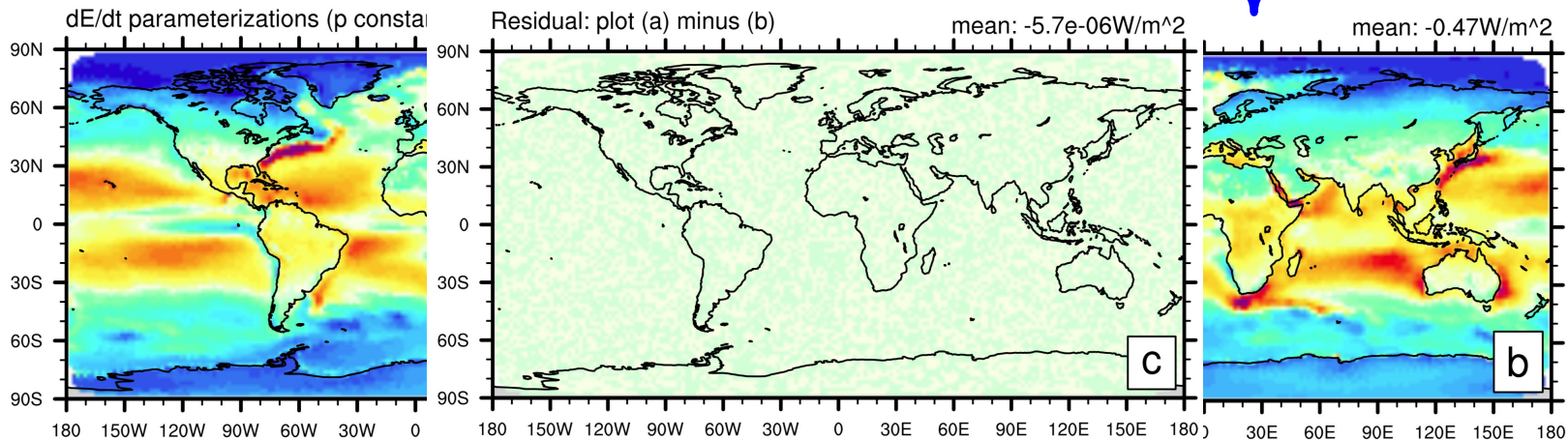
1 year ave, F2000climo, ne30pg3, CAM6

**Diagnostics coded inline in CAM!**

# CAM energy equation

$$\frac{\partial}{\partial t} \iiint \rho^{(d)} \left\{ \left[ m^{(d)} + m_{t=t^n}^{(H_2O)} \right] (K + \Phi_s) + c_p^{(d)} T + m_{t=t^n}^{(H_2O)} c_p^{(d)} T + m^{(wv)} L_{s,00} + m^{(liq)} L_{f,00} \right\} dA dz$$

$$\approx \iint \left\{ L_{s,00} F_{net}^{(wv)} + L_{f,00} F_{net}^{(liq)} + F^{(turb,rad)} \right\} dA. \quad (98)$$



Diagnostics coded inline in CAM!





# CAM energy equation



$$\frac{\partial}{\partial t} \iiint \rho^{(d)} \left\{ \left[ m^{(d)} + m_{t=t^n}^{(H_2O)} \right] (K + \Phi_s) + c_p^{(d)} T + m_{t=t^n}^{(H_2O)} c_p^{(d)} T + m^{(wv)} L_{s,00} + m^{(liq)} L_{f,00} \right\} dA dz$$
$$\approx \iint \left\{ L_{s,00} F_{net}^{(wv)} + L_{f,00} F_{net}^{(liq)} + F^{(turb,rad)} \right\} dA. \quad (98)$$

Consider a physics column (no interaction between columns so equation holds in column!):

- Each parameterization (in theory) satisfies this equation in CAM (see Figure on next slide)

This system is energetically consistent - **and cleverly chosen to keep energetics simple and let the CAM energy fixer restore global energy conservation for processes that we are not accounting for (e.g. enthalpy of hydrometeors leaving/entering the column, kinetic and geopotential energy associated with hydrometeors leaving/entering the column)**



# CAM energy equation



$$\frac{\partial}{\partial t} \iiint \rho^{(d)} \left\{ \left[ m^{(d)} + \underline{m_{t=t^n}^{(H_2O)}} \right] (K + \Phi_s) + c_p^{(d)} T + \underline{m_{t=t^n}^{(H_2O)}} \underline{c_p^{(d)}} T + m^{(wv)} L_{s,00} + m^{(liq)} L_{f,00} \right\} dA dz$$

$$\approx \iint \left\{ L_{s,00} F_{net}^{(wv)} + L_{f,00} F_{net}^{(liq)} + F^{(turb,rad)} \right\} dA. \quad (98)$$

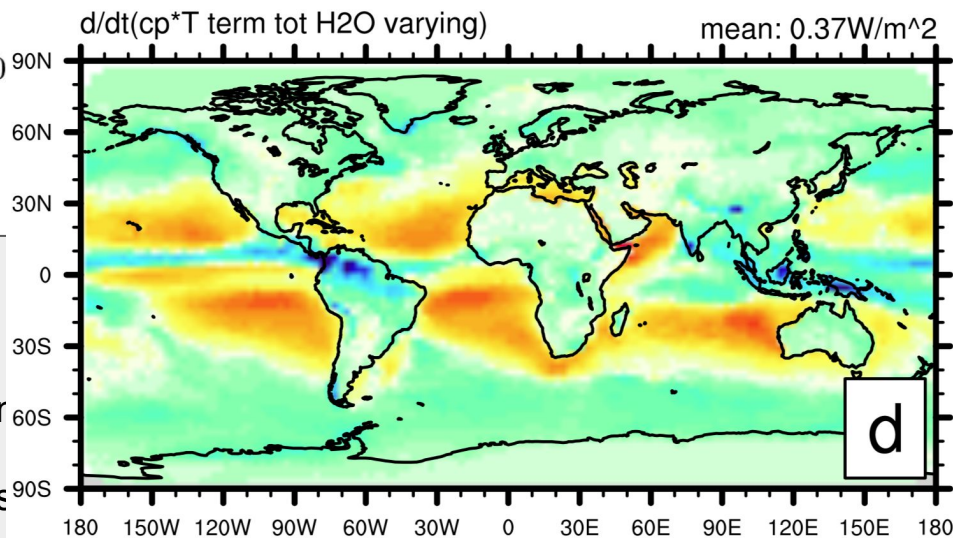
Further simplifications in CAM:

- Assume  $m^{(H_2O)} = m^{(wv)}$   
i.e. condensates do not contribute to kinetic, internal or geopotential energy (they are mass-less!)
- Assume that  $\underline{m^{(H_2O)}}$  during physics updates:  $m^{(H_2O)} = m_{t=t^n}^{(H_2O)}$
- Discard enthalpy flux at the surface  $c_p^{(d)} \tilde{T}_{surf} F_{net}^{(H_2O)}$
- Use heat capacity of dry air for all forms of water

# CAM energy equation

$$\frac{\partial}{\partial t} \iiint \rho^{(d)} \left\{ \left[ m^{(d)} + \underline{m_{t=t^n}^{(H_2O)}} \right] (K + \Phi_s) + c_p^{(d)} T + \underline{m_{t=t^n}^{(H_2O)}} \underline{c_p^{(d)}} T + m^{(wv)} L_{s,00} + m^{(liq)} L_{f,00} \right\} dA dz$$

$$\approx \iint \left\{ L_{s,0} \right\}$$



Further simplifications in CAM:

- Assume  $m^{(H_2O)} = m^{(wv)}$   
i.e. condensates do not contribute to kinetic, ir
- Assume that  $\underline{m^{(H_2O)}}$  during physics updates
- Discard enthalpy flux at the surface  $c_p^{(d)} \tilde{T}_{surf} F_{net}^{(H_2O)}$
- Use heat capacity of dry air for all forms of water



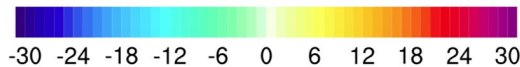
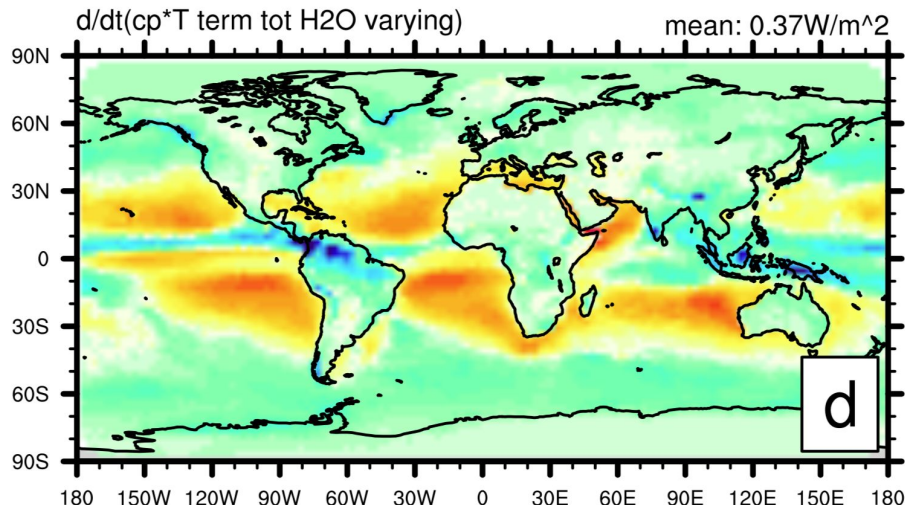
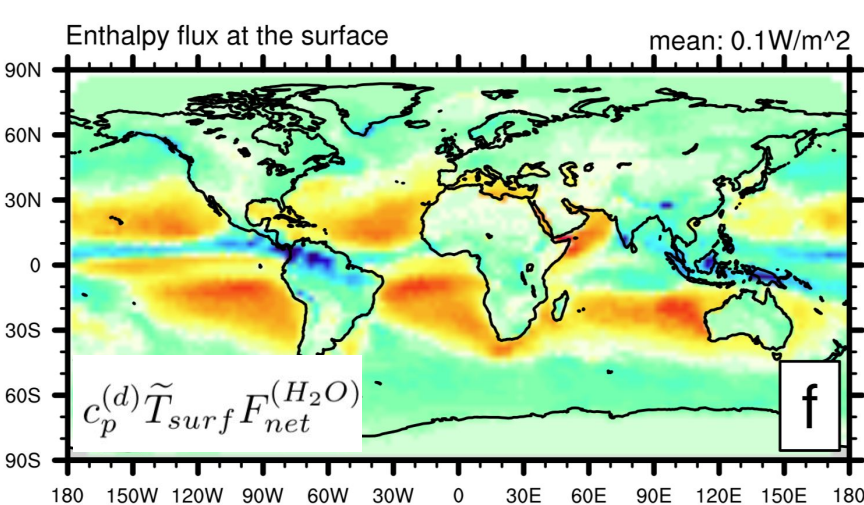
# Missing enthalpy flux at surface



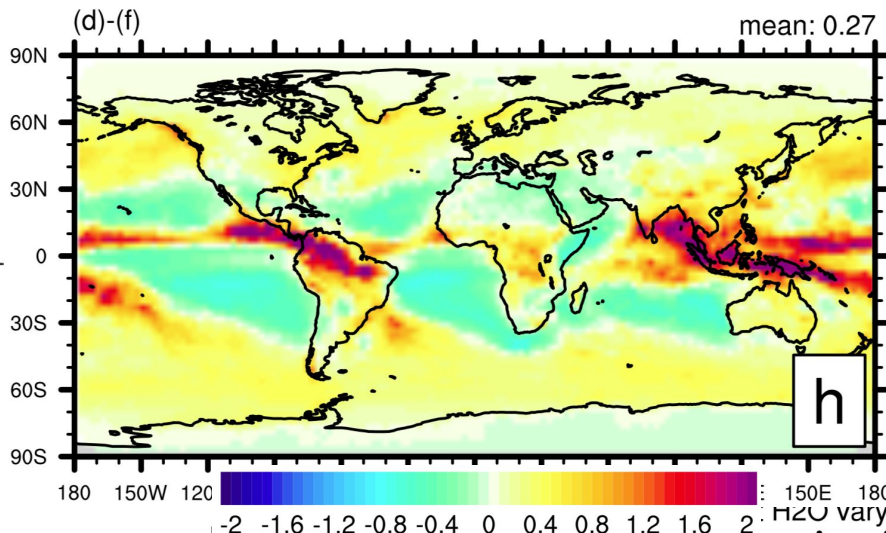
using T at the surface, TS

$$\frac{\partial}{\partial t} \iiint \rho^{(d)} \left\{ \left[ m^{(d)} + \underline{m_{t=t^n}^{(H_2O)}} \right] (K + \Phi_s) + c_p^{(d)} T + \underline{m_{t=t^n}^{(H_2O)}} c_p^{(d)} T + m^{(wv)} L_{s,00} + m^{(liq)} L_{f,00} \right\} dA dz$$

$$\approx \iint \left\{ L_{s,00} F_{net}^{(wv)} + L_{f,00} F_{net}^{(liq)} + F^{(turb,rad)} \right\} dA. \quad (98)$$

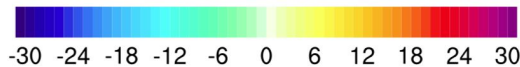
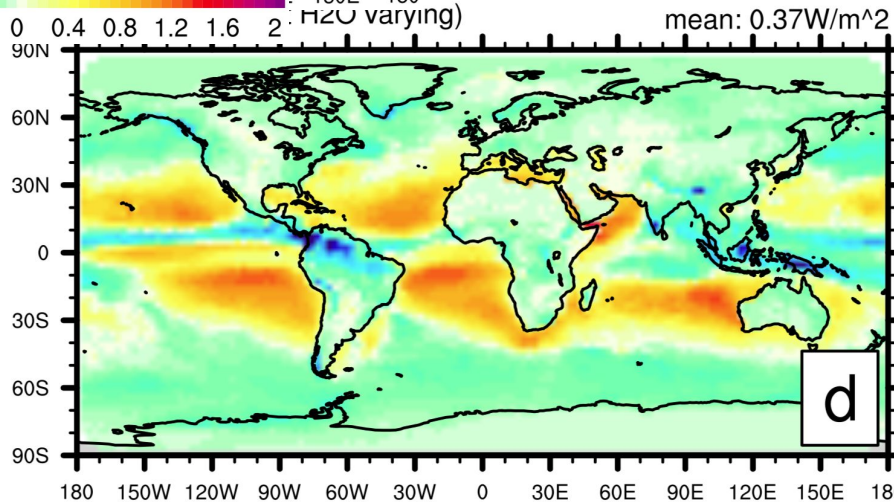
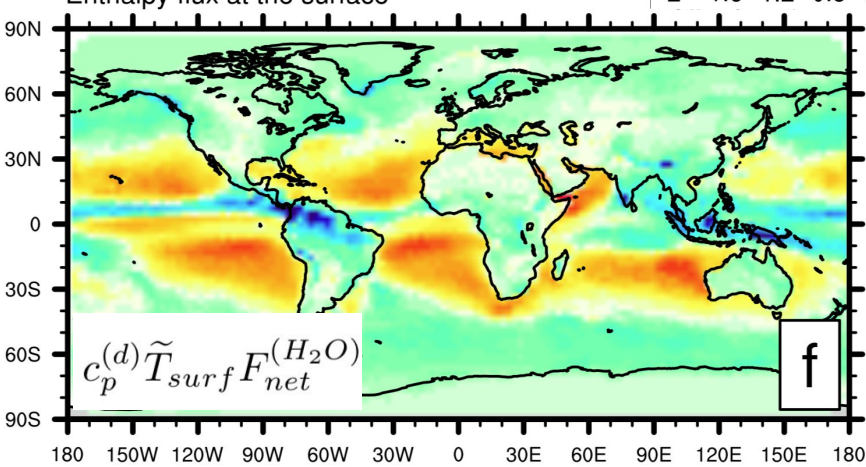






$$\left\{ m^{(d)} + \left[ m^{(d)} + \left( wv \right) L_{s,00} + m^{(liq)} L_{f,00} \right] \right\} dA dz + F^{(turb,rad)} \left\} dA. \quad (98)$$

Enthalpy flux at the surface





# Concluding remarks



- **The current framework will still need an energy fixer but local imbalances can be reduced significantly by including enthalpy flux term!** Danger: if “surface” temperature is larger than the “true” T of water entering/leaving the column the energy that should have stayed in the atmosphere goes to surface component - vice versa for lower “surface” temperature! Use LES simulations to assess?
- **Being rigorous in terms of monitoring energy conservation forces modelers to consider consistency between parameterizations and dynamical core!**
- **CESM3 ocean model (MOM6) requires enthalpy fluxes from atmosphere:**

M. Vertenstein, G. Marques, and F. Bryan have implemented infrastructure in CESM coupler to enable the exchange of enthalpy fluxes between atmosphere and ocean so we can start exploring the effect in coupled climate simulation ...

