

# Are sea ice models iterated to convergence?

Jean-Francois Lemieux

Bruno Tremblay

McGill University

# Main Messages

- We do not iterate sea ice models to convergence
- The residual errors are of the same order of magnitude of the mean sea ice drift.
- The mean residual errors are not random and are also of the same order of magnitude as the mean drift.
- Do we have the same problem in the EVP formulation?

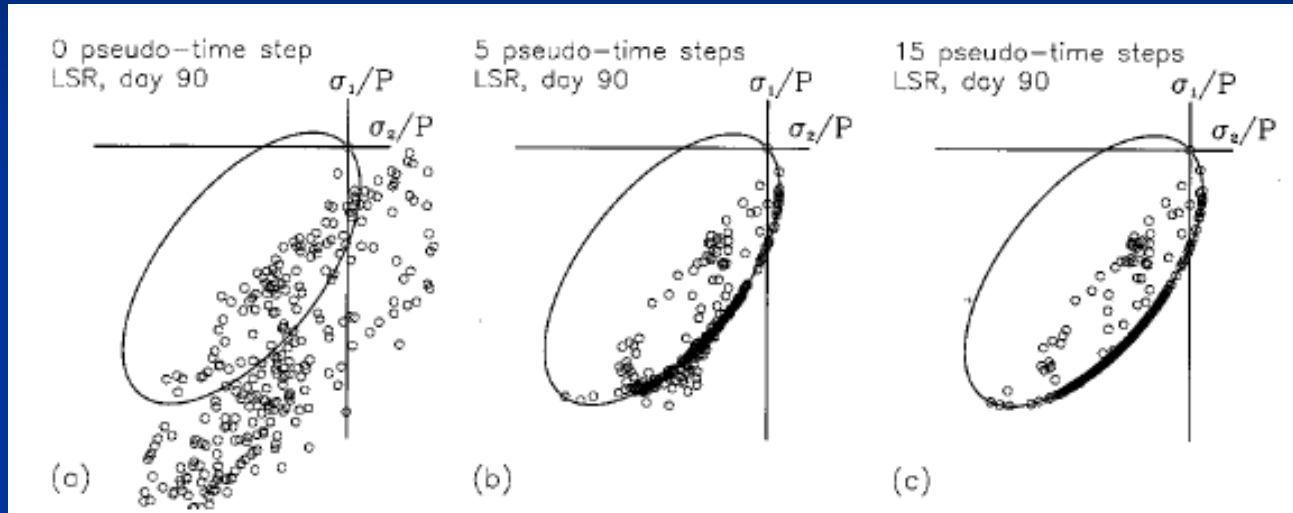
# Stress State vs Pseudo Time Step

0

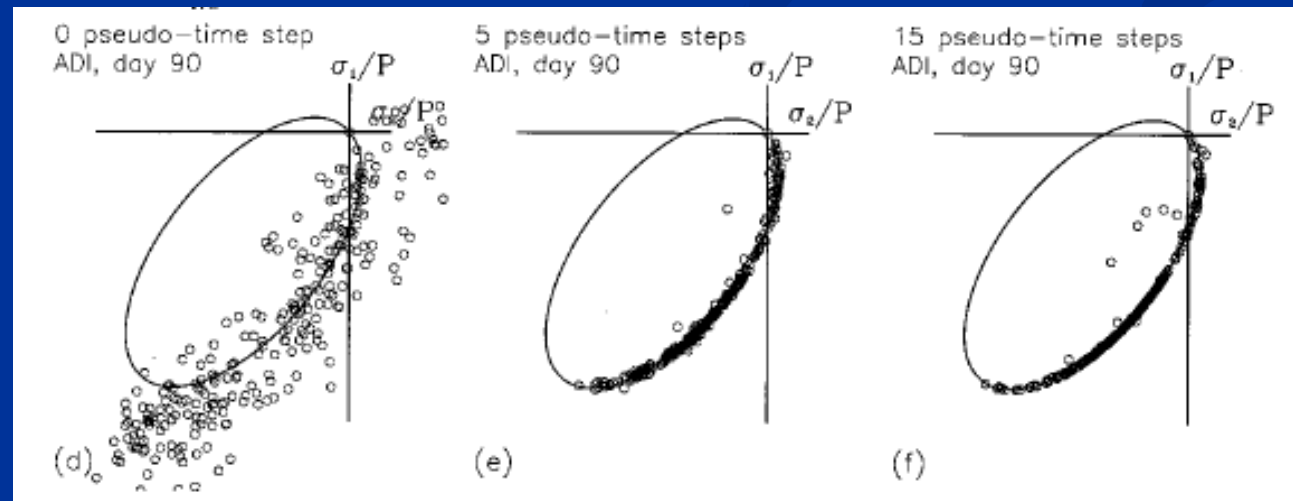
5

15

LSR



ADI



# Calibration of ice model against buoy data

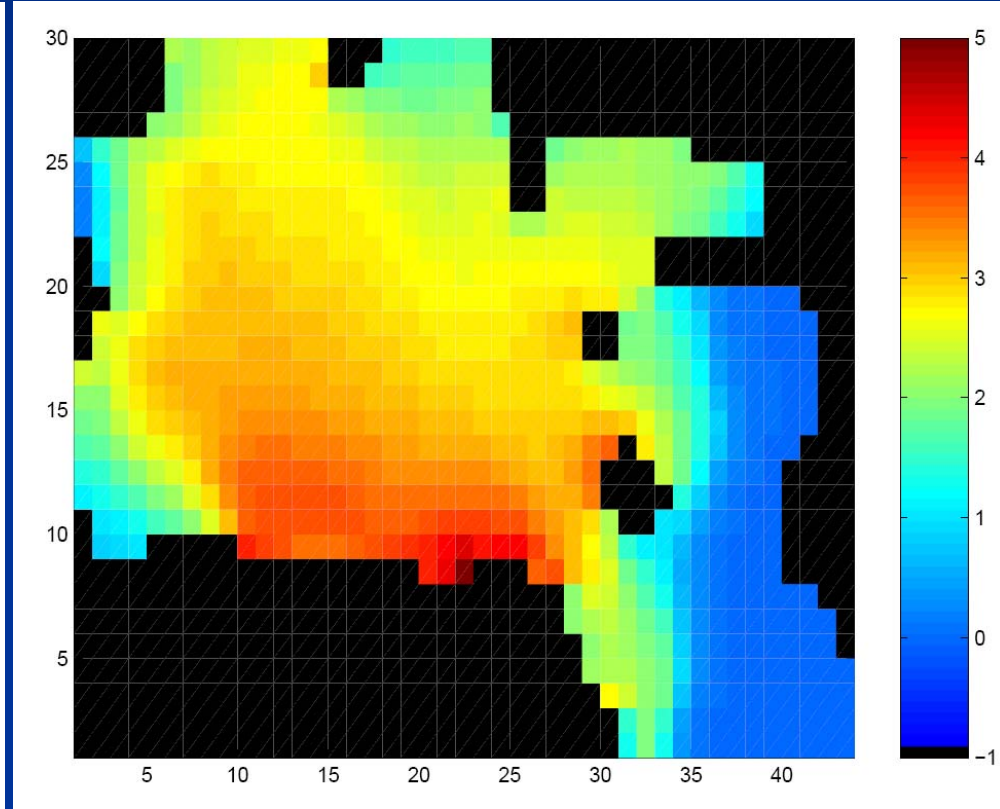
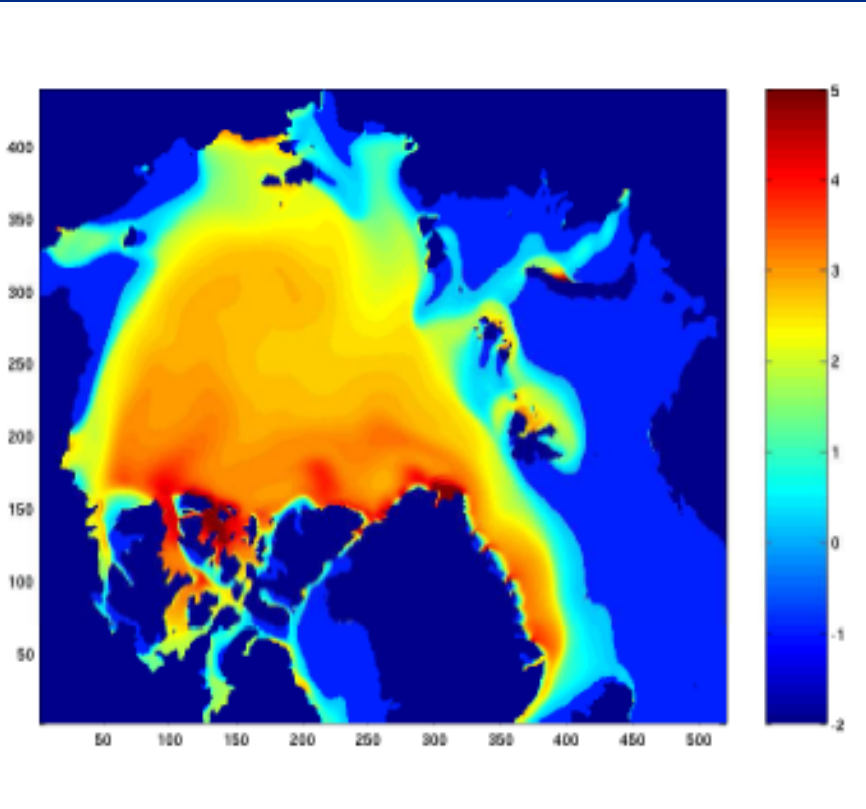
**$P^* = 27.5 \text{ KN/m}^2$**

- Hibler Walsh, 1982
  - $C_{da} = 1.2 \times 10^{-3}$
  - Geostrophic winds

**$P^* = 15 \text{ KN/m}^2$**

- Kreyscher et al, (2000)
  - $C_{da} = 2.75 \times 10^{-3}$
  - Surface winds

# 10 km – 110 km models



**Elliptical yield curve and normal flow rule**

# Numerical Scheme

$$\begin{aligned} \frac{\partial}{\partial x} \left[ (\eta(\mathbf{u}_i^k) + \zeta(\mathbf{u}_i^k)) \frac{\partial u^k}{\partial x} \right] + \frac{\partial}{\partial y} \left[ \eta(\mathbf{u}_i^k) \frac{\partial u^k}{\partial y} \right] + \frac{\partial}{\partial x} \left[ (\zeta(\mathbf{u}_i^k) - \eta(\mathbf{u}_i^k)) \frac{\partial v^k}{\partial y} \right] + \\ \frac{\partial}{\partial y} \left[ \eta(\mathbf{u}_i^k) \frac{\partial v^k}{\partial x} \right] - \frac{1}{2} \frac{\partial P}{\partial x} + \rho_i h f v^k - C_w(\mathbf{u}_i^k) (\cos \theta_w u^k - \sin \theta_w v^k) = b_u \end{aligned}$$

$$\mathbf{u}_i^k = \frac{(\mathbf{u}^{k-1} + \mathbf{u}_i^{k-1})}{2}$$

# Numerical Scheme

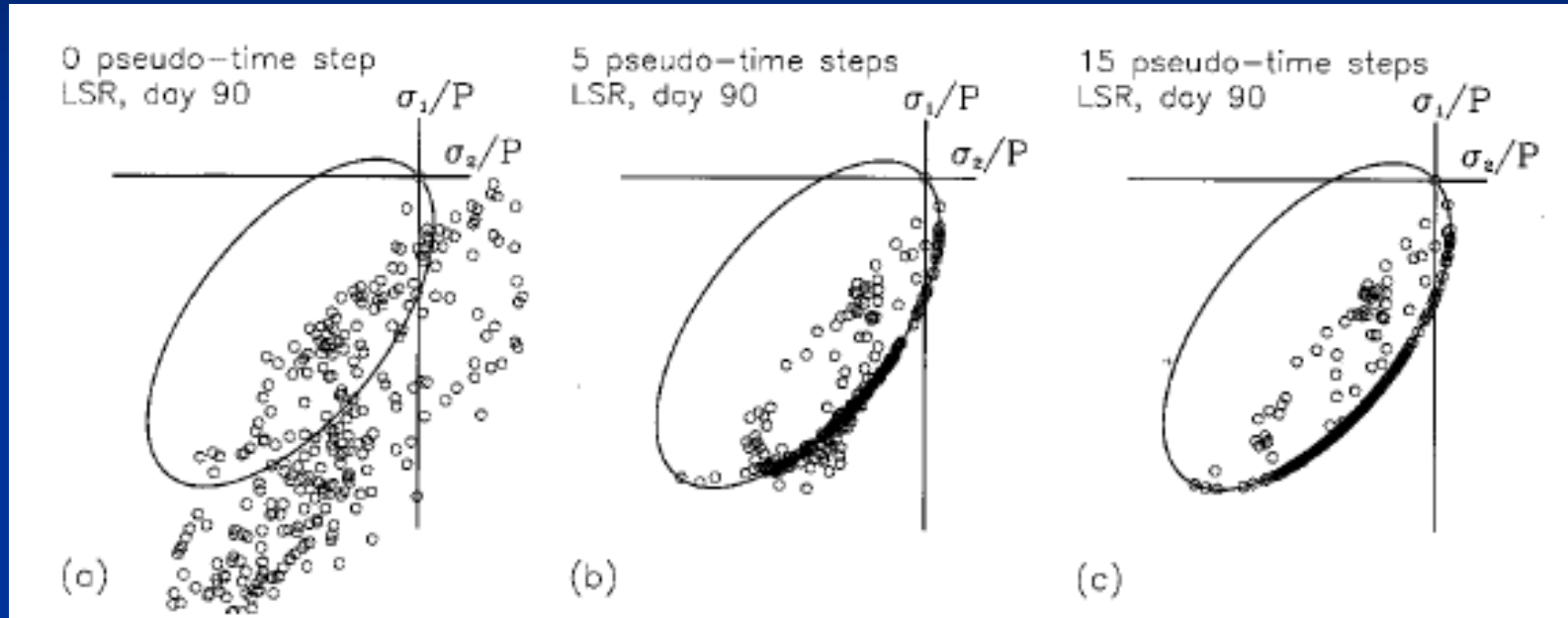
```
1. Start with an initial guess  $\mathbf{u}^0$ 
do k=1,  $k_{max}$ 
  2. Linearize the momentum equation
  3. Solve  $\mathbf{A}\mathbf{u}^k = \mathbf{b}$  with the preconditioned GMRES method
enddo
```

GMRES as other Krylov methods:

- Low storage requirements
- Good convergence properties when preconditioned
- Allows parallelization

Symmetry is not a prerequisite for GMRES.

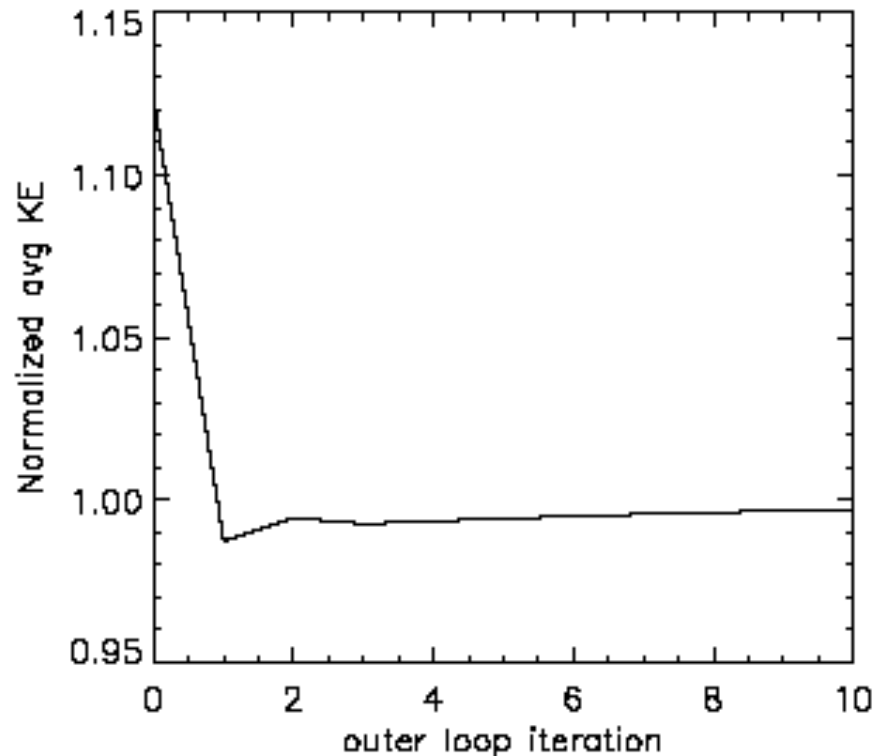
# Convergence criteria





# Convergence Criteria

Criterion: avg KE of the pack is within 1% of the fully converged value



**Figure:** Normalized average KE of the pack on January 1<sup>st</sup> 1997 at 00Z as a function of the outer loop iteration

# Residual error after 8 outerloops

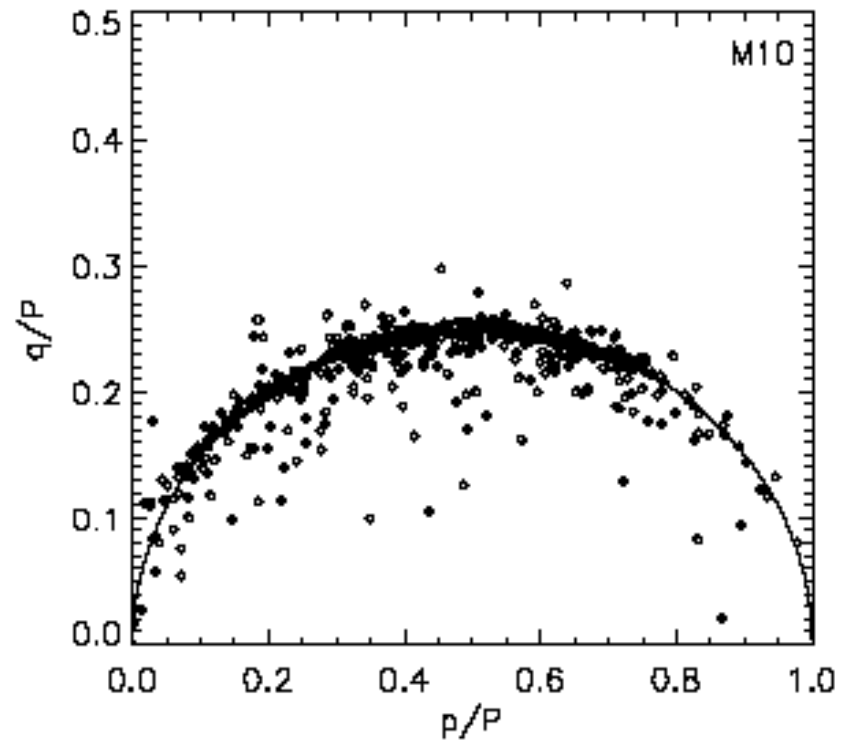
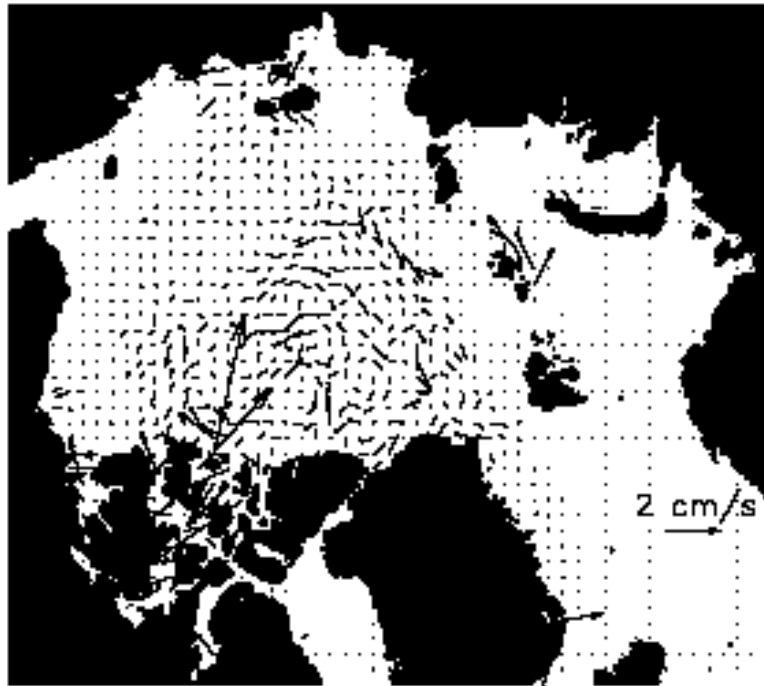


Figure: Left: velocity field difference (8 loops - 250 loops), Right: stress states after 8 outer loop iterations

# After 40 outerloops

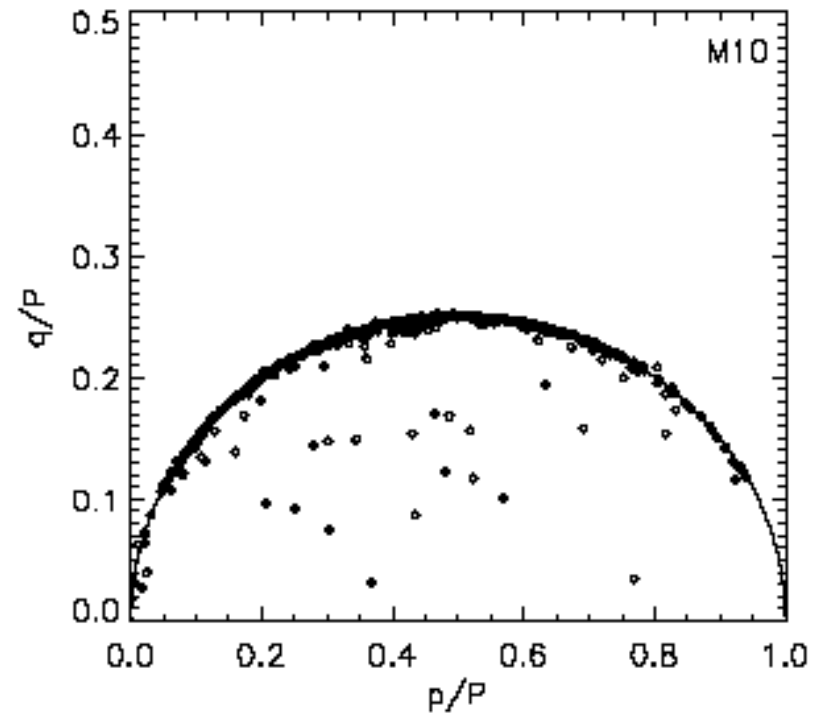
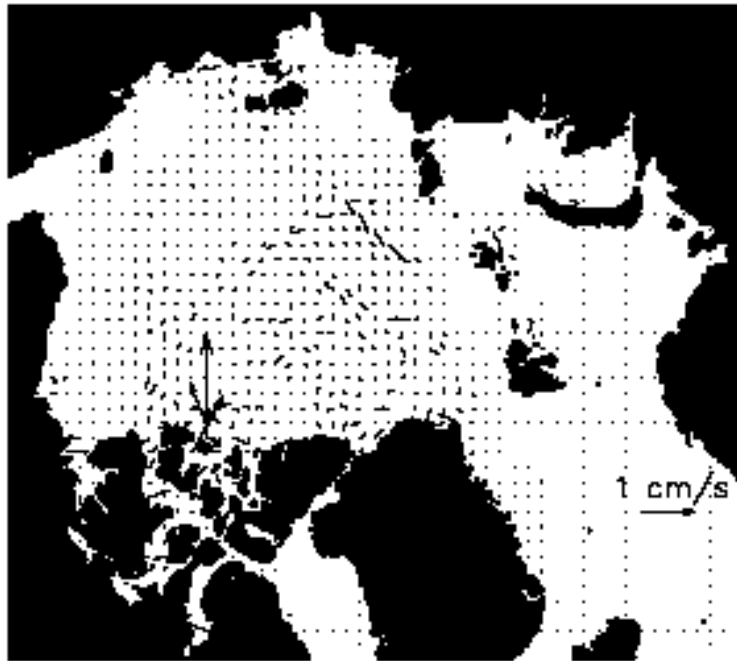


Figure: Left: velocity field difference (40 loops - 250 loops), Right: stress states after 40 outer loop iterations

# Message

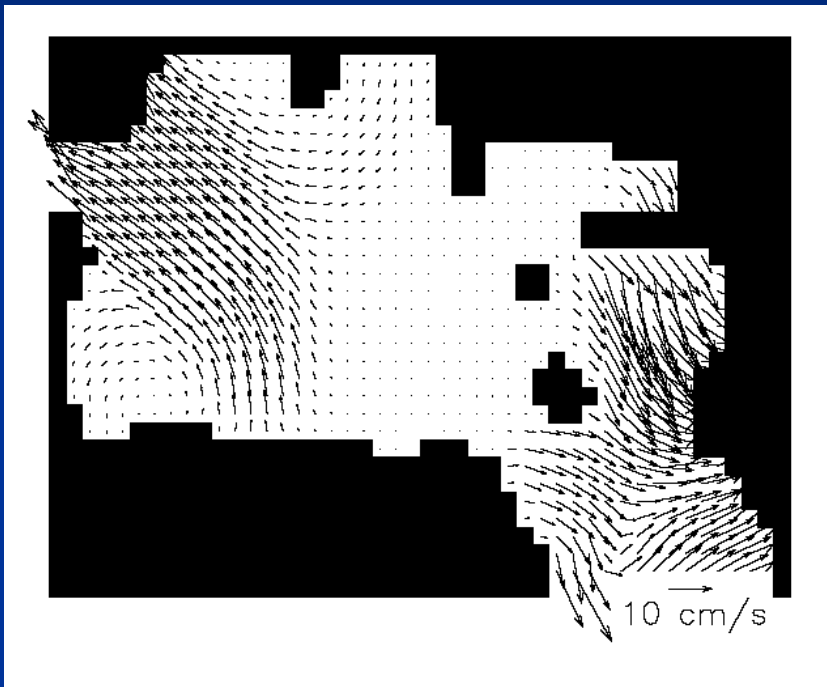
- Stress state can be on the yield curve, there can still be large errors
- Mean kinetic energy of the pack can be within 1% of the converged value, there can still be large errors

# Instantaneous Fields

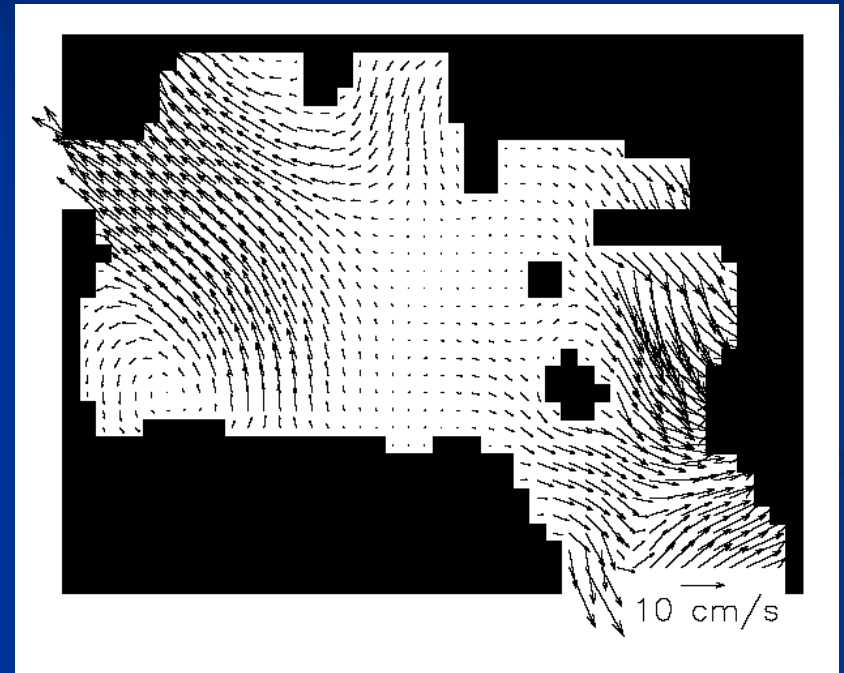


# Instantaneous Fields

Jan 1 - 1997



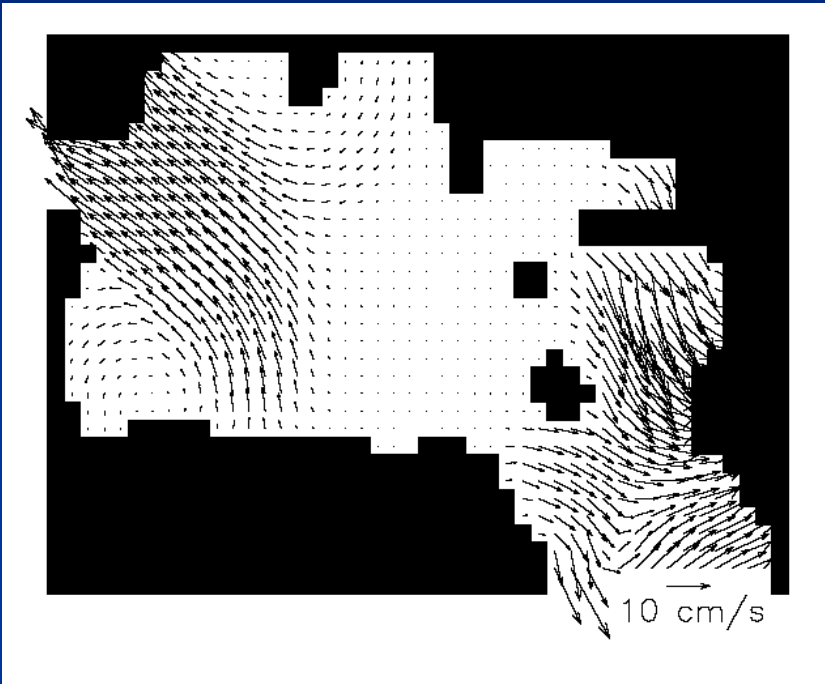
**1000 outerloops**



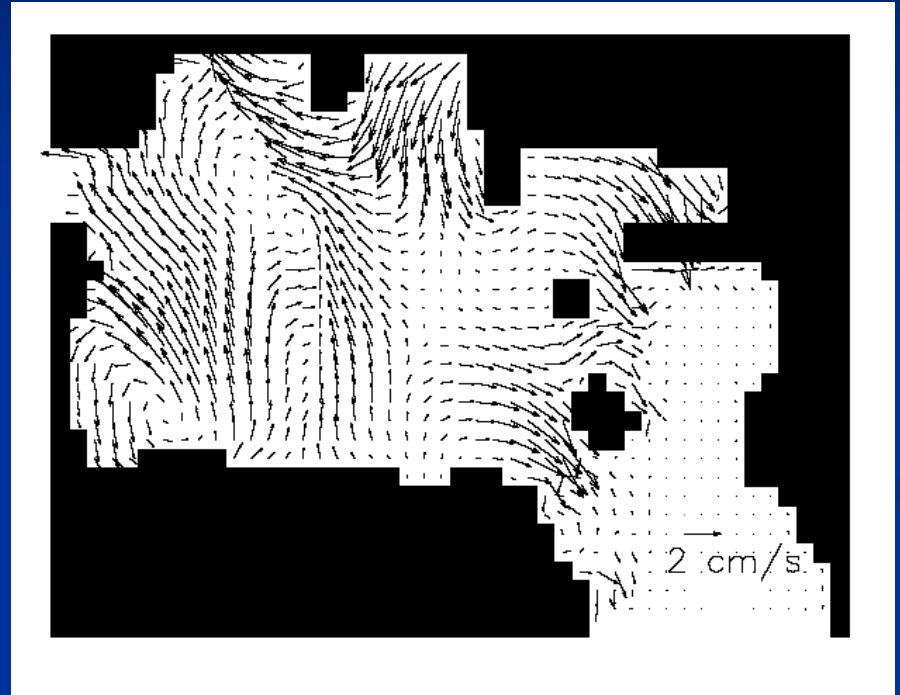
**2 outerloops**

# 2 Superloops

Jan 1, 1997



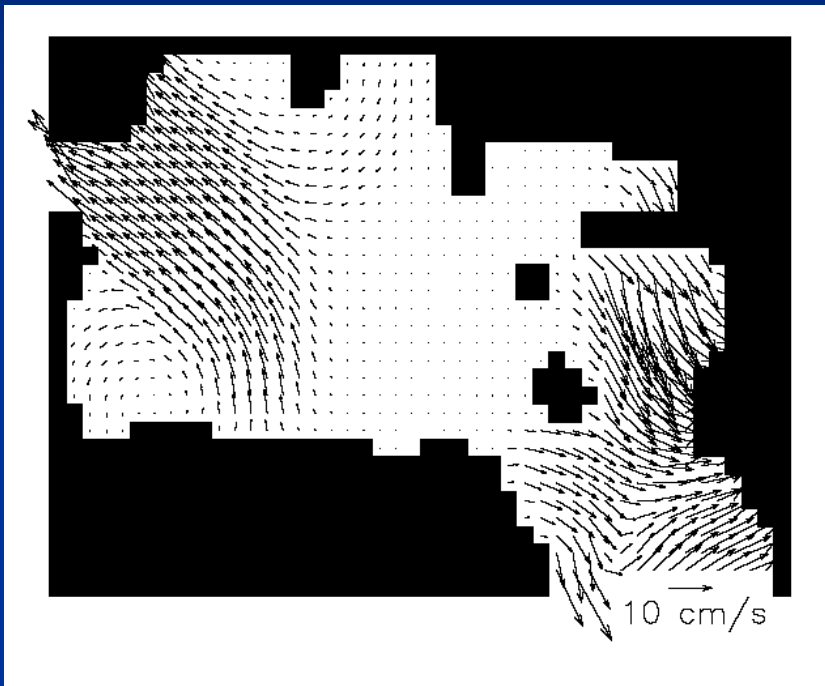
**1000 superloops**



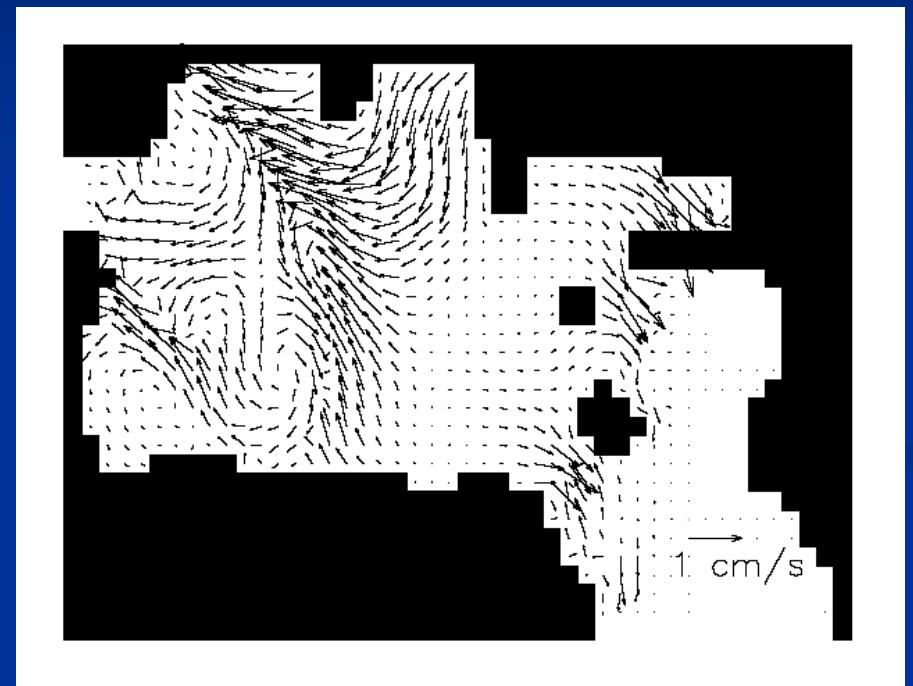
**2 superloops - 1000**

# 10 Outerloops

Jan 1, 1997



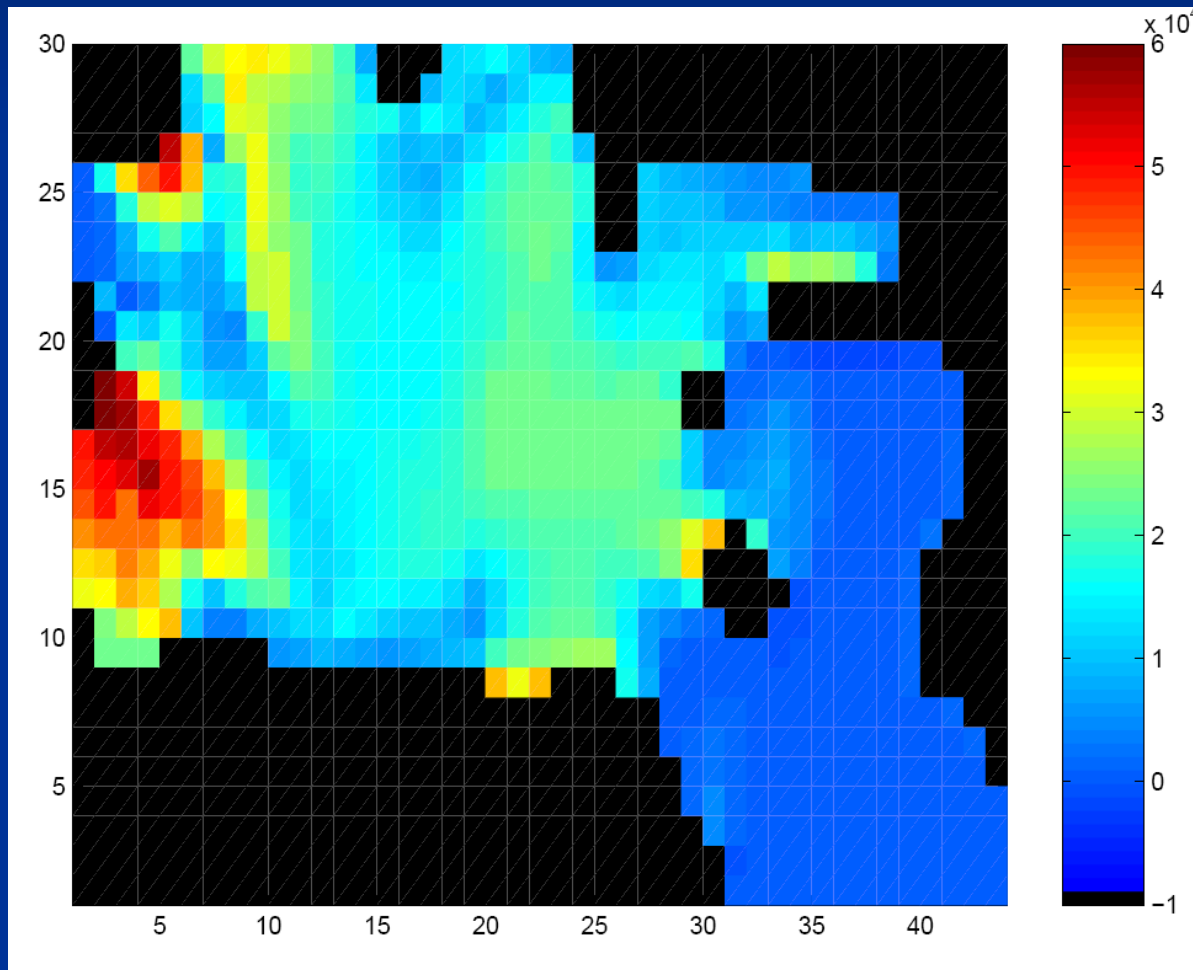
**1000 superloops**



**10 superloops - 1000**



# Sea Ice Pressure jan 1 1997

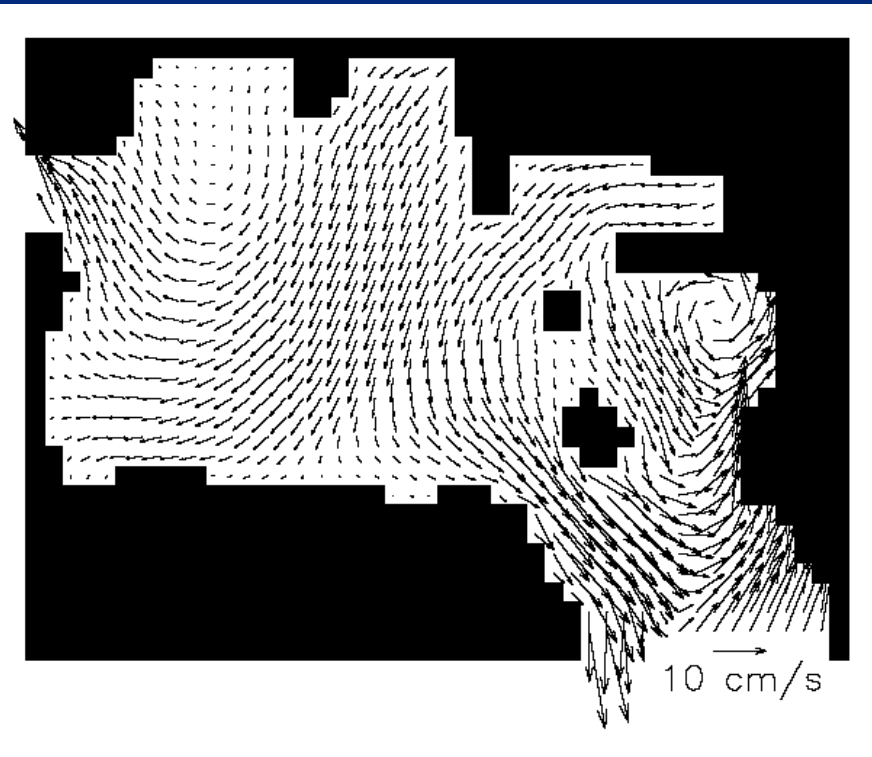


# Mean Quantities

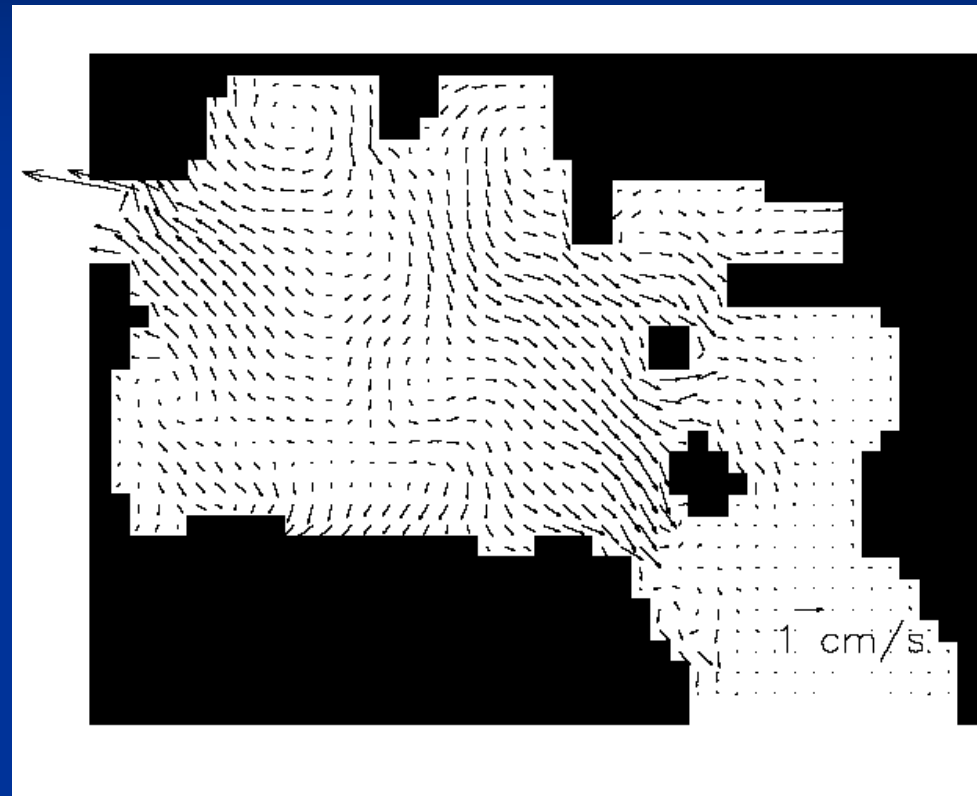
- Are the error random and do they average out with time?

# Mean January 1997

2 outerloops



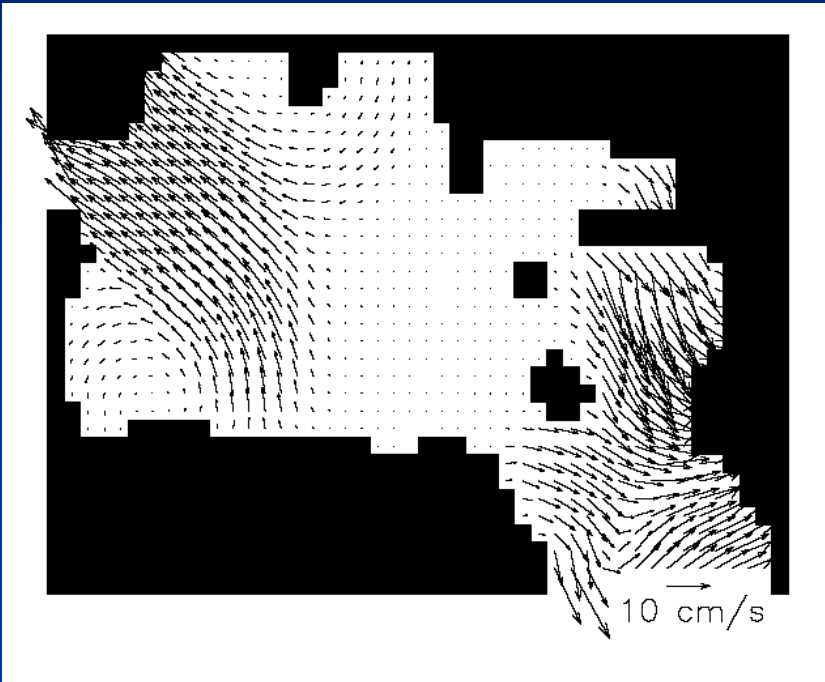
Mean velocity



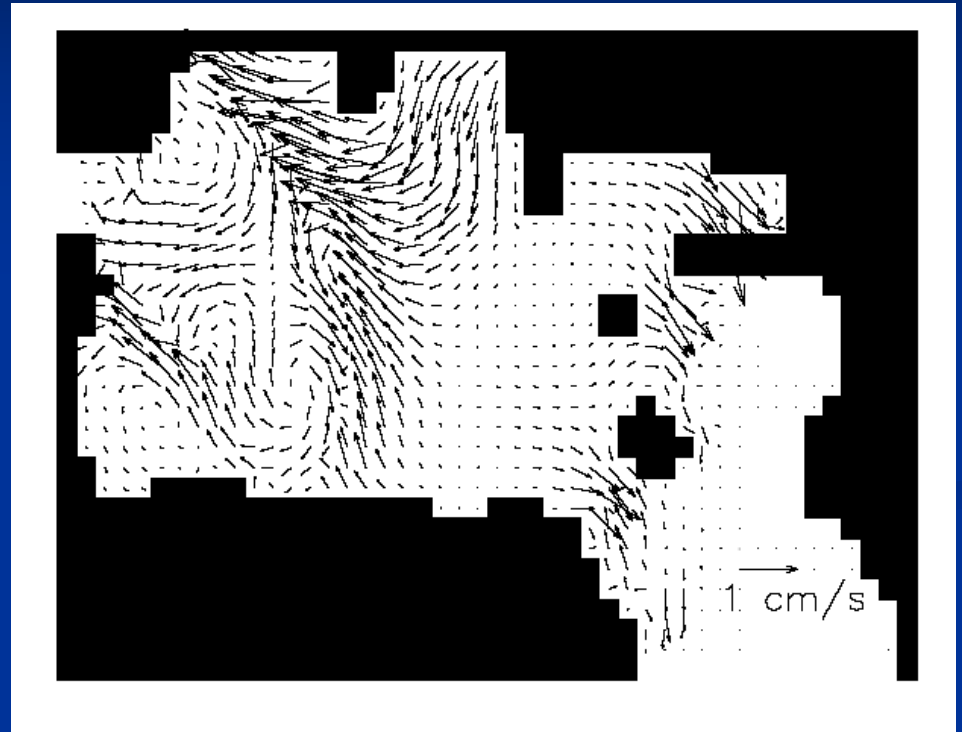
2 superloop - 1000

# 10 superloops

Jan 1, 1997



1000 superloops

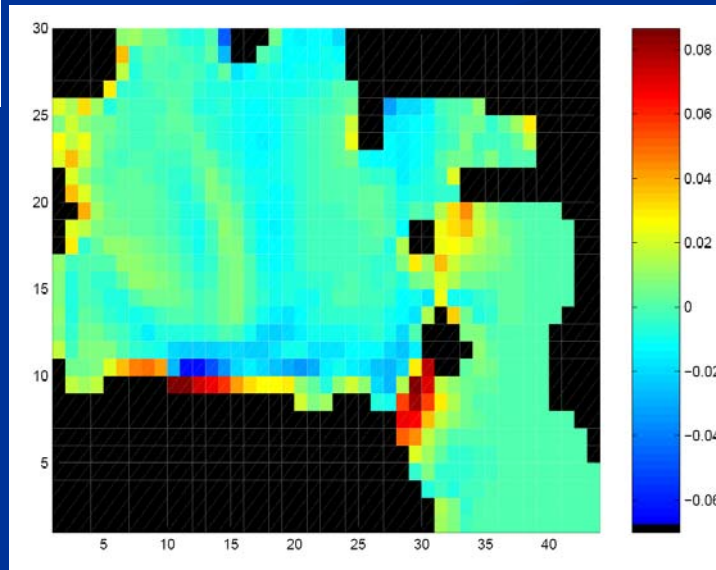
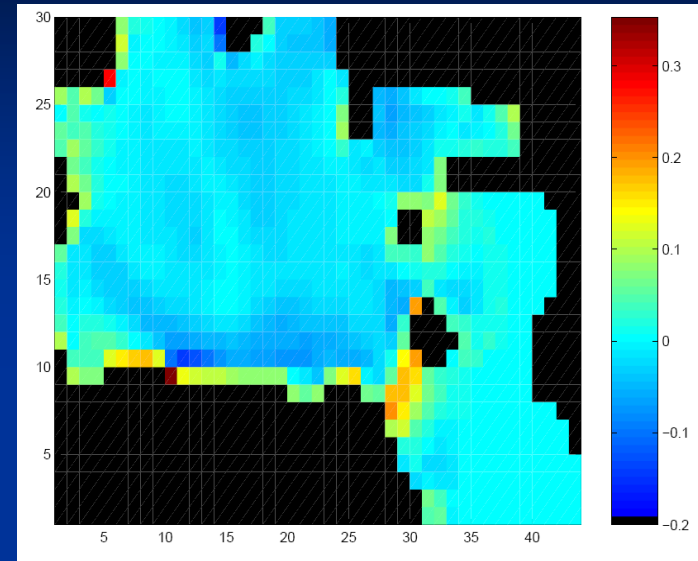
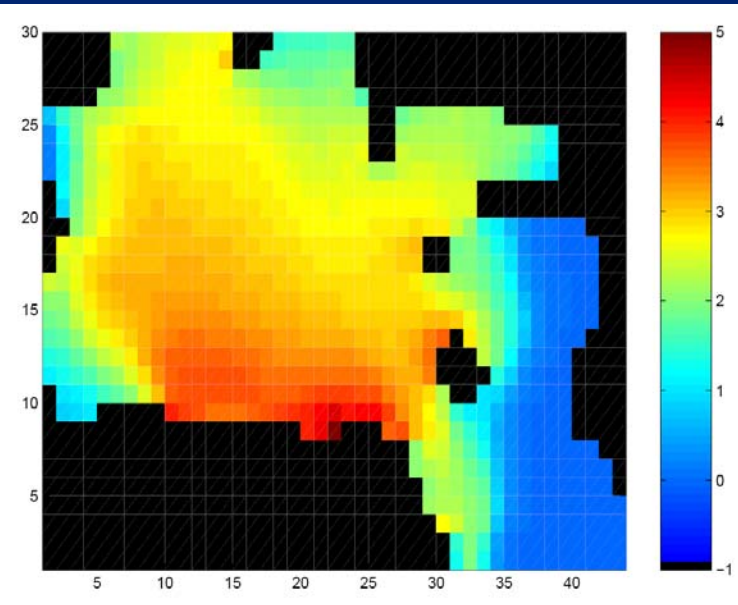


10 superloops - 1000

2 superloop

# Sea Ice Thickness

January - 1997



**2 outerloops**

**10 outerloops**

# Plastic vs Viscous

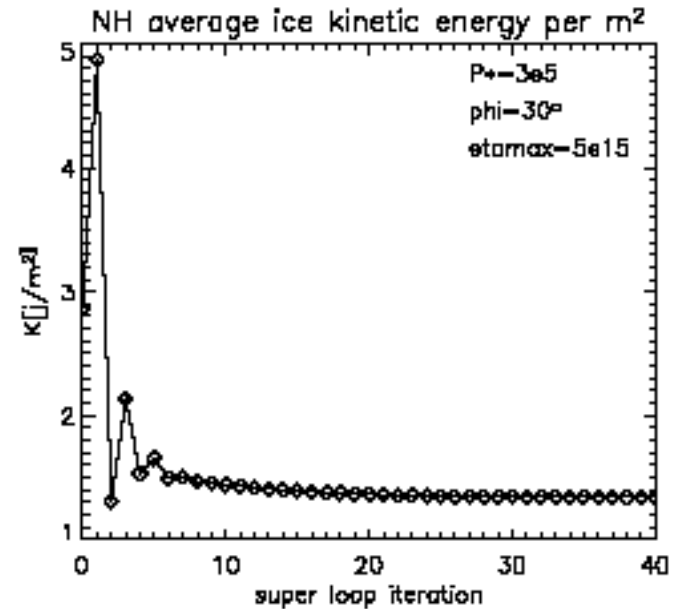
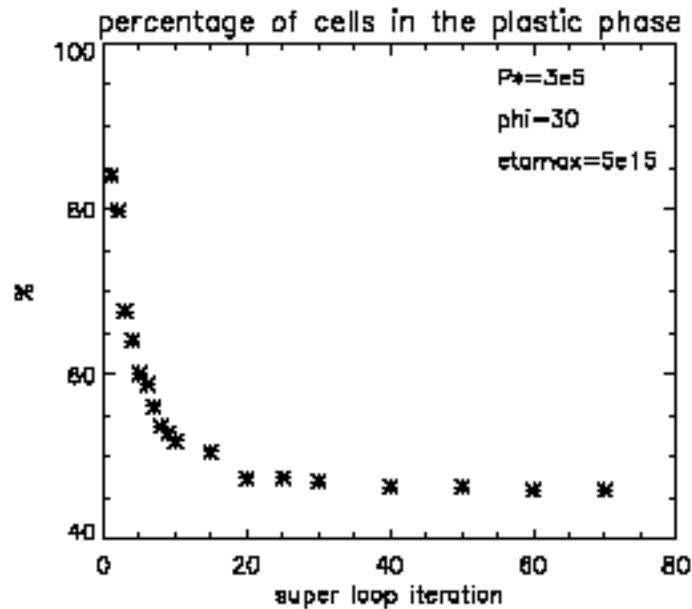


Figure: % of ice-covered grid cells in the plastic phase (left) and Northern Hemisphere average kinetic energy (right) as a function of the number of 'super loop' iterations

# Future Work

- Costly to iterate until convergence
- Develop a Jacobian-free Newton Raphson method
- Parrallelization of the code