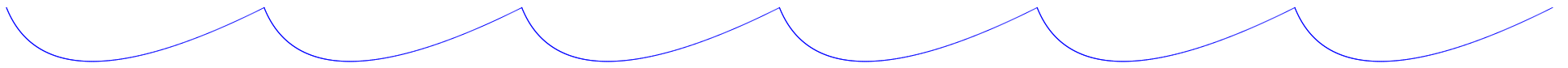




CLIMATE, OCEAN AND SEA ICE MODELING PROGRAM

# High-latitude-friendly lateral mixing parameterizations for global ocean simulations

Elizabeth Hunke, Mat Maltrud, Matthew Hecht

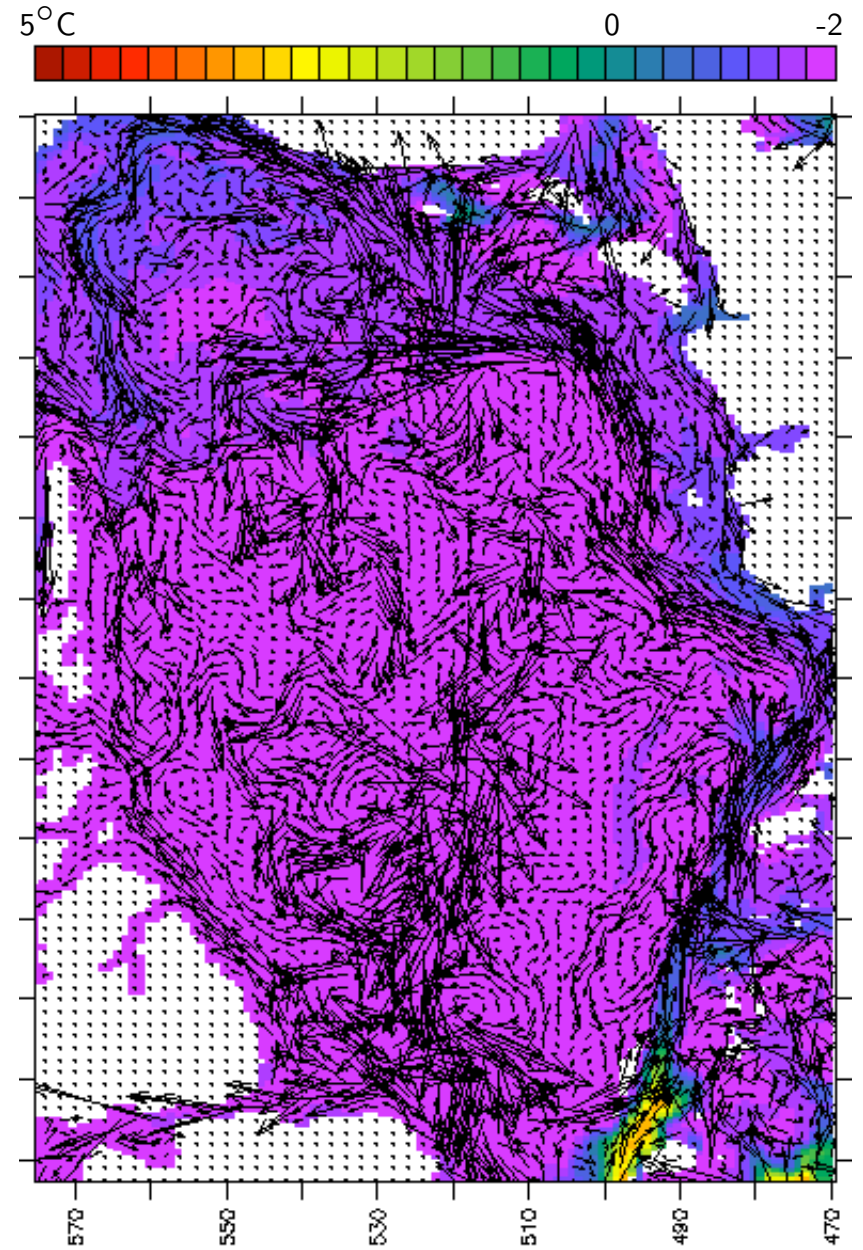
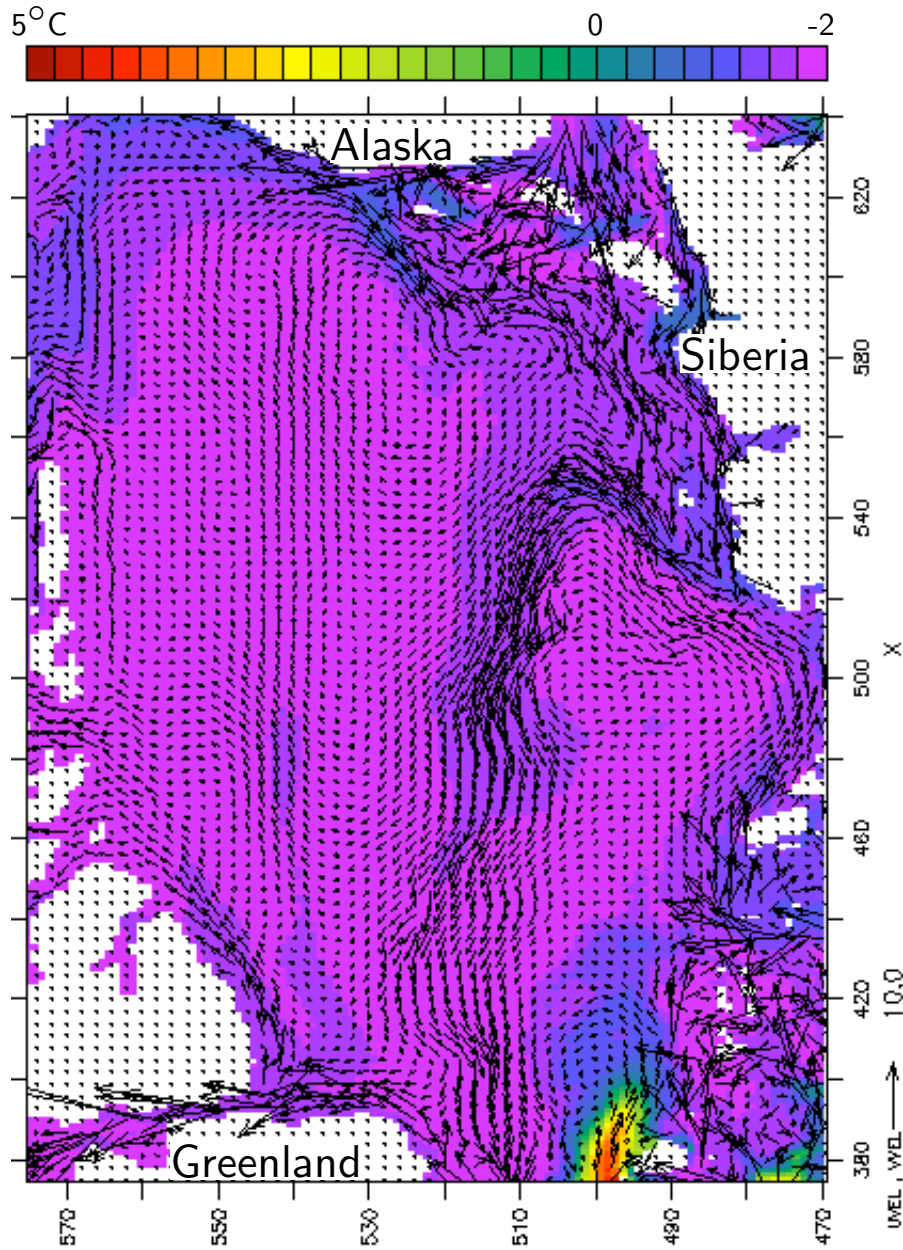


# 1990 Potential Temperature and Velocity

GM

36 m

Biharmonic

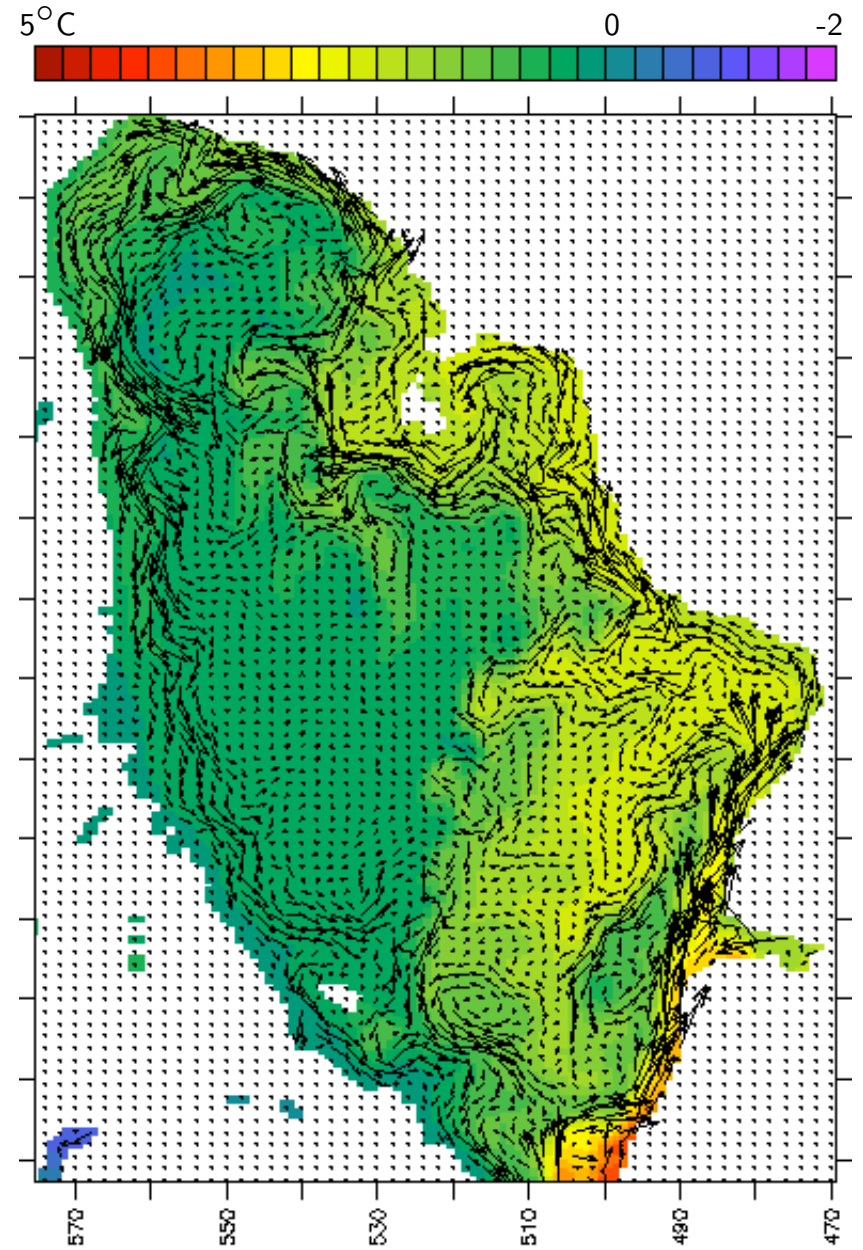
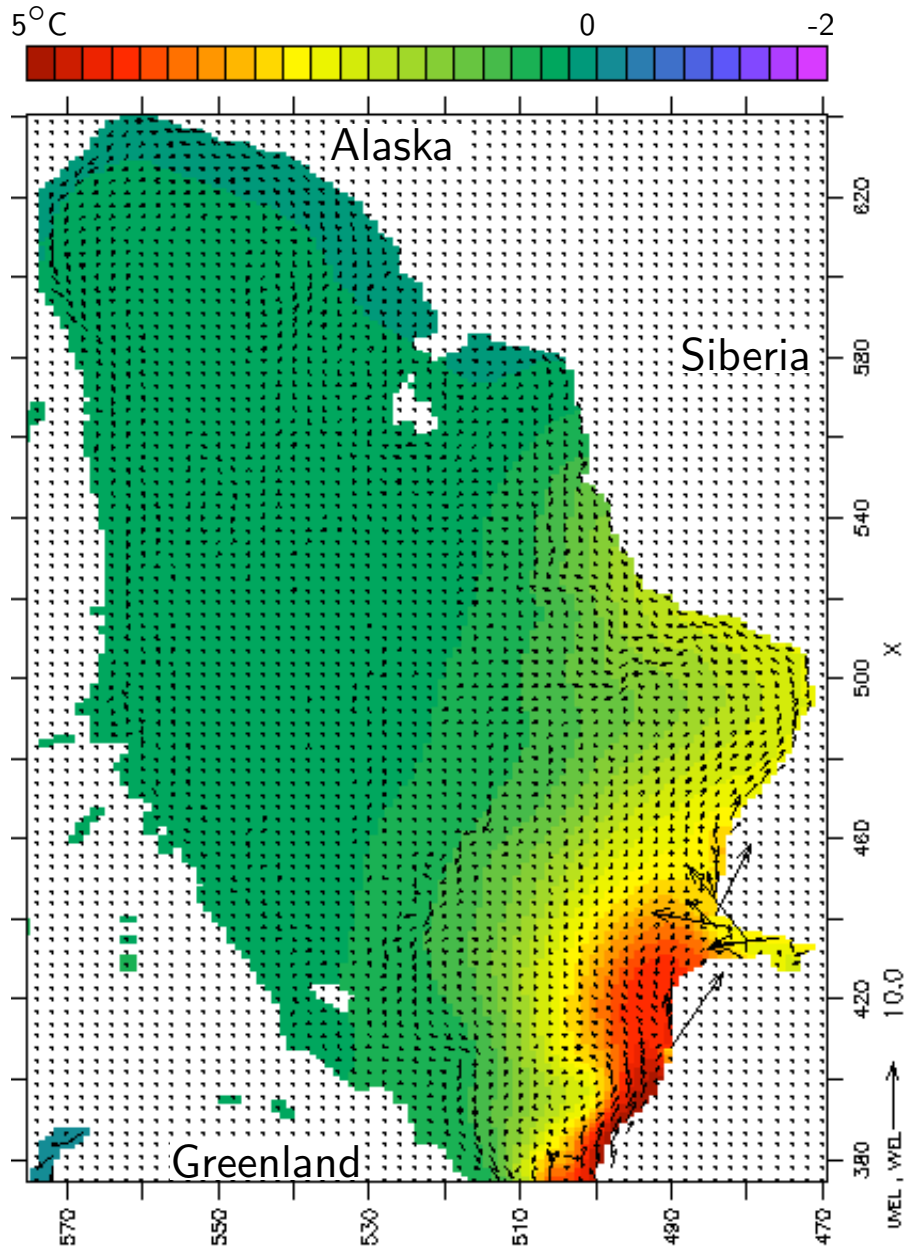


# 1990 Potential Temperature and Velocity

GM

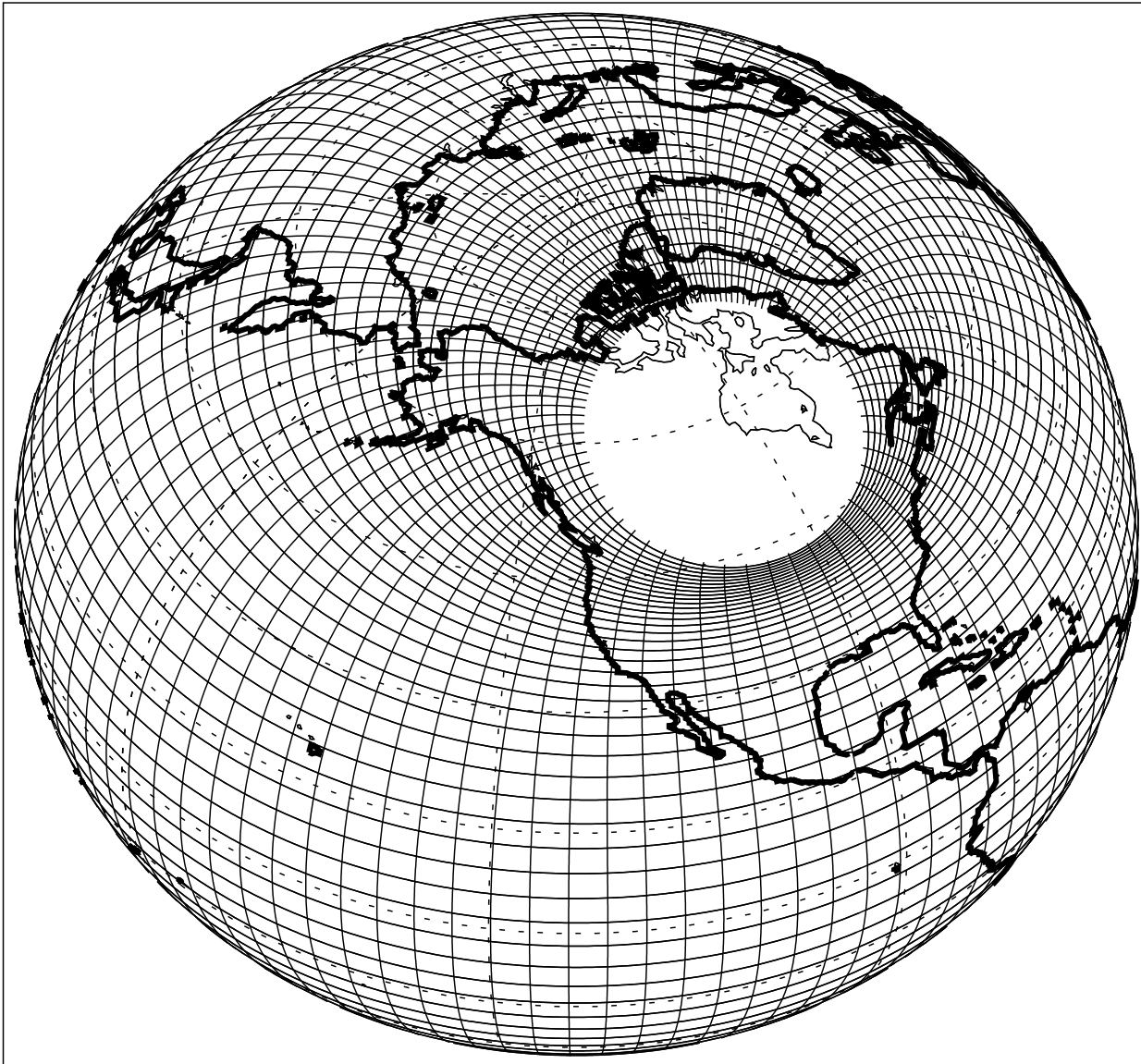
466 m

Biharmonic

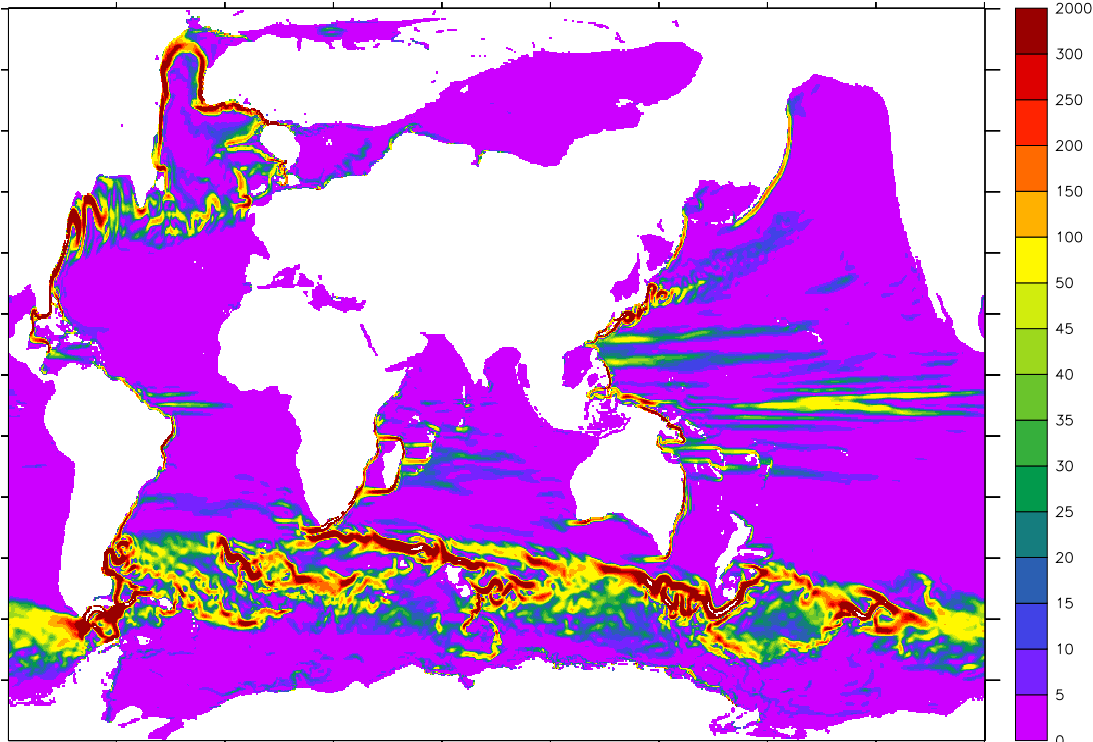




0.4°: 900x600x40

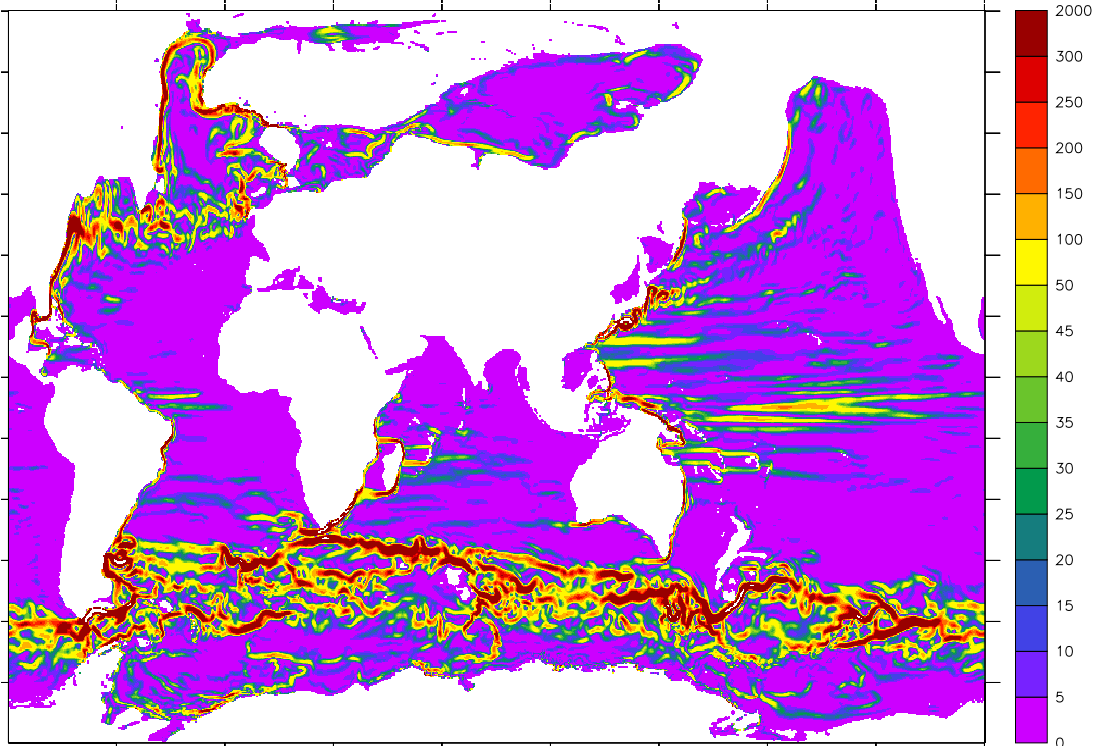


1 of every 100 mesh nodes shown



Kinetic Energy  
1990, 466 m

GM



Biharmonic

# Tracer transport

$$\frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla_2 T + w \frac{\partial T}{\partial z} = D_H(T) + D_V(T)$$

**biharmonic mixing**

$$D_H = \nabla_2^2 (\kappa_o A^{\frac{3}{2}} \nabla_2^2 T)$$

**Gent-McWilliams mixing**

$$D_H = \nabla_3 \cdot \mathbf{K} \cdot \nabla_3 T$$

# I. Damping time scales for simple systems

following Griffies and Hallberg, MWR 2000

Suppose  $T = \gamma(t)e^{ikx_n}$ , where  $x_n = n\Delta$ . Then

(1) for the **Laplacian** case,  $T_t = C T_{xx}$ ,

$$\gamma(t) = \exp \left[ -C \left( \frac{2}{\Delta} \sin \frac{k\Delta}{2} \right)^2 t \right]$$

$$\text{Damping time scale} = \tau_C = \frac{1}{C \left( \frac{2}{\Delta} \sin \frac{k\Delta}{2} \right)^2}$$

(2) for the **biharmonic** case,  $T_t = B T_{xxxx}$ ,

$$\text{Damping time scale} = \tau_B = \frac{1}{B \left( \frac{2}{\Delta} \sin \frac{k\Delta}{2} \right)^4}$$

$$\Rightarrow \frac{\tau_B}{\tau_C} = \frac{C}{B} \left( \frac{2}{\Delta} \sin \frac{k\Delta}{2} \right)^{-2} \sim \frac{C}{Bk^2} \text{ if } \frac{k\Delta}{2} \text{ is small.}$$

# Tracer transport

$$\frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla_2 T + w \frac{\partial T}{\partial z} = D_H(T) + D_V(T)$$

**biharmonic mixing**

$$D_H = \nabla_2^2 (\kappa_o A^{\frac{3}{2}} \nabla_2^2 T)$$

**Gent-McWilliams mixing**

$$D_H = \nabla_3 \cdot \mathbf{K} \cdot \nabla_3 T$$



## II. Scaling argument for grid dependence for simple systems

following Maltrud et al., JGR 1998

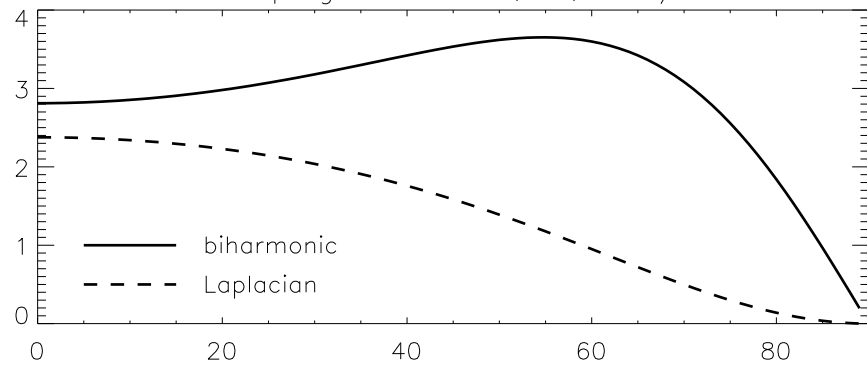
Balance advective and diffusive terms. Then

(1) for the **Laplacian** case,  $U \frac{\partial T}{\partial x} = C \frac{\partial^2 T}{\partial x^2}$ , and  $C$  scales with  $\Delta \sim dx$ ,

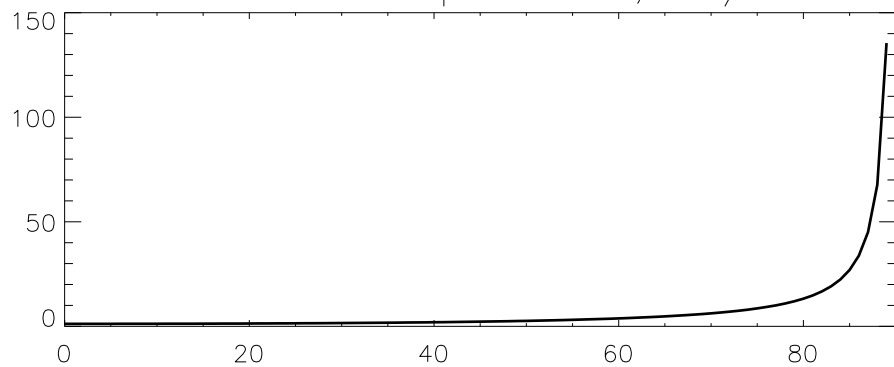
(2) for the **biharmonic** case,  $U \frac{\partial T}{\partial x} = B \frac{\partial^4 T}{\partial x^4}$ , and  $B$  scales with  $\Delta^3$ .

In 2D, grid cell area  $A \sim \Delta^2$ , so  $C \propto A^{1/2}$  and  $B \propto A^{3/2}$ .

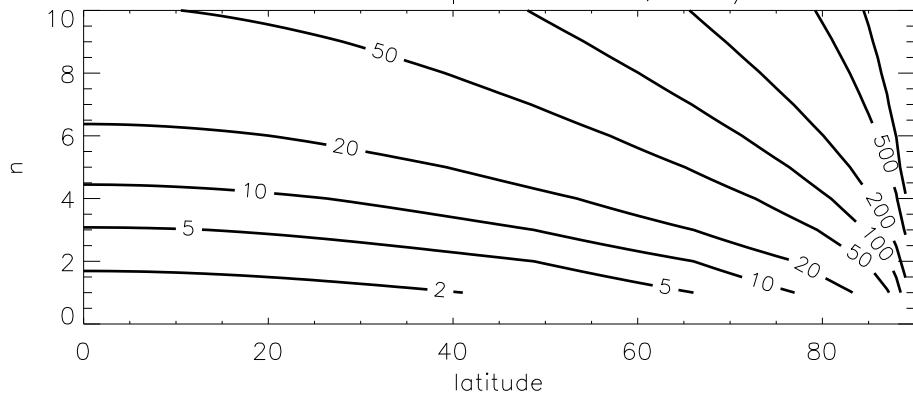
damping timescales, hr,  $k=\pi/\Delta$



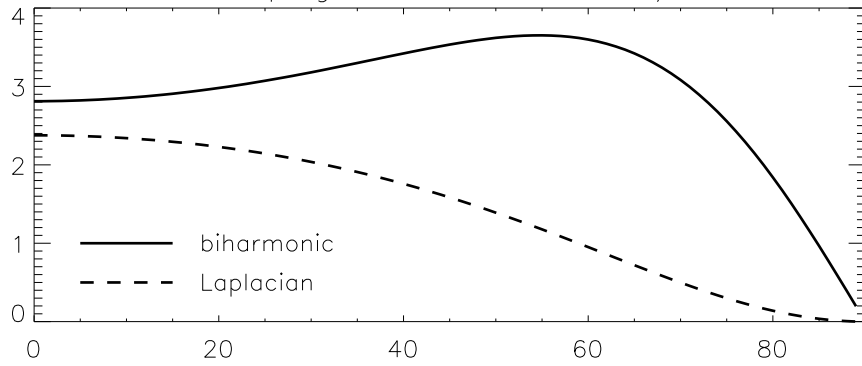
biharmonic : Laplacian ratio,  $k=\pi/\Delta$



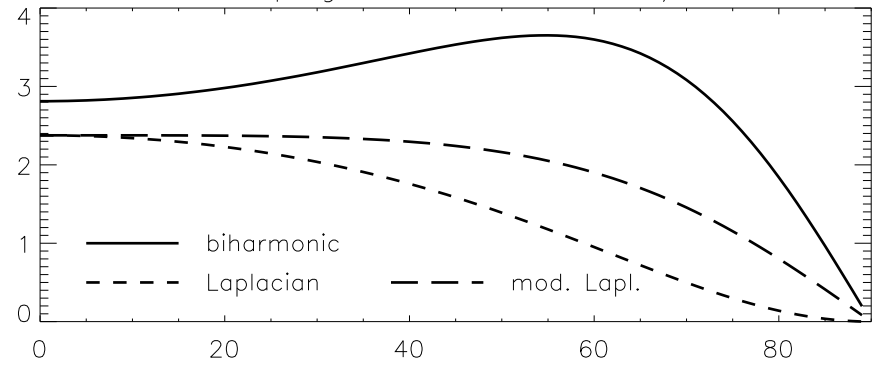
biharmonic : Laplacian ratio,  $k=\pi/n\Delta$



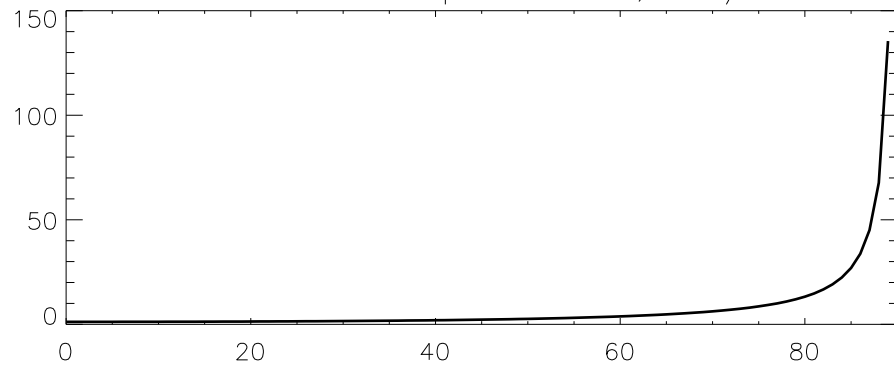
damping timescales, hr,  $k=\pi/\Delta$



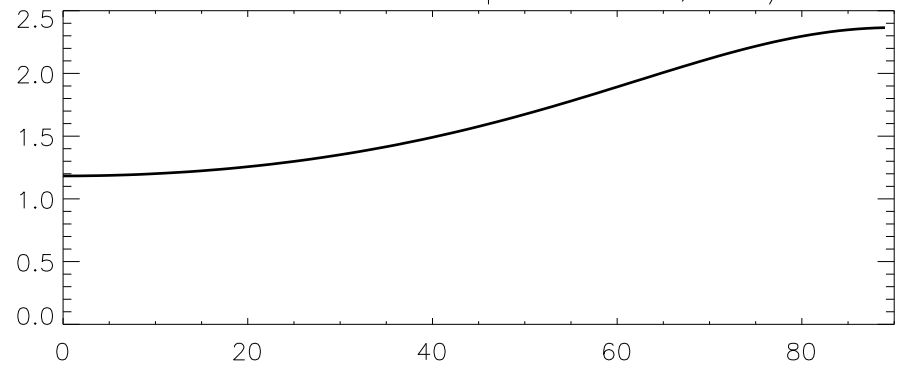
damping timescales, hr,  $k=\pi/\Delta$



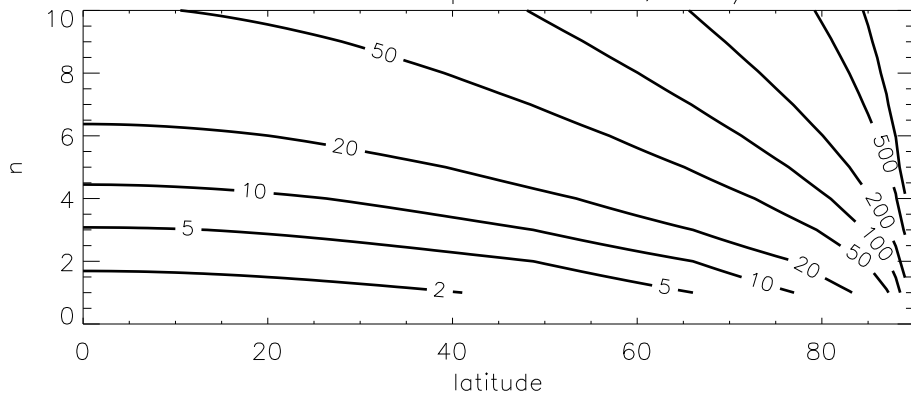
biharmonic : Laplacian ratio,  $k=\pi/\Delta$



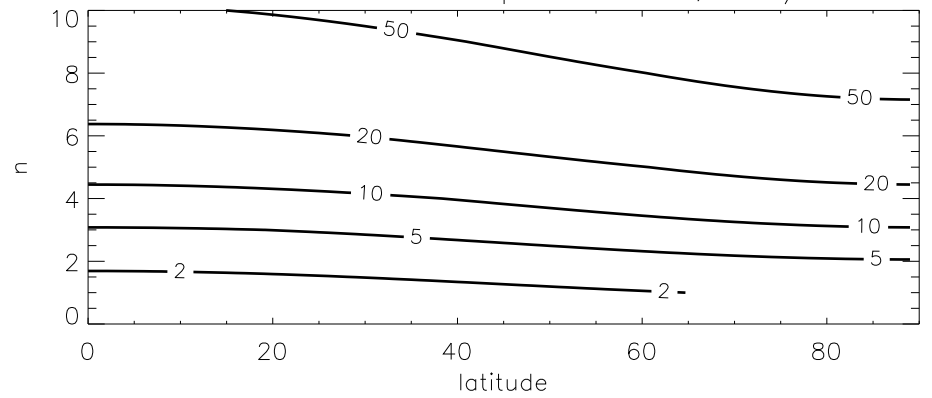
biharmonic : mod. Laplacian ratio,  $k=\pi/\Delta$



biharmonic : Laplacian ratio,  $k=\pi/n\Delta$



biharmonic : mod. Laplacian ratio,  $k=\pi/n\Delta$



# Simulations

POP-CICE coupled model

0.4° grid, modified Large-Yeager atmospheric forcing

10-year “spin-up” runs

biharmonic (includes area scaling)

GM

GM with area scaling (“modified GM”)

anisotropic GM

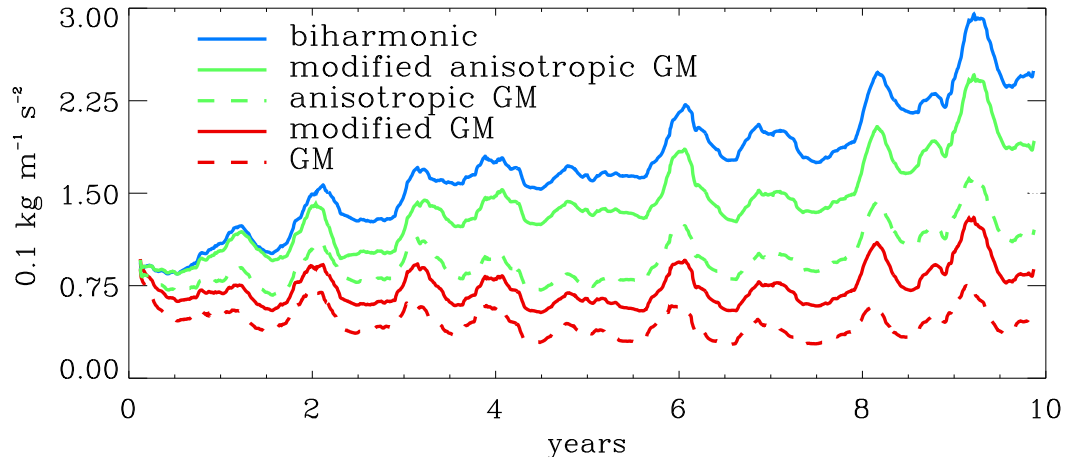
anisotropic GM with area scaling (“modified anisotropic GM”)

1-year “spin-down” runs

from beginning of year 10, biharmonic run

5 runs above, plus Laplacian with and without area scaling

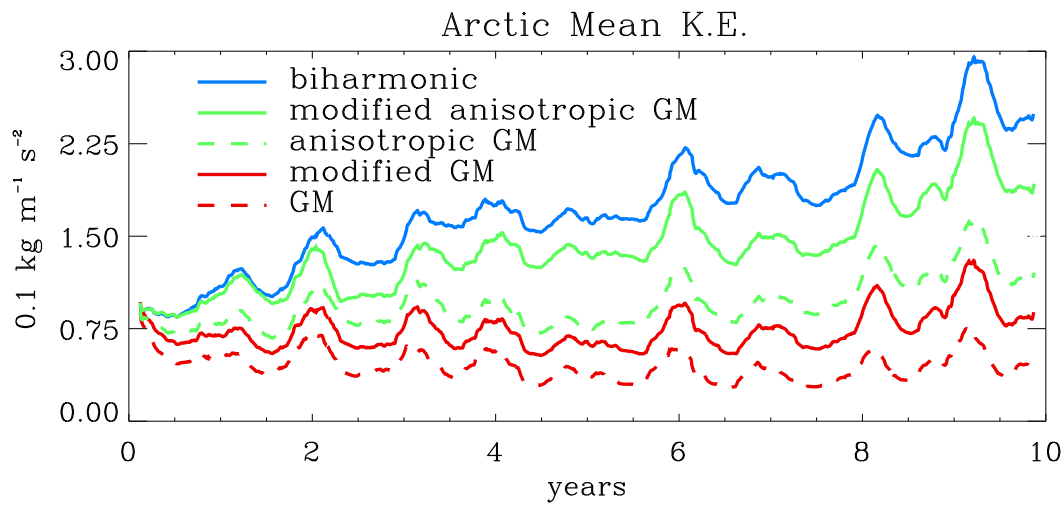
Arctic Mean K.E.



## Mean kinetic energy

averaged over the ocean volume  
north of 80°N

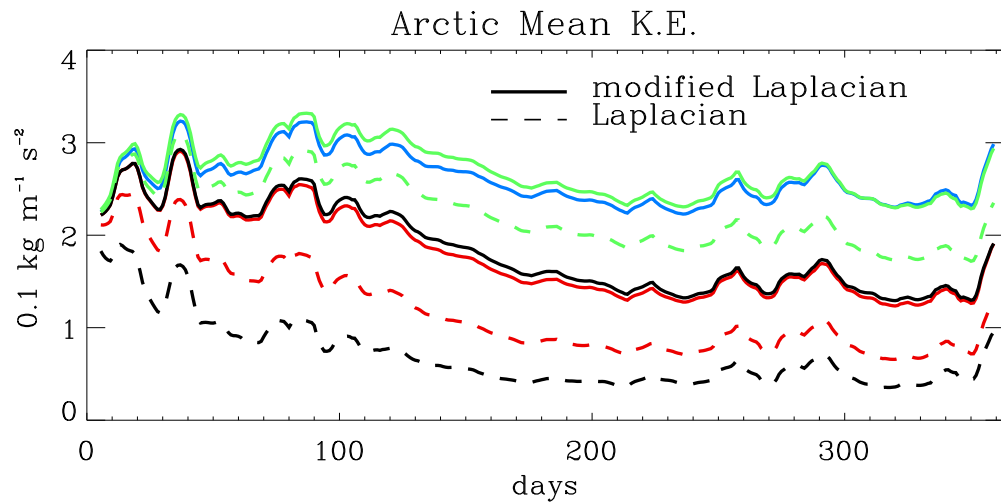
10-year "spin-up" (1981–1990)



## Mean kinetic energy

averaged over the ocean volume  
north of 80°N

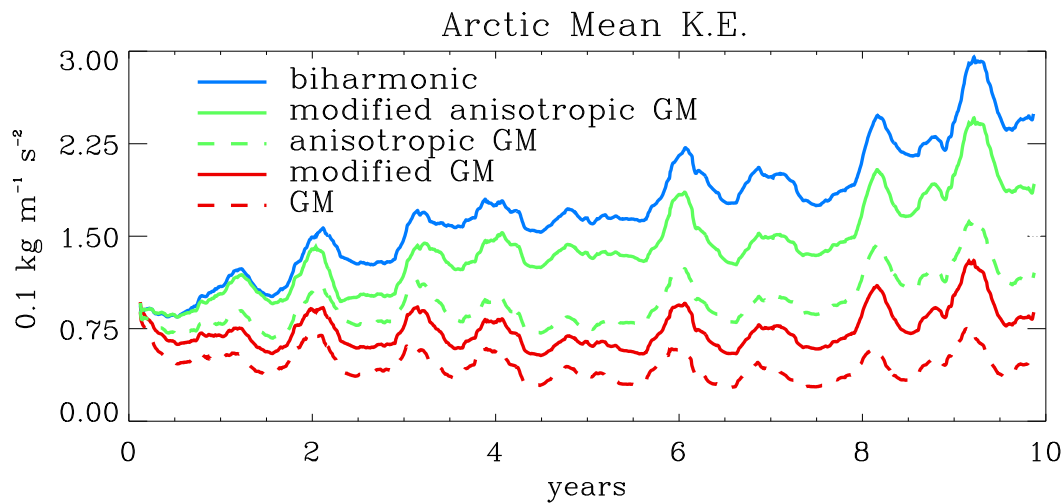
10-year “spin-up” (1981–1990)



1-year “spin-down” (1990)

initialized from biharmonic

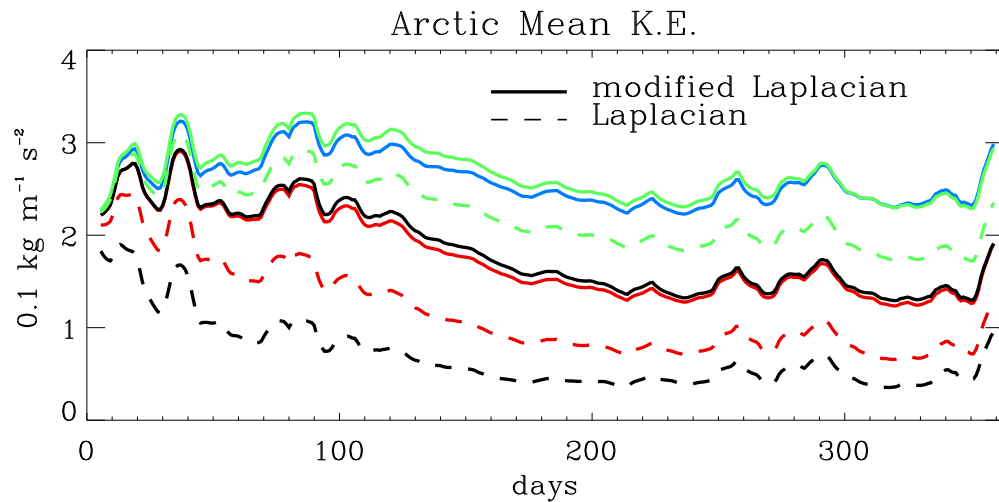




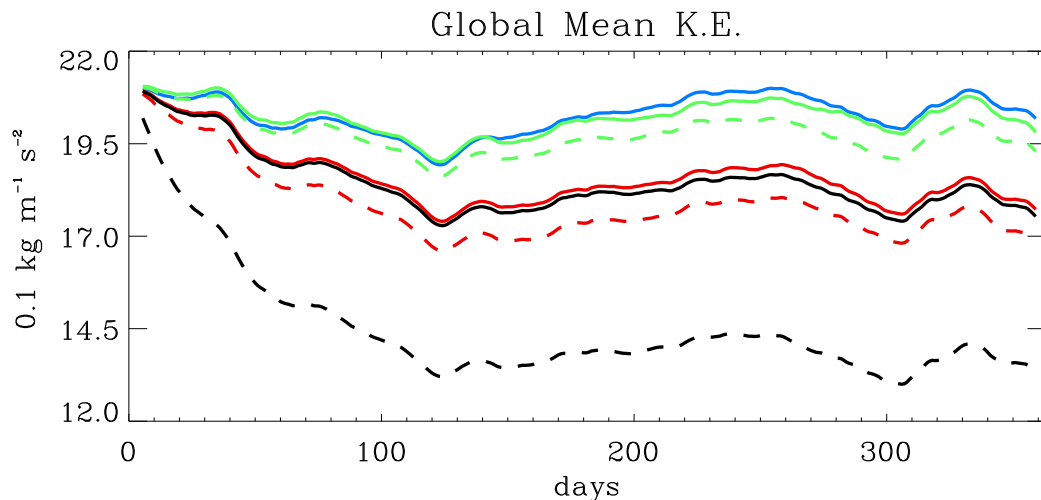
## Mean kinetic energy

averaged over the ocean volume  
north of 80°N

10-year “spin-up” (1981–1990)



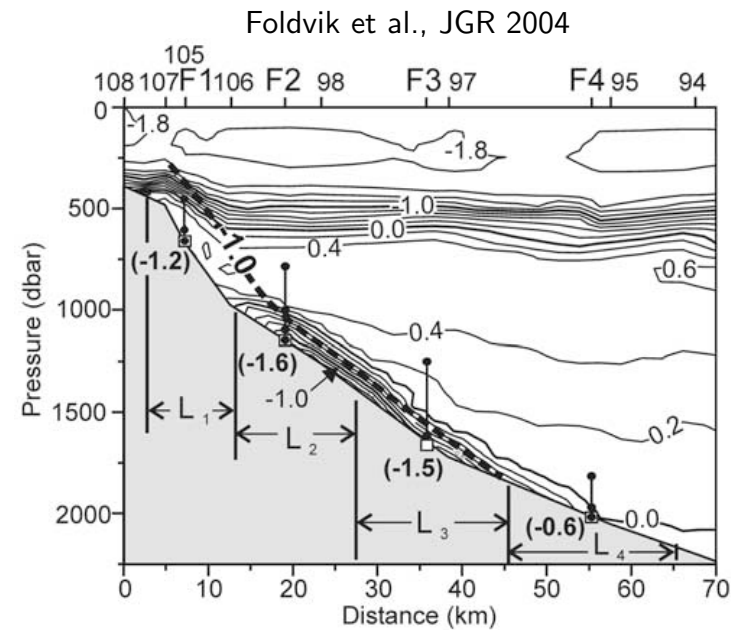
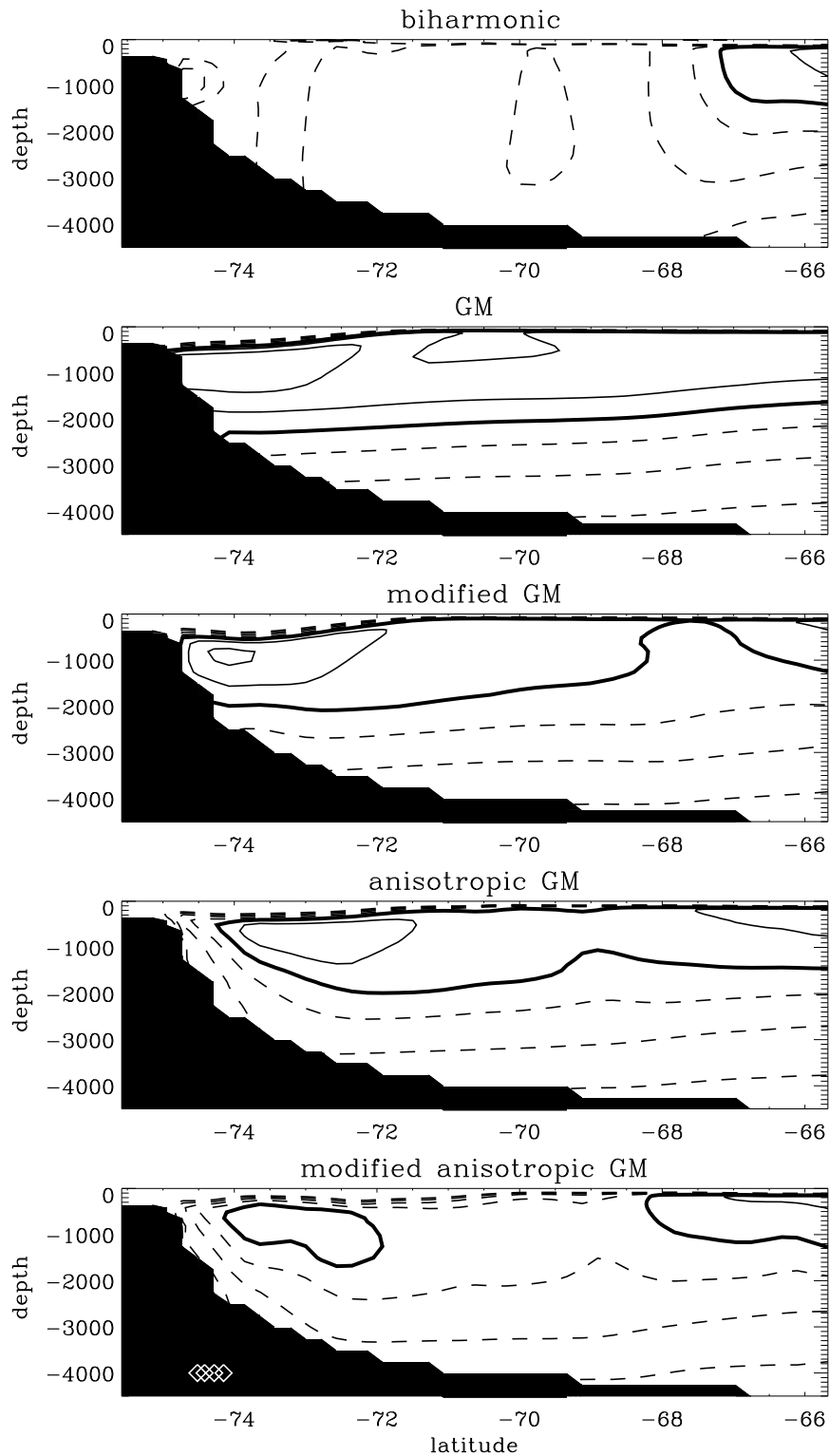
1-year “spin-down” (1990)  
initialized from biharmonic



Global  
1-year “spin-down” (1990)

# Potential Temperature

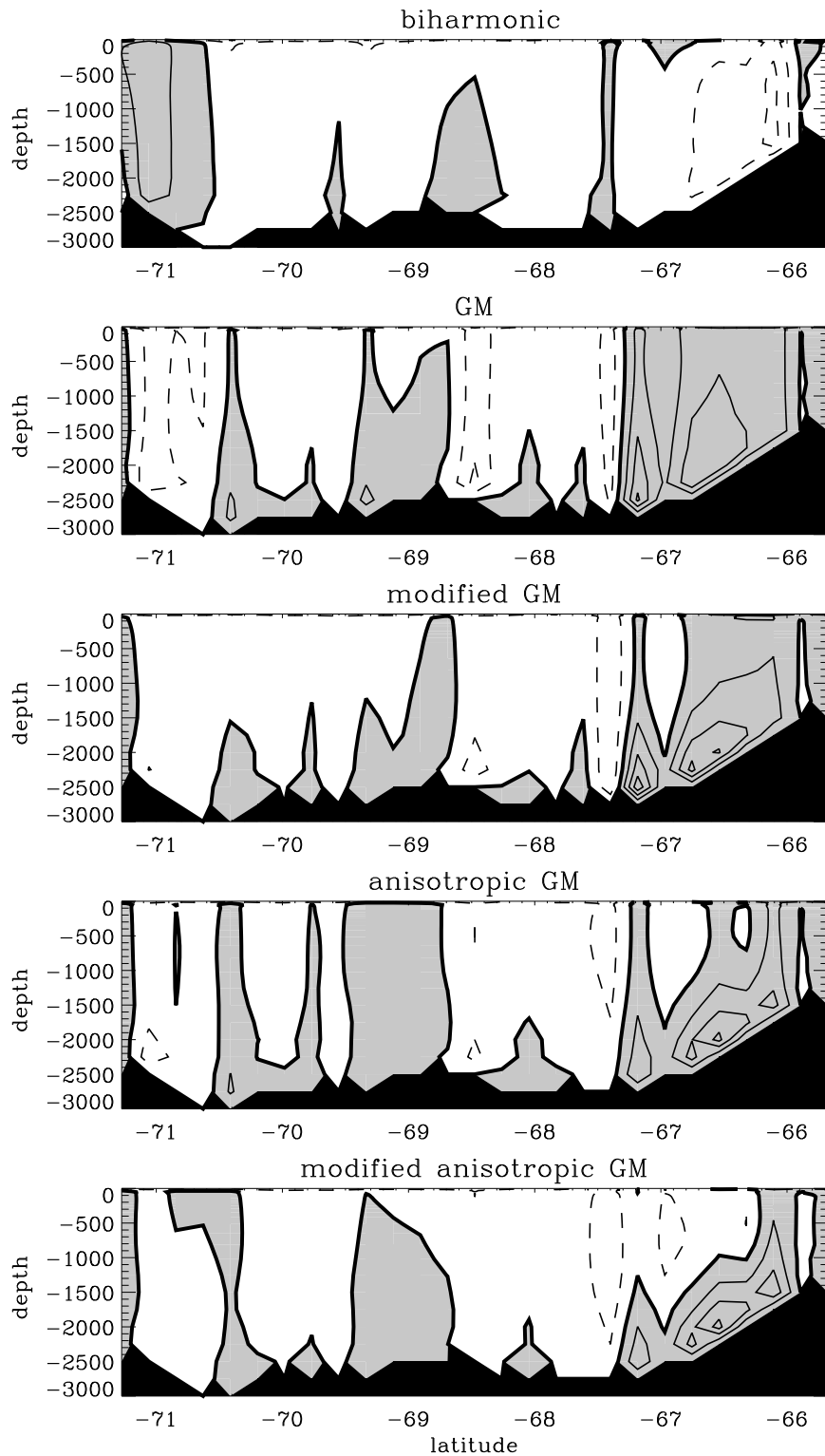
Weddell Sea, 36.4 W  
contour interval  $0.2^{\circ}$



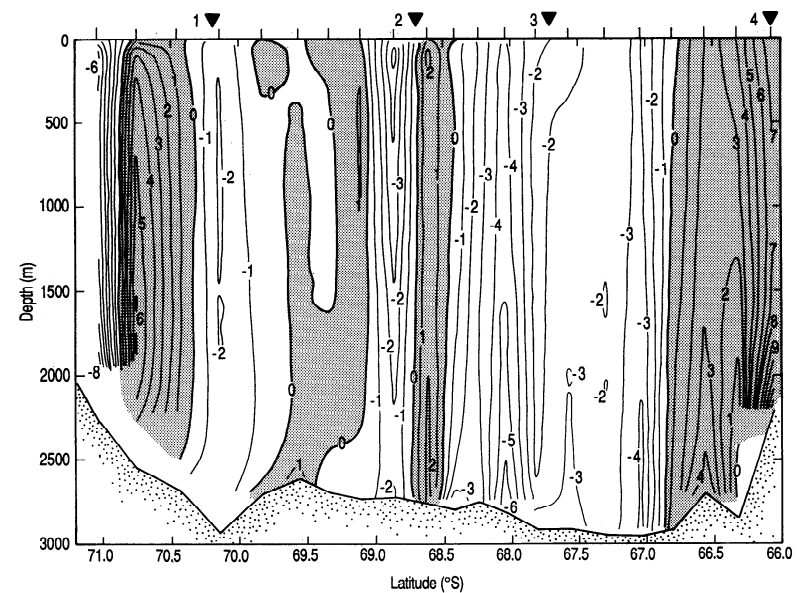
**Figure 6.** Potential temperature distribution at the F-section obtained during the recovery of the moorings (ANT XVI/2). The positions of the current meters (circles) and the MicroCATs (squares) are indicated. The one-year average of the  $-1^{\circ}\text{C}$  isotherm obtained from the current-meters is shown by the thick, dashed curve. The average potential temperatures ( $^{\circ}\text{C}$ ) from the MicroCATs are indicated in parentheses. The horizontal distances  $L_i$  used for transport calculations are indicated. Tick-marks on the top axis mark the location of the CTD casts.

# Current Speed

normal to 54 W (ISW transect)  
contour interval 1 cm/s  
shaded  $\Rightarrow$  eastward



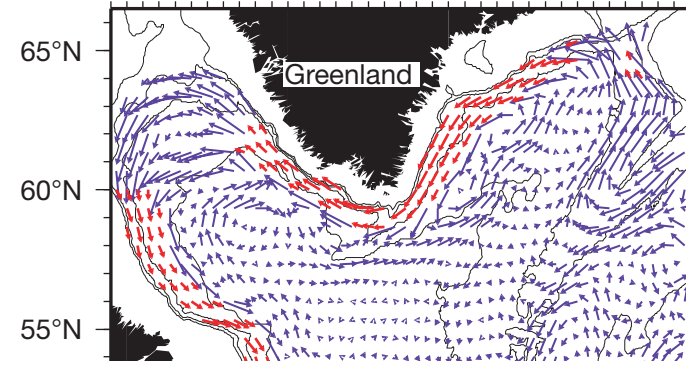
MUENCH AND GORDON: WESTERN WEDDELL CIRCULATION AND TRANSPORT



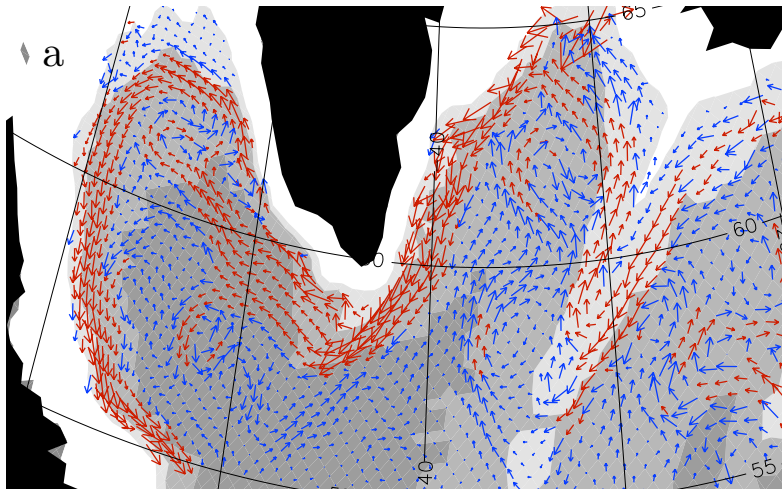
**Figure 7.** Distribution of current speed in centimeters per second normal to the meridional transect described by the northward drift of manned ice camp ISW-1. Positive values (shaded) indicate eastward flow. Numbered arrowheads along the top axis indicate locations where the transect intersected zonal CTD transects 1-4. Currents are referenced to measurements obtained from 50 and 200 m depths at the drifting current meter sites, as discussed in the text.

# Labrador and Irminger Seas

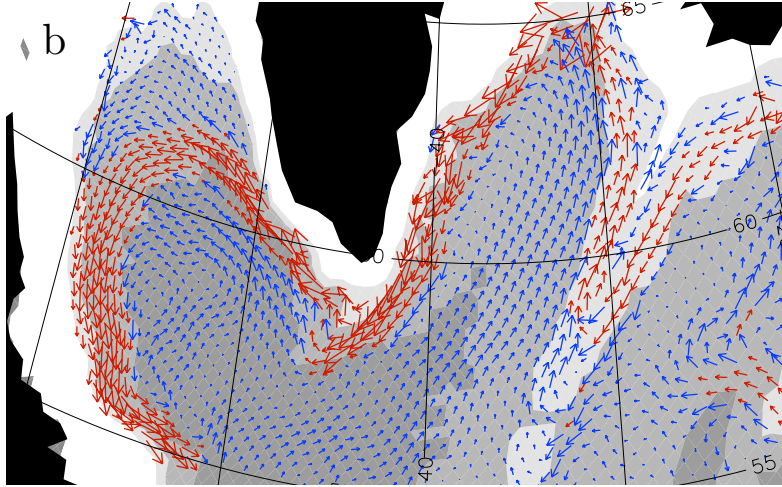
Lavender, et al., Nature, 2000



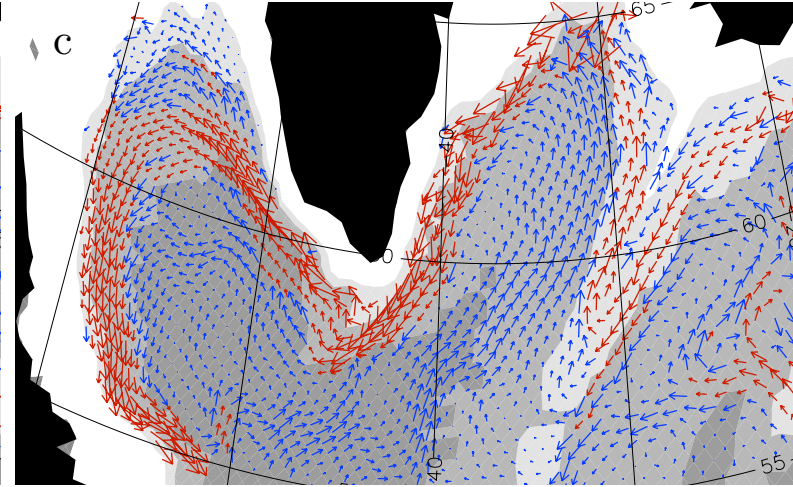
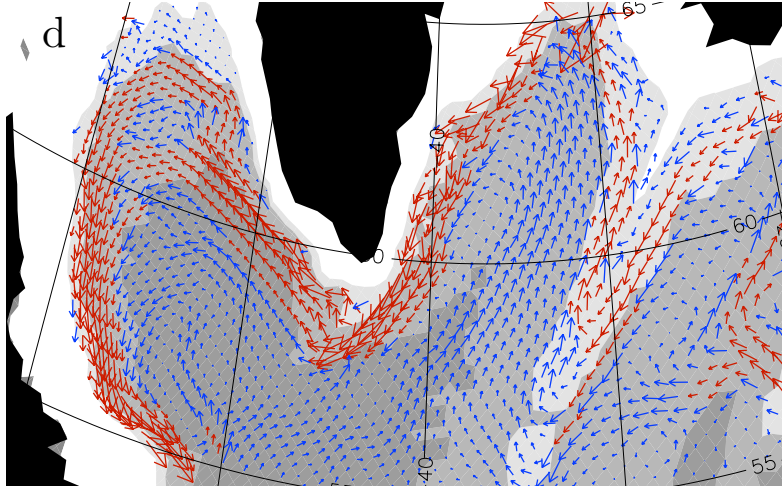
biharmonic



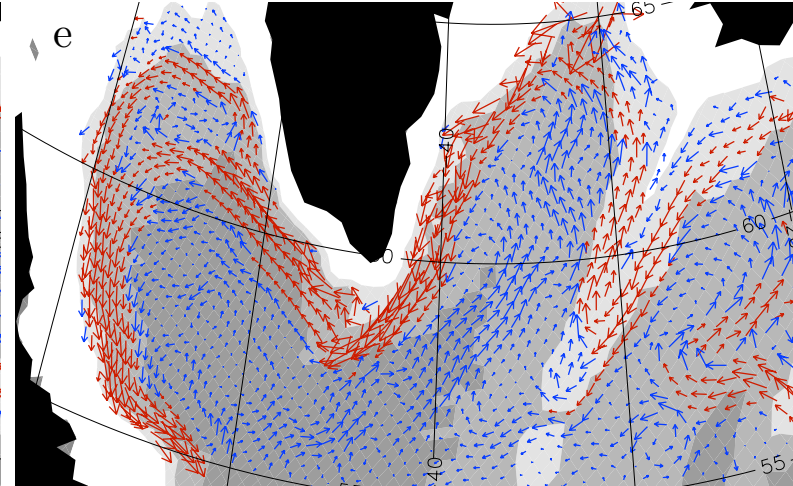
GM



anisotropic GM



modified GM

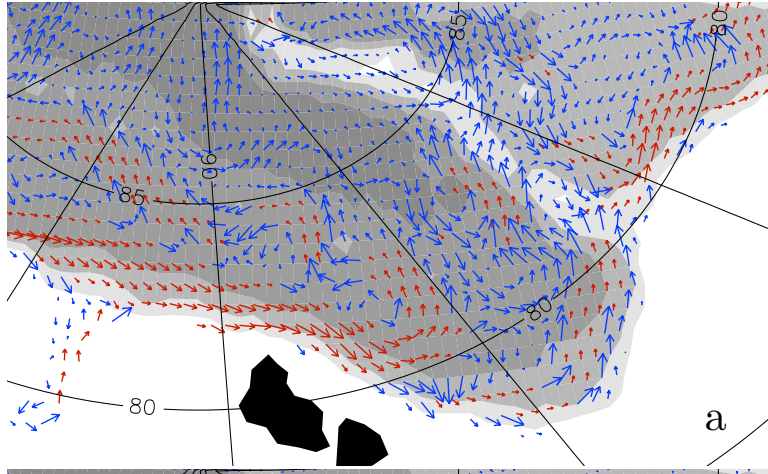


modified aniso GM



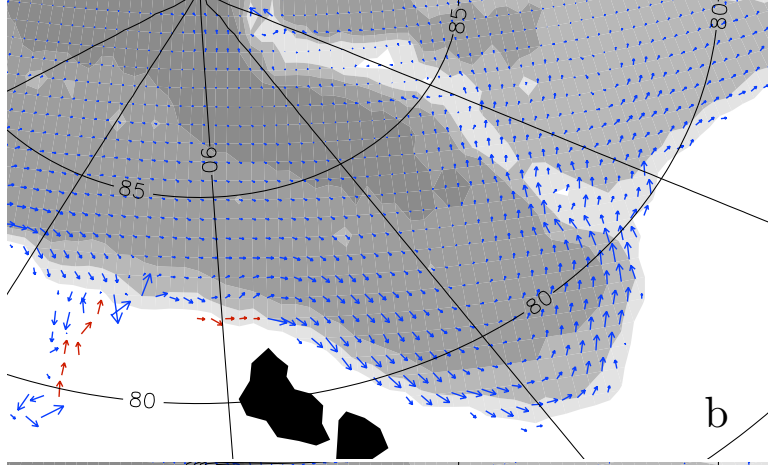
# Eurasian Basin, Arctic

biharmonic



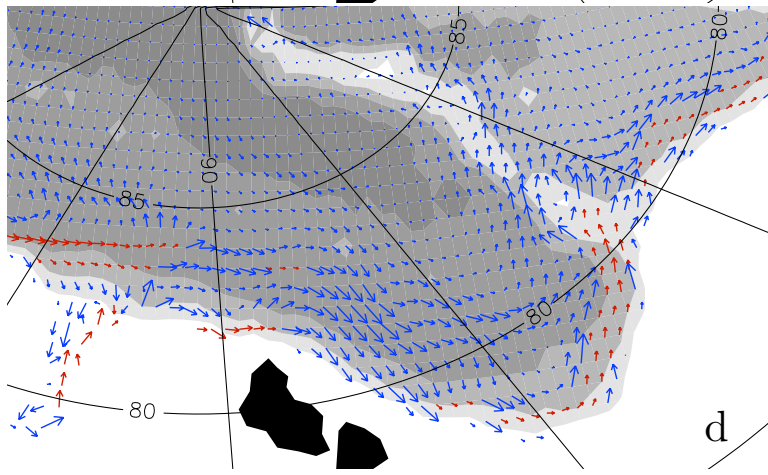
a

GM

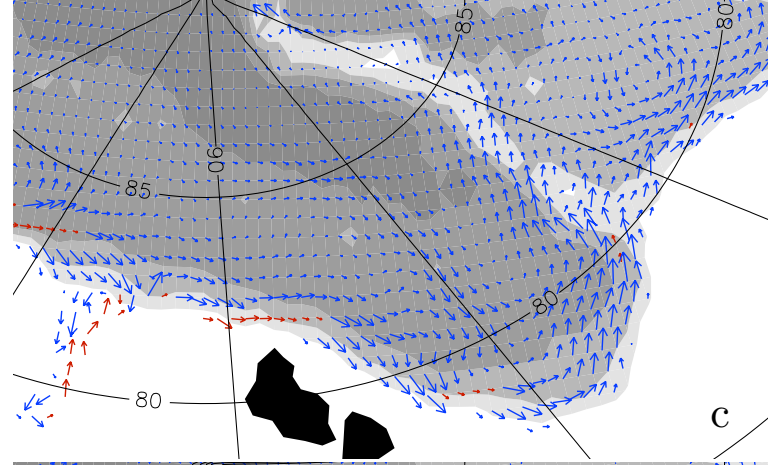


b

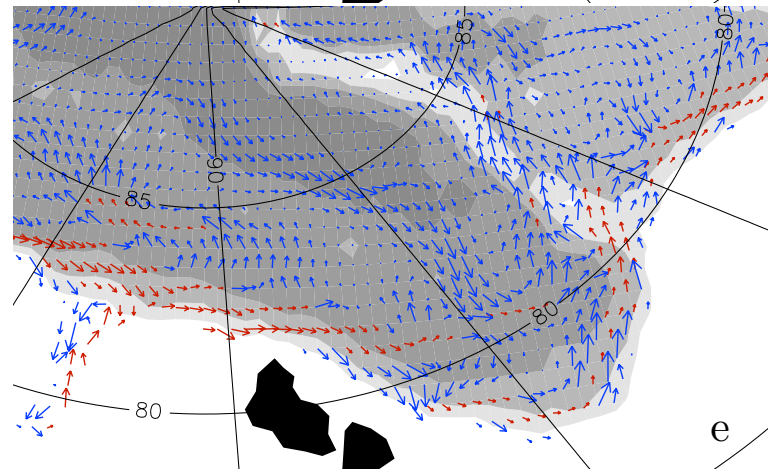
anisotropic GM



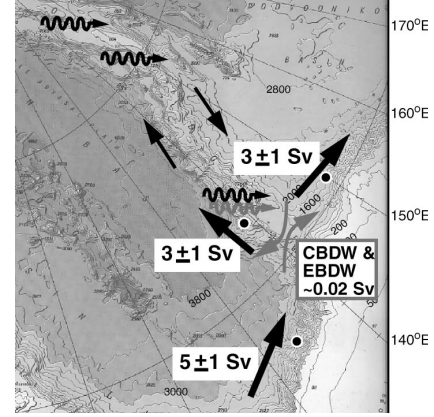
d



c



e



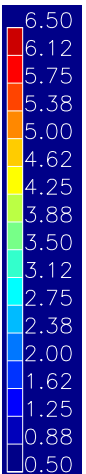
Woodgate, et al.  
DSR, 2001

modified GM

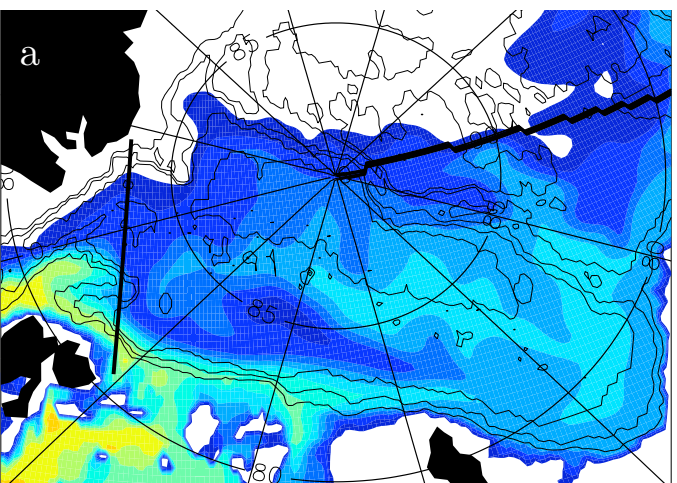
modified aniso GM

# Eurasian Basin, Arctic

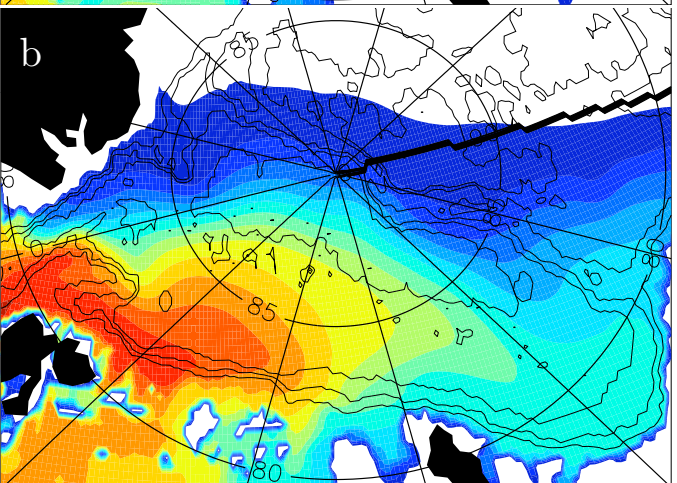
Maximum potential temperature  
in the water column  
1990



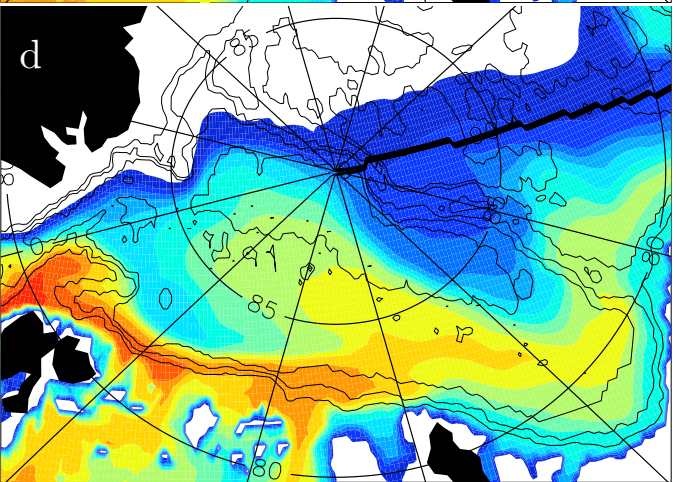
biharmonic



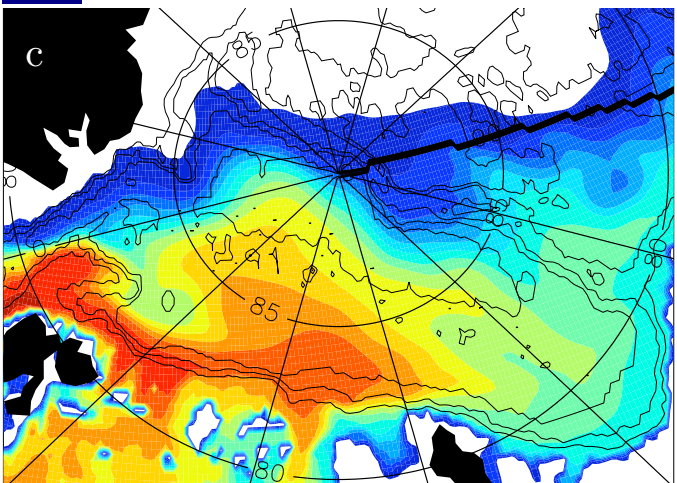
GM



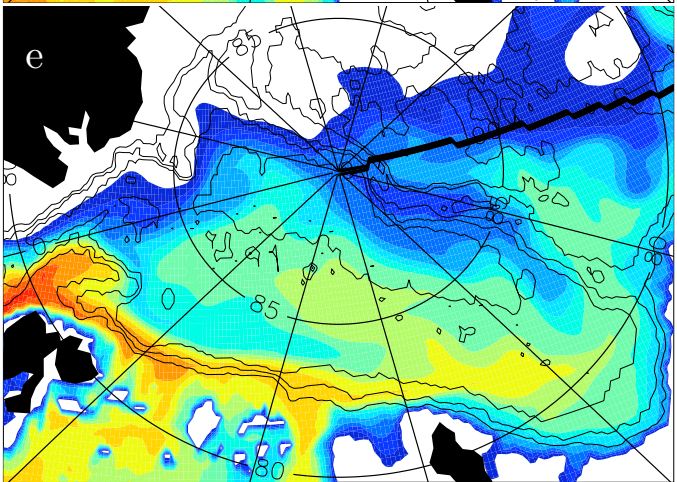
anisotropic GM



modified GM



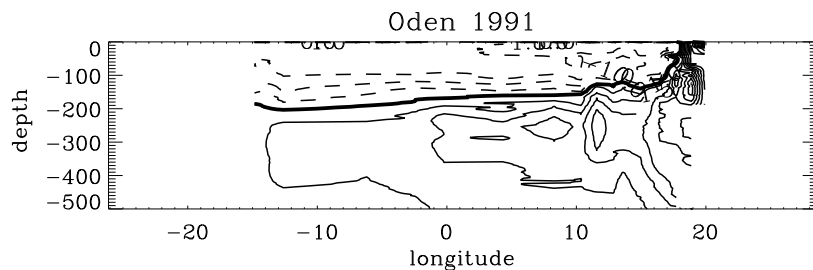
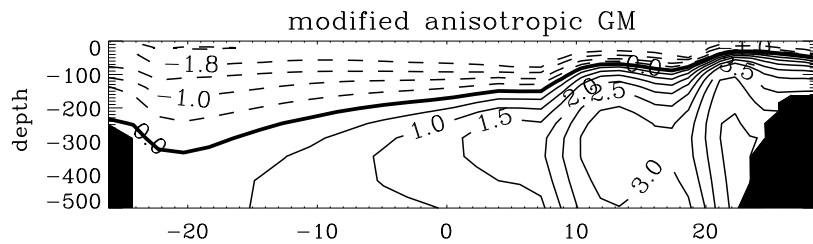
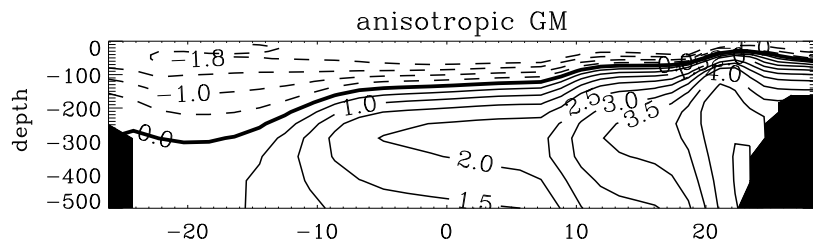
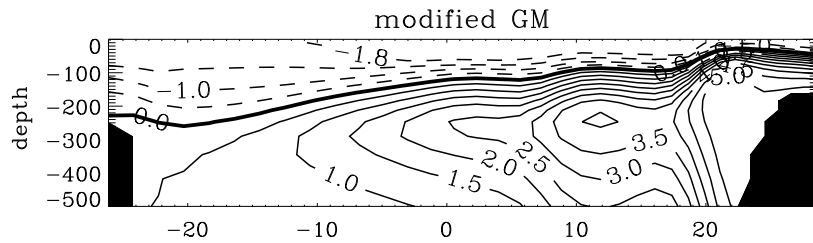
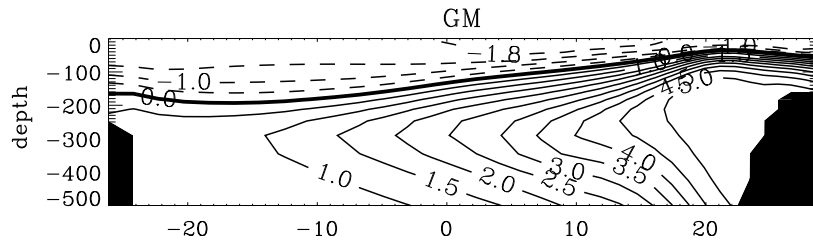
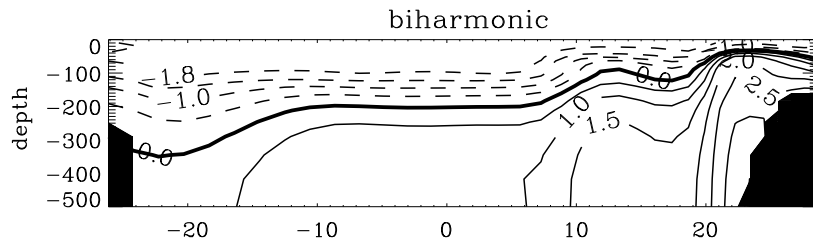
modified aniso GM





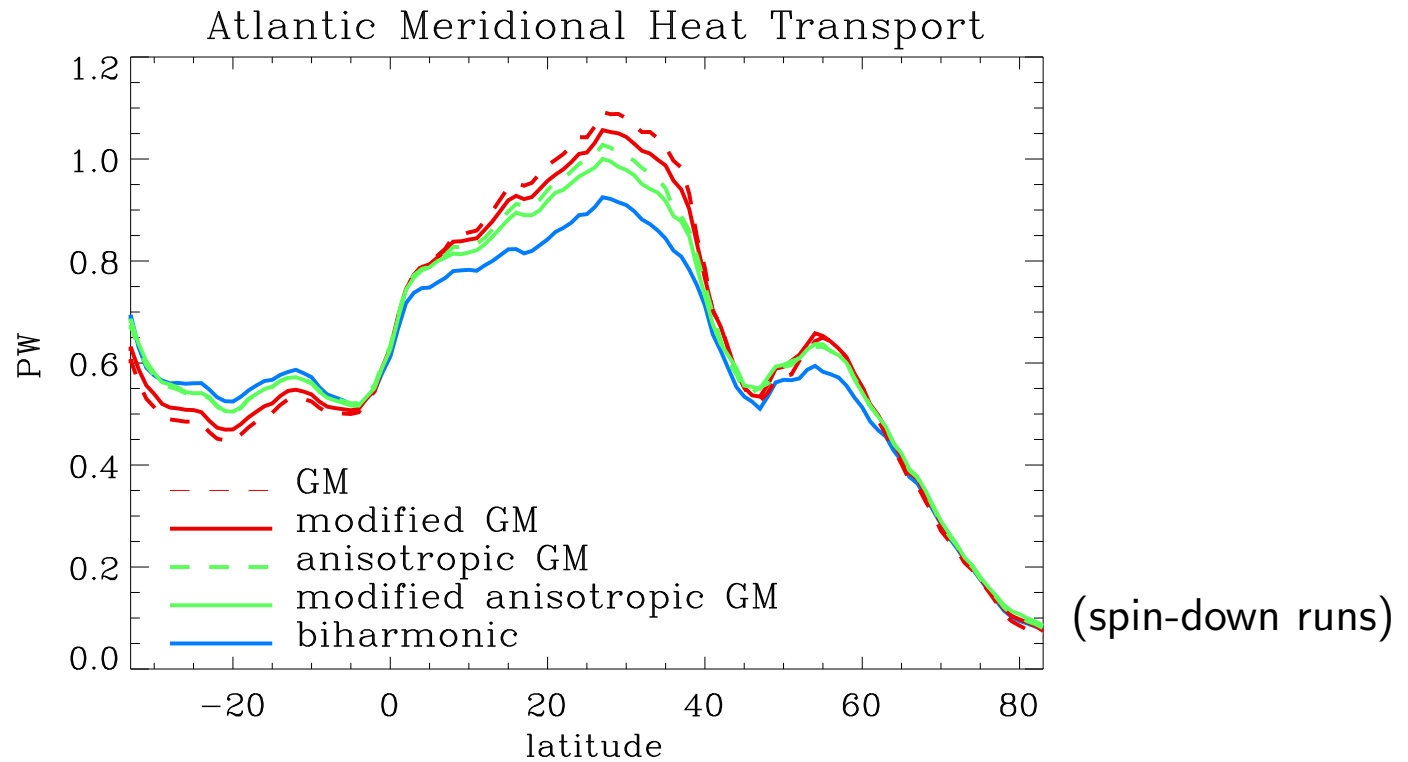
# Potential Temperature

north of Fram Strait, 1990  
(Oden 1991 section)



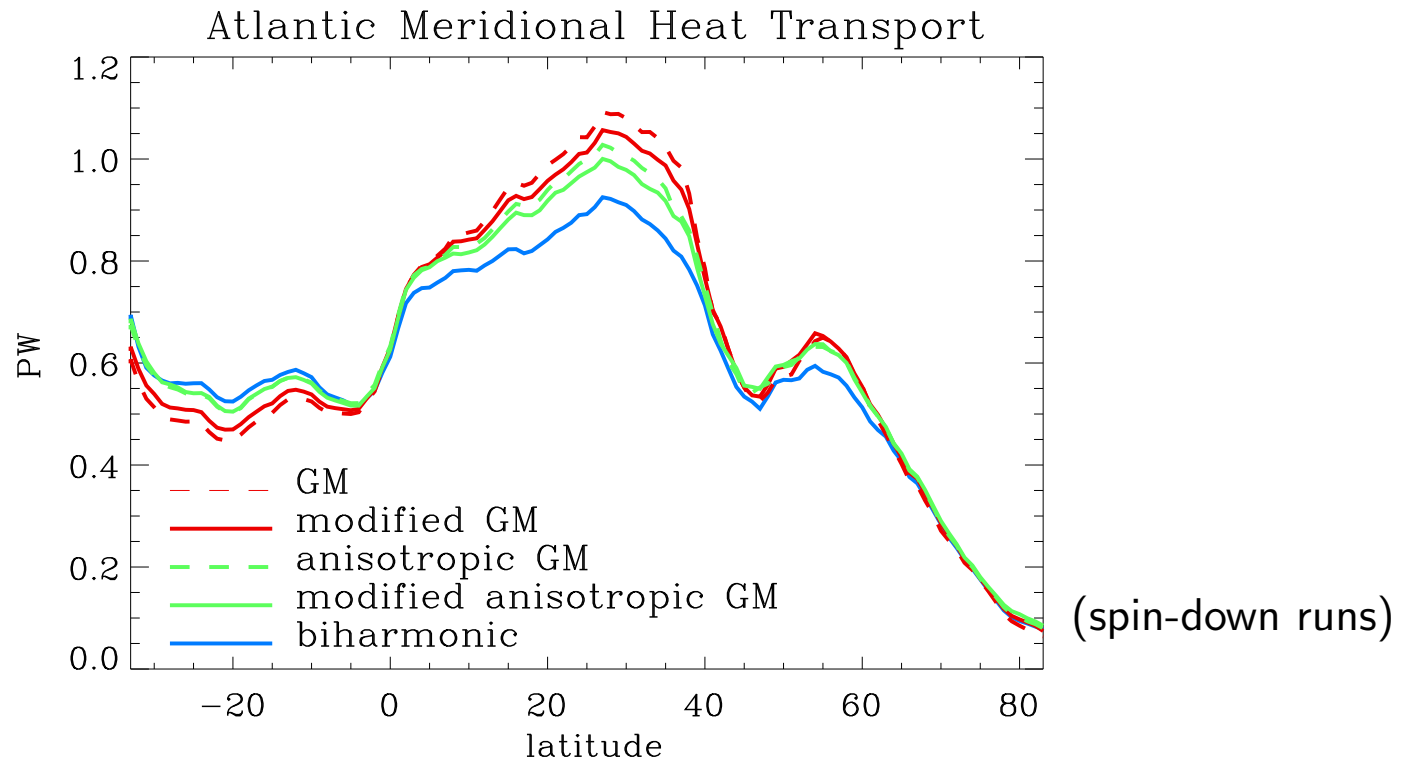


# Global Considerations



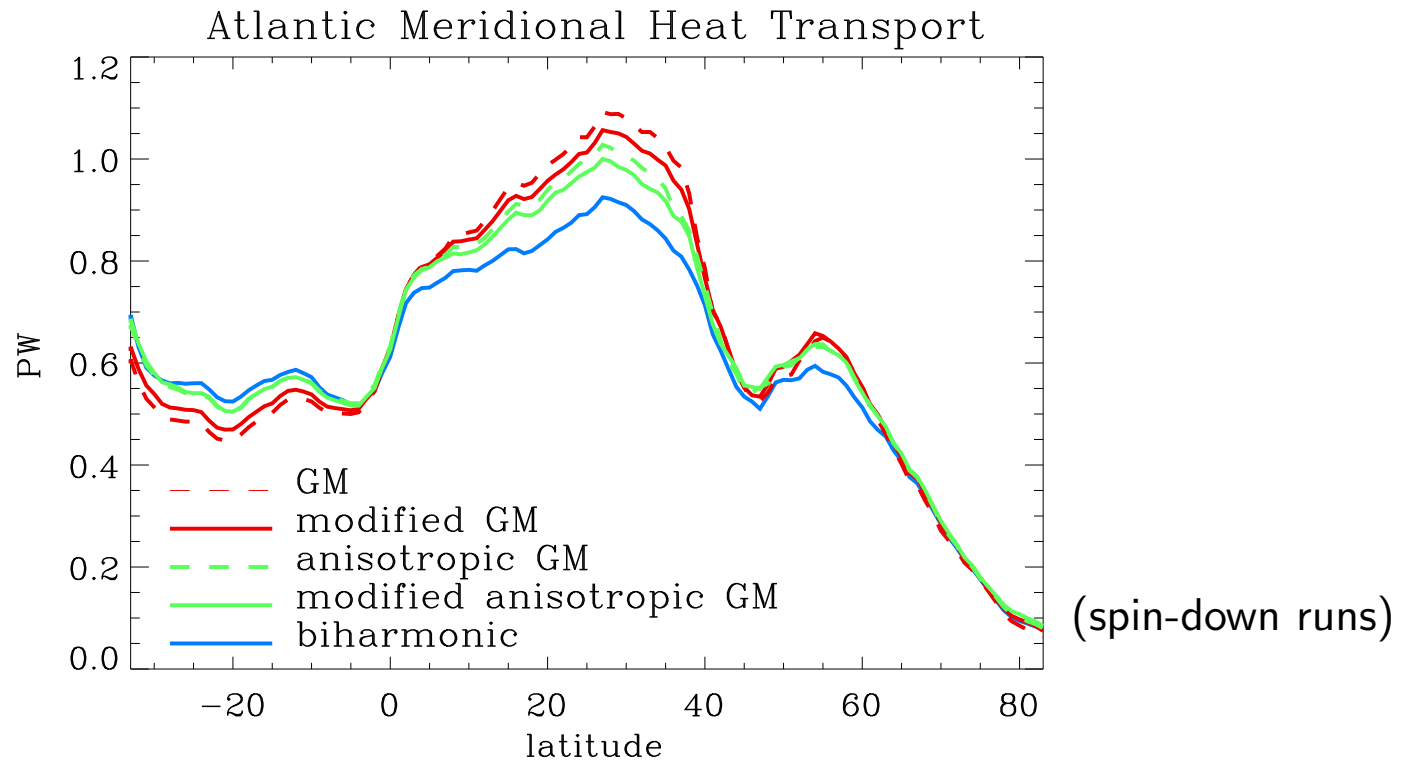
Mass Transport (Sv)	max MOC
GM	16.36
modified GM	15.53
anisotropic GM	14.94
modified anisotropic GM	14.72
biharmonic	14.39

# Global Considerations



Mass Transport (Sv)	max MOC	Drake
GM	16.36	162.0 ± 8.0
modified GM	15.53	160.7 ± 7.5
anisotropic GM	14.94	160.6 ± 7.7
modified anisotropic GM	14.72	159.6 ± 7.2
biharmonic	14.39	158.0 ± 6.8
		$\Delta \sim 2.5\%$

# Global Considerations



Mass Transport (Sv)	max MOC	Drake	Fram
GM	16.36	162.0 ± 8.0	1.04 ± 2.31
modified GM	15.53	160.7 ± 7.5	0.91 ± 2.34
anisotropic GM	14.94	160.6 ± 7.7	0.70 ± 2.37
modified anisotropic GM	14.72	159.6 ± 7.2	0.73 ± 2.36
biharmonic	14.39	158.0 ± 6.8	0.71 ± 2.36
		$\Delta \sim 2.5\%$	$\Delta \sim 40\%$

## Summary

1. Small eddies are not resolved on this  $0.4^\circ$  grid—we need to parameterize their effects yet still admit those that can be resolved. **Scaling diffusivity by grid cell area** is beneficial for relieving excessive high-latitude damping by constant-diffusivity GM.
2. **Tuning GM parameters** via Drake Passage transport (55–60 S) may miss important higher-latitude effects.



3. **Anisotropic GM** forms look quite promising, allowing eddy variability similar to biharmonic diffusion without excessive diapycnal mixing.

## Thanks

Mat Maltrud  
Matthew Hecht  
JoAnn Lysne